In situ and induced stresses

Introduction

Rock at depth is subjected to stresses resulting from the weight of the overlying strata and from locked in stresses of tectonic origin. When an opening is excavated in this rock, the stress field is locally disrupted and a new set of stresses are induced in the rock surrounding the opening. Knowledge of the magnitudes and directions of these in situ and induced stresses is an essential component of underground excavation design since, in many cases, the strength of the rock is exceeded and the resulting instability can have serious consequences on the behaviour of the excavations.

This chapter deals with the question of in situ stresses and also with the stress changes that are induced when tunnels or caverns are excavated in stressed rock. Problems, associated with failure of the rock around underground openings and with the design of support for these openings, will be dealt with in later chapters.

The presentation, which follows, is intended to cover only those topics which are essential for the reader to know about when dealing with the analysis of stress induced instability and the design of support to stabilise the rock under these conditions.

In situ stresses

Consider an element of rock at a depth of 1,000 m below the surface. The weight of the vertical column of rock resting on this element is the product of the depth and the unit weight of the overlying rock mass (typically about 2.7 tonnes/m^3 or 0.027 MN/m^3). Hence the vertical stress on the element is 2,700 tonnes/m^2 or 27 MPa. This stress is estimated from the simple relationship:

$$\sigma_v = \gamma z \tag{1}$$

where σ_v is the vertical stress
γ is the unit weight of the overlying rock and
z is the depth below surface.

Measurements of vertical stress at various mining and civil engineering sites around the world confirm that this relationship is valid although, as illustrated in Figure 1, there is a significant amount of scatter in the measurements.

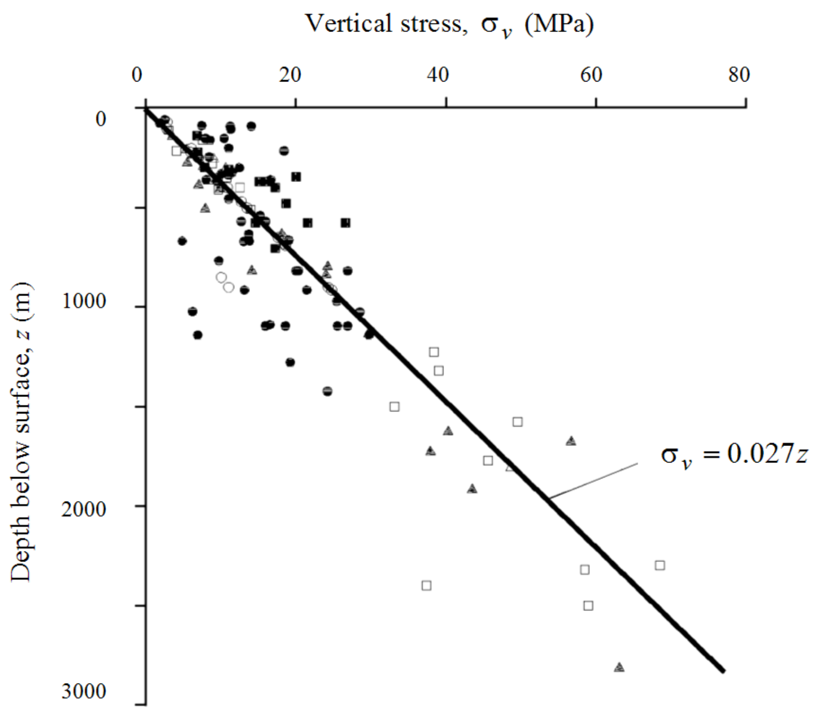


Figure 1: Vertical stress measurements from mining and civil engineering projects around the world. (After Brown and Hoek 1978).

The horizontal stresses acting on an element of rock at a depth *z* below the surface are much more difficult to estimate than the vertical stresses. Normally, the ratio of the average horizontal stress to the vertical stress is denoted by the letter *k* such that:

$$\sigma_h = k\sigma_v = k\gamma z \tag{2}$$

Terzaghi and Richart (1952) suggested that, for a gravitationally loaded rock mass in which no lateral strain was permitted during formation of the overlying strata, the value of *k* is independent of depth and is given by $k = \nu/(1-\nu)$, where ν is the Poisson's ratio of the rock mass. This relationship was widely used in the early days of rock mechanics but, as discussed below, it proved to be inaccurate and is seldom used today.

Measurements of horizontal stresses at civil and mining sites around the world show that the ratio *k* tends to be high at shallow depth and that it decreases at depth (Brown and Hoek, 1978, Herget, 1988). In order to understand the reason for these horizontal stress variations it is necessary to consider the problem on a much larger scale than that of a single site.

Sheorey (1994) developed an elasto-static thermal stress model of the earth. This model considers curvature of the crust and variation of elastic constants, density and thermal expansion coefficients through the crust and mantle. A detailed discussion on Sheorey's model is beyond the scope of this chapter, but he did provide a simplified equation which can be used for estimating the horizontal to vertical stress ratio *k*. This equation is:

(3)

where *z* (m) is the depth below surface and E_h (GPa) is the average deformation modulus of the upper part of the earth's crust measured in a horizontal direction. This direction of measurement is important particularly in layered sedimentary rocks, in which the deformation modulus may be significantly different in different directions.

A plot of this equation is given in Figure 2 for a range of deformation moduli. The curves relating *k* with depth below surface *z* are similar to those published by Brown and Hoek (1978), Herget (1988) and others for measured in situ stresses. Hence equation 3 is considered to provide a reasonable basis for estimating the value of *k*.

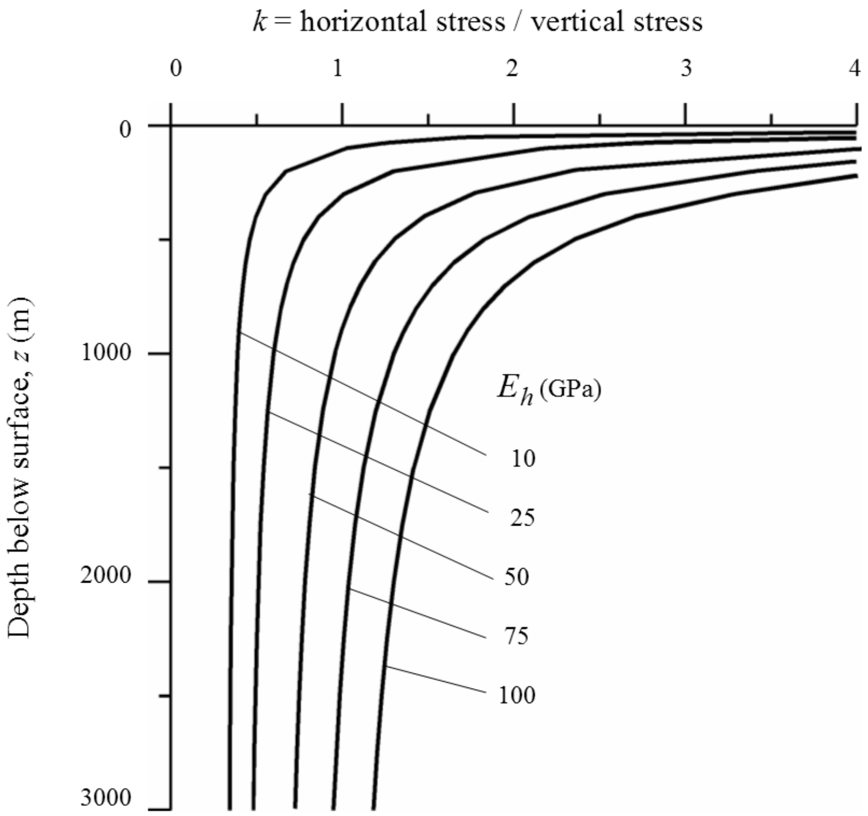


Figure 2: Ratio of horizontal to vertical stress for different deformation moduli based upon Sheorey's equation. (After Sheorey 1994).

