Rock mass properties for underground mines

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1.1   INTRODUCTION

Reliable estimates of the strength and deformation characteristics of rock masses are required for almost any form of analysis used for the design of underground excavations. Hoek and Brown (1980a, 1980b) proposed a method for obtaining estimates of the strength of jointed rock masses, based upon an assessment of the interlocking of rock blocks and the condition of the surfaces between these blocks. This method was modified over the years in order to meet the needs of users who applied it to problems that were not considered when the original criterion was developed (Hoek 1983, Hoek and Brown 1988). The application of the method to very poor quality rock masses required further changes (Hoek, Wood and Shah 1992) and, eventually, the development of a new classification called the Geological Strength Index (Hoek 1994, Hoek, Kaiser and Bawden 1995, Hoek and Brown 1997, Hoek, Marinos and Benissi (1998)). A review of the development of the criterion and of the equations proposed at various stages in this development is given in Hoek and Brown (1997).

   This chapter presents the Hoek-Brown criterion in a form that has been found practical in the field and that appears to provide the most reliable set of results for use as input for methods of analysis currently used in rock engineering.

1.2   GENERALISED HOEK-BROWN CRITERION

The Generalised Hoek-Brown failure criterion for jointed rock masses is defined by:

$$\sigma_1^{'} = \sigma_3^{'} + \sigma_{ci}\left(m_b\,\frac{\sigma_3^{'}}{\sigma_{ci}} + s \right)^a \tag{1.1}$$

where $\sigma_1^{'}$ and $\sigma_3^{'}$ are the maximum and minimum effective stresses at failure,

   m_b is the value of the Hoek-Brown constant m for the rock mass,

   s and a are constants which depend upon the rock mass characteristics, and

   σ_{ci} is the uniaxial compressive strength of the intact rock pieces.

   The Mohr envelope, relating normal and shear stresses, can be determined by the method proposed by Hoek and Brown (1980a). In this approach, equation 1.1 is used

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to generate a series of triaxial test values, simulating full scale field tests, and a statistical curve fitting process is used to derive an equivalent Mohr envelope defined by the equation:

$$\tau = A\sigma_{ci}\left(\frac{\sigma_n' - \sigma_{tm}}{\sigma_{ci}}\right)^B \tag{1.2}$$

where *A* and *B* are material constants

σ_n' is the normal effective stress, and

σ_{tm} is the 'tensile' strength of the rock mass.

In order to use the Hoek-Brown criterion for estimating the strength and deformability of jointed rock masses, three 'properties' of the rock mass have to be estimated. These are

1. the uniaxial compressive strength σ_{ci} of the intact rock elements,
2. the value of the Hoek-Brown constant m_i for these intact rock elements, and
3. the value of the Geological Strength Index *GSI* for the rock mass.

1.3   INTACT ROCK PROPERTIES

For the intact rock pieces that make up the rock mass equation 1.1 simplifies to:

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_i\frac{\sigma_3'}{\sigma_{ci}} + 1\right)^{0.5} \tag{1.3}$$

The relationship between the principal stresses at failure for a given rock is defined by two constants, the uniaxial compressive strength σ_{ci} and a constant m_i. Wherever possible the values of these constants should be determined by statistical analysis of the results of a set of triaxial tests on carefully prepared core samples. When laboratory tests are not possible, Table 1.1 and Table 1.2 can be used to obtain estimates of σ_{ci} and m_i.

In the case of mineralised rocks, the effects of alteration can have a significant impact on the properties of the intact rock components and this should be taken into account in estimating the values of σ_{ci} and m_i. For example, the influence of quartz-seritic alteration of andesite and porphyry is illustrated in the Figure 1.1, based upon data provided by Karzulovic (2000). Similar trends have been observed for other forms of alteration and, where this type of effect is considered likely, the geotechnical engineer would be well advised to invest in a program of laboratory testing to establish the appropriate properties for the intact rock.
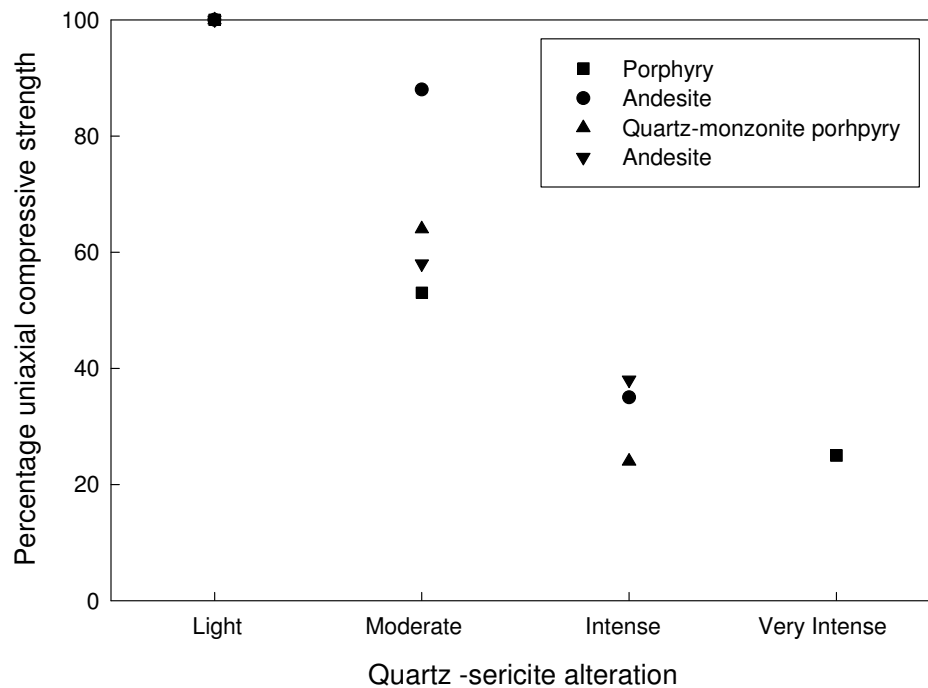
Figure 1.1: Influence of quartz-seritic alteration on the uniaxial compressive strength of "intact" specimens of andesite and porphyry. (After Karzulovic, 2000)

The Hoek-Brown failure criterion, which assumes isotropic rock and rock mass behaviour, should only be applied to those rock masses in which there are a sufficient number of closely spaced discontinuities, with similar surface characteristics, that isotropic behaviour involving failure on multiple discontinuities can be assumed. When the structure being analysed is large and the block size small in comparison, the rock mass can be treated as a Hoek-Brown material.

