Back Analysis of Material Properties

1. Introduction

This tutorial will demonstrate how to perform back analysis of material properties using sensitivity analysis or probabilistic analysis features in Slide2. Back analysis can be used to determine material strength at slope failure, or the required material strength to achieve a given safety factor.

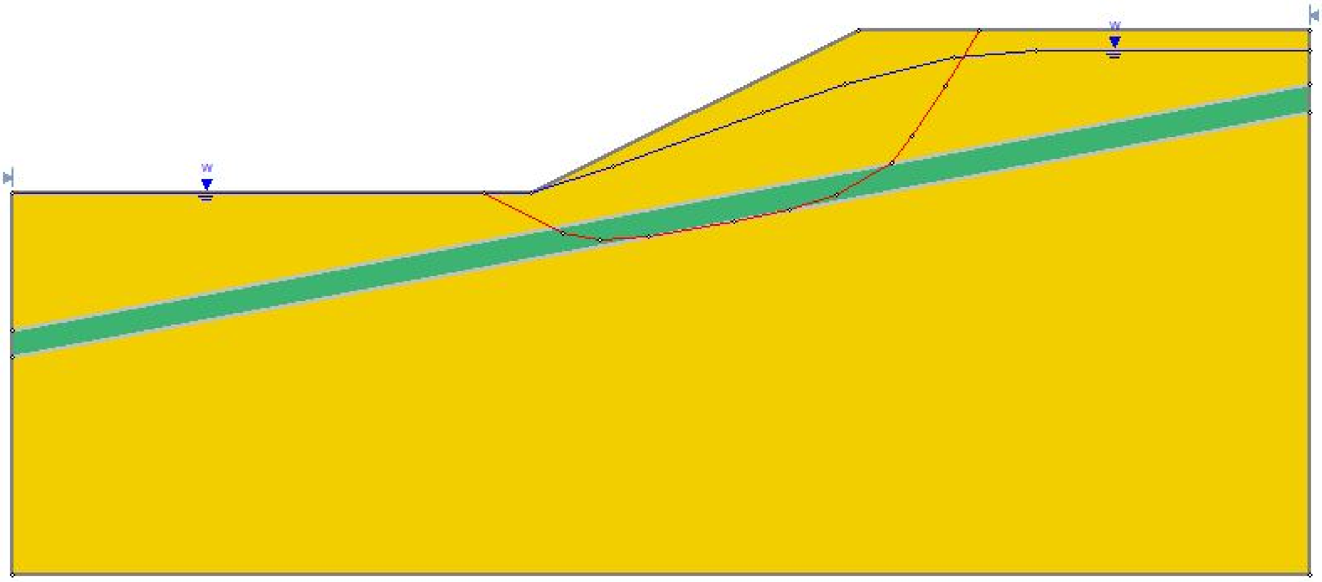
Sensitivity analysis can be used for back analysis of individual variables. Probabilistic analysis can be used for back analysis of multiple variables.

The tutorial can be found in the *Tutorial 23 Back Analysis Material Properties.slmd* data file. All tutorial files installed with Slide2 can be accessed by selecting **File > Recent Folders > Tutorials Folder** from the Slide2 main menu.

2. Model

Start the Slide2 Model program. Select **File > Recent Folders > Tutorials Folder** from the Slide2 main menu, and open the *Tutorial 23 Back Analysis Material Properties.slmd* file.

You should see the following model.



For this model, a slope failure is assumed to have occurred. The actual failure surface is given by the red line shown in the above figure. The failure surface is non-circular and passes through a weak material layer (green material).

The material above and below the weak layer (yellow material) is significantly stronger than the weak layer, and has the following strength properties: cohesion = 17.5 kPa and friction angle = 30 degrees.

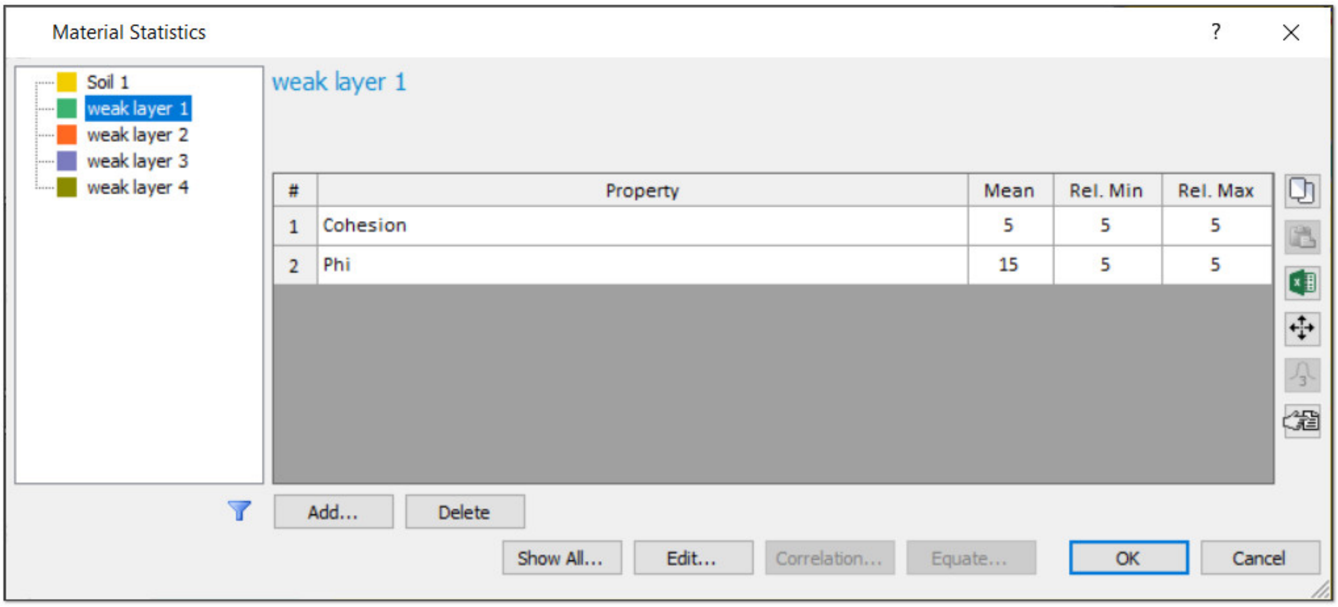
We have initially assigned the weak layer the strength properties: cohesion = 5 kPa and friction angle = 15 degrees. Additionally, we have already pre-defined two variables for sensitivity analysis:

