Spatial Variability

1. Introduction

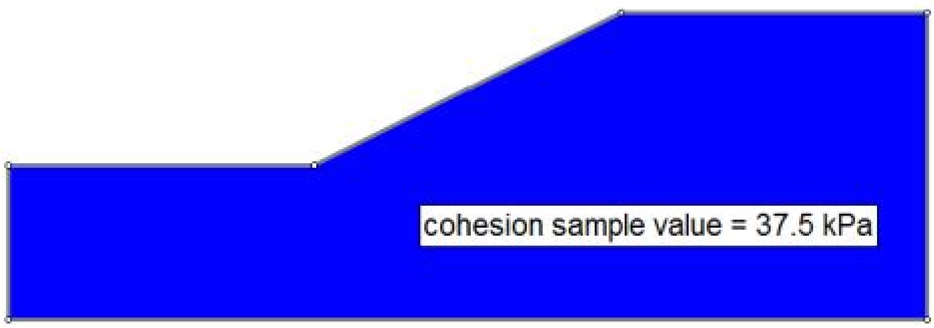
Spatial variability analysis is a sub-option of the probabilistic analysis in Slide2, which allows you to simulate the variability of soil properties such as strength and unit weight, with location within a soil mass. This tutorial will demonstrate the spatial variability option using a simple one-material slope with cohesive strength only.

WHAT IS SPATIAL VARIABILITY ANALYSIS?

Most real soil materials have properties which vary, to some extent, with the location within the soil mass. A traditional probabilistic slope stability analysis does not account for this type of variability. In a traditional probabilistic analysis:

• a statistical distribution is defined for a parameter (e.g. cohesion)

• for each simulation, the entire soil mass is assigned a single random value



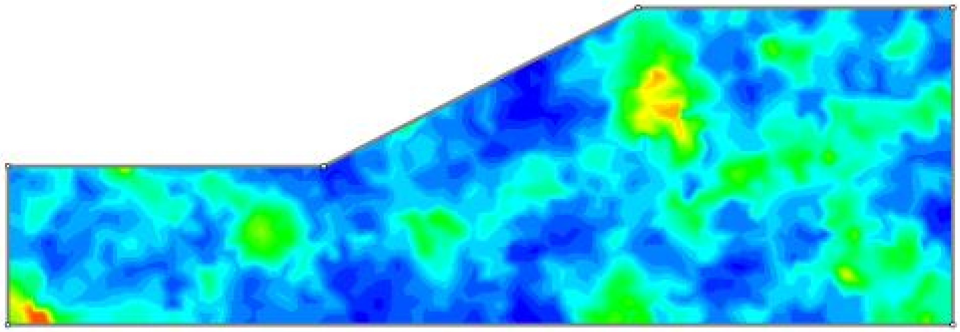
Single random sample value (cohesion) applied to the entire soil region

With spatial variability analysis:

• a statistical distribution is defined for a parameter

• correlation lengths are defined in the x and y directions

• for each simulation, a random field of values is generated for the soil mass



Random field of spatially variable cohesion

With spatial variability, rather than assigning a single randomly generated sample value to a soil region, a random field of values is generated for each sampling based on the statistical distribution and the correlation length parameters of a material. This creates a spatial distribution of values (e.g. cohesion) throughout the material, for each sampling, as shown in the above figure.

During the slope stability analysis, any slip surface which passes through a spatially variable material will encounter variability of properties along the slip surface.

WHY USE SPATIAL VARIABILITY ANALYSIS?

The premise of using spatial variability analysis is that real soil materials have properties which vary with location throughout the material. Therefore, an analysis which tries to account for this type of variability should provide a more realistic assessment of the mean factor of safety and probability of failure of a slope.

The use of spatially variable analysis has been shown to affect the calculated probability of failure of slope models. For example, it has been shown that for slope models with a mean factor of safety greater than 1, accounting for spatial variability of material properties (e.g. cohesion and unit weight) results in a LOWER probability of failure, compared to the same analysis without using spatial variability (Ref. 1). A probabilistic analysis that does not consider spatial variability has been shown to result in unrealistic and overly conservative probabilities of failure.

OVERVIEW OF STEPS FOR SPATIAL VARIABILITY ANALYSIS

The general procedure for using spatial variability in Slide2 is as follows.

1. Select **Project Settings > Statistics > Probabilistic Analysis**, and select the **Spatial Variability Analysis** checkbox.

> **Note**
>
> The Overall Slope Analysis Type is automatically selected. Due to the nature of a spatial variability analysis, you must use the Overall Slope probabilistic analysis type since a new random field is generated with each sample for each spatial material.

2. In the Material Statistics dialog, define the Correlation Lengths in the X and Y direction for each material you are defining as spatially variable.

> **Note**
>
> Correlation lengths are defined per material.

3. Preview the generated fields in the Property Viewer to ensure the field has been correctly defined.

4. When you select Compute, the Overall Slope probabilistic analysis method is used. In general, this type of analysis will be longer than a typical Slide2 analysis, due to the repetition of the critical slip surface search required by the Overall Slope analysis.

5. When you view results in Interpret, the presentation of results is mostly the same as for a non-spatial probabilistic analysis, with additional features of the Property Viewer.

6. The Property Viewer option allows you to view the actual random fields generated by the analysis. This option is available in both the Slide2 Model and Interpret programs. In the Interpret program, you can view random fields and results (slip surfaces) on the same plot, allowing you to see the effect of changing random fields on the analysis results.

PROPERTIES AVAILABLE FOR SPATIAL ANALYSIS

At present the spatial variability analysis in Slide2 only works in conjunction with the following strength models:

- Mohr-Coulomb
- Undrained (constant or linearly increasing)

And the following material parameters:

- Cohesion
- Friction Angle
- Unit Weight

Spatial variability cannot be applied to any other material parameters.

NUMBER OF SAMPLES / SAMPLING METHOD

For spatial variability analysis, a large number of samples, and the Latin Hypercube sampling method is recommended. A minimum of 1000 samples should be used or higher (e.g. 2000 to 10,000 depending on the model complexity).

To save some computation time for this tutorial example, we have used only 500 samples so that you can quickly view results and vary input parameters.

The **Number of Samples** is specified in **Project Settings > Statistics**.

NUMBER OF SLICES

For spatial variability analysis, it is not necessary to increase the number of slices. Slide2 uses a sub-sampling method to sample the bottom of each slice, and hence properly sample all the cells at the bottom of each slice.

2. Overall Slope Analysis (non-spatial)

Select **File > Recent Folders> Tutorials** Folder and open the file *Tutorial 33 Spatial Variability – starting file.slmd*.