Where the block size is of the same order as that of the structure being analysed or when one of the discontinuity sets is significantly weaker than the others, the Hoek-Brown criterion should not be used. In these cases, the stability of the structure should be analysed by considering failure mechanisms involving the sliding or rotation of blocks and wedges defined by intersecting structural features. Figure 1.2 summarises these statements in a graphical form.

1.4   GEOLOGICAL STRENGTH INDEX

The strength of a jointed rock mass depends on the properties of the intact rock pieces and also upon the freedom of these pieces to slide and rotate under different stress conditions. This freedom is controlled by the geometrical shape of the intact rock pieces as well as the condition of the surfaces separating the pieces. Angular rock pieces with clean, rough discontinuity surfaces will result in a much stronger rock mass than one which contains rounded particles surrounded by weathered and altered material.

Table 1.1: Field estimates of uniaxial compressive strength.

Grade*	Term	Uniaxial Comp. Strength (MPa)	Point Load Index (MPa)	Field estimate of strength	Examples
R6	Extremely Strong	> 250	>10	Specimen can only be chipped with a geological hammer	Fresh basalt, chert, diabase, gneiss, granite, quartzite
R5	Very strong	100 - 250	4 - 10	Specimen requires many blows of a geological hammer to fracture it	Amphibolite, sandstone, basalt, gabbro, gneiss, granodiorite, peridotite , rhyolite, tuff
R4	Strong	50 - 100	2 - 4	Specimen requires more than one blow of a geological hammer to fracture it	Limestone, marble, sandstone, schist
R3	Medium strong	25 - 50	1 - 2	Cannot be scraped or peeled with a pocket knife, specimen can be fractured with a single blow from a geological hammer	Concete, phyllite, schist, siltstone
R2	Weak	5 - 25	**	Can be peeled with a pocket knife with difficulty, shallow indentation made by firm blow with point of a geological hammer	Chalk, claystone, potash, marl, siltstone, shale, rocksalt,
R1	Very weak	1 - 5	**	Crumbles under firm blows with point of a geological hammer, can be peeled by a pocket knife	Highly weathered or altered rock, shale
R0	Extremely weak	0.25 - 1	**	Indented by thumbnail	Stiff fault gouge

* Grade according to Brown (1981).
** Point load tests on rocks with a uniaxial compressive strength below 25 MPa are likely to yield highly ambiguous results.

Table 1.2: Values of the constant m_i for intact rock, by rock group. Note that values in parenthesis are estimates.

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerates (21 ± 3) Breccias (19 ± 5)	Sandstones 17 ± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Marls (7 ± 2)
	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
		Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
		Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6		
	Foliated*		Gneiss 28 ± 5	Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite Diorite 32 ± 3 25 ± 5 Granodiorite (29 ± 3)			
		Dark	Gabbro 27 ± 3 Norite 20 ± 5	Dolerite (16 ± 5)		
	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5	Dacite (25 ± 3) Basalt (25 ± 5)	Obsidian (19 ± 3)
		Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

* These values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if failure occurs along a weakness plane.
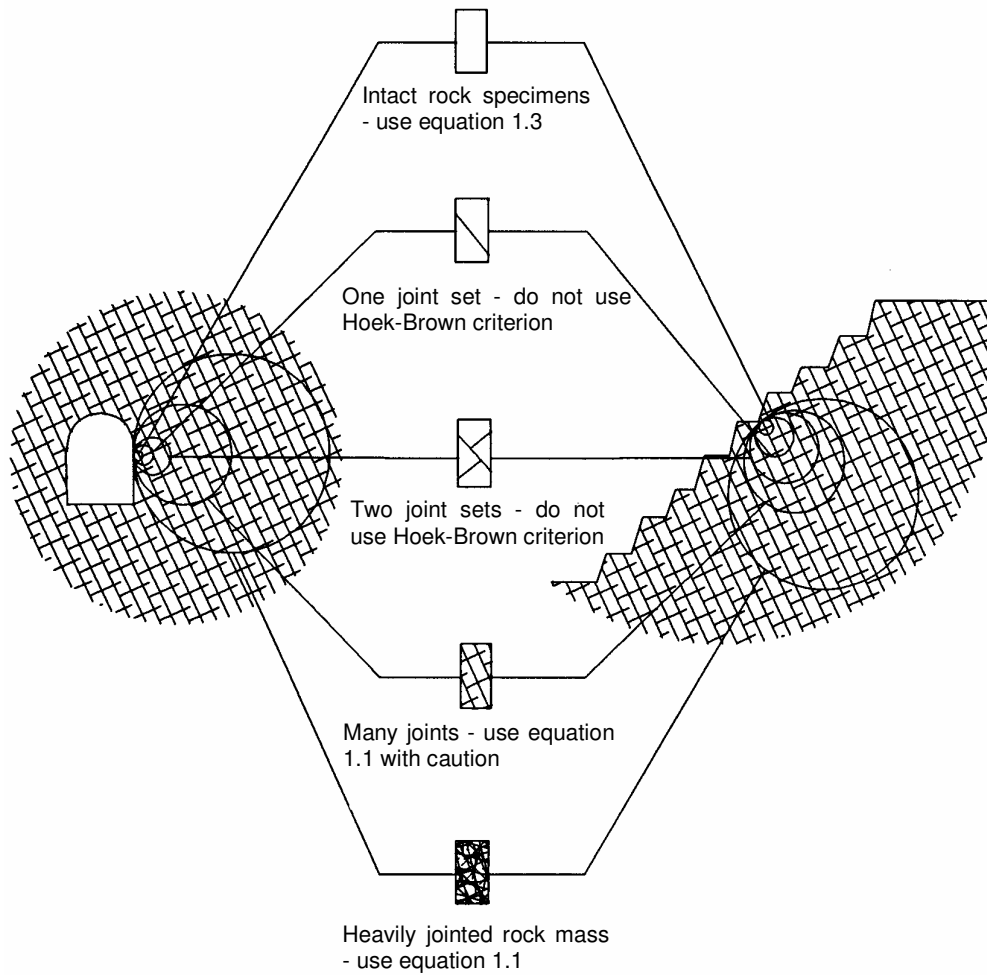
Figure 1.2: Idealised diagram showing the transition from intact to a heavily jointed rock mass with increasing sample size.