• Weak layer cohesion

• Weak layer friction angle

There are four scenarios defined in this model. Each has different properties for the weak layer. Let's look at **Group 1 – Master Scenario** now.

In Group 1, right-click on any material and select **Statistical Properties**. Look at the weak layer 1 material and you should see the following properties:

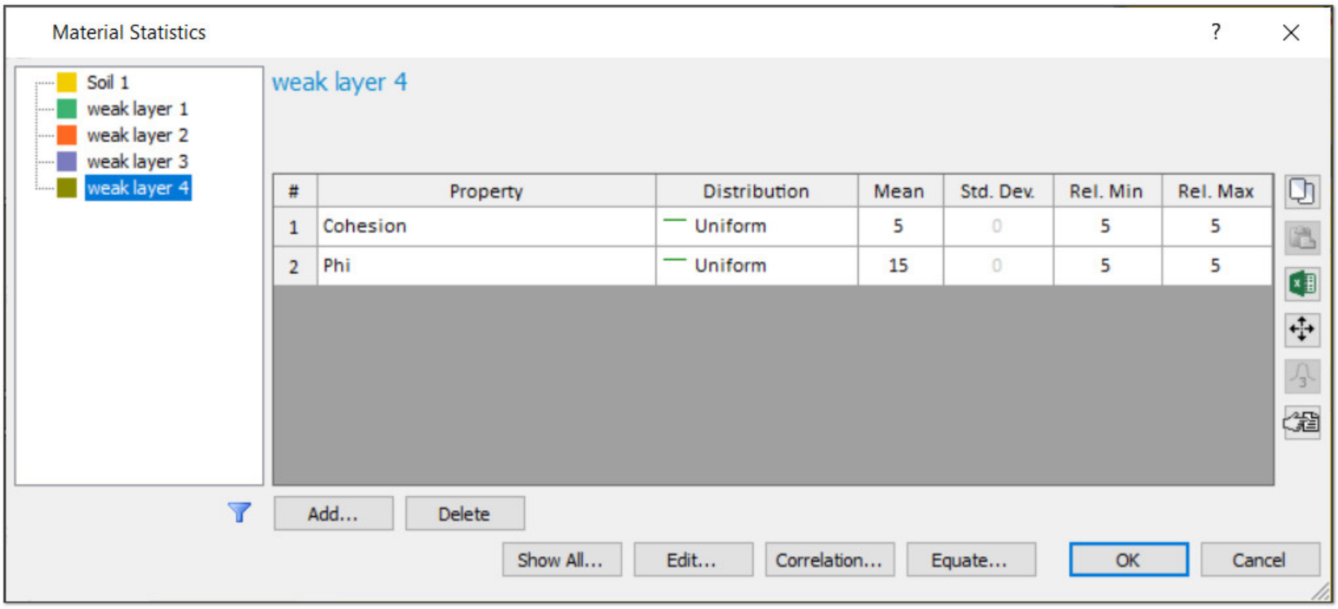


The statistical material properties for all scenarios in Group 1 are outlined below.

Scenario	Property	Mean	Relative Min	Relative Max
Master Scenario (weak layer 1)	Cohesion	5	5	5
	Phi	15	5	5
Master Scenario (weak layer 2)	Cohesion	10	10	10
	Phi	15	5	5

Master Scenario (weak layer 3)	Cohesion	5	5	5
	Phi	15	10	10

In Group 2, where **weak layer 4** is used, we have turned on the probabilistic analysis option in the Statistics tab of the Project Settings. For the statistical material properties we have the following:



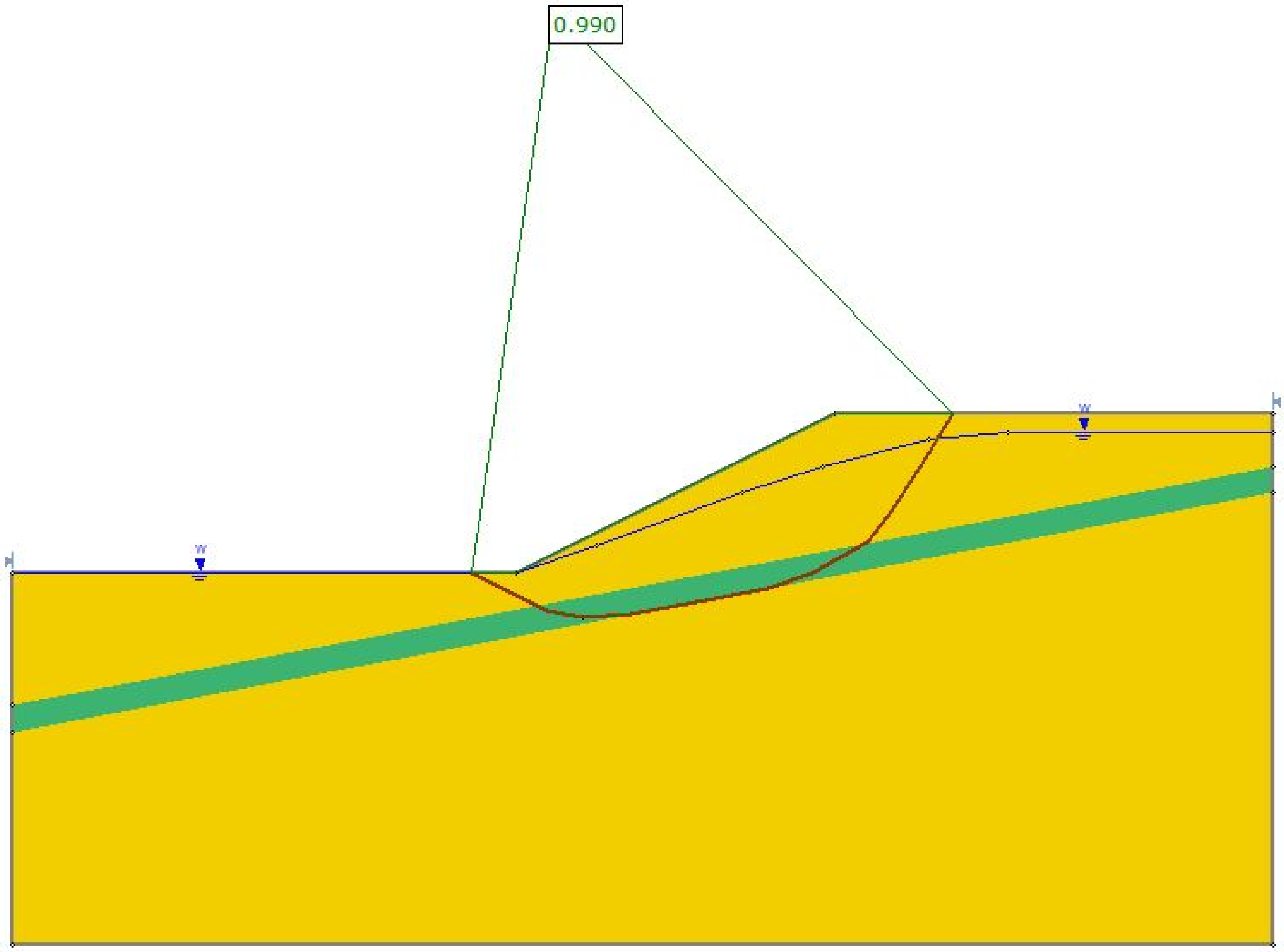
3. Compute

Select the Compute option

4. Interpret

Select **Analysis > Interpret.**

Make sure you are in Group 1 – Master Scenario. You should see the following:



For the Spencer analysis method, the safety factor of this slip surface is 0.99. This is consistent with an actual failure surface since the safety factor is approximately 1.

We have arrived at this result using specific values of cohesion and friction angle for both materials (strong material and weak layer).

SENSITIVITY ANALYSIS RESULTS

Click on Scenario 2 now. In this case, let's assume that the material strength parameters of the weak are not fully known. We'll assume the cohesion is unknown, but the friction angle is known.

