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Empirical Strength Criterion for Rock Masses

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Introduction

The design of an excavation in rock requires an assessment of the likely response of the rock mass to a set of induced stresses. In order to predict this response, a knowledge is required of the complete stress-strain behavior and strength characteristics of the rock mass and of the influence of time on these properties. This paper is concerned with only part of this total response, the peak strength developed under a given set of stresses; the influence of time is not considered.

In many cases such as slopes (30) or shallow tunnels (29), instability may be associated with the structural features in the rock mass, and the shear strengths of these discontinuities will be required for use in design calculations. In some deep underground excavations, stability may depend on the relationship between the induced stresses and the strength of the intact rock (27). The processes of drilling and blasting and excavation by tunnelling machinery are also strongly influenced by the strength of the intact rock material (41,47).

There is a further class of rock engineering problem in which the overall stability of a deep surface cut or the components of a system of underground excavations will be determined by the mass behavior of the rock mass surrounding the excavation. In some cases, the rock mass may be heavily jointed so that, on the scale of the problem, it can be regarded as an isotropic assembly of interlocking angular particles. The transition from intact rock to heavily jointed rock mass with increasing sample size in a hypothetical rock mass surrounding an underground excavation is shown in Fig. 1. Which model will apply in a given case will depend on the size of the excavation relative to the discontinuity

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spacing, the boundary conditions, the imposed stress level, and the orientations and strengths of discontinuities.

A significant proportion of past rock mechanics research effort has been devoted to the search for strength criteria for use in one or more of the classes of design problem previously described. Two general classes of strength criterion have been used—empirical criteria (3,5,25) and criteria based on mechanistic or physical models of the deformation or fracture processes involved (22,26,28,38), or both. Clearly, a fundamental mechanistic approach is to be preferred. However, it must be acknowledged that despite the attention that the subject has received and the volume of the resulting literature, a basic rock mass strength criterion suitable for general practical application, has not been developed as yet.

Because of this, the writers have reexamined the available experimental data in an attempt to develop an empirical strength criterion for rocks and rock masses for use in excavation design. Ideally, such a criterion should:

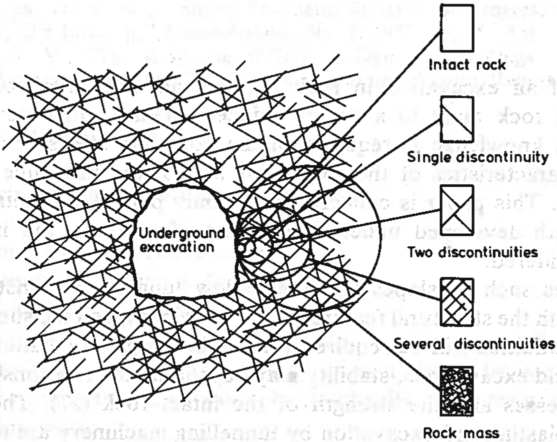

FIG. 1.—Transition from Intact Rock to Heavily Jointed Rock Mass with Increasing Sample Size

1. Adequately describe the response of intact rock material to the full range of stress conditions likely to be encountered in practice.
2. Be capable of accounting for anisotropic strength behavior associated with the existence of planes of weakness.
3. Provide some indication, even if approximate, of the likely strength of a full-scale rock mass containing several sets of discontinuities.

General Empirical Strength Criterion

Examination of a wide range of experimental data for intact rock and rock discontinuities, and the very much more limited range of data available for jointed rock masses, shows that the relationships between major and minor principal stresses and between shear and normal stresses at failure (defined here as the attainment of peak stress), are generally nonlinear. By analogy with the nonlinear failure envelope predicted by classical Griffith crack theory for plane compression (28) and by using a process of trial and error, the writers have developed the following empirical relationship between the principal stresses at failure:

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \sqrt{m\frac{\sigma_3}{\sigma_c} + s} \quad \ldots \ldots (1)$$

in which σ_1 = the major principal stress at failure; σ_3 = the minor principal stress; σ_c = the uniaxial compressive strength of the intact rock material; and m and s = constants that depend on the properties of the rock and on the extent to which it had been broken before being subjected to the failure stresses σ_1 and σ_3.

For conciseness, Eq. 1 may be rewritten in the form

$$\sigma_{1n} = \sigma_{3n} + \sqrt{m\sigma_{3n} + s} \quad \ldots \ldots (2)$$

in which σ_{1n} and σ_{3n} are the values of the principal stresses normalized with respect to $\sigma_c(\sigma_{1n} = \sigma_1/\sigma_c, \sigma_{3n} = \sigma_3/\sigma_c)$.

By putting $\sigma_3 = 0$ in Eq. 1, the uniaxial compressive strength of the rock is obtained as

$$\sigma_{cs} = \sqrt{s\,\sigma_c^2} \quad \ldots \ldots (3)$$

For intact rock material, $s = 1.0$ and $\sigma_{cs} = \sigma_c$ as required. For previously broken rock, $s < 1$ and the compressive strength at zero, confining pressure is given by Eq. 3. It must be emphasized that σ_c is the uniaxial compressive strength of the *intact* rock material making up the specimen. For a completely granulated specimen or a rock aggregate, $s = 0$.

The uniaxial tensile strength of a specimen is found by putting $\sigma_1 = 0$ in Eq. 1 and solving the resulting equation for $\sigma_3 = \sigma_t$ to obtain

$$\sigma_t = \frac{\sigma_c}{2}(m - \sqrt{m^2 + 4s}) \quad \ldots \ldots (4)$$

When $s = 0$, $\sigma_t = 0$ as would be expected for completely broken material. For intact rock material with $s = 1.0$ and $m >> 1$, $m \simeq \sigma_c/|\sigma_t|$. However, because of the difficulty involved in adopting the uniaxial tensile strength as a fundamental rock property (23), it is preferable to treat m simply as an empirical curve fitting parameter. The value of m will decrease as the degree of prior fracturing of a specimen increases.