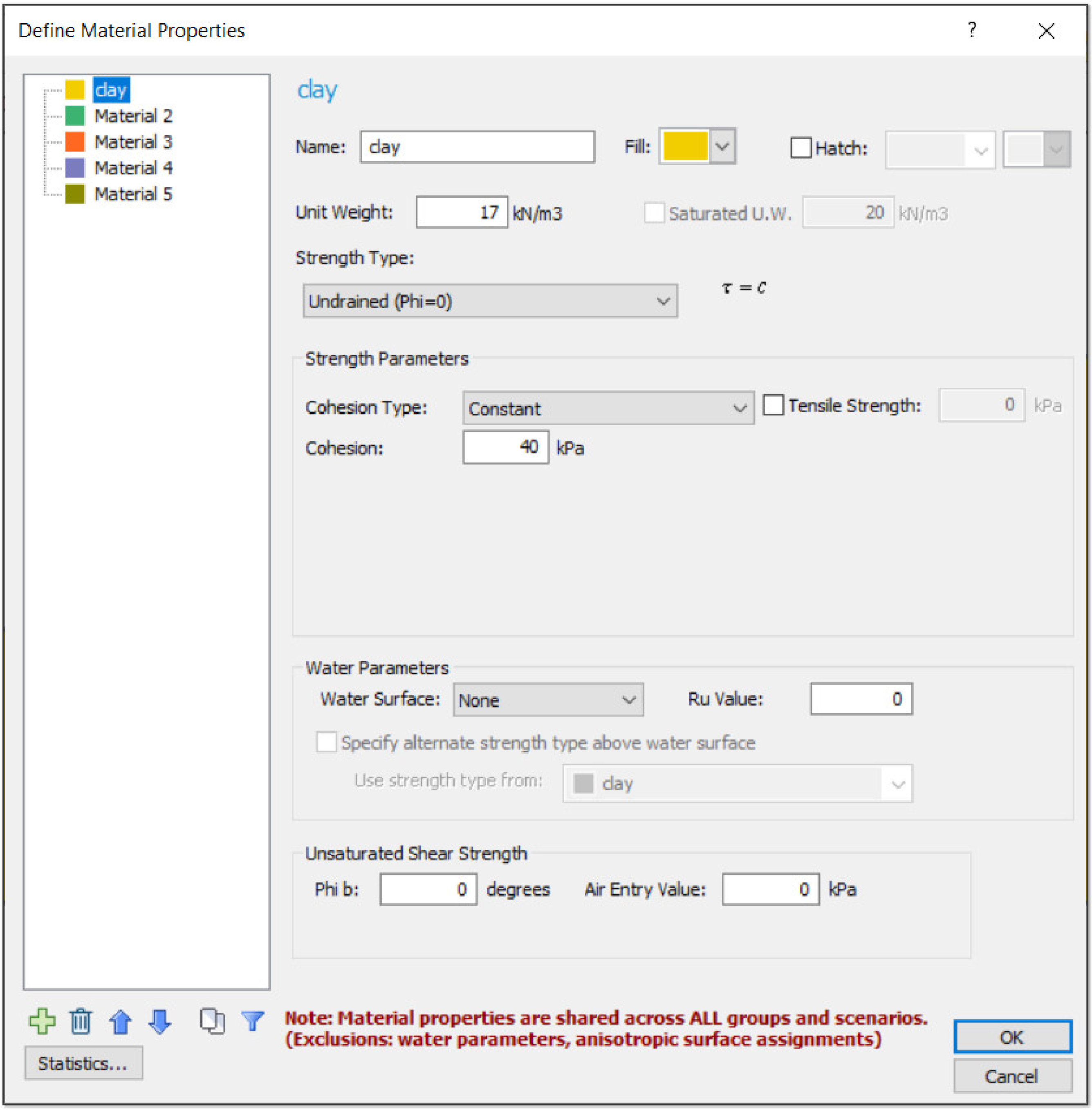
This model is a simple slope with a cohesion-only material. We will first run the model WITHOUT spatial variability, using:

• The Overall Slope probabilistic analysis type with 500 samples

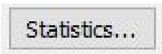
• Circular slip surfaces

• Auto Refine Search method

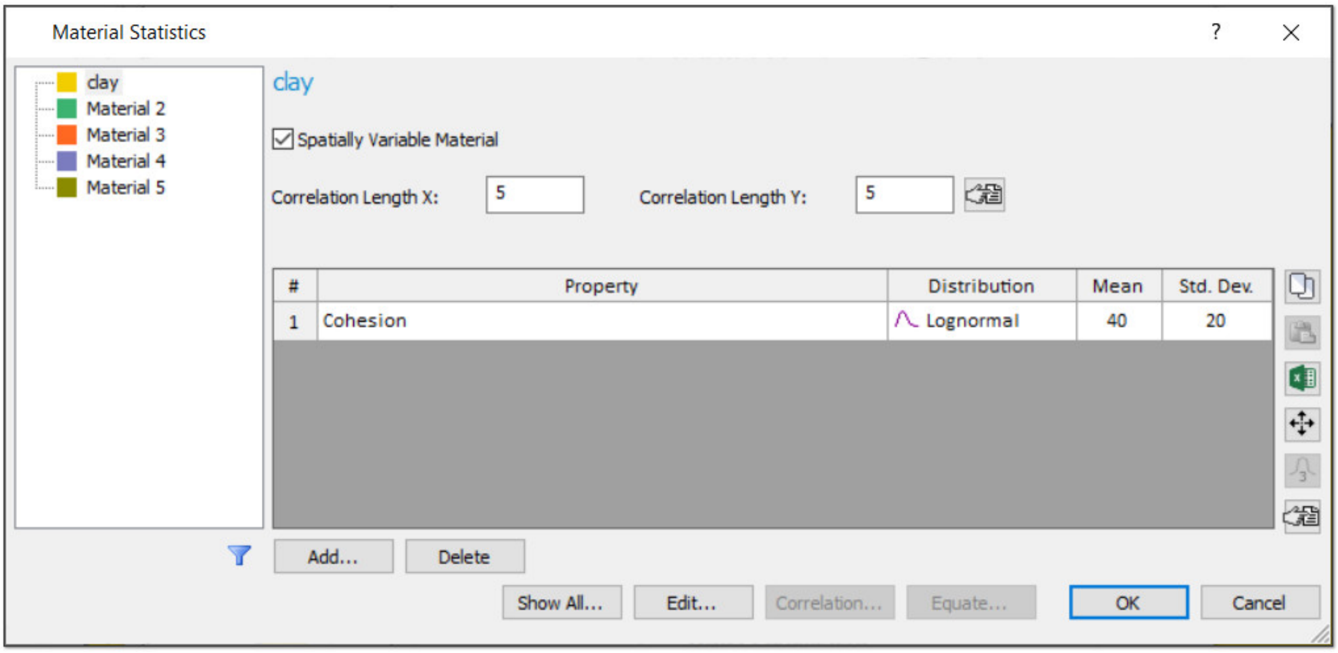
Let’s have a look at our material properties. Select **Properties > Define Materials**. As you can see we have a simple **Undrained** material with **Cohesion = 40 kPa**



Let's view our statistical input. Select the at the bottom of the **Define Materials** dialog.