The Geological Strength Index (GSI), introduced by Hoek (1994) and Hoek, Kaiser and Bawden (1995) provides a system for estimating the reduction in rock mass strength for different geological conditions. This system is presented in Table 1.3, for blocky rock masses, and Table 1.4 for schistose metamorphic rocks.

Once the Geological Strength Index has been estimated, the parameters that describe the rock mass strength characteristics, are calculated as follows:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28}\right) \tag{1.4}$$

For GSI > 25, i.e. rock masses of good to reasonable quality:

$$s = \exp\left(\frac{GSI - 100}{9}\right) \quad (1.5)$$

and

$$a = 0.5 \quad (1.6)$$

For GSI < 25, i.e. rock masses of very poor quality:

$$s = 0 \quad (1.7)$$

and

$$a = 0.65 - \frac{GSI}{200} \quad (1.8)$$

For better quality rock masses (GSI > 25), the value of GSI can be estimated directly from the 1976 version of Bieniawski's Rock Mass Rating, with the Groundwater rating set to 10 (dry) and the Adjustment for Joint Orientation set to 0 (very favourable) (Bieniawski 1976). For very poor quality rock masses the value of RMR is very difficult to estimate and the balance between the ratings no longer gives a reliable basis for estimating rock mass strength. Consequently, Bieniawski's RMR classification should not be used for estimating the GSI values for poor quality rock masses (RMR < 25) and the GSI charts should be used directly.

If the 1989 version of Bieniawski's RMR classification (Bieniawski 1989) is used, then GSI = RMR_{89}' - 5 where RMR_{89}' has the Groundwater rating set to 15 and the Adjustment for Joint Orientation set to zero.

1.5 MOHR-COULOMB PARAMETERS

Most geotechnical software is written in terms of the Mohr-Coulomb failure criterion in which the rock mass strength is defined by the cohesive strength c' and the angle of friction ϕ'. The linear relationship between the major and minor principal stresses, σ_1' and σ_3', for the Mohr-Coulomb criterion is

$$\sigma_1' = \sigma_{cm} + k\sigma_3' \quad (1.9)$$

where σ_{cm} is the uniaxial compressive strength of the rock mass and k is the slope of the line relating σ_1' and σ_3'. The values of ϕ' and c' can be calculated from

$$\sin\phi' = \frac{k - 1}{k + 1} \quad (1.10)$$

$$c' = \frac{\sigma_{cm}(1 - \sin\phi')}{2\cos\phi'} \quad (1.11)$$

Table 1.3: Characterisation of a blocky rock masses on the basis of particle interlocking and discontinuity condition. After Hoek, Marinos and Benissi (1998).
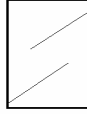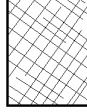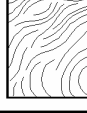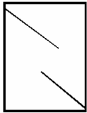
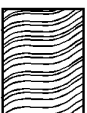| GEOLOGICAL STRENGTH INDEX FOR BLOCKY JOINTED ROCKS From a description of the structure and surface conditions of the rock mass, pick an appropriate box in this chart. Estimate the average value of GSI from the contours. Do not attempt to be too precise. Quoting a range from 36 to 42 is more realistic than stating that GSI = 38. It is also important to recognize that the Hoek-Brown criterion should only be applied to rock masses where the size of individual blocks or pieces is small compared with the size of the excavation under consideration. When the individual block size is more than about one quarter of the excavation size, the failure will be structurally controlled and the Hoek-Brown criterion should not be used. STRUCTURE	SURFACE CONDITIONS	VERY GOOD Very rough, fresh unweathered surfaces	GOOD Rough, slightly weathered, iron stained surfaces	FAIR Smooth, moderately weathered and altered surfaces	POOR Slickensided, highly weathered surfaces with compact coatings or fillings or angular fragments	VERY POOR Slickensided, highly weathered surfaces with soft clay coatings or fillings
		DECREASING SURFACE QUALITY ⟹				
INTACT OR MASSIVE - intact rock specimens or massive in situ rock with few widely spaced discontinuities	DECREASING INTERLOCKING OF ROCK PIECES	90 / 80		N/A	N/A	N/A
BLOCKY - well interlocked un-disturbed rock mass consisting of cubical blocks formed by three intersecting discontinuity sets			70 / 60			
VERY BLOCKY- interlocked, partially disturbed mass with multi-faceted angular blocks formed by 4 or more joint sets				50		
BLOCKY/DISTURBED - folded and/or faulted with angular blocks formed by many intersecting discontinuity sets				40	30	
DISINTEGRATED - poorly inter-locked, heavily broken rock mass with mixture of angular and rounded rock pieces						20
FOLIATED/LAMINATED - folded and tectonically sheared. Lack of blockiness due to schistosity prevailing over other discontinuities		N/A	N/A			10