Although for many design applications, it will be appropriate to express the rock strength criterion in terms of principal stresses, for cases involving shear failure on a single surface or in a narrow zone, a criterion expressed in terms of the shear and normal stresses on the slip surface may be required. Using the relationship between normal and shear stresses on the critical plane derived by Balmer (2) for a general nonlinear failure criterion, it is found that the normal stress, σ, and the shear strength, τ, defined in Fig. 2, can be written as

$$\sigma = \sigma_3 + \frac{\tau_m^2}{\tau_m + \frac{m\sigma_c}{8}} \quad \ldots \ldots (5)$$

$$\text{and} \quad \tau = (\sigma - \sigma_3)\sqrt{1 + \frac{m\sigma_c}{4\tau_m}} \quad \text{. . . . . (6)}$$

$$\text{in which} \quad \tau_m = \frac{1}{2}(\sigma_1 - \sigma_3) \quad \text{. . . . . (7)}$$

From Fig. 2 it can be seen that the normalized values of σ and τ, $\sigma_n = \sigma/\sigma_c$ and $\tau_n = \tau/\sigma_c$, may be related by an equation of the form

$$\tau_n = A(\sigma_n - \sigma_{tn})^B \quad \text{. . . . . (8)}$$

in which σ_{tn} = the normalized tensile strength of the rock given by

$$\sigma_{tn} = \frac{1}{2}(m - \sqrt{m^2 + 4s}) \quad \text{. . . . . (9)}$$

and A and B = constants depending upon the value of m.

In developing the criterion, it has been assumed that the intermediate principal stress will have a negligible influence on failure conditions. This may be less

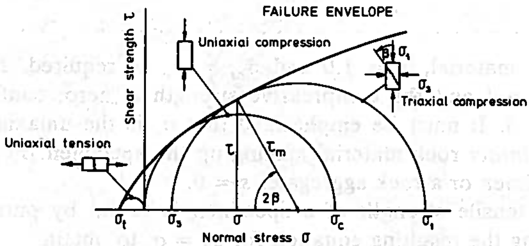

FIG. 2.—Failure Envelope Showing Relationships between Stress Components

acceptable for jointed rock than it is for intact isotropic rock material (49). The criterion has been expressed in terms of total or applied stresses. The effective stress form of the criterion that allows pore—or joint—water pressures to be accounted for can be obtained simply by substituting the effective stresses σ_1' and σ_3' for the total stresses σ_1 and σ_3 in the preceding equations. The parameters m and s determined from the effective stress form of the strength criterion will not be necessarily the same as those evaluated in terms of total stresses.

Application to Isotropic Rock Material

In order to check the applicability of the empirical strength criterion to isotropic rock material, published triaxial test data for a variety of rock types were analyzed. Only those data that were found to be well supported by details of the experimental techniques used were included in the analysis.

It is well recognized that the behavior of most rocks changes from brittle to ductile at some elevated confining pressure, defined by Byerlee (15) as the brittle-ductile transition pressure. Not only does the shape of the stress-strain curve change at this pressure, but the mechanism of deformation also changes in the transition zone (54). Therefore, it cannot be expected that a strength criterion developed for application in the brittle range should be equally applicable in the ductile range. Mogi (42) found that the brittle-ductile transition for most rocks could be defined approximately by the intersection of the line

$$\sigma_1 = 3.4\,\sigma_3 \qquad (10)$$

with the principal stress failure envelope. Accordingly, only those data sets containing five or more experimental points well spaced in the stress space defined by $\sigma_t < \sigma_3 < \sigma_1/3.4$ were accepted for analysis. All data analyzed were for oven- or air-dried samples except where otherwise stated in Table 1.

For intact rock material, s = 1.0, and the normalized form of the strength

TABLE 1.—Values of Strength Parameter m for Intact Rock Materials (s = 1.0)

Rock type (1)	Data from reference numbers (2)	Number of data points (3)	Range of σ_c, in pounds per square inch (4)	m (5)	Coefficient of determination, r^2 (6)
Limestone	21, 41, 51	84	6,380–29,200	5.4	0.68
Dolomite	9, 43	25	21,500–73,400	6.8	0.90
Mudstone	5, 41	34	—[a]	7.3	0.82
Marble	11, 12, 21, 36, 51, 54	105	7,210–19,300	10.6	0.90
Sandstone	1, 5, 8, 21, 32, 36, 41, 48, 50, 51	375	5,790–57,800	14.3	0.87
Dolerite	9, 21, 27	51	42,600–83,000	15.2	0.97
Quartzite	5, 27, 41	59	32,900–47,500	16.8	0.84
Chert	27	24	84,000	20.3	0.93
Norite	5	17	—[a]	23.2	0.97
Quartz-diorite	12	10	27,300 (saturated)–35,200 (dry)	23.4	0.98
Gabbro	12	10	29,700 (saturated)–50,900 (dry)	23.9	0.97
Gneiss	12	10	26,700 (parallel foliation)–25,900 (perpendicular foliation)	24.5	0.91
Amphibolite	36	10	21,300 (parallel foliation)–29,200 (perpendicular foliation)	25.1	0.98
Granite	9, 21, 23, 41, 43, 51, 53	109	16,900–50,000	27.9	0.99

[a]Source data in Ref. 5 given in normalized form.
Note: 1 psi = 6.9 kPa.

criterion becomes

$$\sigma_{1n} = \sigma_{3n} + \sqrt{m\sigma_{3n} + 1.0} \quad \text{(11)}$$

A linear regression analysis was used to determine the values of σ_c and m giving the best fit of Eq. 11 to each individual set of data obtained by a given writer for a particular rock type. These individual sets of data were then grouped according to rock type. Each data set was normalized using its own value of σ_c, and a further linear regression analysis was carried out to determine the value of m giving the best overall fit for each rock type.

Table 1 summarizes the results obtained from this analysis. Typical normalized plots of principal stresses at failure (taken to correspond with the peak value of σ_1) are shown in Figs. 3 and 4 for sandstones and granites, respectively. The grouping and analysis of data according to rock type has obvious disadvan-

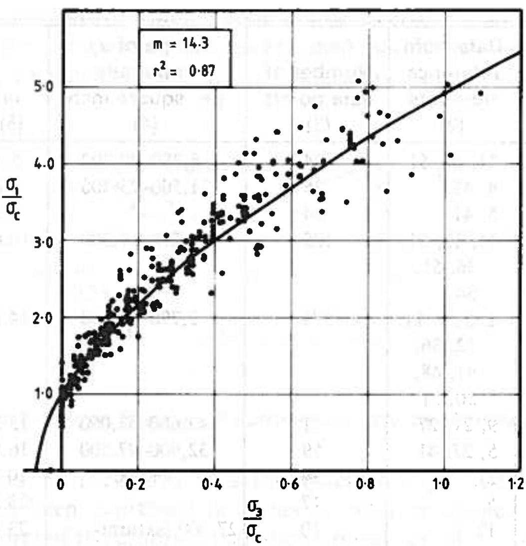

FIG. 3.—Normalized Failure Envelope for Sandstones

tages. Detailed studies of rock strength and fracture indicate that factors such as mineral composition, grain size and angularity, grain packing patterns and the nature of cementing materials between grains, all influence the manner in which fracture initiates and propagates (45). If these factors are relatively uniform within a given rock type, then it might be expected that a single curve would give a good fit to the normalized strength data with a correspondingly high value of the coefficient of determination, r^2. Such a result is shown for granite in Fig. 4. If, on the other hand, these factors are quite variable from one occurrence of a given rock type to another, then a wider scatter of data points and a poorer fit by a single curve might be anticipated. Table 1 shows that for sandstone (Fig. 3) where grain size, porosity and the nature of the cementing material can vary widely, and for limestone that is a name given to a wide variety of carbonate rocks, the values of r^2 are, indeed, quite low.