As pointed out by Sheorey, his work does not explain the occurrence of measured vertical stresses that are higher than the calculated overburden pressure, the presence of very high horizontal stresses at some locations or why the two horizontal stresses are seldom equal. These differences are probably due to local topographic and geological features that cannot be taken into account in a large scale model such as that proposed by Sheorey.

Where sensitivity studies have shown that the in situ stresses are likely to have a significant influence on the behaviour of underground openings, it is recommended that the in situ stresses should be measured. Suggestions for setting up a stress measuring programme are discussed later in this chapter.

The World stress map

The World Stress Map project, completed in July 1992, involved over 30 scientists from 18 countries and was carried out under the auspices of the International Lithosphere Project (Zoback, 1992). The aim of the project was to compile a global database of contemporary tectonic stress data.

The World Stress Map (WSM) is now maintained and it has been extended by the Geophysical Institute of Karlsruhe University as a research project of the Heidelberg Academy of Sciences and Humanities. The 2005 version of the map contains approximately 16,000 data sets and various versions of the map for the World, Europe, America, Africa, Asia and Australia can be downloaded from the Internet. The WSM is an open-access database that can be accessed at www.world-stress-map.org (Reinecker et al, 2005)

The 2005 World Stress Map is reproduced in Figure 3 while a stress map for the Mediterranean is reproduced in Figure 4.

The stress maps display the orientations of the maximum horizontal compressive stress. The length of the stress symbols represents the data quality, with A being the best quality. Quality A data are assumed to record the orientation of the maximum horizontal compressive stress to within 10°-15°, quality B data to within 15°-20°, and quality C data to within 25°. Quality D data are considered to give questionable tectonic stress orientations.

The 1992 version of the World Stress Map was derived mainly from geological observations on earthquake focal mechanisms, volcanic alignments and fault slip interpretations. Less than 5% of the data was based upon hydraulic fracturing or overcoring measurements of the type commonly used in mining and civil engineering projects. In contrast, the 2005 version of the map includes a significantly greater number of observations from borehole break-outs, hydraulic fracturing, overcoring and borehole slotting. It is therefore worth considering the relative accuracy of these measurements as compared with the geological observations upon which the original map was based.

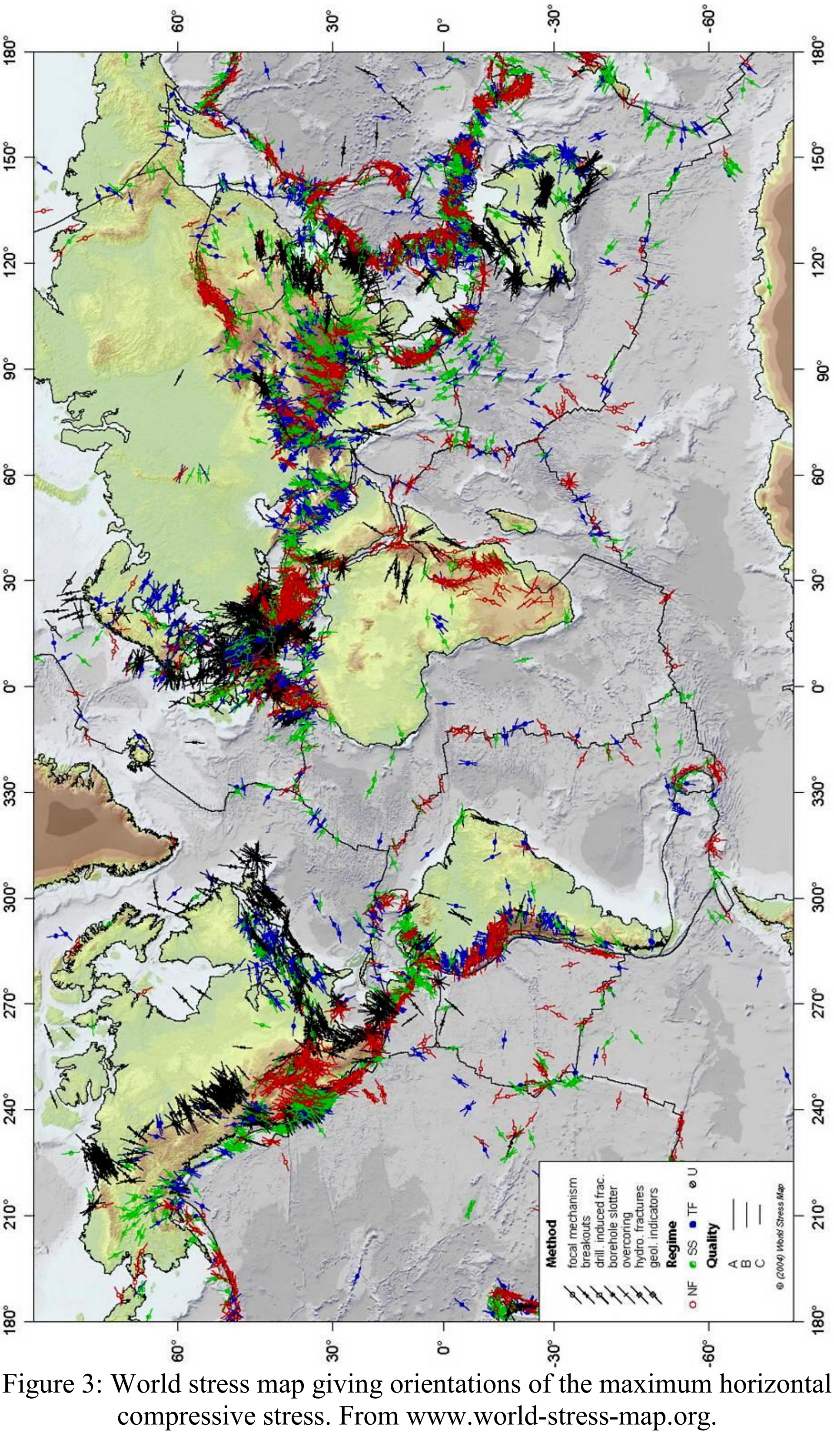


Figure 3: World stress map giving orientations of the maximum horizontal compressive stress. From www.world-stress-map.org.