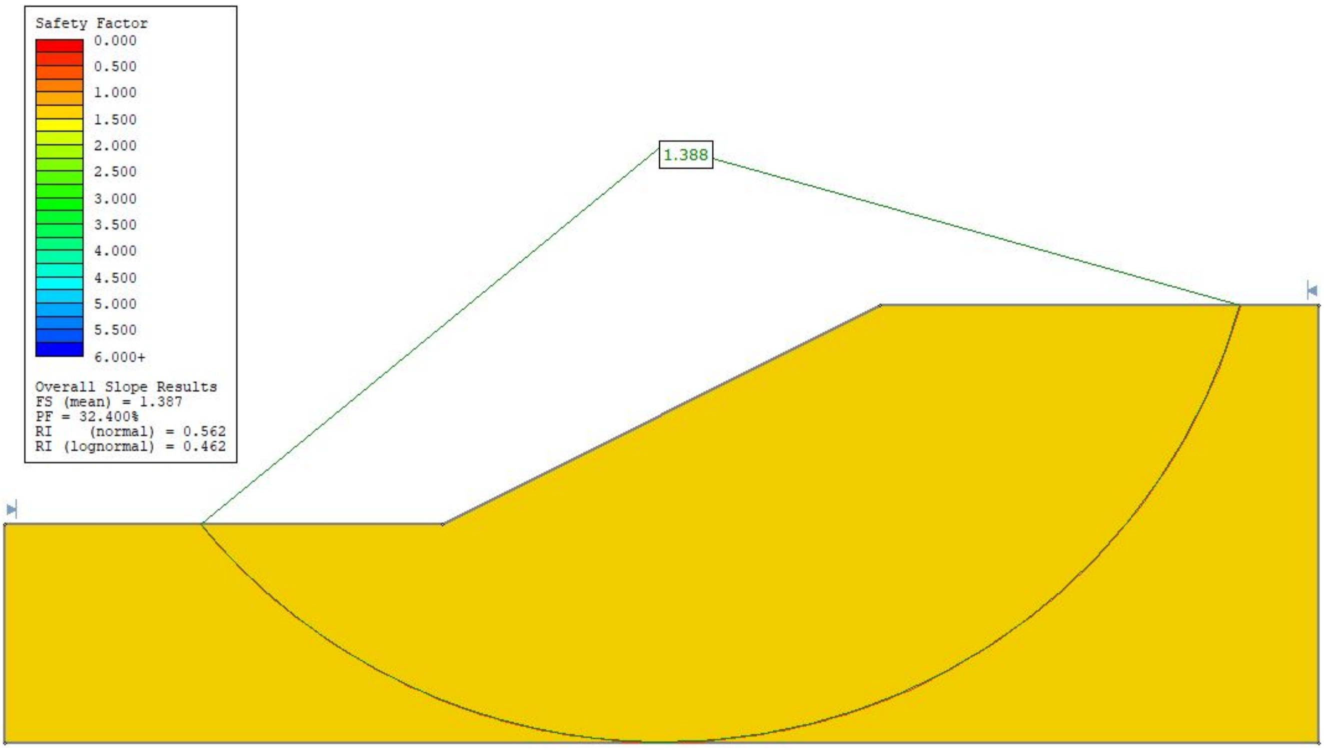


For the cohesion, we have defined a **Lognormal Distribution** with a large standard deviation (COV coefficient of variation = 50 percent).



Select **Cancel** in both dialogs.

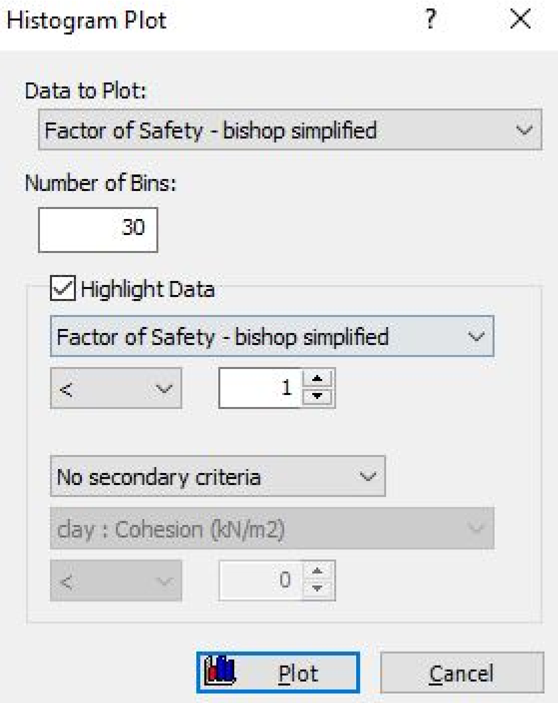
Select **Compute** and then Interpret. You should see the following results.



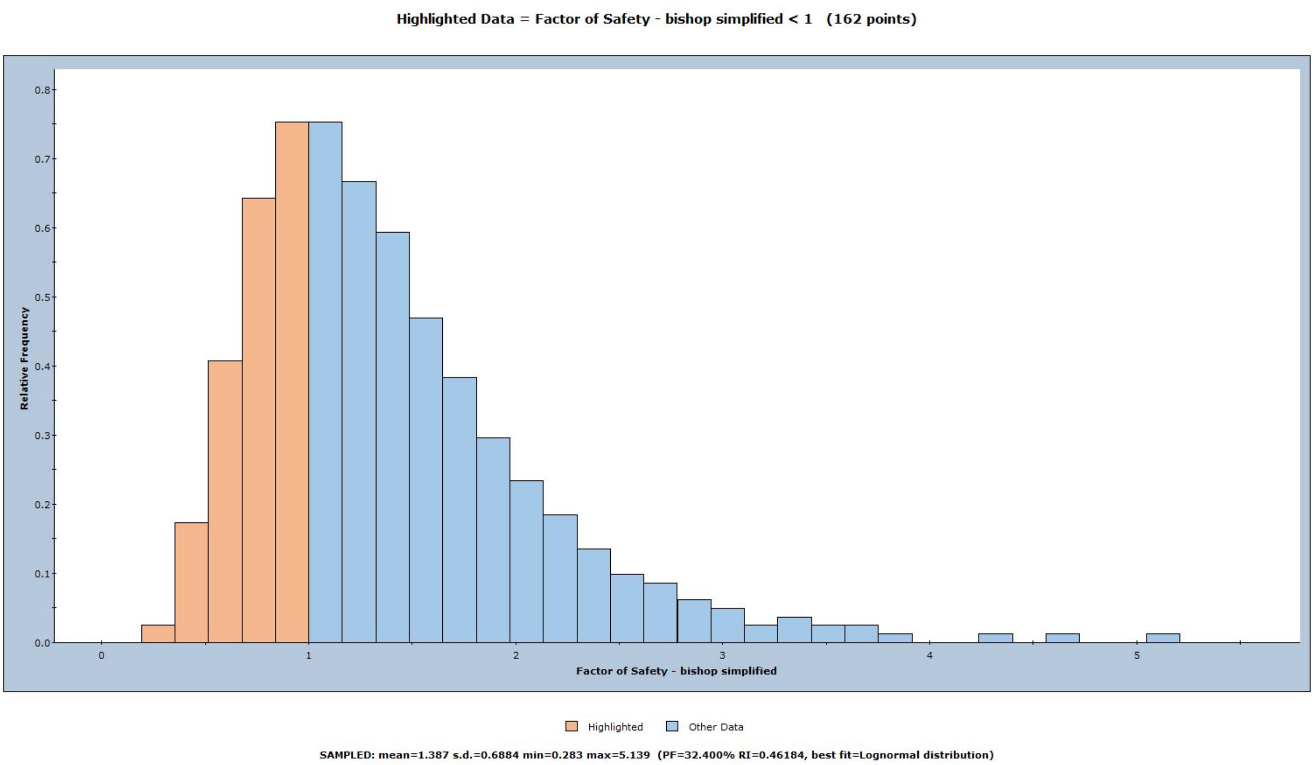
The primary results for the overall slope analysis (mean safety factor and probability of failure) are shown in the legend.

```
Overall Slope Results
FS (mean) = 1.387
PF = 32.400%
RI     (normal) = 0.562
RI (lognormal) = 0.462
```