Table 1.4: Characterisation of a schistose metamorphic rock masses on the basis of foliation and discontinuity condition. (After M. Truzman, 1999)

GEOLOGICAL STRENGTH INDEX FOR SCHISTOSE METAMORPHIC ROCKS	SURFACE CONDITIONS	VERY GOOD — Very rough, fresh unweathered surfaces	GOOD — Rough, slightly weathered, aperture < 1 mm hard filling	FAIR — Slightly rough, moderately weathered, aperture 1 – 5 mm, hard and soft filling	POOR — Smooth, highly weathered surfaces, aperture > 5 mm, predominantly soft fillings	VERY POOR — Slickensided, highly weathered surfaces, aperture > 5 mm, soft fillings
From a description of the structure and surface conditions of the rock mass, pick an appropriate box in this chart. Estimate the average value of GSI from the contours. Do not attempt to be too precise. Quoting a range from 36 to 42 is more realistic than stating that GSI = 38. It is also important to recognize that the Hoek-Brown criterion should only be applied to rock masses where the size of individual blocks or pieces is small compared with the size of the excavation under consideration. When the individual block size is more than about one quarter of the excavation size, the failure will be structurally controlled and the Hoek-Brown criterion should not be used.						
STRUCTURE		DECREASING SURFACE QUALITY ⟹				
INTACT OR MASSIVE - complete lack of foliation and very few widely spaced discontinuities	DECREASING INTERLOCKING OF ROCK PIECES ⟱	90 80			N/A	N/A	N/A
SPARSELY FOLIATED - partially fractured, massive intervals prevail over foliated intervals			70 60			
MODERATELY FOLIATED - fractured rock mass formed by massive and foliated intervals in similar proportions				50 40		
FOLIATED - folded and/or faulted rock mass with occasional massive intervals					30	
VERY FOLIATED - folded and/or faulted rock mass, highly fractured, formed by foliated rocks only					20	
FAULTED/SHEARED - very folded and faulted, tectonically disturbed rock mass		N/A	N/A			10

There is no direct correlation between equation 1.9 and the non-linear Hoek-Brown criterion defined by equation 1.1. Consequently, determination of the values of c' and ϕ' for a rock mass that has been evaluated as a Hoek-Brown material is a difficult problem.

Having considered a number of possible approaches, it has been concluded that the most practical solution is to treat the problem as an analysis of a set of full-scale triaxial strength tests. The results of such tests are simulated by using the Hoek-Brown equation 1.1 to generate a series of triaxial test values. Equation 1.9 is then fitted to these test results by linear regression analysis and the values of c' and ϕ' are determined from equations 1.11 and 1.10. A full discussion on the steps required to carry out this analysis is presented in the Appendix, together with a spreadsheet for implementing this analysis.

The range of stresses used in the curve fitting process described above is very important. For the confined conditions surrounding tunnels at depths of more than about 30 m, the most reliable estimates are given by using a confining stress range from zero to $0.25\sigma_{ci}$, where σ_{ci} is the uniaxial compressive strength of the intact rock elements. A series of plots showing the uniaxial compressive strength of the rock mass σ_{cm}, the cohesive strength c and the friction angle ϕ are given in Figures 1.3 and 1.4.
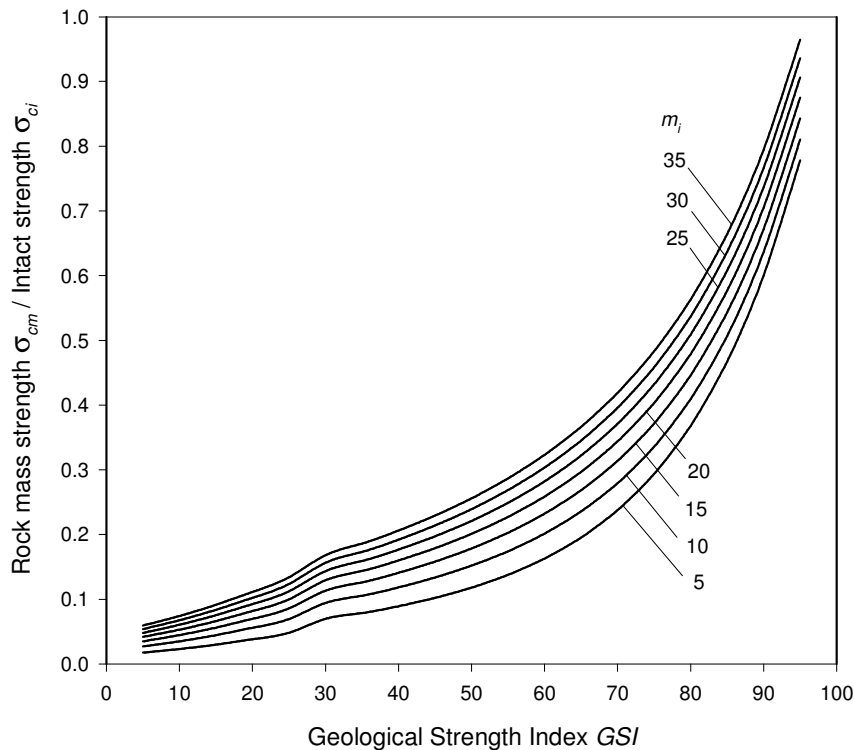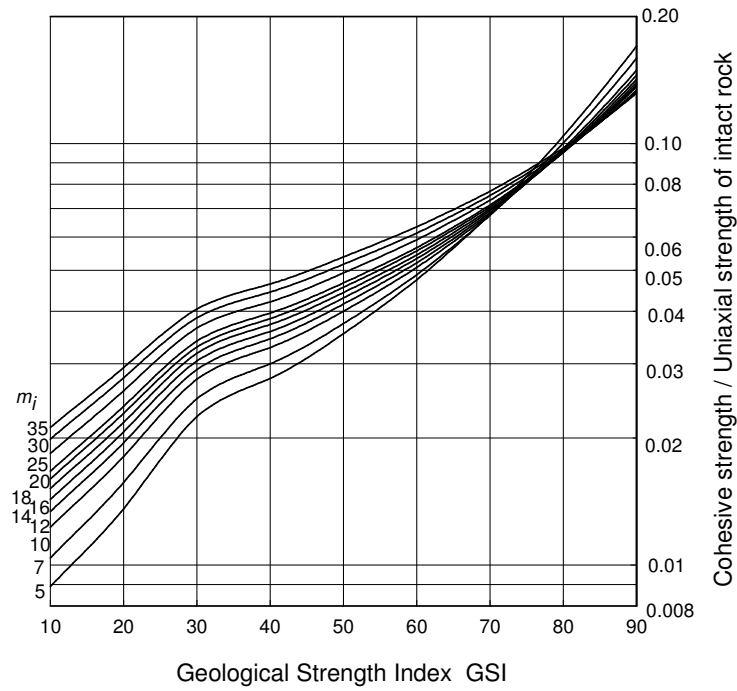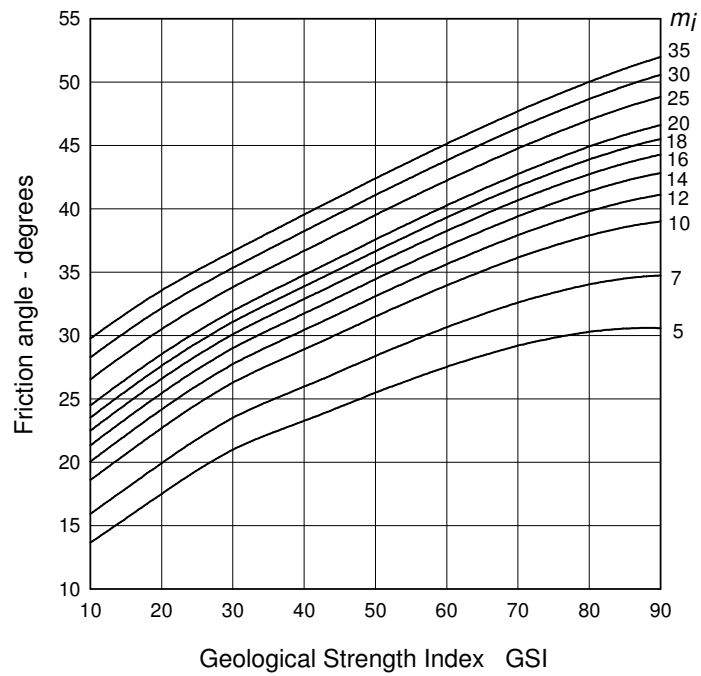