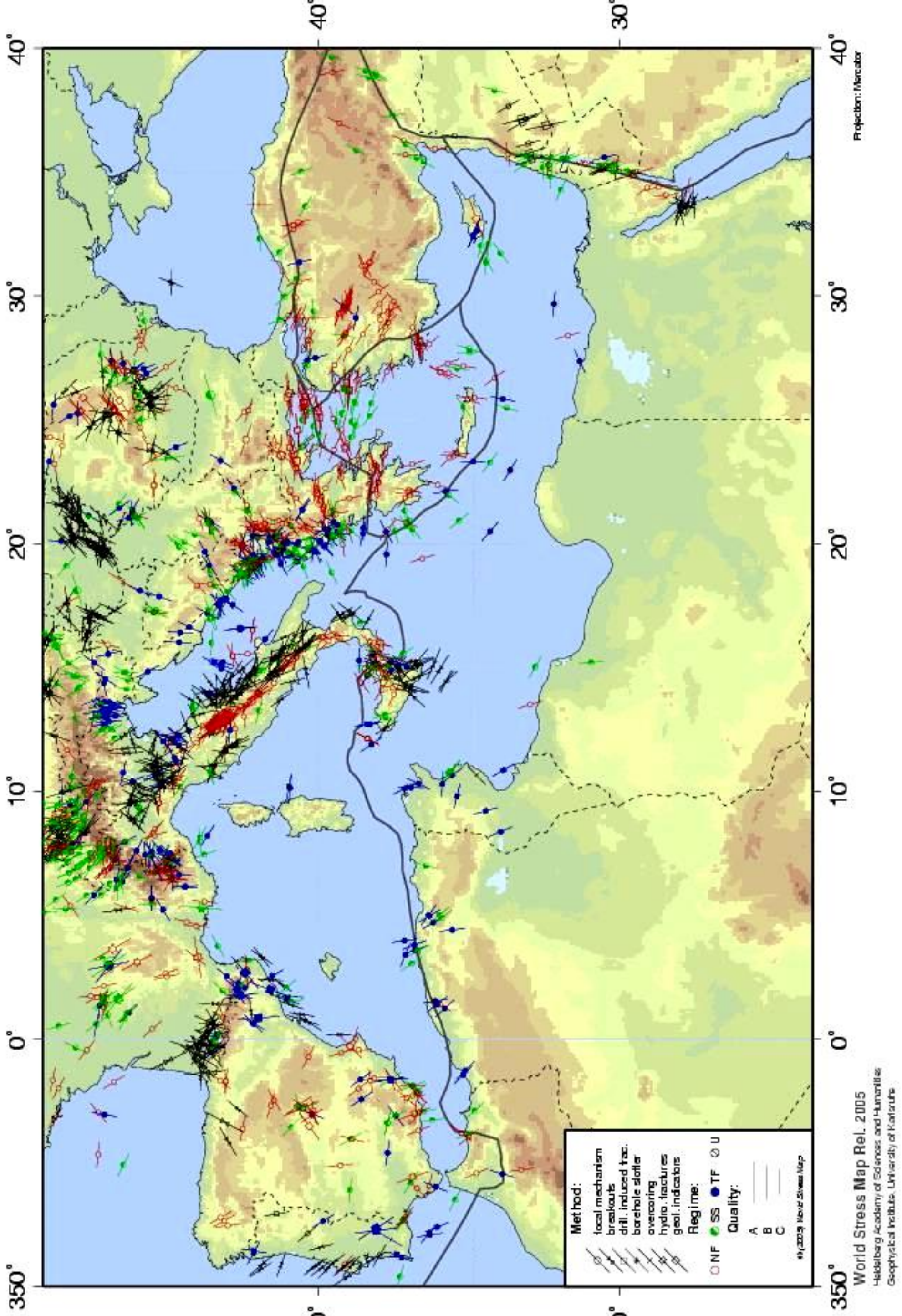


Figure 4: Stress map of the Mediterranean giving orientations of the maximum horizontal compressive stress. From www.world-stress-map.org.

In discussing hydraulic fracturing and overcoring stress measurements, Zoback (1992) has the following comments:

> 'Detailed hydraulic fracturing testing in a number of boreholes beginning very close to surface (10-20 m depth) has revealed marked changes in stress orientations and relative magnitudes with depth in the upper few hundred metres, possibly related to effects of nearby topography or a high degree of near surface fracturing.
>
> Included in the category of 'overcoring' stress measurements are a variety of stress or strain relief measurement techniques. These techniques involve a three-dimensional measurement of the strain relief in a body of rock when isolated from the surrounding rock volume; the three-dimensional stress tensor can subsequently be calculated with a knowledge of the complete compliance tensor of the rock. There are two primary drawbacks with this technique which restricts its usefulness as a tectonic stress indicator: measurements must be made near a free surface, and strain relief is determined over very small areas (a few square millimetres to square centimetres). Furthermore, near surface measurements (by far the most common) have been shown to be subject to effects of local topography, rock anisotropy, and natural fracturing (Engelder and Sbar, 1984). In addition, many of these measurements have been made for specific engineering applications (e.g. dam site evaluation, mining work), places where topography, fracturing or nearby excavations could strongly perturb the regional stress field.'

Obviously, from a global or even a regional scale, the type of engineering stress measurements carried out in a mine or on a civil engineering site are not regarded as very reliable. Conversely, the World Stress Map versions presented in Figures 3 and 4 can only be used to give first order estimates of the stress directions which are likely to be encountered on a specific site. Since both stress directions and stress magnitudes are critically important in the design of underground excavations, it follows that a stress measuring programme may be required in any major underground mining or civil engineering project.

Developing a stress measuring programme

Consider the example of a tunnel to be driven a depth of 1,000 m below surface in a hard rock environment. The depth of the tunnel is such that it is probable that in situ and induced stresses will be an important consideration in the design of the excavation. Typical steps that could be followed in the analysis of this problem are:

The World Stress Map for the area under consideration will give a good first indication of the possible complexity of the regional stress field and possible directions for the maximum horizontal compressive stress.

1. During preliminary design, the information presented in equations 1 and 3 can be used to obtain a first rough estimate of the vertical and average horizontal

stress in the vicinity of the tunnel. For a depth of 1,000 m, these equations give the vertical stress σ_v = 27 MPa, the ratio k = 1.3 (for E_h = 75 GPa) and hence the average horizontal stress σ_h= 35.1 MPa. A preliminary analysis of the stresses induced around the proposed tunnel shows that these induced stresses are likely to exceed the strength of the rock and that the question of stress measurement must be considered in more detail. Note that for many openings in strong rock at shallow depth, stress problems may not be significant and the analysis need not proceed any further.