Because this is a cohesion-only slope, the Overall Slope Analysis finds the same global minimum surface for all analysis runs (i.e. all random sample values of cohesion give the same global minimum surface). Let's view the safety factor distribution. Select **Statistics > Histogram**, choose **Data to Plot = Factor of Safety**, select **Highlight Data Factor of Safety < 1** and select **Plot**.



Notice the lognormal distribution of safety factors (corresponding to our lognormal distribution of cohesion), and the red bars representing safety factor < 1.

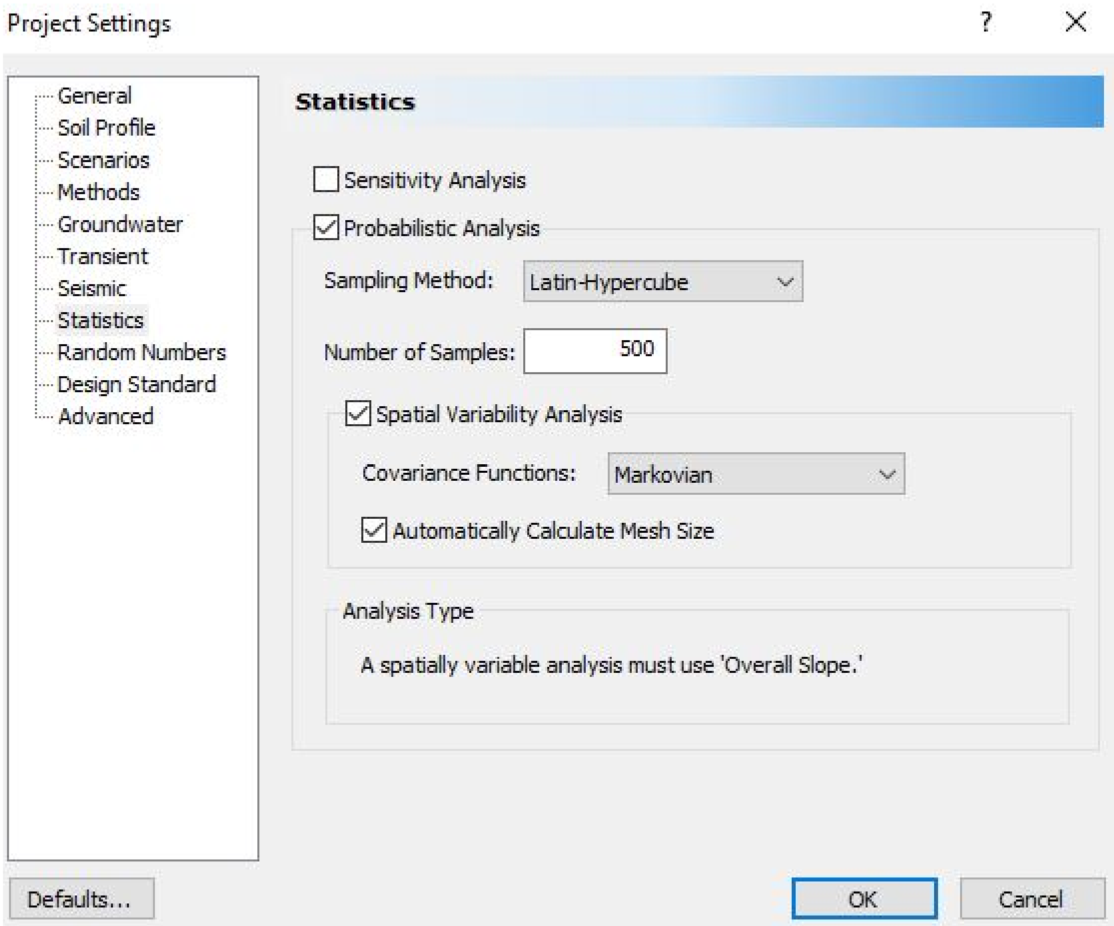


Return to the Slide2 Modeller.

3. Overall Slope Analysis with Spatial Variability

Save the file with a new file name.

Select **Project Settings > Statistics** and turn on the **Spatial Variability Analysis** checkbox. This will enable spatial variability analysis and allow you to define correlation lengths for spatial random variables. The **Automatically Calculate Mesh Size** option should be left on (see note below). Select **OK**.