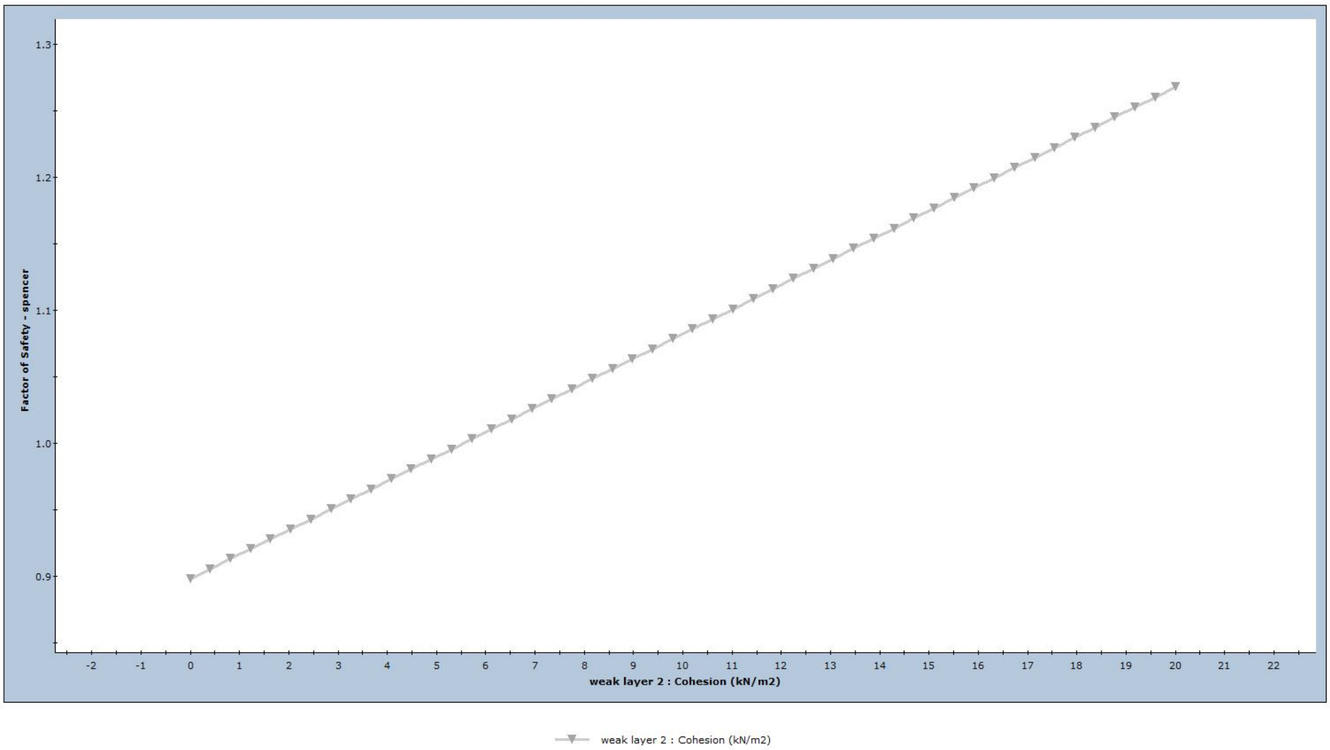
Using sensitivity analysis, and the fact that we have a known failure surface (safety factor = 1), we can easily determine a value of weak layer cohesion at failure.

The material properties we have assigned for the weak layer in Scenario 2 are:

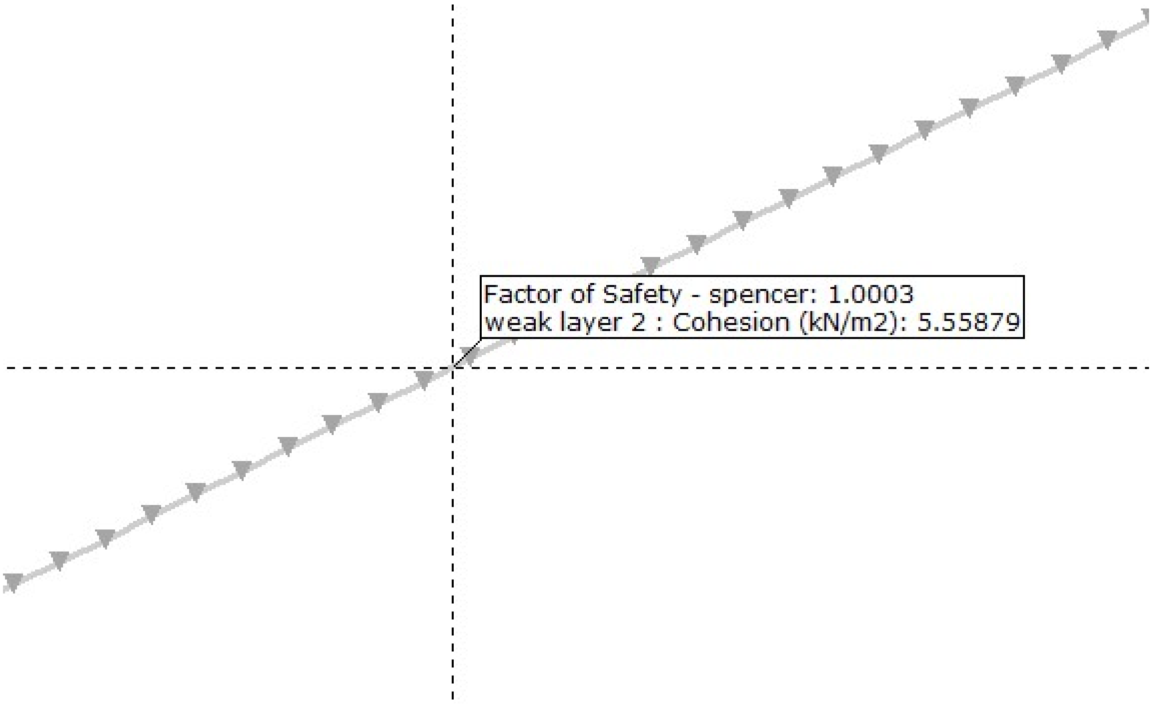
Scenario	Property	Mean	Relative Min	Relative Max
Scenario 2 (weak layer 2)	Cohesion	10	10	10
	Phi	15	5	5

The FS is 1.082. This is because we changed the mean cohesion from 5kPa to 10kPa. Since the mean values of all parameters are used to calculate the deterministic safety factor during a sensitivity or probabilistic analysis, any change to the mean values will affect the deterministic safety factor.