For this particular case, stress problems are considered to be important. A typical next step would be to search the literature in an effort to determine whether the results of in situ stress measurement programmes are available for mines or civil engineering projects within a radius of say 50 km of the site. With luck, a few stress measurement results will be available for the region in which the tunnel is located and these results can be used to refine the analysis discussed above.

Assuming that the results of the analysis of induced stresses in the rock surrounding the proposed tunnel indicate that significant zones of rock failure are likely to develop, and that support costs are likely to be high, it is probably justifiable to set up a stress measurement project on the site. These measurements can be carried out in deep boreholes from the surface, using hydraulic fracturing techniques, or from underground access using overcoring methods. The choice of the method and the number of measurements to be carried out depends upon the urgency of the problem, the availability of underground access and the costs involved in the project. Note that very few project organisations have access to the equipment required to carry out a stress measurement project and, rather than purchase this equipment, it may be worth bringing in an organisation which has the equipment and which specialises in such measurements.

2. Where regional tectonic features such as major faults are likely to be encountered the in situ stresses in the vicinity of the feature may be rotated with respect to the regional stress field. The stresses may be significantly different in magnitude from the values estimated from the general trends described above. These differences can be very important in the design of the openings and in the selection of support and, where it is suspected that this is likely to be the case, in situ stress measurements become an essential component of the overall design process.

Analysis of induced stresses

When an underground opening is excavated into a stressed rock mass, the stresses in the vicinity of the new opening are re-distributed. Consider the example of the stresses induced in the rock surrounding a horizontal circular tunnel as illustrated in Figure 5, showing a vertical slice normal to the tunnel axis.

Before the tunnel is excavated, the in situ stresses σ_v, σ_{h1} and σ_{h2} are uniformly distributed in the slice of rock under consideration. After removal of the rock from within the tunnel, the stresses in the immediate vicinity of the tunnel are changed and new stresses are induced. Three principal stresses σ_1, σ_2 and σ_3 acting on a typical element of rock are shown in Figure 5.

The convention used in rock engineering is that *compressive* stresses are always *positive* and the three principal stresses are numbered such that σ_1 is the largest compressive stress and σ_3 is the smallest compressive stress or the largest tensile stress of the three.

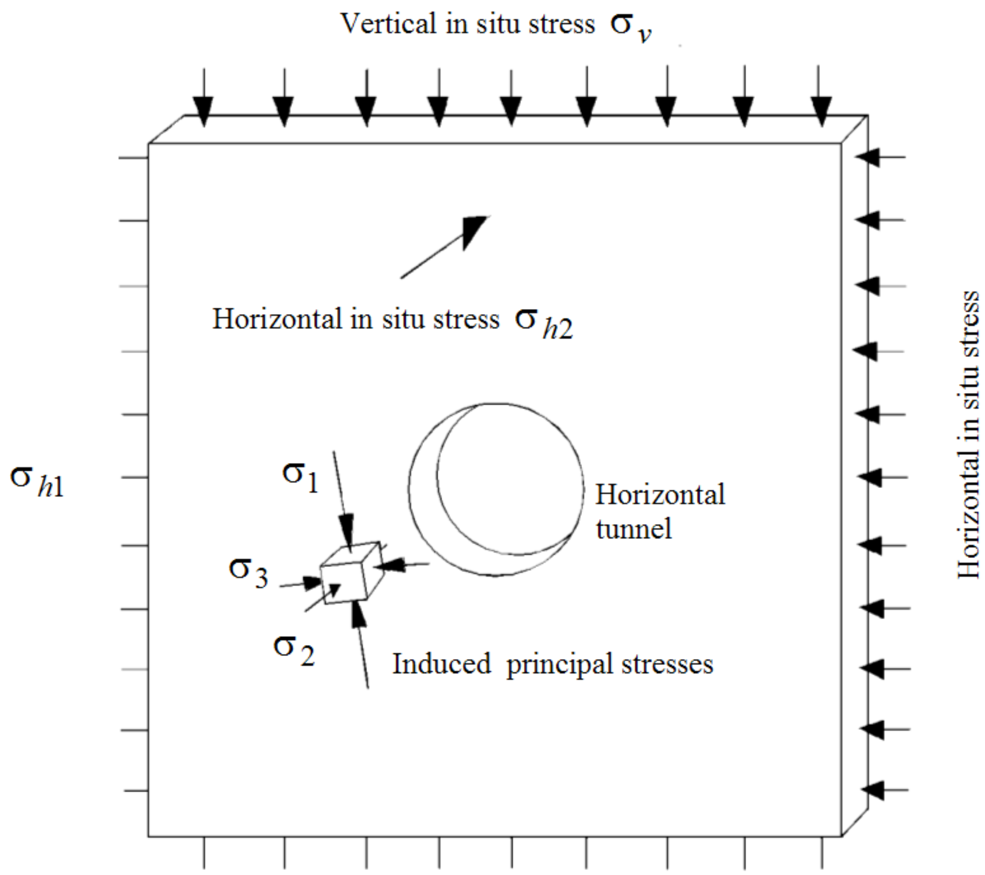


Figure 5: Illustration of principal stresses induced in an element of rock close to a horizontal tunnel subjected to a vertical in situ stress σ_v, a horizontal in situ stress σ_{h1} in a plane normal to the tunnel axis and a horizontal in situ stress σ_{h2} parallel to the tunnel axis.

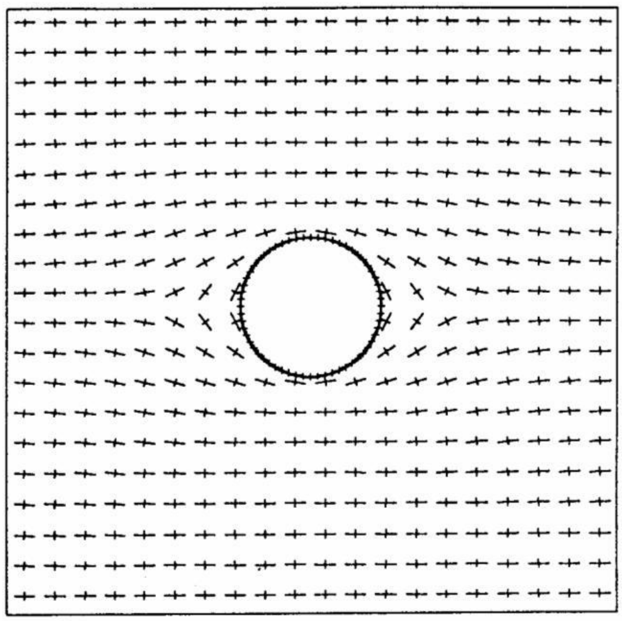


Figure 6: Principal stress directions in the rock surrounding a horizontal tunnel subjected to a horizontal in situ stress σ_{h1} equal to $3\sigma_v$, where σ_v is the vertical in situ stress.