Despite these difficulties and the sometimes arbitrary allocation of a particular name to a given rock, the results shown in Table 1 do serve an important practical purpose. By using the approximate value of m found to apply for a particular rock type, it may be possible to carry out preliminary design calculations on the basis of no testing other than a determination of a suitable value of σ_c made using a simple test such as the point load test (5). A value of σ_c is required as a scaling factor to determine the strength of a particular sample of rock. Thus, although the same value of m may apply to the granites tested by Schwartz (51) and Brace (9), respectively, the strengths of the two rocks at a given confining pressure can differ by a factor of three.

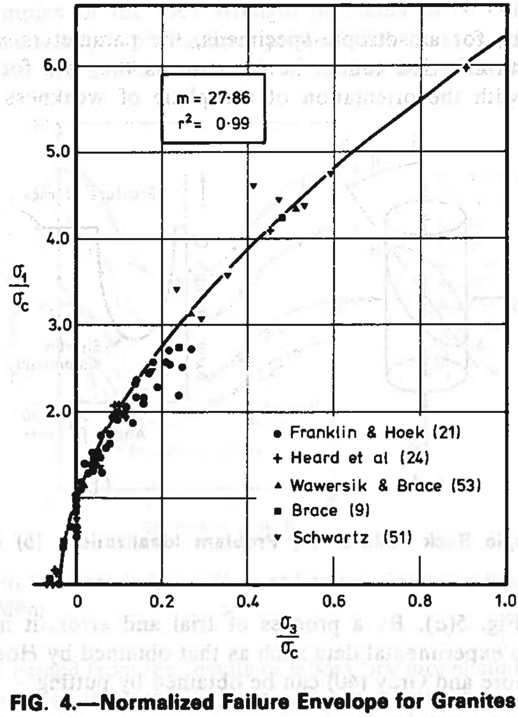

FIG. 4.—Normalized Failure Envelope for Granites

Rock types are listed in Table 1 in increasing order of the value of the parameter m. There is clearly some general pattern to the relationship between intact rock type and m. These and other results not included in the tabulation because of their limited range suggest that m increases with rock type in the following general way: (1) $m \simeq 7$—Carbonate rocks with well developed crystal cleavage (dolomite, limestone, marble); (2) $m \simeq 10$—lithified argillaceous rocks (mudstone, siltstone, shale, slate); (3) $m \simeq 15$—arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone, quartzite); (4) $m \simeq 17$—fine-grained polymineralllic igneous crystalline rocks (andesite, dolerite, diabase, rhyolite); and (5) $m \simeq 25$—coarse-grained polymineralllic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, norite, quartz-diorite).

Application to Anisotropic Rock

The influence of single discontinuities or planes of weakness on the strength of otherwise intact rock has been widely investigated. Jaeger's single plane of weakness theory (31), developed for the case shown in Fig. 5(*a*), postulates two independent failure modes, slip on the discontinuity and shear fracture of the intact rock material, depending on the orientation of the discontinuity to the principal stress directions; see Fig. 5(*b*). This behavior is reasonably well represented in specimens containing a single open joint or an artifically induced discontinuity (31). The behavior of anisotropic rock such as slate or shale is more complex than that allowed for by this theory, with specimen strengths varying continuously with the orientation of the planes of weakness (18,26,40).

It is clear that, for anisotropic specimens, the parameters m and s in the empirical strength criterion cannot be constant as they are for isotropic rock but must vary with the orientation of the plane of weakness as defined by

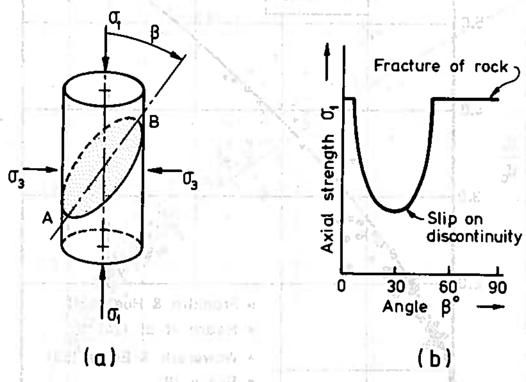

FIG. 5.—Anisotropic Rock Failure: (*a*) Problem Idealization; (*b*) $\sigma_1 - \beta$ Failure Characteristic

the angle β in Fig. 5(*a*). By a process of trial and error, it has been found that a good fit to experimental data such as that obtained by Hoek (26), Donath (18) and McLamore and Gray (40) can be obtained by putting

$$m = m_i\,[1 - N_1 \exp(-\theta)^4] \quad \text{(12)}$$

and $$s = 1 - P_1 \exp(-\zeta)^4 \quad \text{(13)}$$

in which m_i = the value of m for intact rock determined for $\beta = 90°$; and

$$\theta = \frac{\beta - \beta_m}{N_2 + N_3\beta} \quad \text{(14a)}$$

$$\zeta = \frac{\beta - \beta_s}{P_2 + P_3\beta} \quad \text{(14b)}$$

where β_m = the value of β at which m is a minimum; β_s = the value of β at which s is a minimum; and N_1, N_2, N_3, P_1, P_2, and P_3 = constants.

An example of the application of this approach to the triaxial test data obtained for a slate by McLamore and Gray (40) is shown in Fig. 6. It is hardly surprising that a reasonable fit to the data can be obtained given the large number of parameters and constants involved in the anisotropic formulation of the empirical strength criterion. Indeed, it is unlikely that this approach would be particularly useful in practice given that reasonable fits to experimental data can be obtained with less complicated approaches (26,31,40). However, the results obtained do show that the general criterion can be modified to take account of anisotropic strength behavior.