Now select the **Sensitivity Plot** option from the **Statistics** menu or the toolbar. Make sure that only the **Cohesion** checkbox is selected and the friction angle checkbox is cleared.



We want to know the value of cohesion which corresponds to safety factor = 1. We can do this as follows. Right-click on the plot and select **Sampler > Edit Sampler**. Two dotted crossing lines should appear. Scroll along the plot line until you obtain a factor of safety as close to 1.0 as possible. The cursor should snap along the plot line.



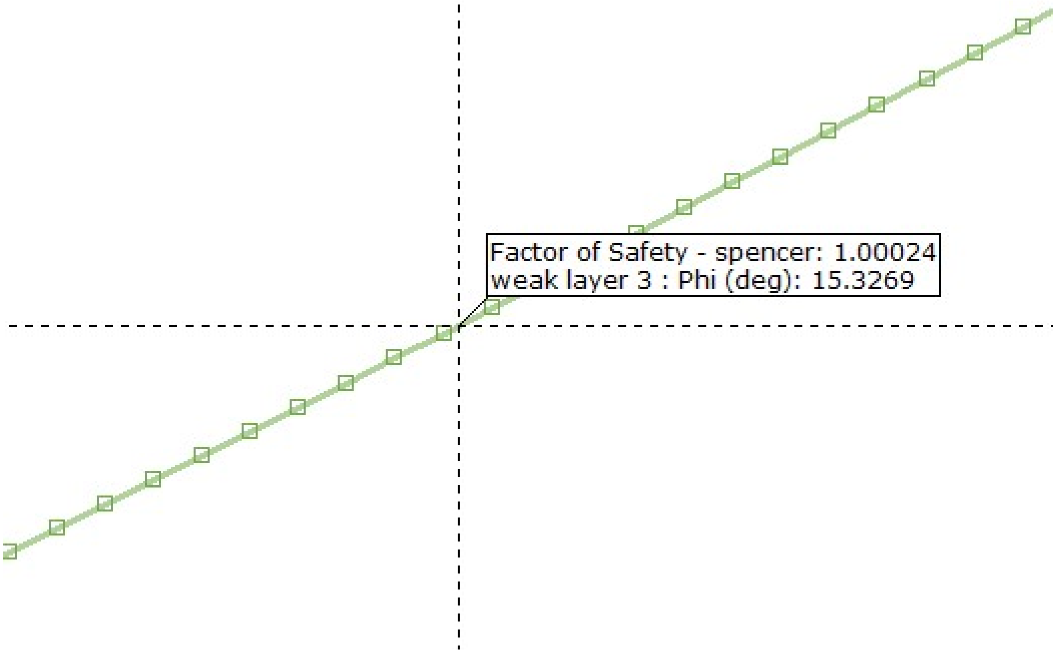
We see that a value of weak layer cohesion = 5.5 kPa corresponds to safety factor = 1. This assumes that all other slope parameters are exactly known and equal to their mean values (i.e. for a friction angle = 15 degrees, the cohesion at failure = 5.5 kPa).

Now let’s assume that the cohesion is known and the friction angle is unknown.

Click on Scenario 3. The material properties we have assigned for the weak layer in Scenario 3 are:

Scenario	Property	Mean	Relative Min	Relative Max
Scenario 3 (weak layer 3)	Cohesion	5	5	5
	Phi	15	10	10

Select the Sensitivity Analysis Plot and plot only friction angle. Using the sampler we can find the friction angle corresponding to a FS of 1.



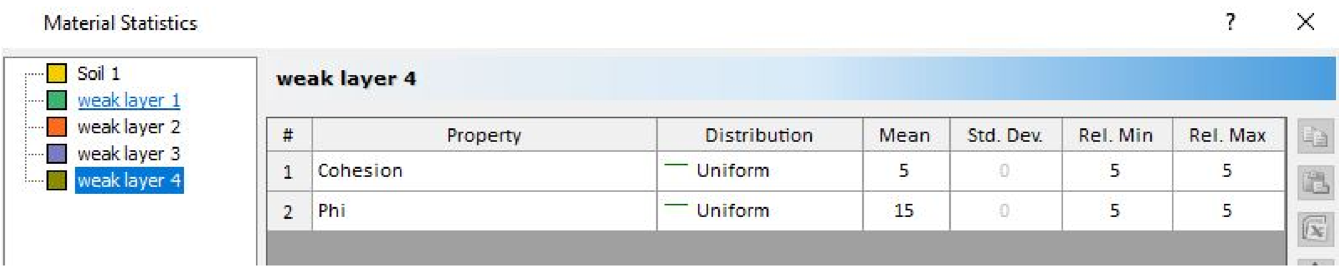
A friction angle = 15.3 degrees gives safety factor = 1. This assumes all other parameters are equal to their mean values (i.e. cohesion = 5 kPa).

To summarize: sensitivity analysis can easily be used for the back analysis of individual variables. Just remember, when you are viewing a sensitivity plot of an individual variable, all other analysis variables are assumed to be constant and equal to their mean value. In general, this procedure may require some trial and error, for example, if the desired safety factor is not within the output range of the sensitivity plot, you will have to adjust the allowable range of values of the variable and re-run the analysis.