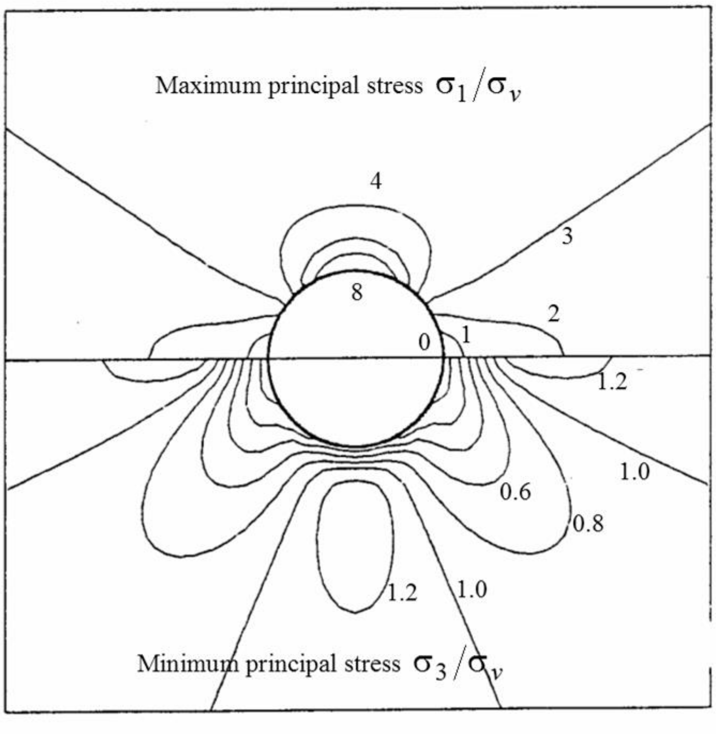


Figure 7: Contours of maximum and minimum principal stress magnitudes in the rock surrounding a horizontal tunnel, subjected to a vertical in situ stress of σ_v and a horizontal in situ stress of $3\sigma_v$.

The three principal stresses are mutually perpendicular but they may be inclined to the direction of the applied in situ stress. This is evident in Figure 6 which shows the directions of the stresses in the rock surrounding a horizontal tunnel subjected to a horizontal in situ stress σ_{h1} equal to three times the vertical in situ stress σ_v. The longer bars in this figure represent the directions of the maximum principal stress σ_1, while the shorter bars give the directions of the minimum principal stress σ_3 at each element considered. In this particular case, σ_2 is coaxial with the in situ stress σ_{h2}, but the other principal stresses σ_1 and σ_3 are inclined to σ_{h1} and σ_v in the immediate vicinity of the tunnel.

Contours of the magnitudes of the maximum principal stress σ_1 and the minimum principal stress σ_3 are given in Figure 7. This figure shows that the redistribution of stresses is concentrated in the rock close to the tunnel and that, at a distance of say three times the radius from the centre of the hole, the disturbance to the in situ stress field is negligible.

An analytical solution for the stress distribution in a stressed elastic plate containing a circular hole was published by Kirsch (1898) and this formed the basis for many early studies of rock behaviour around tunnels and shafts. Following along the path pioneered by Kirsch, researchers such as Love (1927), Muskhelishvili (1953) and Savin (1961) published solutions for excavations of various shapes in elastic plates. A useful summary of these solutions and their application in rock mechanics was published by Brown in an introduction to a volume entitled *Analytical and Computational Methods in Engineering Rock Mechanics* (1987).

Closed form solutions still possess great value for conceptual understanding of behaviour and for the testing and calibration of numerical models. For design purposes, however, these models are restricted to very simple geometries and material models. They are of limited practical value. Fortunately, with the development of computers, many powerful programs that provide numerical solutions to these problems are now readily available. A brief review of some of these numerical solutions is given below.

Numerical methods of stress analysis

Most underground excavations are irregular in shape and are frequently grouped close to other excavations. These groups of excavations can form a set of complex three-dimensional shapes. In addition, because of the presence of geological features such as faults and dykes, the rock properties are seldom uniform within the rock volume of interest. Consequently, closed form solutions are of limited value in calculating the stresses, displacements and failure of the rock mass surrounding underground excavations. A number of computer-based numerical methods have been developed over the past few decades and these methods provide the means for obtaining approximate solutions to these problems.

Numerical methods for the analysis of stress driven problems in rock mechanics can be divided into two classes:

- *Boundary discretization methods*, in which only the boundary of the excavation is divided into elements and the interior of the rock mass is represented mathematically as an infinite continuum. These methods are normally restricted to elastic analyses.
- *Domain discretization methods*, in which the interior of the rock mass is divided into geometrically simple elements each with assumed properties. The collective behaviour and interaction of these simplified elements model the more complex overall behaviour of the rock mass. In other words domain methods allow consideration of more complex material models than boundary methods. *Finite element* and *finite difference* methods are domain techniques which treat the rock mass as a continuum. The *distinct element* method is also a domain method which models each individual block of rock as a unique element.

These two classes of analysis can be combined in the form of *hybrid models* in order to maximise the advantages and minimise the disadvantages of each method.

It is possible to make some general observations about the two types of approaches discussed above. In domain methods, a significant amount of effort is required to create the mesh that is used to divide the rock mass into elements. In the case of complex models, such as those containing multiple openings, meshing can become extremely difficult. In contrast, boundary methods require only that the excavation boundary be discretized and the surrounding rock mass is treated as an infinite continuum. Since fewer elements are required in the boundary method, the demand on computer memory and on the skill and experience of the user is reduced. The availability of highly optimised mesh-generators in many domain models has narrowed this difference to the point where most users of domain programs would be unaware of the mesh generation problems discussed above and hence the choice of models can be based on other considerations.

In the case of domain methods, the outer boundaries of the model must be placed sufficiently far away from the excavations in order that errors, arising from the interaction between these outer boundaries and the excavations, are reduced to an acceptable minimum. On the other hand, since boundary methods treat the rock mass as an infinite continuum, the far field conditions need only be specified as stresses acting on the entire rock mass and no outer boundaries are required. The main strength of boundary methods lies in the simplicity achieved by representing the rock mass as a continuum of infinite extent. It is this representation, however, that makes it difficult to incorporate variable material properties and discontinuities such as joints and faults. While techniques have been developed to allow some boundary element modelling of variable rock properties, these types of problems are more conveniently modelled by domain methods.

Before selecting the appropriate modelling technique for particular types of problems, it is necessary to understand the basic components of each technique.

Boundary Element Method

The boundary element method derives its name from the fact that only the boundaries of the problem geometry are divided into elements. In other words, only the excavation surfaces, the free surface for shallow problems, joint surfaces where joints are considered explicitly and material interfaces for multi-material problems are divided into elements. In fact, several types of boundary element models are collectively referred to as 'the boundary element method' (Crouch and Starfield, 1983). These models may be grouped as follows:

Indirect (Fictitious Stress) method, so named because the first step in the solution is to find a set of fictitious stresses that satisfy prescribed boundary conditions. These stresses are then used in the calculation of actual stresses and displacements in the rock mass.

Direct method, so named because the displacements are solved directly for the specified boundary conditions.