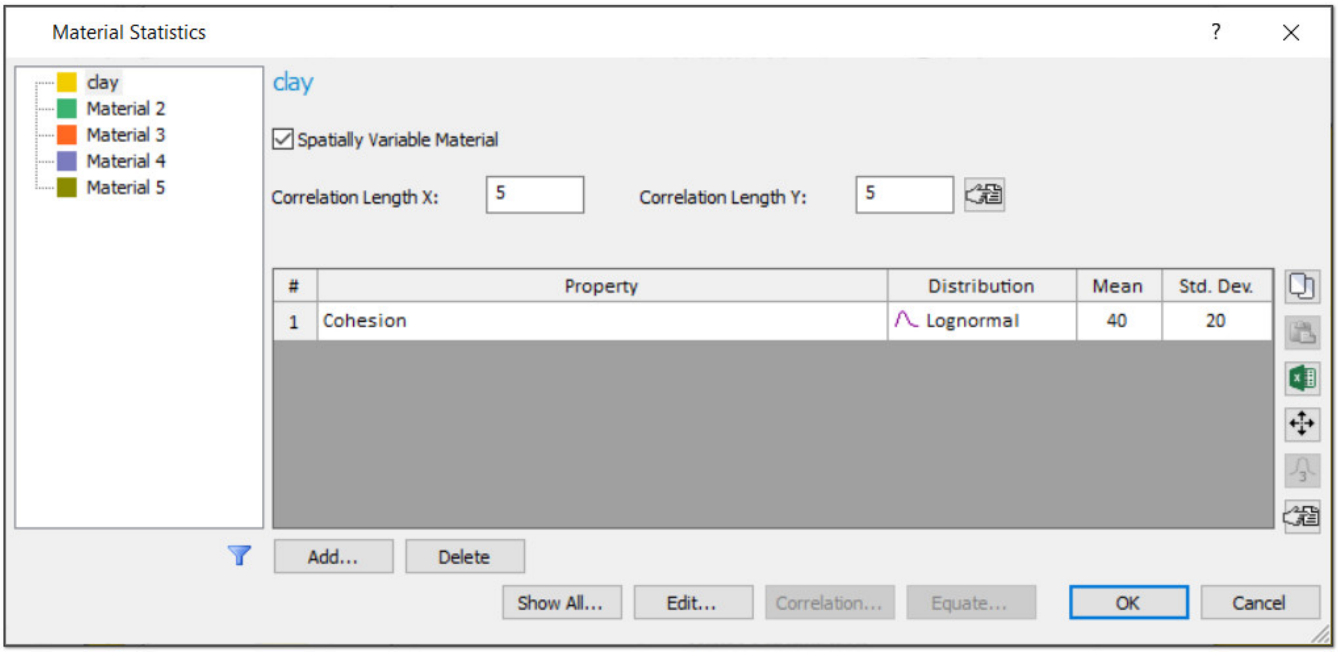
AUTOMATICALLY CALCULATE MESH SIZE

The mesh size in a random field must meet two requirements: 1) it is recommended that it be at least half of the smaller correlation length (X or Y), and 2) each material layer must have a sufficient number of cells to correctly represent the variability (for example a thin horizontal layer should not have just one row of cells because this would not take into account the vertical variability). Slide2 uses an algorithm that automatically calculates the mesh size for each layer in order to satisfy these requirements. It is recommended that this checkbox is not turned off. However, the user may turn off the checkbox if they want to define their own mesh size for spatially variable materials in the Material Statistics dialog.

DEFINING CORRELATION LENGTH

Select **Statistics > Materials**. To run a spatial variability analysis, you must define at least one material property to be spatially variable, and define the correlation lengths.

In the Material Statistics dialog, for the “clay” material, select the **Spatially Variable Material** checkbox, and enter **Correlation Length = 5** for both the X and Y directions.



The **Correlation Length** represents the distance within a material over which values of a spatially random variable (e.g. cohesion) are expected to be highly correlated (i.e. similar in magnitude). In practice, two samples taken from adjacent locations in the soil profile tend to have very close values and as the sampling distance increases this tendency decreases. Small values of spatial correlation length mean that soil properties highly fluctuate; on the other hand, large values of spatial correlation length mean that soil properties vary smoothly within the soil profile. If the Correlation lengths are equal in the X and Y directions, this will generate isotropic random fields (i.e. no preferred direction, same spatial variability in the horizontal and vertical directions).

Select OK in the Material Statistics dialog.

PROPERTY VIEWER

Before computing, it is good practice to preview the fields in the **Property Viewer.** The Property Viewer options are covered at the end of the tutorial.

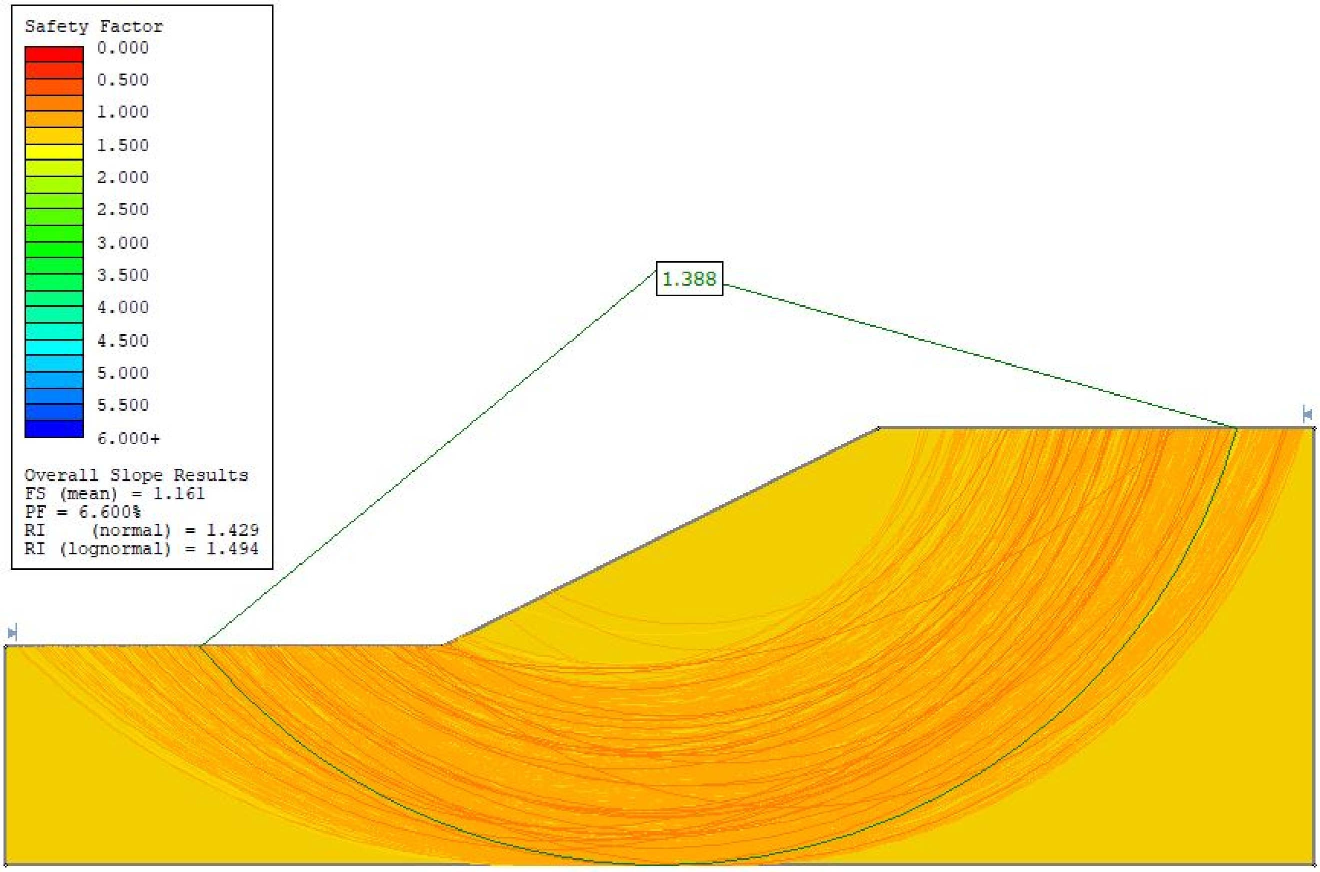
4. Compute

Save the file and select **Compute**.