PROBABILISTIC ANALYSIS RESULTS

The probabilistic analysis option in Slide2 can be used to carry out a back analysis of two variables simultaneously. For example, determine all possible values of cohesion AND friction angle for a specified factor of safety. This is what we have set up in Group 2.

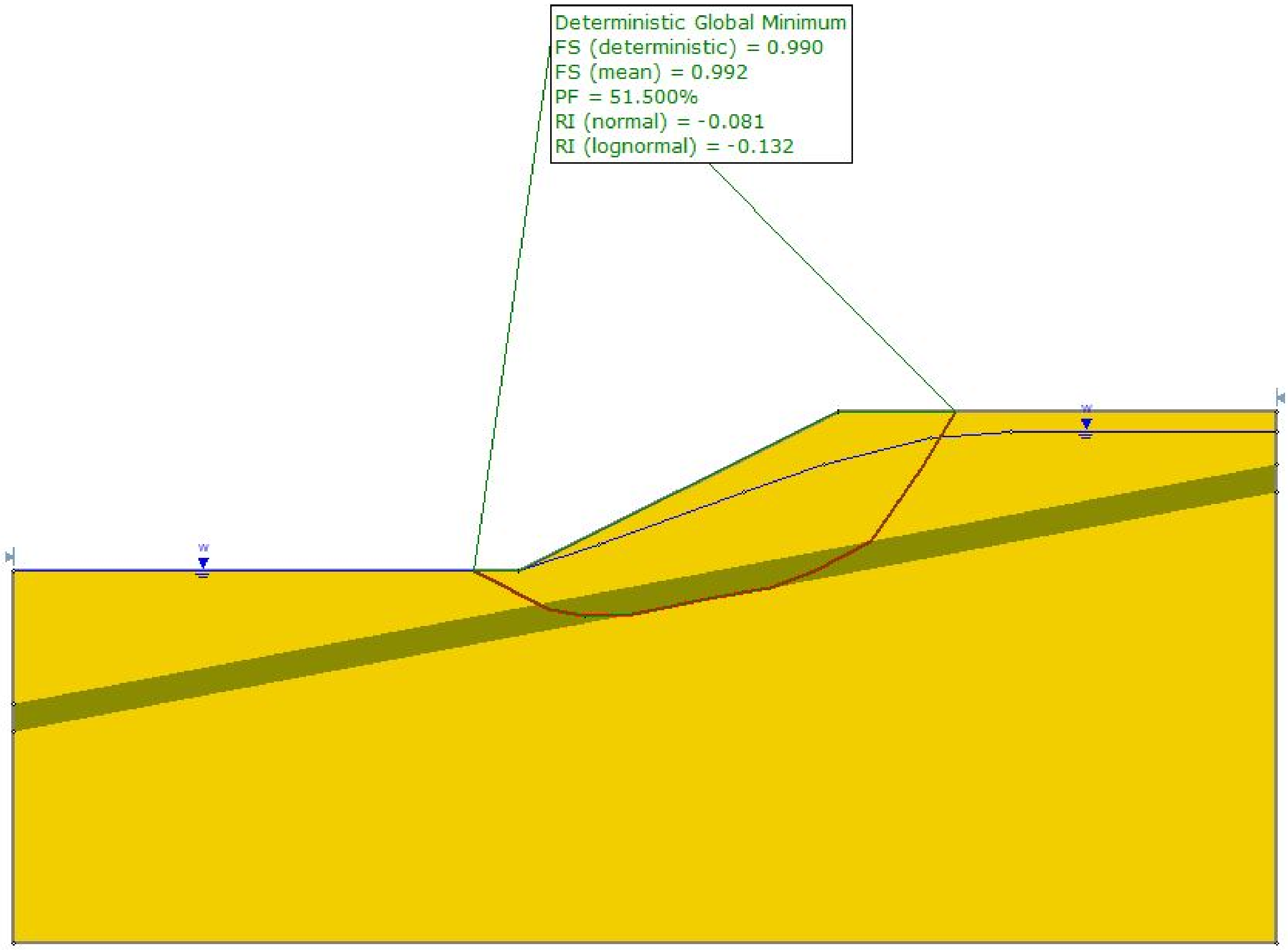
Recall that these are the material properties defined for Group 2.



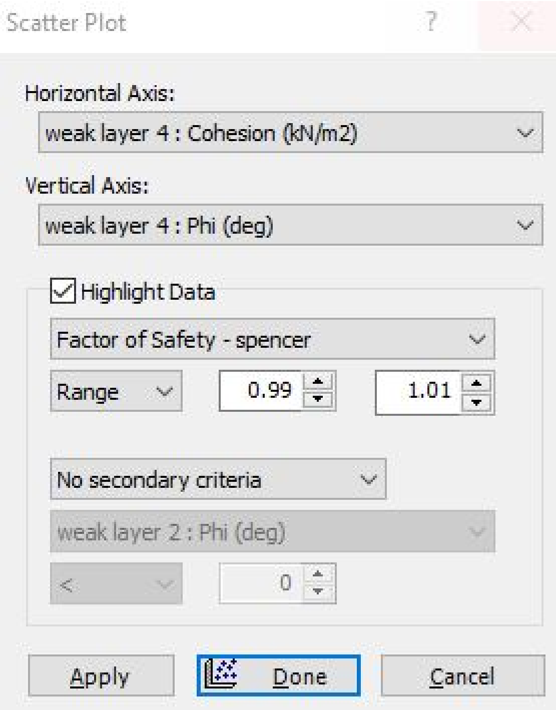
We have used a Uniform Distribution for both Cohesion and Friction Angle. Since we are assuming that BOTH cohesion and friction angle are unknown variables, a simple Uniform distribution will ensure that random samples are generated uniformly over the entire range of each variable. Note that for a Uniform distribution, the Standard Deviation is not applicable and is therefore disabled.