Figure 1.3: Ratio of uniaxial compressive strength of rock mass to intact rock versus Geological Strength Index GSI.

a. Plot of ratio of cohesive strength c' to uniaxial compressive strength σ_{ci}.

b. Plot of friction angle ϕ'

Figure 1.4: Cohesive strengths and friction angles for different GSI and m_i values.

1.6 DEFORMATION MODULUS

Serafim and Pereira (1983) proposed a relationship between the in situ modulus of deformation and Bieniawski's RMR classification. This relationship is based upon back analysis of dam foundation deformations and it has been found to work well for better quality rocks. However, for many of the poor quality rocks it appears to predict deformation modulus values that are too high. Based upon practical observations and back analysis of excavation behaviour in poor quality rock masses, the following modification to Serafim and Pereira's equation is proposed for $\sigma_{ci} < 100$:

$$E_m = \sqrt{\frac{\sigma_{ci}}{100}} 10^{\left(\frac{GSI-10}{40}\right)}$$ (1.12)

Note that GSI has been substituted for RMR in this equation and that the modulus E_m is reduced progressively as the value of σ_{ci} falls below 100. This reduction is based upon the reasoning that the deformation of better quality rock masses is controlled by the discontinuities while, for poorer quality rock masses, the deformation of the intact rock pieces contributes to the overall deformation process.

Based upon measured deformations, equation 1.12 appears to work reasonably well in those cases where it has been applied. However, as more field evidence is gathered it may be necessary to modify this relationship.
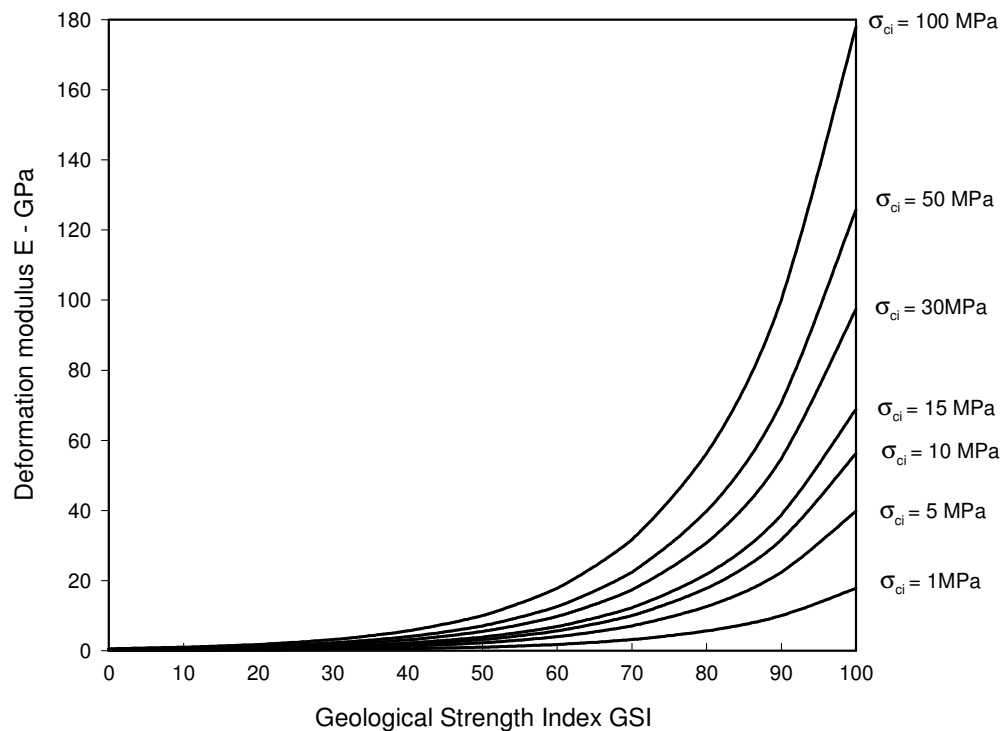