Displacement Discontinuity method, so named because the solution is based on the superposition of the fundamental solution of an elongated slit in an elastic continuum and shearing and normal displacements in the direction of the slit.

The differences between the first two methods are not apparent to the program user. The direct method has certain advantages in terms of program development, as will be discussed later in the section on Hybrid approaches.

The fact that a boundary element model extends 'to infinity' can also be a disadvantage. For example, a heterogeneous rock mass consists of regions of finite, not infinite, extent. Special techniques must be used to handle these situations. Joints are modelled explicitly in the boundary element method using the displacement discontinuity approach, but this can result in a considerable increase in computational effort. Numerical convergence is often found to be a problem for models incorporating many joints. For these reasons, problems, requiring explicit consideration of several joints and/or sophisticated modelling of joint constitutive behaviour, are often better handled by one of the domain methods such as finite elements.

A widely-used application of displacement discontinuity boundary elements is in the modelling of tabular ore bodies. Here, the entire ore seam is represented as a 'discontinuity' which is initially filled with ore. Mining is simulated by reduction of the ore stiffness to zero in those areas where mining has occurred, and the resulting stress redistribution to the surrounding pillars may be examined (Salamon, 1974, von Kimmelmann et al., 1984).

Finite element and finite difference methods

In practice, the finite element method is usually indistinguishable from the finite difference method; thus, they will be treated here as one and the same. For the boundary element method, it was seen that conditions on a domain boundary could be related to the state at *all* points throughout the remaining rock, even to infinity. In comparison, the finite element method relates the conditions at a few points within the rock (nodal points) to the state within a finite closed region formed by these points (the element). In the finite element method the physical problem is modelled numerically by dividing the entire problem region into elements.

The finite element method is well suited to solving problems involving heterogeneous or non-linear material properties, since each element explicitly models the response of its contained material. However, finite elements are not well suited to modelling infinite boundaries, such as occur in underground excavation problems. One technique for handling infinite boundaries is to discretize beyond the zone of influence of the excavation and to apply appropriate boundary conditions to the outer edges. Another approach has been to develop elements for which one edge extends to infinity i.e. so-called 'infinity' finite elements. In practice, efficient pre- and post-processors allow the user to perform parametric analyses and assess the influence of approximated far-field boundary conditions. The time required for this process is negligible compared to the total analysis time.

Joints can be represented explicitly using specific 'joint elements'. Different techniques have been proposed for handling such elements, but no single technique has found universal favour. Joint interfaces may be modelled, using quite general constitutive relations, though possibly at increased computational expense depending on the solution technique.

Once the model has been divided into elements, material properties have been assigned and loads have been prescribed, some technique must be used to redistribute any unbalanced loads and thus determine the solution to the new equilibrium state. Available solution techniques can be broadly divided into two classes - implicit and explicit. Implicit techniques assemble systems of linear equations that are then solved using standard matrix reduction techniques. Any material non-linearity is accounted for by modifying stiffness coefficients (secant approach) and/or by adjusting prescribed variables (initial stress or initial strain approach). These changes are made in an iterative manner such that all constitutive and equilibrium equations are satisfied for the given load state.

The response of a non-linear system generally depends upon the sequence of loading. Thus it is necessary that the load path modelled be representative of the actual load path experienced by the body. This is achieved by breaking the total applied load into load increments, each increment being sufficiently small, so that solution convergence for the increment is achieved after only a few iterations. However, as the system being modelled becomes increasingly non-linear and the load increment represents an ever smaller portion of the total load, the incremental solution technique becomes similar to modelling the quasi-dynamic behaviour of the body, as it responds to gradual application of the total load.

In order to overcome this, a 'dynamic relaxation' solution technique was proposed (Otter et al., 1966) and first applied to geomechanics modelling by Cundall (1971). In this technique no matrices are formed. Rather, the solution proceeds explicitly - unbalanced forces, acting at a material integration point, result in acceleration of the mass associated with the point; applying Newton's law of motion expressed as a difference equation yields incremental displacements, applying the appropriate constitutive relation produces the new set of forces, and so on marching in time, for each material integration point in the model. This solution technique has the advantage that both geometric and material non-linearities are accommodated, with relatively little additional computational effort as compared to a corresponding linear analysis, and computational expense increases only linearly with the number of elements used. A further practical advantage lies in the fact that numerical divergence usually results in the model predicting obviously anomalous physical behaviour. Thus, even relatively inexperienced users may recognise numerical divergence.

Most commercially available finite element packages use implicit (i.e. matrix) solution techniques. For linear problems and problems of moderate non-linearity, implicit techniques tend to perform faster than explicit solution techniques. However, as the degree of non-linearity of the system increases, imposed loads must be applied in smaller increments which implies a greater number of matrix re-formations and reductions, and hence increased computational expense. Therefore, highly non-linear problems are best handled by packages using an explicit solution technique.

Distinct Element Method

In ground conditions conventionally described as blocky (i.e. where the spacing of the joints is of the same order of magnitude as the excavation dimensions), intersecting joints form wedges of rock that may be regarded as rigid bodies. That is, these individual pieces of rock may be free to rotate and translate, and the deformation that takes place at block contacts may be significantly greater than the deformation of the intact rock. Hence, individual wedges may be considered rigid. For such conditions it is usually necessary to model many joints explicitly. However, the behaviour of such systems is so highly non-linear, that even a jointed finite element code, employing an explicit solution technique, may perform relatively inefficiently.

An alternative modelling approach is to develop data structures that represent the blocky nature of the system being analysed. Each block is considered a unique free body that may interact at contact locations with surrounding blocks. Contacts may be represented by the overlaps of adjacent blocks, thereby avoiding the necessity of unique joint elements. This has the added advantage that arbitrarily large relative displacements at the contact may occur, a situation not generally tractable in finite element codes.

Due to the high degree of non-linearity of the systems being modelled, explicit solution techniques are favoured for distinct element codes. As is the case for finite element codes employing explicit solution techniques, this permits very general constitutive modelling of joint behaviour with little increase in computational effort and results in computation time being only linearly dependent on the number of elements used. The use of explicit solution techniques places fewer demands on the skills and experience than the use of codes employing implicit solution techniques.

Although the distinct element method has been used most extensively in academic environments to date, it is finding its way into the offices of consultants, planners and designers. Further experience in the application of this powerful modelling tool to practical design situations and subsequent documentation of these case histories is required, so that an understanding may be developed of where, when and how the distinct element method is best applied.

Hybrid approaches

The objective of a hybrid method is to combine the above methods in order to eliminate undesirable characteristics while retaining as many advantages as possible. For example, in modelling an underground excavation, most non-linearity will occur close to the excavation boundary, while the rock mass at some distance will behave in an elastic fashion. Thus, the near-field rock mass might be modelled, using a distinct element or finite element method, which is then linked at its outer limits to a boundary element model, so that the far-field boundary conditions are modelled exactly. In such an approach, the direct boundary element technique is favoured as it results in increased programming and solution efficiency.