Click on **Group 2** in **Interpret.**

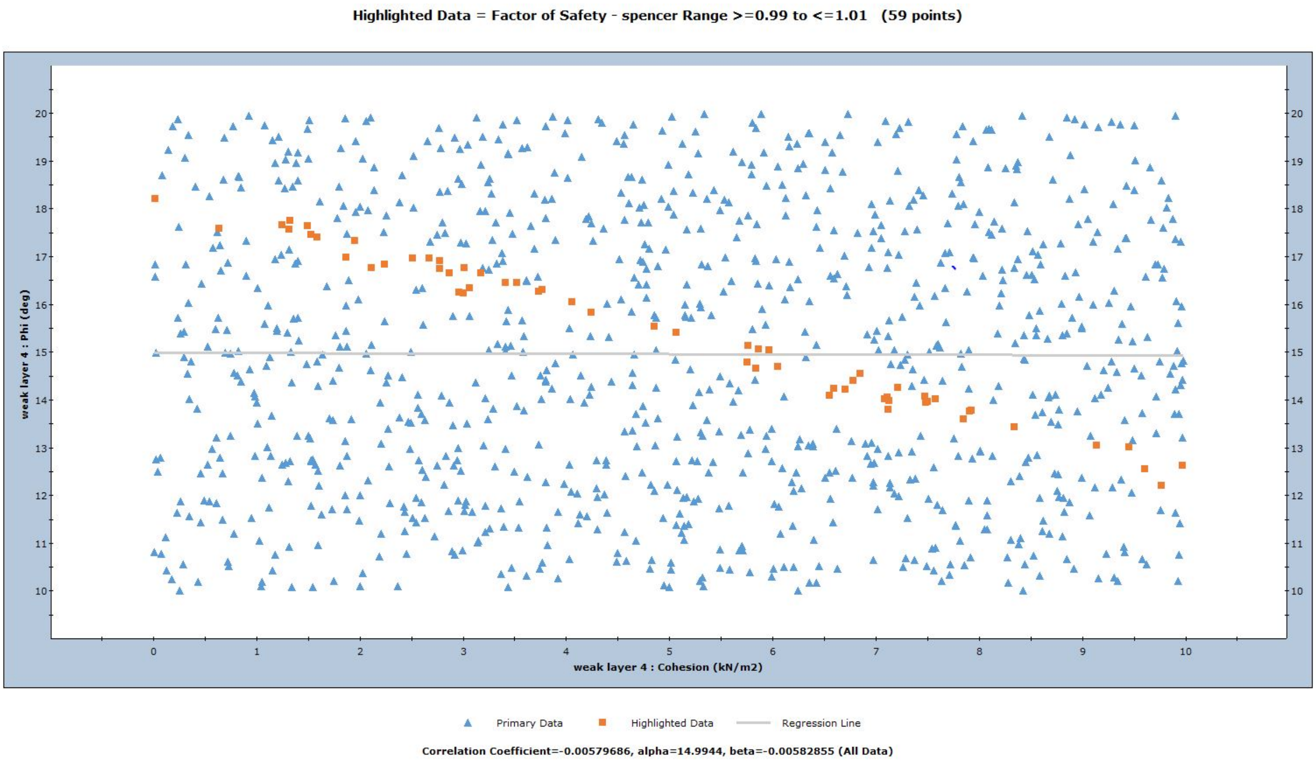
You should see the following results. Note that the deterministic safety factor is still 0.99. A summary of probabilistic results is also listed (PF = probability of failure = 51.5%, i.e. more than half of all outcomes result in slope failure. This is as expected since the distribution of safety factors is approximately centered about the mean value of 1).



Now select the **Scatter Plot** option from the **Statistics** menu. In the Scatter Plot dialog make the following selections. Plot weak layer Cohesion versus Phi. Select the **Highlight Data** checkbox. Select **Factor of Safety – Spencer**. Select the **Range** option and set a range of 0.99 to 1.01. Select the **Plot** button.