The computation time will be about the same as an Overall Slope analysis without spatial variability.

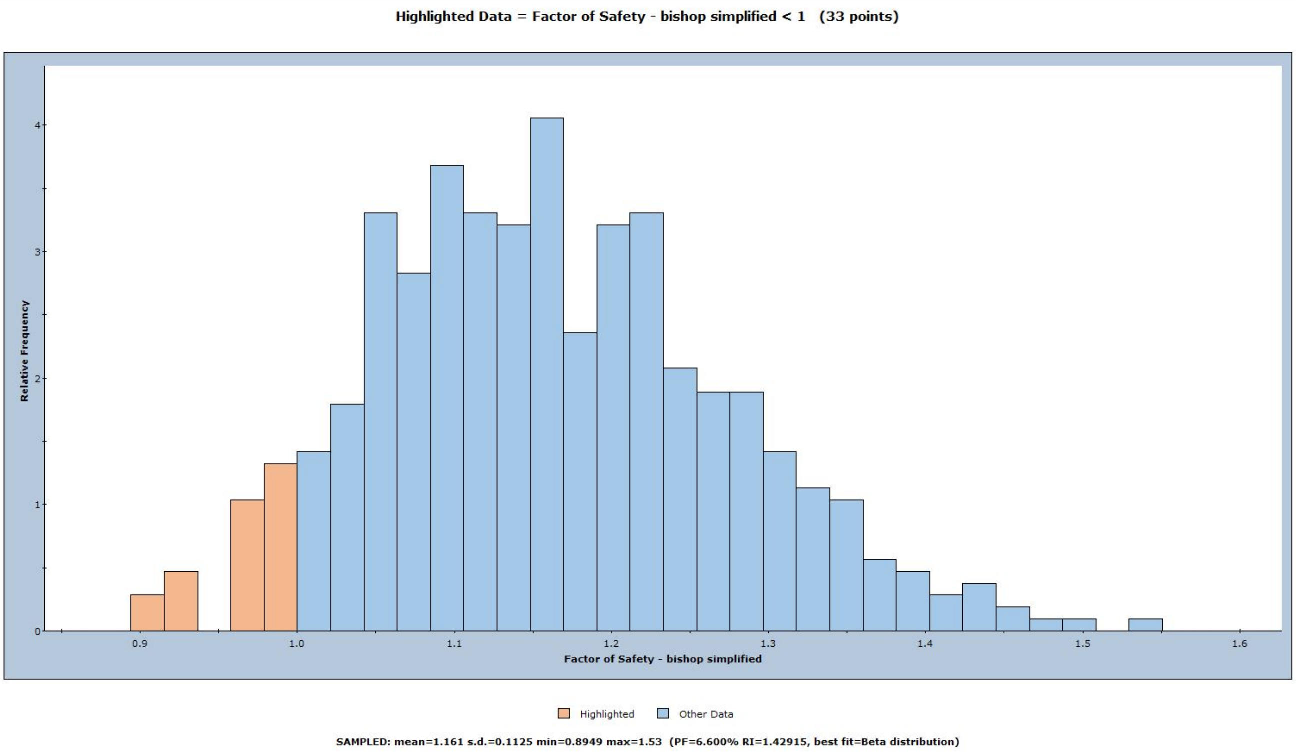
5. Interpret

When computing is finished select **Interpret**. You should see the following result. Notice that the spatial analysis generates many different global minimum surfaces, compared to the non-spatial analysis (which for this example generated only a single global minimum surface).




```
Overall Slope Results
FS (mean) = 1.161
PF = 6.600%
RI     (normal) = 1.429
RI (lognormal) = 1.494
```

Let's view the safety factor distribution. Select **Statistics > Histogram**, choose **Data to Plot = Factor of Safety,** select **Highlight Data Factor of Safety < 1** and select **Plot.**



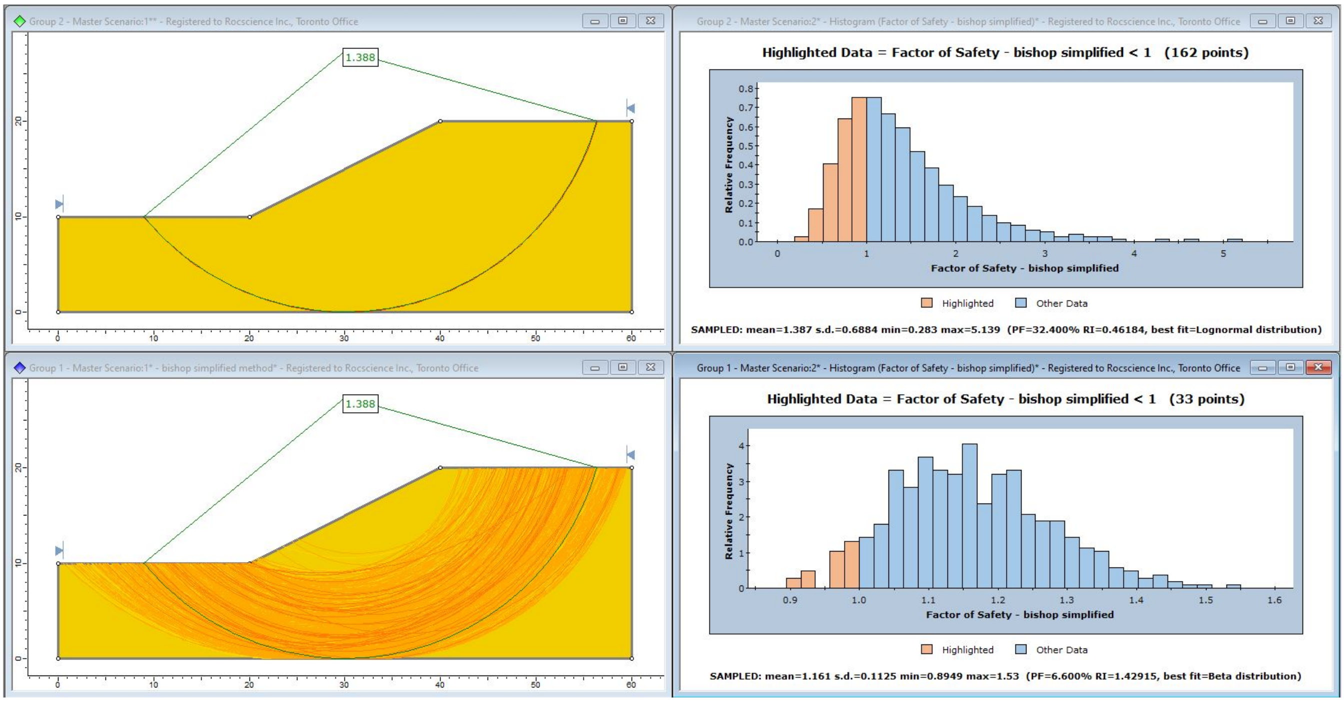
COMPARISON WITH NON-SPATIAL ANALYSIS

Let's compare the primary results (Mean FS and Prob. of Failure) for the spatial and nonspatial analysis.