Application to Multiply Jointed Rock

The most complex of the rock strength problems to be considered here is that of the mass strength of multiply jointed rock masses. Very little direct experimental data are available for this case, but some understanding of the

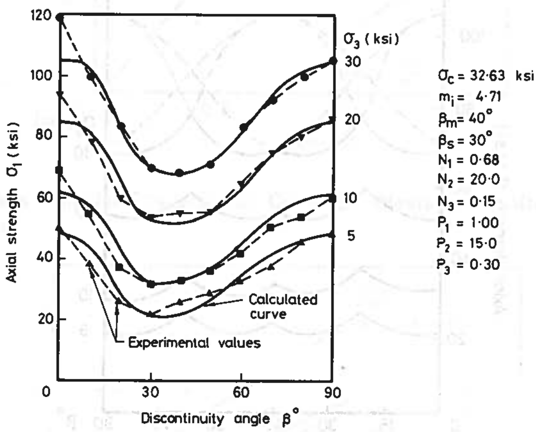

FIG. 6.—Strength Characteristics of Slate at Various Confining Pressures; Note Ref. 40 (1 ksi = 6.9 MPa)

problem can be gained from the results of laboratory model studies (13,14,16,19, 35,38). These studies show that a great number of failure modes are possible in jointed rock (13,14), and that the internal distribution of stresses within a jointed rock mass can be highly complex (16). Of particular interest is the finding that, even where failure takes place not along the discontinuities but through the intact rock material, the strength of a jointed mass may be considerably less than that of the unjointed material tested under otherwise identical conditions (13,19). The relative strength reduction measured in triaxial compression tests decreases with increasing confining pressure (13). The quantification of these effects in natural rock masses remains an outstanding problem in rock engineering.

The influence of multiple discontinuities on rock mass strength in a somewhat different case can be illustrated by reconsidering the data for a slate presented in Fig. 6. Assume that, instead of containing one set of planes of weakness, the specimens contain a number of identical sets oriented at equal angles to each other. It is possible to apply Jaeger's theory in several parts to construct a set of σ_1 versus β characteristics for the composite specimens for the values of σ_3 represented in Fig. 6. The solution for the case of four identical sets of planes of weakness is shown in Fig. 7. In this case, it is found that failure will always take place by slip on one of the planes of weakness. It will be noted that, to a very good first approximation, the strength of this hypothetical rock mass may be considered to be isotropic. A composite failure envelope for this case, constructed using the minimum strength developed for each value of σ_3, yields the parameters $m = 1.48$ and $s = 0.007$ with $\sigma_c = 32{,}630$ psi (225 MPa) measured for $\beta = 90°$; for the intact rock material, $m = 4.71$ and $s = 1.0$.

The result shown in Fig. 7 suggests that if a rock mass contains four or more sets of discontinuities having similar properties, it may be possible to

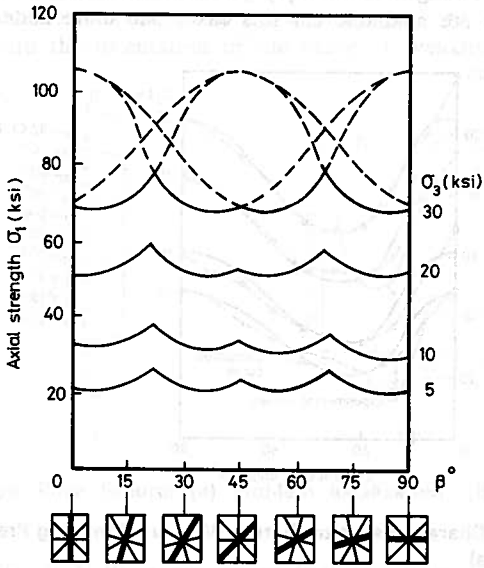

FIG. 7.—Hypothetical Composite Strength Characteristics for Slate Containing Four Identical Sets of Discontinuities (1 ksi = 6.9 MPa)

assume the strength of the rock mass to be isotropic. This will not be the case when one of the discontinuities has a more pronounced influence than the others because of a greater persistence or a lower shear strength due to the presence of gouge, for example.

It is to be expected that the presence of one or more sets of discontinuities in a rock mass will cause a reduction in the values of both m and s. Unfortunately, relatively few sets of reliable triaxial test data for jointed rock are known to exist, and so the choice of m and s for a given rock mass must be based on the very few sets of data available as well as back-analysis of documented cases of rock mass failure and a good measure of judgment. Such data as are available to the writers will be considered in some detail since the lessons to be learned from them are vital to the development of an understanding of the way in which the parameters m and s vary with rock mass characteristics.

The presence of fractures in a rock mass results in a decrease in both m and s below the intact values because of the greater freedom of movement made available to individual pieces of rock material. This effect can be seen

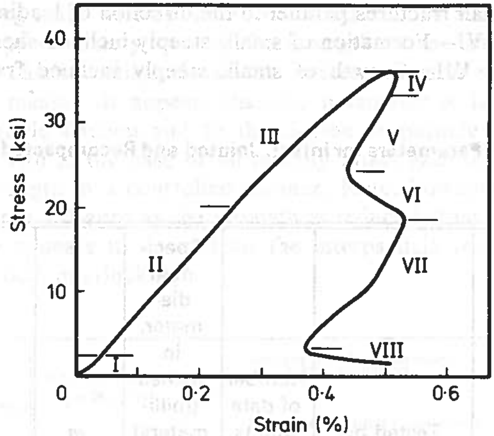

FIG. 8.—Complete Uniaxial Stress-Strain Curve for Westerly Granite; Note Ref. 53 (1 ksi = 6.9 MPa)

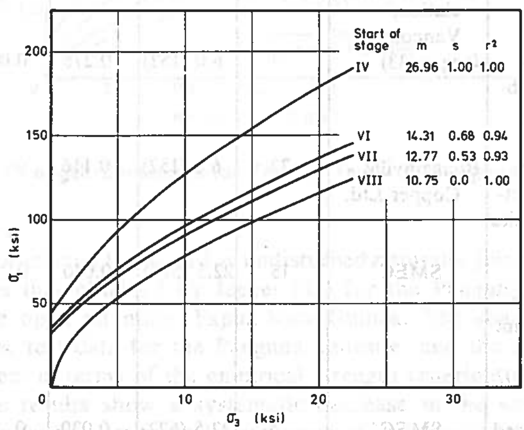

FIG. 9.—Strength Envelopes for Various Stages of Triaxial Tests on Westerly Granite (1 ksi = 6.9 MPa)

in the results of a series of triaxial compression tests carried out on Westerly granite by Wawersik and Brace (53). A stiff testing machine and special testing techniques were used so that the progressive failure of the specimens could be studied. A complete uniaxial stress-strain curve for Westerly granite is shown in Fig. 8. Wawersik and Brace identified a number of well-defined stages in the breakdown process on this and other curves. The same series of stages could be identified in triaxial compression tests carried out at seven different confining pressures, although the relative importance of the various stages changed with the confining pressure.