Figure 1.5: Deformation modulus versus Geological Strength Index GSI.

1.7 POST-FAILURE BEHAVIOUR

When using numerical models to study the progressive failure of rock masses, estimates of the post-peak or post-failure characteristics of the rock mass are required. In some of these models, the Hoek-Brown failure criterion is treated as a yield criterion and the analysis is carried out using plasticity theory. No definite rules for dealing with this problem can be given but, based upon experience in numerical analysis of a variety of practical problems, the post-failure characteristics illustrated in Figures 1.6 to 1.8 are suggested as a starting point.

1.7.1 Very good quality hard rock masses

For very good quality hard rock masses, such as massive granites or quartzites, the analysis of spalling around highly stressed openings (Hoek, Kaiser and Bawden 1995) suggests that the rock mass behaves in an elastic brittle manner as shown in Figure 1.6. When the strength of the rock mass is exceeded, a sudden strength drop occurs. This is associated with significant dilation of the broken rock pieces. If this broken rock is confined, for example by rock support, then it can be assumed to behave as a rock fill with a friction angle of approximately $\phi' = 38°$ and zero cohesive strength.

Typical properties for this very good quality hard rock mass may be as shown in Table 1.7. Note that, in some numerical analyses, it may be necessary to assign a very small cohesive strength in order to avoid numerical instability.
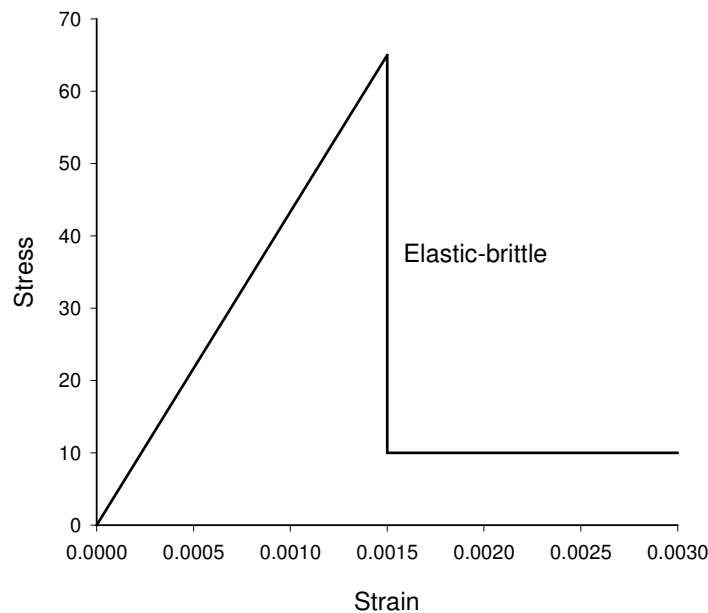

Figure 1.6: Very good quality hard rock mass

Table 1.7: Typical properties for a very good quality hard rock mass

Intact rock strength	σ_{ci}	150 MPa
Hoek-Brown constant	m_i	25
Geological Strength Index	GSI	75
Friction angle	ϕ'	46°
Cohesive strength	c'	13 MPa
Rock mass compressive strength	σ_{cm}	64.8 MPa
Rock mass tensile strength	σ_{tm}	-0.9 MPa
Deformation modulus	E_m	42000 MPa
Poisson's ratio	ν	0.2
Dilation angle	α	$\phi'/4 = 11.5°$
Post-peak characteristics		
Friction angle	ϕ_f'	38°
Cohesive strength	c_f'	0
Deformation modulus	E_{fm}	10000 MPa

1.7.2 Average quality rock mass

In the case of an average quality rock mass it is reasonable to assume that the post-failure characteristics can be estimated by reducing the GSI value from the in situ value to a lower value which characterises the broken rock mass.

The reduction of the rock mass strength from the in situ to the broken state corresponds to the strain softening behaviour illustrated in Figure 1.7. In this figure it has been assumed that post failure deformation occurs at a constant stress level, defined by the compressive strength of the broken rock mass. The validity of this assumption is uncertain.

Typical properties for this average quality rock mass may be as shown in Table 1.8.

Table 1.8: Typical properties for an average rock mass.