	Non-Spatial Analysis	Spatial Analysis
Mean Factor of Safety	1.387	1.161
Probability of Failure (%)	32.4	6.6

Including spatial variability for this example, has significantly DECREASED both the Mean Factor of Safety and the Probability of Failure, compared to the non-spatial probabilistic analysis.

If we plot the safety factor distribution for the spatial and non-spatial analysis at the same scale, and tile the views, this graphically illustrates the effect of including spatial variability in the analysis. The distribution of safety factors from the spatial analysis is much narrower. Even though the mean safety factor is lower, the probability of failure is also lower, due to the narrower distribution of safety factors for the spatial analysis, and the different shape of the distribution (lognormal vs beta).



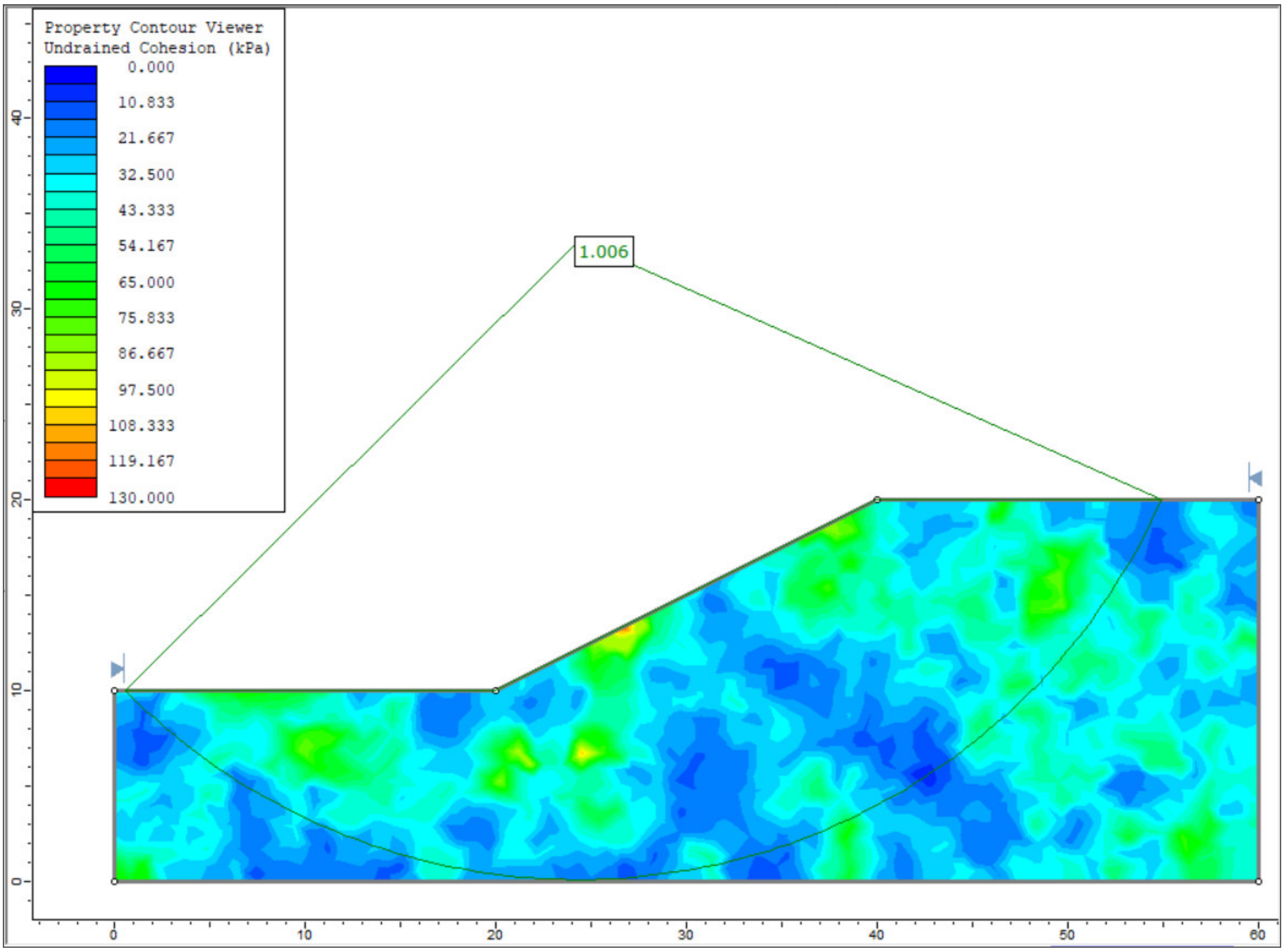
PROPERTY VIEWER

Return to the spatial analysis file.

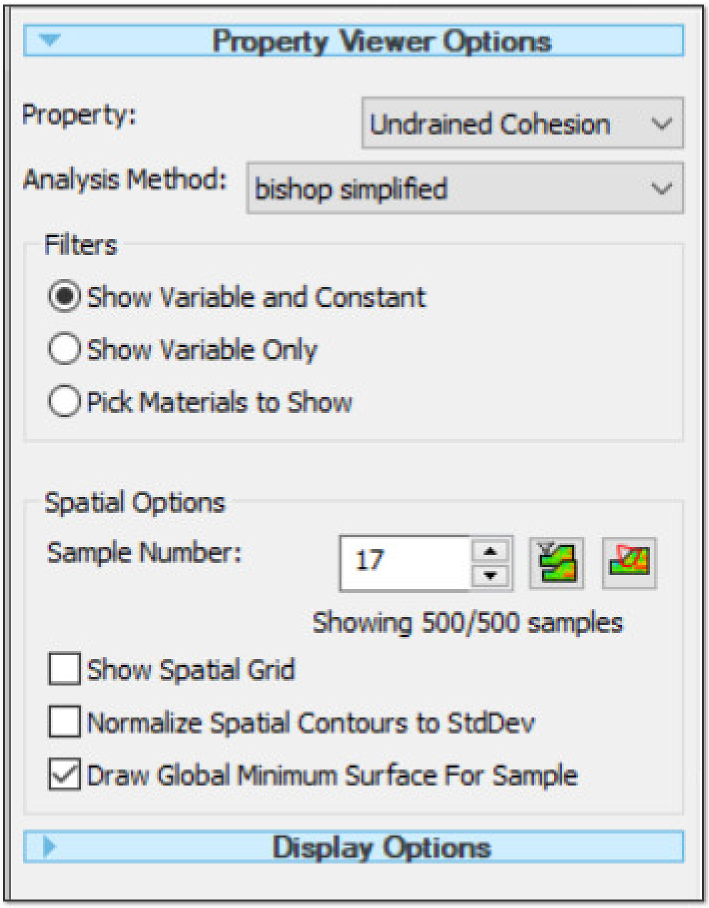
Select the **Property Viewer** option from the **Analysis** menu or the toolbar.