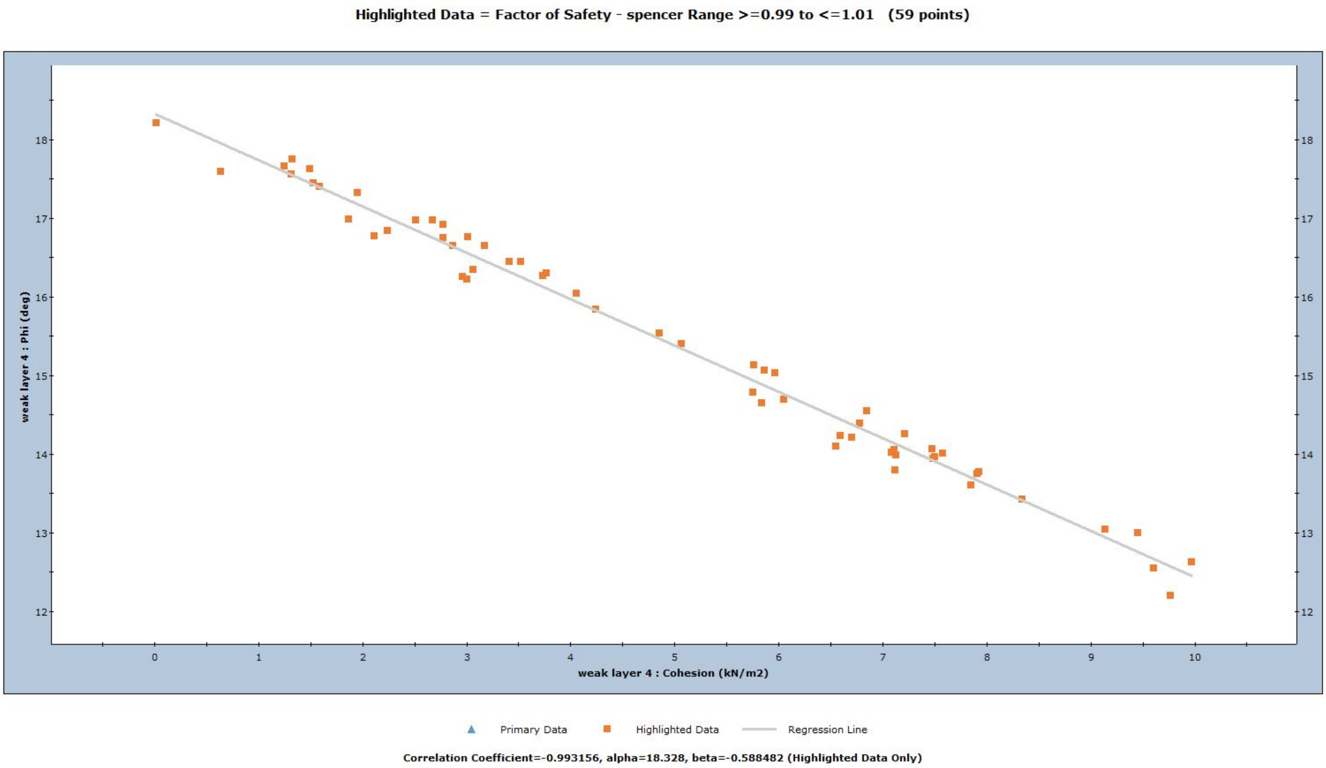
You should see the scatter plot



Since we chose a Uniform distribution for both Cohesion and Phi, and we did not use Correlation, you can see that samples have been uniformly generated over the entire specified range of both variables (i.e. cohesion 0 to 10 and friction angle 10 to 20).

Notice the data points highlighted in red. These represent pairs of cohesion and friction angle, which result in a safety factor between 0.99 and 1.01 (as we specified in the scatter plot dialog).

To view the highlighted data only, right-click on the plot and select “Highlighted Data Only” from the popup menu. The plot should look as follows.



This plot shows the relationship between cohesion and friction angle for a safety factor of approximately 1. As you can see, the relationship is linear over the range of values plotted, for this example.

The parameters of the linear best fit line through the highlighted data, are given at the bottom of the plot. The parameter alpha is the y-intercept and beta is the slope of the line. In this case:

Friction angle = -0.58 *cohesion + 18.2

You could use this equation to determine values of cohesion and friction angle over this range of values. You can also do this graphically with the sampler. If you right-click on the plot and select Sampler > Edit Sampler, you can graphically drag the sampler along the line with the mouse, and the coordinates (cohesion, phi) will be displayed interactively, as before.

Any pair of cohesion, friction angle values which fall on this line, should give you a safety factor of about 1. For example: using the sampler we can determine that the data pair of cohesion = 2 and friction angle = 17 falls on this line. If you input this data as the strength of the weak layer and re-run the analysis, you will see that the safety factor does indeed = 1.

Although the relationship is linear in this example, this will not always necessarily be the case. Depending on the range of values, variables plotted and the nature of your model, a scatter plot might exhibit any type of functional relationship between two variables.

The above procedure can be used for any range of safety factor. For example, if you wanted to find values of cohesion and friction angle corresponding to some other value of safety factor, simply define a small range which brackets the safety factor (e.g. 1.19 to 1.21) using the Highlight Data option in the Scatter plot dialog.

EXPORT TO EXCEL

If you wish to do further processing of data using other applications, you can easily export data to Excel. For example, right-click on the Scatter plot and select Plot in Excel from the popup menu, and the graph and data will automatically be exported to Excel.

5. Back Analysis of Other Variables

In this tutorial, we have only looked at the back analysis of material strength parameters, in particular, cohesion and friction angle of a Mohr-Coulomb material. In Slide2 nearly all input variables can be defined for use in sensitivity or probabilistic analysis. The procedures described in this tutorial could be applied to other strength parameters and strength models (e.g. Hoek-Brown or anisotropic materials) or to any other random input parameter in Slide2 (e.g. support properties). The user is encouraged to experiment with the options available.

This concludes the back analysis of material properties tutorial.