| Intact rock strength	σ_{ci}	80 MPa
Hoek-Brown constant	m_i	12
Geological Strength Index	GSI	50
Friction angle	ϕ'	33°
Cohesive strength	c'	3.5 MPa
Rock mass compressive strength	σ_{cm}	13 MPa
Rock mass tensile strength	σ_{tm}	-0.15
Deformation modulus	E_m	9000 MPa
Poisson's ratio	ν	0.25
Dilation angle	α	$\phi'/8 = 4°$
Post-peak characteristics		
Broken rock mass strength	σ_{fcm}	8 MPa
Deformation modulus	E_{fm}	5000 MPa
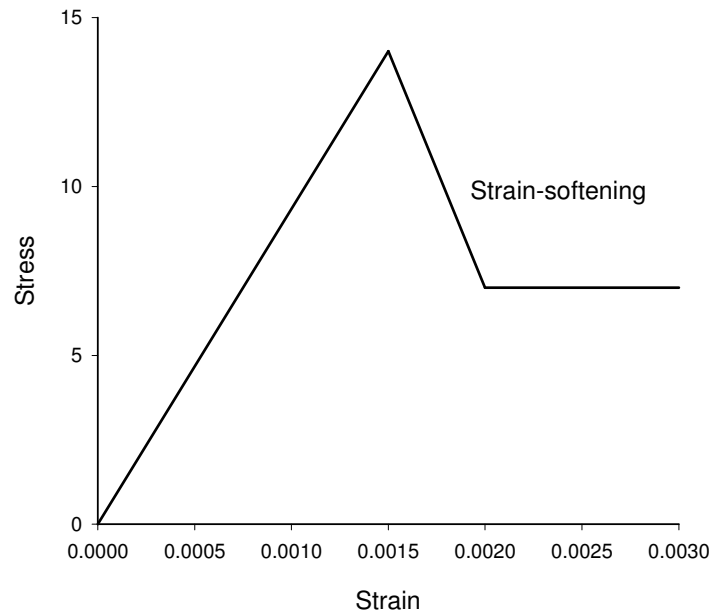
Figure 1.7: Average quality rock mass

1.7.3 Very poor quality rock mass

Analysis of the progressive failure of very poor quality rock masses surrounding tunnels suggests that the post-failure characteristics of the rock are adequately represented by assuming that it behaves perfectly plastically. This means that it continues to deform at a constant stress level and that no volume change is associated with this ongoing failure. This type of behaviour is illustrated in Figure 1.8.

Typical properties for this very poor quality rock mass may be as shown in Table 1.9:

Table 1.9: Typical properties for a very poor quality rock mass

| Intact rock strength	σ_{ci}	20 MPa
Hoek-Brown constant	m_i	8
Geological Strength Index	GSI	30
Friction angle	ϕ'	24°
Cohesive strength	c'	0.55 MPa
Rock mass compressive strength	σ_{cm}	1.7 MPa
Rock mass tensile strength	σ_{tm}	-0.01 MPa
Deformation modulus	E_m	1400 MPa
Poisson's ratio	ν	0.3
Dilation angle	α	zero
Post-peak characteristics		
Broken rock mass strength	σ_{fcm}	1.7 MPa
Deformation modulus	E_{fm}	1400 MPa
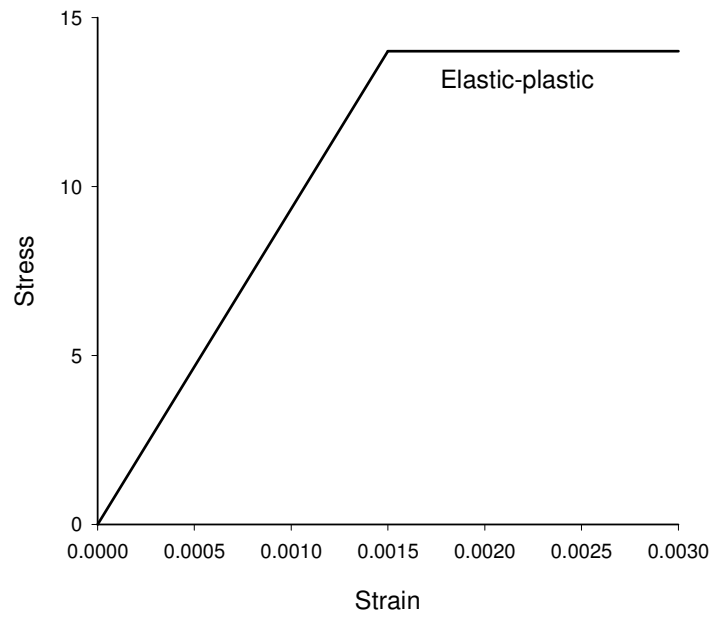
Figure 1.8: Very poor quality soft rock mass

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1.9 APPENDIX – DETERMINATION OF MOHR COULOMB CONSTANTS

The steps required to determine the parameters A, B, c' and ϕ' are given below. A spreadsheet for carrying out this analysis, with a listing of all the cell formulae, is given in Figure 1.9.

 The relationship between the normal and shear stresses can be expressed in terms of the corresponding principal effective stresses as suggested by Balmer (1952):

$$\sigma'_n = \sigma'_3 + \frac{\sigma'_1 - \sigma'_3}{\partial \sigma'_1 / \partial \sigma'_3 + 1} \tag{1.13}$$

$$\tau = (\sigma'_1 - \sigma'_3)\sqrt{\partial \sigma'_1 / \partial \sigma'_3} \tag{1.14}$$

For the *GSI* > 25, when *a* = 0.5:

$$\frac{\partial \sigma'_1}{\partial \sigma'_3} = 1 + \frac{m_b \sigma_{ci}}{2(\sigma'_1 - \sigma'_3)} \tag{1.15}$$

For *GSI* < 25, when *s* = 0:

$$\frac{\partial \sigma'_1}{\partial \sigma'_3} = 1 + a m_b^a \left(\frac{\sigma'_3}{\sigma_{ci}}\right)^{a-1} \tag{1.16}$$

The tensile strength of the rock mass is calculated from:

$$\sigma_{tm} = \frac{\sigma_{ci}}{2}\left(m_b - \sqrt{m_b^2 + 4s}\right) \tag{1.17}$$

The equivalent Mohr envelope, defined by equation 1.2, may be written in the form

$$Y = \log A + BX \tag{1.18}$$

where

$$Y = \log\left(\frac{\tau}{\sigma_{ci}}\right), \; X = \log\left(\frac{\sigma'_n - \sigma_{tm}}{\sigma_{ci}}\right) \tag{1.19}$$

Using the value of σ_{tm} calculated from equation 1.17 and a range of values of τ and σ'_n calculated from equations 1.13 and 1.14 the values of *A* and *B* are determined by linear regression where :

$$B = \frac{\sum XY - (\sum X \sum Y)/T}{\sum X^2 - (\sum X)^2/T} \tag{1.20}$$

$$A = 10^{\wedge}\left(\sum Y/T - B(\sum X/T)\right) \qquad (1.21)$$

and T is the total number of data pairs included in the regression analysis.