For the present analysis, the following stages of each test were used:

1. Start of stage IV—Maximum stress associated with the formation of a large number of small fractures parallel to the direction of loading.
2. Start of stage VI—Formation of small, steeply inclined shear fractures.
3. Start of stage VII—Growth of small, steeply inclined fractures into an open fault.

TABLE 2.—Strength Parameters for Intact, Jointed and Recompacted Panguna Andesite

Specimen description (1)	Tested by (2)	Number of data points (3)	Specimen diameter, in inches (millimeters) (4)	*m* (5)	*s* (6)	Coefficient of determination r^2 (7)
Intact fresh rock material	Jaeger (33) Golder Association, Vancouver	5	2.0 (51) 4.0 (102)	18.9	1.00	0.85
Undisturbed closely jointed samples obtained using triple tube core barrel	Jaeger (33)	7	6.0 (152)	0.278	0.0002	0.99
Graded mine bench samples recompacted to close to in situ unit weight	Bougainville Copper Ltd.	72	6.0 (152)	0.116	0	—
Fresh to slightly weathered rock recompacted to a unit weight of 127.5–129.4 pcf (2.04–2.07 t/m^3)	SMEC[a]	15	22.5 (572)	0.040	0	—
Moderately weathered rock recompacted to a unit weight of 124.3 pcf (1.99 t/m^3)	SMEC[a]	5	22.5 (572)	0.030	0	—
Highly weathered rock recompacted to a unit weight of 101.5 pcf (1.63 t/m^3)	SMEC[a]	3	22.5 (572)	0.012	0	—

[a]Snowy Mountains Engineering Corporation

4. Start of stage VIII—Attainment of the residual strength of loose, broken material held together by friction between the particles.

Each of these points was identified on the stress-strain curve obtained at each confining pressure and the corresponding relationship between the principal stresses plotted (Fig. 9). As shown on Fig. 9, both *m* and *s* decrease as the degree of fracturing of the initially intact rock increases. The value of *m* decreases by a factor of approximately two as the strength of the rock reduces from peak to residual. This is a relatively modest decrease compared with that found for natural rock masses. It appears that the parameter *m* is sensitive to the angle of interparticle friction and to the degree of particle interlocking that is still relatively high in the case of an initially intact granite specimen loaded to its residual strength in a controlled manner. Note, however, that the value of *s* decreases from 1.0–zero as the strength is reduced from peak to residual. The parameter *s* appears to depend on the interparticle tensile strength and the degree of particle interlocking.

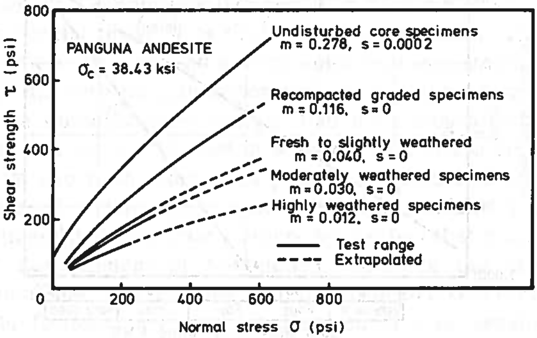

FIG. 10.—Shear Strength Envelopes for Triaxial Tests on Panguna Andesite (1 psi = 6.9 kPa)

The only set of triaxial test data for undisturbed naturally jointed rock known to the writers is that obtained by Jaeger (33) for the Panguna andesite from the Bougainville open pit mine, Papua-New Guinea. The sources and nature of this and other test data for the Panguna andesite, and the results obtained by analyzing them in terms of the empirical strength criterion, are summarized in Table 2. The results show a systematic decrease in the values of *m* and *s* with increasing fragmentation and weathering of the samples (Fig. 10).

A decrease in values of *m* and *s* with increased jointing intensity was also found by reanalyzing the results of laboratory tests on models of jointed rock (13,19,35) and the case history data presented by Brady (10). Although insufficient data are presented to enable complete analyses to be carried out, John (34), Manev and Avramova-Tacheva (39) and Muller (44) all present information for natural rock masses that indicate similar trends.

It is apparent that, although trends are clearly indicated, the test data available are insufficient to permit the determination of a set of parameters that could be used to quantify the full range of rock mass strength behavior. Indeed, it is most unlikely that, because of the complexity and expense of obtaining it, such data will ever become available. Accordingly, some other aid must be sought in the development of a means of predicting rock mass strengths. One possible approach is to use the well-known rock mass quality classification schemes developed by Barton, Lien and Lunde (4) of the Norwegian Geotechnical Institute (NGI) and Bieniawski (6), lately of the South African Council for Scientific and Industrial Research (CSIR). Fundamental objections can be raised to the use of these classification schemes in engineering design in that they do not address the mechanics of engineering problems. However they have the advantage of providing reasonably consistent quantification of rock mass quality based on data that can be collected or estimated in the site investigation phase of a construction project.

Fig. 11 shows plots of the parameter s and the ratio m/m_i, in which m_i is the value of the parameter m for intact rock material, against the NGI and

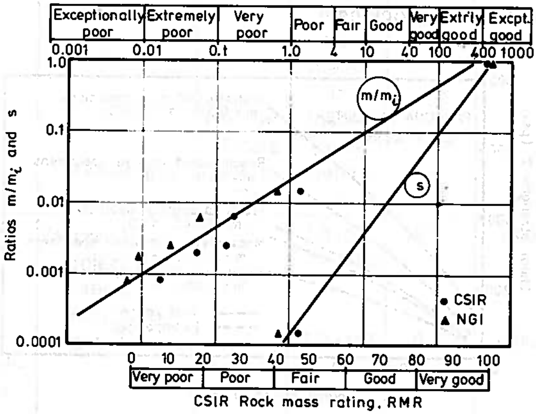

FIG. 11.—Plots of m/m_i and s for Panguna Andesite against Rock Mass Classifications

CSIR classification ratings estimated for the various categories of Panguna andesite listed in Table 2. Bieniawski (7) proposed the following relationship between the NGI quality index, Q, and the CSIR rock mass rating, RMR:

$$\text{RMR} = 9 \ln Q + 44 \quad (15)$$

This relationship was used in positioning the scales in Fig. 11.