Lorig and Brady (1984) used a hybrid model consisting of a discrete element model for the near field and a boundary element model for the far field in a rock mass surrounding a circular tunnel.

Two-dimensional and three-dimensional models

A two-dimensional model, such as that illustrated in Figure 5, can be used for the analysis of stresses and displacements in the rock surrounding a tunnel, shaft or borehole, where the length of the opening is much larger than its cross-sectional dimensions. The stresses and displacements in a plane, normal to the axis of the opening, are not influenced by the ends of the opening, provided that these ends are far enough away.

On the other hand, an underground powerhouse or crusher chamber has a much more equi-dimensional shape and the effect of the end walls cannot be neglected. In this case, it is much more appropriate to carry out a three-dimensional analysis of the stresses and displacements in the surrounding rock mass. Unfortunately, this switch from two to three dimensions is not as simple as it sounds and there are relatively few good three-dimensional numerical models, which are suitable for routine stress analysis work in a typical engineering design office.

EXAMINE3D (www.rocscience.com) is a three-dimensional boundary element program that provides a starting point for an analysis of a problem in which the three-dimensional geometry of the openings is important. Such three-dimensional analyses provide clear indications of stress concentrations and of the influence of three-dimensional geometry. In many cases, it is possible to simplify the problem to two-dimensions by considering the stresses on critical sections identified in the three-dimensional model.

More sophisticated three-dimensional finite element models such as FLAC3D (www.itascacg.com) are available, but the definition of the input parameters and interpretation of the results of these models would stretch the capabilities of all but the most experienced modellers. It is probably best to leave this type of modelling in the hands of these specialists.

It is recommended that, where the problem being considered is obviously three-dimensional, a preliminary elastic analysis be carried out by means of one of the three-dimensional boundary element programs. The results can then be used to decide whether further three-dimensional analyses are required or whether appropriate two-dimensional sections can be modelled using a program such as PHASE2 (www.rocscience.com), a powerful but user-friendly finite element program that generally meets the needs of most underground excavation design projects.

Examples of two-dimensional stress analysis

A boundary element program called EXAMINE2D is available as a free download from www.rocscience.com. While this program is limited to elastic analyses it can provide a very useful introduction for those who are not familiar with the numerical stress analysis methods described above. The following examples demonstrate the use of this program to explore some common problems in tunnelling.

Tunnel shape

Most contractors like a simple horseshoe shape for tunnels since this gives a wide flat floor for the equipment used during construction. For relatively shallow tunnels in good quality rock this is an appropriate tunnel shape and there are many hundreds of kilometres of horseshoe shaped tunnels all over the world.

In poor quality rock masses or in tunnels at great depth, the simple horseshoe shape is not a good choice because of the high stress concentrations at the corners where the sidewalls meet the floor or invert. In some cases failures initiating at these corners can lead to severe floor heave and even to failure of the entire tunnel perimeter as shown in Figure 8.

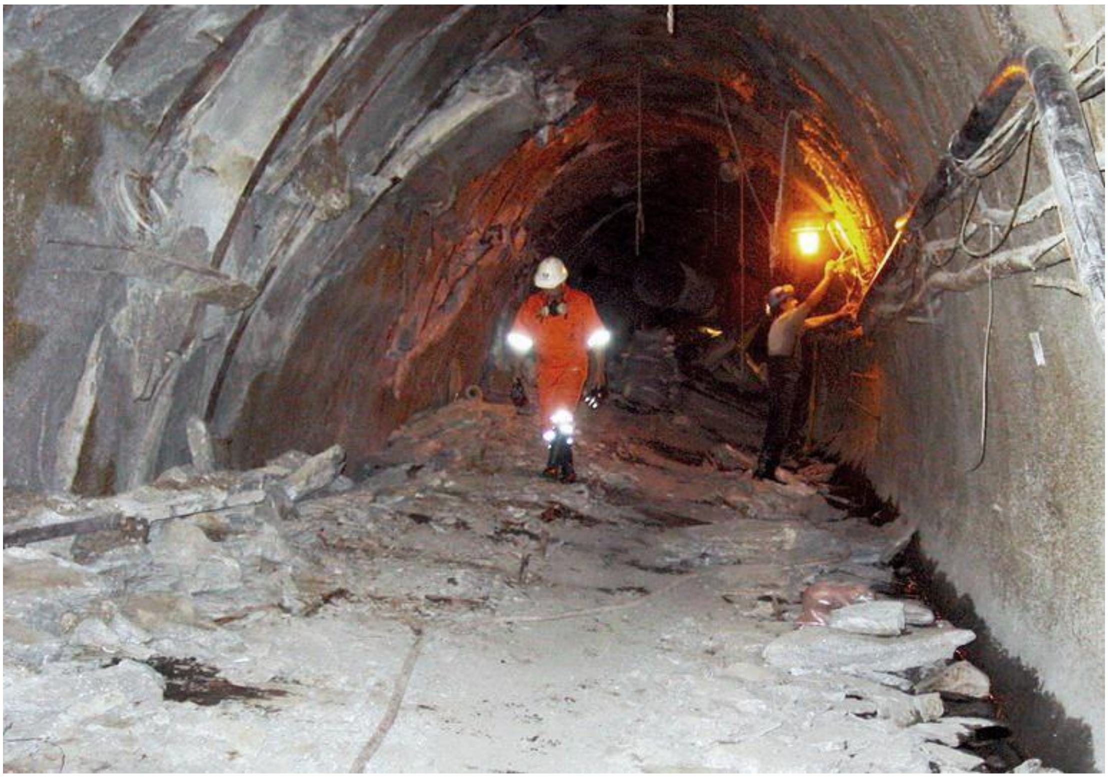


Figure 8: Failure of the lining in a horseshoe shaped tunnel in a highly stressed poor quality rock mass. This failure initiated at the corners where the invert meets the sidewalls.