The most critical step in this process is the selection of the range of σ_3' values. As far as the author is aware, there are no theoretically correct methods for choosing this range and a trial and error method, based upon practical compromise, has been used for selecting the range included in the spreadsheet presented in Figure 1.9.

For a Mohr envelope defined by equation 1.2, the friction angle ϕ_i' for a specified normal stress σ_{ni}' is given by:

$$\phi_i' = \arctan\left(AB\left(\frac{\sigma_{ni}' - \sigma_{tm}}{\sigma_{ci}} \right)^{B-1} \right) \qquad (1.22)$$

The corresponding cohesive strength c_i' is given by:

$$c_i' = \tau - \sigma_{ni}' \tan \phi_i' \qquad (1.23)$$

and the corresponding uniaxial compressive strength of the rock mass is :

$$\sigma_{cmi} = \frac{2c_i' \cos\phi_i'}{1 - \sin\phi_i'} \qquad (1.24)$$

The values of c' and ϕ' obtained from this analysis are very sensitive to the range of values of the minor principal stress σ_3' used to generate the simulated full-scale triaxial test results. On the basis of trial and error, it has been found that the most consistent results for deep excavations (depth > 30 m below surface) are obtained when 8 equally spaced values of σ_3' are used in the range $0 < \sigma_3' < 0.25\sigma_{ci}$.

Figure 1.9 Spreadsheet for calculation of Hoek-Brown and equivalent Mohr-Coulomb parameters for excavations deeper than 30 m.

Hoek-Brown and equivalent Mohr-Coulomb failure criteria

Input:	sigci =	60	MPa	mi =	19		GSI =	50

Output:	mb =	3.19		s = 0.0039			a =	0.5	
	sigtm =	-0.0728	MPa	A =	0.6731		B =	0.7140	
	k =	4.06		phi =	37.20	degrees	coh =	2.930	MPa
	sigcm =	11.80	MPa	E =	7746.0	MPa			

Calculation:

									Sums
sig3	1E-10	2.14	4.29	6.4	8.57	10.71	12.86	15.00	60.00
sig1	3.73	22.72	33.15	41.68	49.22	56.12	62.57	68.68	337.88
ds1ds3	26.62	5.64	4.31	3.71	3.35	3.10	2.92	2.78	52.45
sign	0.14	5.24	9.72	13.91	17.91	21.78	25.53	29.20	123.43
tau	0.70	7.36	11.28	14.42	17.10	19.49	21.67	23.68	115.69
x	-2.46	-1.05	-0.79	-0.63	-0.52	-0.44	-0.37	-0.31	-6.58
y	-1.93	-0.91	-0.73	-0.62	-0.55	-0.49	-0.44	-0.40	-6.07
xy	4.76	0.96	0.57	0.39	0.29	0.21	0.16	0.13	7.47
xsq	6.05	1.11	0.62	0.40	0.27	0.19	0.14	0.10	8.88
sig3sig1	0.00	48.69	142.07	267.95	421.89	601.32	804.50	1030.15	3317
sig3sq	0.00	4.59	18.37	41.33	73.47	114.80	165.31	225.00	643
taucalc	0.71	7.15	11.07	14.28	17.09	19.63	21.99	24.19	
sig1sig3fit	11.80	20.50	29.19	37.89	46.58	55.28	63.97	72.67	
signtaufit	3.03	6.91	10.31	13.49	16.53	19.46	22.31	25.09	

Cell formulae:

mb = mi*EXP((GSI-100)/28)

s = IF(GSI>25,EXP((GSI-100)/9),0)

a = IF(GSI>25,0.5,0.65-GSI/200)

sigtm = 0.5*sigci*(mb-SQRT(mb^2+4*s))

sig3 = Start at 1E-10 (to avoid zero errors) and increment in 7 steps of sigci/28 to 0.25*sigci

sig1 = sig3+sigci*(((mb*sig3)/sigci)+s)^a

ds1ds3 = IF(GSI>25,(1+(mb*sigci)/(2*(sig1-sig3))),1+(a*mb^a)*(sig3/sigci)^(a-1))

sign = sig3+(sig1-sig3)/(1+ds1ds3)

tau = (sign-sig3)*SQRT(ds1ds3)

x = LOG((sign-sigtm)/sigci)

y = LOG(tau/sigci)

xy = x*y   x sq = x^2

A =   acalc =   10^(sumy/8 - bcalc*sumx/8)

B =   bcalc =   (sumxy - (sumx*sumy)/8)/(sumxsq - (sumx^2)/8)

k = (sumsig3sig1 - (sumsig3*sumsig1)/8)/(sumsig3sq-(sumsig3^2)/8)

phi = ASIN((k-1)/(k+1))*180/PI()

coh = sigcm/(2*SQRT(k))

sigcm = sumsig1/8 - k*sumsig3/8

E = IF(sigci>100,1000*10^((GSI-10)/40),SQRT(sigci/100)*1000*10^((GSI-10)/40))

phit = (ATAN(acalc*bcalc*((signt-sigtm)/sigci)^(bcalc-1)))*180/PI()

coht = acalc*sigci*((signt-sigtm)/sigci)^bcalc-signt*TAN(phit*PI()/180)

sig3sig1= sig3*sig1   sig3sq = sig3^2

taucalc = acalc*sigci*((sign-sigtm)/sigci)^bcalc

s3sifit = sigcm+k*sig3

sntaufit = coh+sign*TAN(phi*PI()/180)