Despite the paucity and approximate nature of the data, straight lines were drawn on Fig. 11 to give approximate relationships between s and m/m_i and the classification ratings. Using these relationships and the limited experimental data available as guides, approximate relationships between rock type, rock quality and normalized strength were developed (Table 3). In any given case the uniaxial compressive strength of approx 2.0 in. (51 mm) diameter samples of fresh intact rock material will be required to apply to the calculated normalized strength as a scaling factor. The rock mass strengths so obtained are intended to include an allowance for the scale effect in rock material strength.

It must be strongly emphasized that the relationships set out in Table 3 are based on very sparse data and are therefore very approximate; they should be used only as rough guides in preliminary design calculations. Where rock or rock mass failure is likely to be an important consideration in the design of a structure, every attempt should be made to determine the required strength parameters by laboratory and in situ testing and by observations of the full-scale performance of the rock mass around trial excavations (10). The strength relationships given in Table 3 should not be used in cases in which slip is considered likely to take place on one or two dominant planes of weakness.

Practical Applications

The empirical strength criterion has been used successfully in a number of projects involving slopes and underground excavations in rock. Programs for digital computers and programmable calculators have been readily modified to incorporate the nonlinear strength criterion. Generally, the approach has been used for preliminary design calculations in cases in which rock mass strength parameters have had to be estimated from Table 3 using site investigation data to assess rock mass quality. An exception to this general rule was a slope stability analysis for the Bougainville open pit mine in which strength parameters obtained from triaxial test results (Table 2) could be used. The application of the criterion in slope stability and underground excavation analyses will follow.

The criterion has also been incorporated into the ground-support interaction analyses used in tunnel support design. Ladanyi's closed-form solution for a concrete lining to a circular tunnel in a hydrostatic stress field (37) has been modified so that short- and long-term peak and residual strengths of the rock mass are expressed in terms of the new criterion. Ladanyi's approach to the problem is distinguished from many others by the fact that it includes a realistic model of the development of volumetric strains in the failing rock mass surrounding the tunnel. The resulting solution has been coded for a programmable calculator. The formulations used by Daemen (17) in developing numerical solutions to a number of more complex ground-support interaction problems have also been modified to incorporate the empirical rock mass strength criterion.

Slope Stability.—The nonlinear criterion can be incorporated into standard slope stability programs such as those that analyze circular or noncircular slips using Bishop's or Janbu's methods. Since this class of slope stability problem involves shear failure of the rock mass along a continuous slip surface, the shear strength-normal stress form of the criterion (Eq. 8) is used. In modifying existing programs, it is most convenient to define instantaneous values of cohesion, c_i, and friction angle, ϕ_i, from the tangent to the shear strength (τ)—normal stress (σ) curve at the appropriate value of normal stress. These values are given by

$$\tan \phi_i = AB\left(\frac{\sigma}{\sigma_c} - \frac{\sigma_t}{\sigma_c}\right)^{B-1} \quad (16)$$

and $c_i = \tau - \sigma \tan \phi_i$ . . . . (17)

in which σ_t and τ can be calculated from Eqs. 9 and 8, respectively, for the assumed values of A, B and σ_c. Rather than being constants for the jointed rock mass as assumed by Bieniawski (6) and Stimpson and Ross-Brown (52), c_i and ϕ_i will vary with stress level and so must be calculated individually

TABLE 3.—Approximate Strength Criteria

Rock quality (1)	Carbonate rocks with well developed crystal cleavage (dolomite, limestone and marble) (2)	Lithified argillaceous rocks, (mudstone, siltstone, shale and slate norite and quartz-diorite) (3)
Intact rock samples—laboratory size rock specimens free from structural defects (CSIR rating 100+; NGI rating 500)	$\sigma_{1n} = \sigma_{3n} + \sqrt{7\sigma_{3n} + 1.0}$ $\tau_n = 0.816(\sigma_n + 0.140)^{0.658}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{10\sigma_{3n} + 1.0}$ $\tau_n = 0.918(\sigma_n + 0.099)^{0.677}$
Very good quality rock mass—tightly interlocking undisturbed rock with unweathered joints spaced at 3 m ± (CSIR rating 85; NGI rating 100)	$\sigma_{1n} = \sigma_{3n} + \sqrt{3.5\sigma_3 + 0.1}$ $\tau_n = 0.651(\sigma_n + 0.028)^{0.679}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{5\sigma_{3n} + 0.1}$ $\tau_n = 0.739(\sigma_n + 0.020)^{0.692}$
Good quality rock mass—fresh to slightly weathered rock, slightly disturbed with joints spaced at 1 m–3 m (CSIR rating 65; NGI rating 10)	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.7\sigma_{3n} + 0.004}$ $\tau_n = 0.369(\sigma_n + 0.006)^{0.669}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{1.0\sigma_{3n} + 0.004}$ $\tau_n = 0.427(\sigma_n + 0.004)^{0.683}$
Fair quality rock mass—several sets of moderately weathered joints spaced at 0.3 m–1 m (CSIR rating 44; NGI rating 1.0)	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.14\sigma_{3n} + 0.0001}$ $\tau_n = 0.198(\sigma_n + 0.0007)^{0.662}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.20\sigma_{3n} + 0.0001}$ $\tau_n = 0.234(\sigma_n + 0.0005)^{0.675}$
Poor quality rock mass—numerous weathered joints spaced at 30 mm–500 mm with some gouge filling/clean waste rock (CSIR rating 23; NGI rating 0.1)	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.04\sigma_{3n} + 0.00001}$ $\tau_n = 0.115(\sigma_n + 0.0002)^{0.646}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.05\sigma_{3n} + 0.00001}$ $\tau_n = 0.129(\sigma_n + 0.0002)^{0.655}$
Very poor quality rock mass—numerous heavily weathered joints spaced less than 50 mm with gouge filling/waste rock with fines (CSIR rating 3; NGI rating 0.01)	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.007\sigma_{3n} + 0}$ $\tau_n = 0.042(\sigma_n)^{0.534}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.010\sigma_{3n} + 0}$ $\tau_n = 0.050(\sigma_n)^{0.539}$

for each slice in slope stability analyses.