The Property Viewer allows you to plot the actual distributions of random fields generated for the spatial random variables; it also allows you to see the global minimum slip surface corresponding to any sample instance of the random fields.

Press F2 to Zoom All. By default, you are viewing contours of the Undrained Cohesion spatial variable for Sample 1 of the Overall Slope Analysis.

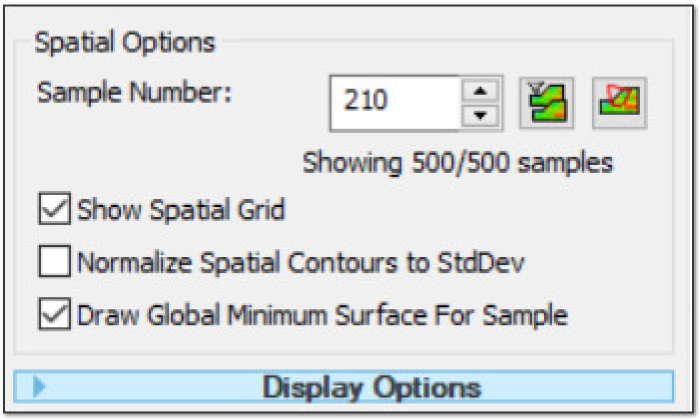


In the sidebar, you will see the controls for the **Property Viewer.** If you click in the **Sample Number** edit box and scroll the mouse wheel, you can view the random field of cohesion generated for each random sampling of the Overall Slope Analysis. The Global Minimum slip surface corresponding to the current sample number will also be displayed.



Scroll through the sample numbers and notice how the global minimum surface has a tendency to pass through the LOW values of cohesion generated in each random field (in this case the dark blue contour areas).

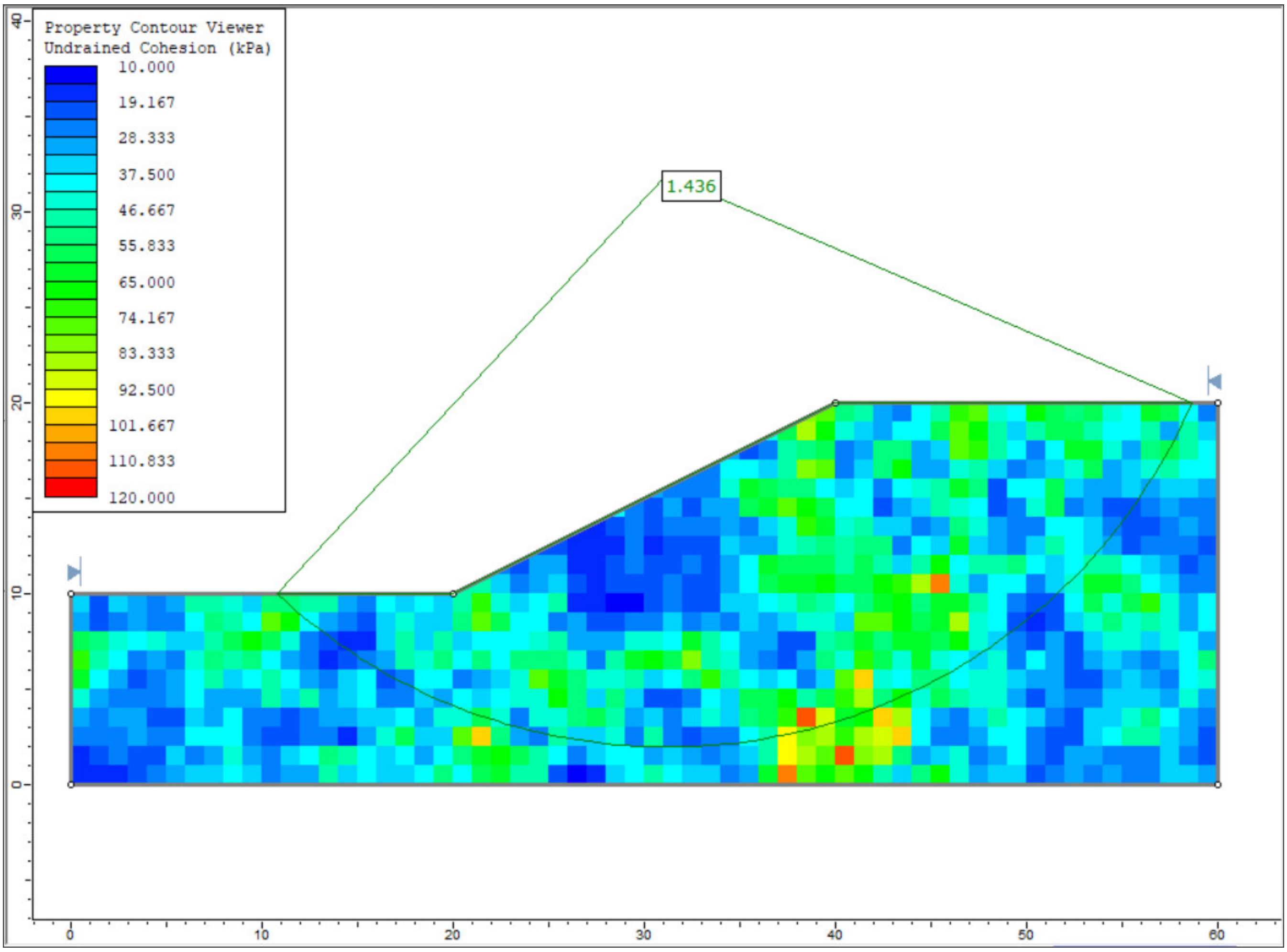
In the sidebar, select the **Show Spatial Grid** checkbox.



This will display the actual values generated at the grid points of the random field, as shown in the following figure. This is the raw data used to generate the contours. You may prefer this representation of the random field.

Note

The Property Viewer is also available in the Slide2 model program, and allows you to view spatial data on the model, without having to run the analysis. This allows you to check your input (e.g. correlation length, standard deviation) as you enter data.



6. Additional Exercises

1. Re-run the spatial analysis with Correlation Length values of 20 meters and 40 meters, and compare results with the Correlation length of 5 meters.

2. Re-run the spatial analysis with differing values of correlation length in the X and Y directions, and observe the random fields. For soils, the X correlation length is usually much larger (e.g. 10 times) than the Y correlation length. This means that the random fields tend to have relatively small spatial variability in the horizontal direction and relatively large spatial variability in the vertical direction.

3. Re-run the analysis with very large values of correlation length (e.g. 500 for both X and Y correlation length). As the correlation lengths become very large, you will notice that the results (i.e. probability of failure) approach that of a non-spatial analysis (i.e. the random fields become more uniform)

7. References

Javankhoshdel, S., Luo, N. and Bathurst, R.J., 2017. *Probabilistic analysis of simple slopes with cohesive soil strength using RLEM and RFEM.* Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 11(3), pp.231-246.