As an example, consider a fair quality argillaceous rock mass for which Table 3 gives the normalized shear strength criterion as

$$\tau_n = 0.234(\sigma_n + 0.0005)^{0.675} \quad (18)$$

In this case, $A = 0.234$; $B = 0.675$; and $\sigma_t/\sigma_c = -0.0005$. The resulting normalized shear strength envelope is shown in Fig. 12(*a*). Note that the unconfined compressive strength of the rock mass is estimated as 0.01 σ_c. Fig. 12(*b*) shows the variation in instantaneous cohesion and instantaneous angle of friction with normal stress for σ_c = 5,000 psi (34.5 MPa).

In the stability analyses carried out to date, parameters given in Table 3 have been used for both total and effective stress analyses. As previously noted, it is likely that the total and effective stress values of the parameters *m*, *s*, *A* and *B* will differ. Because of the lack of adequate field or laboratory data,

for Intact Rock and Jointed Rock Masses

Arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone and quartzite) (4)	Fine grained polyminerallic igneous crystalline rocks, (andesite, dolerite, diabase and rhyolite (5)	Coarse grained polyminerallic igneous and metamorphic crystalline rocks (amphibolite, gabbro, gneiss, granite, norite and quartz-diorite) (6)
$\sigma_{1n} = \sigma_{3n} + \sqrt{15\sigma_{3n} + 1.0}$ $\tau_n = 1.044(\sigma_n + 0.067)^{0.692}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{17\sigma_{3n} + 1.0}$ $\tau_n = 1.086(\sigma_n + 0.059)^{0.696}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{25\sigma_{3n} + 1.0}$ $\tau_n = 1.220(\sigma_n + 0.040)^{0.705}$
$\sigma_{1n} = \sigma_{3n} + \sqrt{7.5\sigma_{3n} + 0.1}$ $\tau_n = 0.848(\sigma_n + 0.013)^{0.702}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{8.5\sigma_{3n} + 0.1}$ $\tau_n = 0.883(\sigma_n + 0.012)^{0.705}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{12.5\sigma_{3n} + 0.1}$ $\tau_n = 0.998(\sigma_n + 0.008)^{0.712}$
$\sigma_{1n} = \sigma_{3n} + \sqrt{1.5\sigma_{3n} + 0.004}$ $\tau_n = 0.501(\sigma_n + 0.003)^{0.695}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{1.7\sigma_{3n} + 0.004}$ $\tau_n = 0.525(\sigma_n + 0.002)^{0.698}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{2.5\sigma_{3n} + 0.004}$ $\tau_n = 0.603(\sigma_n + 0.002)^{0.707}$
$\sigma_{1n} = \sigma_{3n} + \sqrt{0.30\sigma_{3n} + 0.0001}$ $\tau_n = 0.280(\sigma_n + 0.0003)^{0.688}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.34\sigma_{3n} + 0.0001}$ $\tau_n = 0.295(\sigma_n + 0.0003)^{0.691}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.50\sigma_{3n} + 0.0001}$ $\tau_n = 0.346(\sigma_n + 0.0002)^{0.700}$
$\sigma_{1n} = \sigma_{3n} + \sqrt{0.8\sigma_{3n} + 0.00001}$ $\tau_n = 0.162(\sigma_n + 0.0001)^{0.672}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.09\sigma_{3n} + 0.00001}$ $\tau_n = 0.172(\sigma_n + 0.0001)^{0.676}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.13\sigma_{3n} + 0.00001}$ $\tau_n = 0.203(\sigma_n + 0.0001)^{0.686}$
$\sigma_{1n} = \sigma_{3n} + \sqrt{0.015\sigma_{3n} + 0}$ $\tau_n = 0.061(\sigma_n)^{0.546}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.017\sigma_{3n} + 0}$ $\tau_n = 0.065(\sigma_n)^{0.548}$	$\sigma_{1n} = \sigma_{3n} + \sqrt{0.025\sigma_{3n} + 0}$ $\tau_n = 0.078(\sigma_n)^{0.556}$

it has not been possible to determine appropriate effective stress parameters. Until this can be done, the values given in Table 3 should be used with σ_c always being measured for field moisture conditions.

Over-Stressed Zones Around Underground Excavations.—The criterion has been used in conjunction with elastic stress analyses to determine the likely extent of overstressed zones around underground excavations. This approach has been particularly useful in the early stages of the design of underground power stations, e.g., when little information about the properties and behavior of the rock mass is available, but design decisions have to be made about excavation shapes and dimensions.

For this purpose, the principal stress form of the criterion has been incorporated into a computer program that uses an indirect formulation of the boundary element method to calculate the plane strain elastic stresses and displacements around underground excavations of any shape (20). The program allows solution of problems involving deep excavations for which the stress fields may be considered constant with depth, and shallow excavations in which the stress fields are influenced by gravity. Having determined the principal stresses at a series of boundary and internal points, the program calculates the available strength at each point in terms of a major principal stress at failure from Eq. 2 or compressive failure and Eq. 4 or tensile failure, using the computed values of σ_3 and the values of σ_c, m and s chosen as input data. The rock mass strengths that can be developed at each point are then compared with the computed values of σ_1 (compression) or σ_3 (tension) to give a series of strength/stress ratios from which contours can be drawn and potentially over-stressed zones identified.

This approach uses a plane strain elastic stress analysis and does not allow for progressive failure or the post-peak stress-strain behavior of the rock mass. Accordingly, it cannot be expected to provide accurate solutions to all practical problems. Nevertheless, it has been found useful as a quick and inexpensive means of obtaining some indication of the likely behavior of a proposed excavation, particularly in the preliminary stages of design when a series of trial analyses may be required.

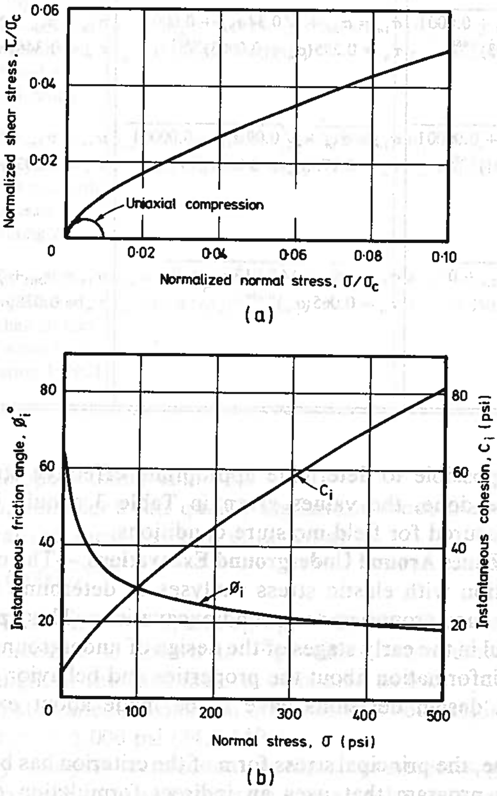

FIG. 12.—(*a*)Normalized Shear Strength Envelope for Fair Quality Argillaceous Rock Mass; (*b*) Variation of c_i and ϕ_i with Normal Stress (1 psi = 6.9 kPa)

As an example of the use of this simple approach, consider a powerhouse cavern to be excavated at a depth of 1210 ft (370 m) in a good quality gneiss. Site investigation data show that the rock material has an unconfined compressive strength of $\sigma_c = 21{,}750$ psi (150 MPa), the unit weight of rock mass is $\gamma =$ 170 pcf (0.027 MN/m^3), and the ratio of horizontal to vertical in situ stress at cavern depth is $k = 0.5$. For this rock mass, strength parameters of $m =$ 2.5 and $s = 0.004$ are selected from Table 3.

Fig. 13 shows the strength/stress ratio contours calculated for three trial cavern shapes for these conditions. Fig. 13(*a*) shows a design popular in the 1950's and 1960's in which the cavern roof is supported by a full concrete arch. The arch reaction and the crane beams are supported by notched haunches cut into the cavern walls. The zone of overstressed rock indicated for this

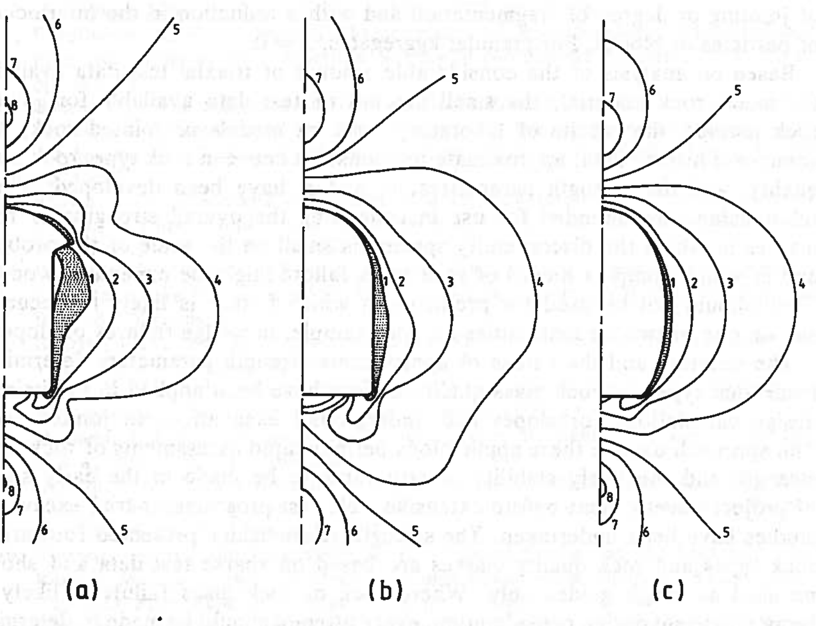

FIG. 13.—Influence of Cavern Shape on Strength/Stress Contours

case is unacceptably large. Long rockbolts could be used to stabilize the walls or, alternatively, the cavern shape could be changed to improve the induced stress distribution. By using rockbolts rather than a concrete arch to support the roof, and by supporting the crane beams on columns, the notch in the wall can be eliminated. As shown in Fig. 13(*b*), this results in a significant reduction in the volume of overstressed rock adjacent to the cavern wall. Further improvement can be achieved by slightly curving the walls as shown in Fig. 13(*c*). In this case, the relatively narrow strip of overstressed rock could probably be supported by short rockbolts or shotcrete, or both. If the cavern walls were curved as in Fig. 13(*c*), it would be necessary that the crane beams be anchored to the cavern walls. This approach offers many constructional advantages and is being increasingly used in cavern design.

Summary and Conclusions

A nonlinear empirical peak strength criterion for rocks and rock masses has been developed. The criterion uses the uniaxial compressive strength of the intact rock material as a scaling parameter, and introduces two dimensionless strength parameters, m and s. The parameter m appears to vary with rock type, the angle of interparticle or interblock friction, and the degree of particle interlocking within the rock mass. The parameter s appears to depend on the interparticle tensile strength and the degree of particle interlocking.

For isotropic intact rock material, $s = 1.0$ and m is a constant depending on rock type. For anisotropic rock, both m and s vary with the orientations of the planes of weakness to the principal stress directions. For highly jointed or fragmented rock, both m and s decrease with an increase in the intensity of jointing or degree of fragmentation and with a reduction in the interlocking of particles or blocks. For granular aggregates, $s = 0$.

Based on analysis of the considerable amount of triaxial test data available for intact rock material, the small amount of test data available for jointed rock masses, the results of laboratory tests on models of jointed rock, and some case history data, approximate relationships between rock type, rock mass quality, and the strength parameters, m and s, have been developed. These relationships are intended for use in estimating the overall strengths of rock masses in which the discontinuity spacing is small on the scale of the problem and in which complex modes of rock mass failure might be expected to occur. They should not be used for problems in which failure is likely to occur by slip on one or two discontinuities as, for example, in wedge failures of slopes.

The criterion and the values of approximate strength parameters determined from rock type and rock mass quality indices have been applied in preliminary design calculations for slopes and underground excavations in jointed rock. The approach used in these applications permits rapid assessments of rock mass strength and the likely stability of structures to be made in the early stages of project development before extensive field test programs or trial excavation studies have been undertaken. The strength relationships presented for various rock types and rock quality classes are based on sparse test data and should be used as rough guides only. Where rock or rock mass failure is likely to be an important design consideration, every attempt should be made to determine the required strength parameters by laboratory or in situ testing and by observations of the full-scale performance of the rock mass around trial excavations.

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