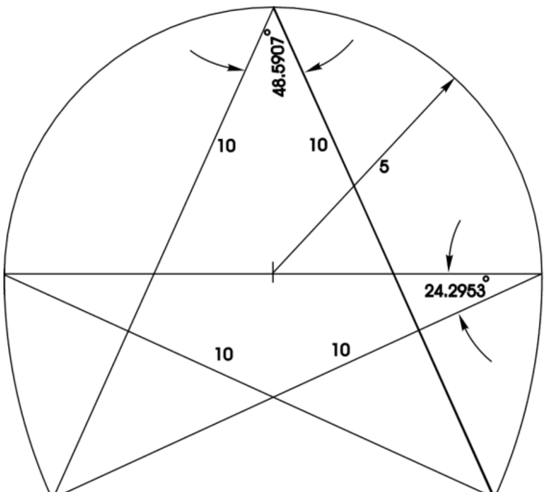
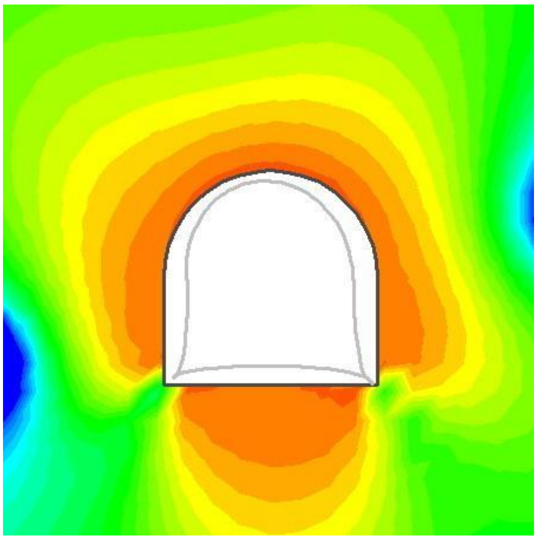
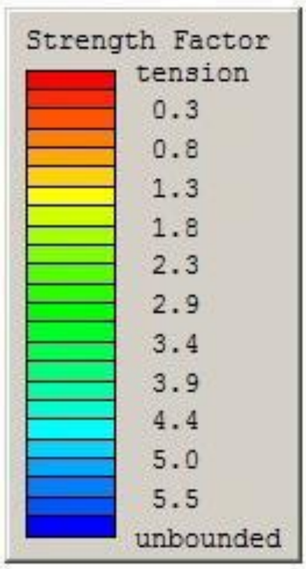
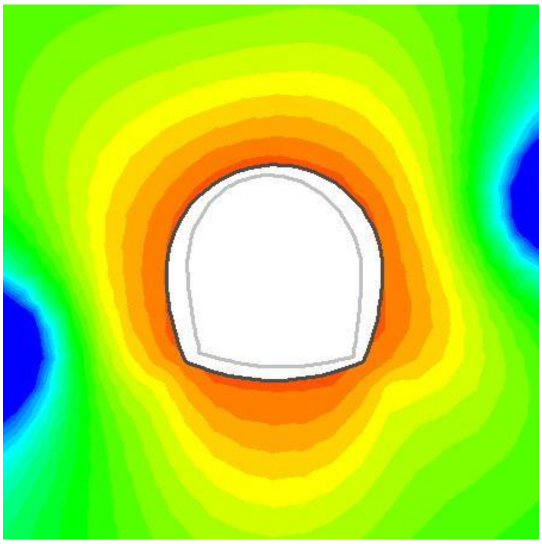


Figure 9: Dimensions of a 10 m span modified horseshoe tunnel shape designed to overcome some of the problems illustrated in Figure 8.

The stress distribution in the rock mass surrounding the tunnel can be improved by modifying the horseshoe shape as shown in Figure 9. In some cases this can eliminate or minimise the types of failure shown in Figure 8 while, in other cases, it may be necessary to use a circular tunnel profile.







In situ stresses:

Major principal stress σ_1 = 10 MPa
Minor principal stress σ_3 = 7 MPa
Intermediate principal stress σ_2 = 9 MPa
Inclination of major principal stress to the horizontal axis = 15º

Rock mass properties:

Friction angle ϕ = 35º
Cohesion c = 1 MPa
Tensile strength = zero
Deformation modulus E = 4600 MPa

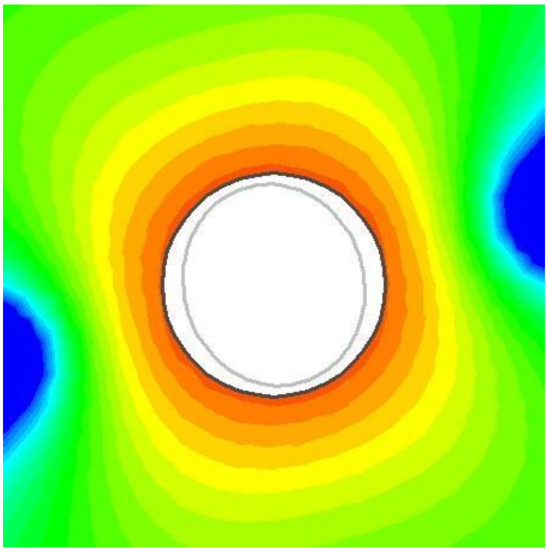


Figure 10: Comparison of three tunnel excavation profiles using EXAMINE2D. The contours are for the Strength Factor defined by the ratio of rock mass strength to the induced stress at each point. The deformed boundary profile (exaggerated) is shown inside each excavation.

The application of the program EXAMINE2D to compare three tunnel shapes is illustrated in Figure 10. Typical "average" in situ stresses and rock mass properties were used in this analysis and the three figures compare Strength Factor contours and deformed excavation profiles (exaggerated) for the three tunnel shapes.

It is clear that the flat floor of the horseshoe tunnel (top figure) allows upward displacement or heaving of the floor. The sharp corners at the junction between the floor and the tunnel sidewalls create high stress concentrations and also generate large bending moments in any lining installed in the tunnel. Failure of the floor generally initiates at these corners as illustrated in Figure 8.

Floor heave is reduced significantly by the concave curvature of the floor of the modified horseshoe shape (middle figure). In marginal cases these modifications to the horseshoe shape may be sufficient to prevent or at least minimise the type of damage illustrated in Figure 8. However, in severe cases, a circular tunnel profile is invariably the best choice, as shown by the smooth Strength Factor contours and the deformed tunnel boundary shape in the bottom figure in Figure 10.

Large underground caverns

A typical underground complex in a hydroelectric project has a powerhouse with a span of 20 to 25 m and a height of 40 to 50 m. Four to six turbine-generator sets are housed in this cavern and a cutaway sketch through one of these sets is shown in Figure 11. Transformers are frequently housed in a chamber or gallery parallel to the powerhouse. Ideally these two caverns should be as close as possible in order to minimise the length of the bus-bars connecting the generators and transformers. This has to be balanced against the size and hence the stability of the pillar between the caverns. The relative location and distance between the caverns is explored in the series of EXAMINE2D models shown in Figure 12, using the same in situ stresses and rock mass properties as listed in Figure 10.

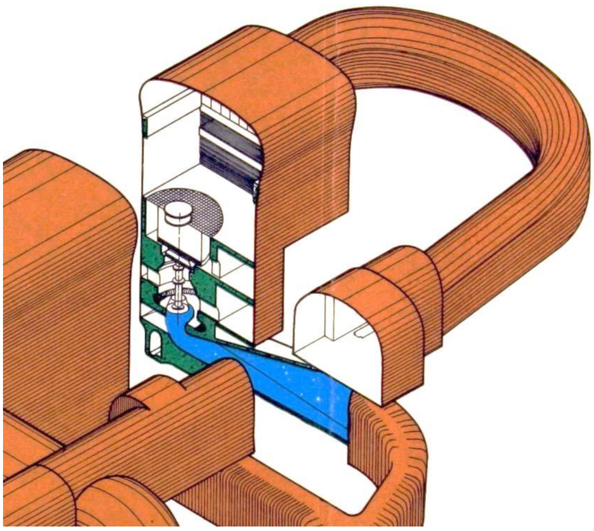
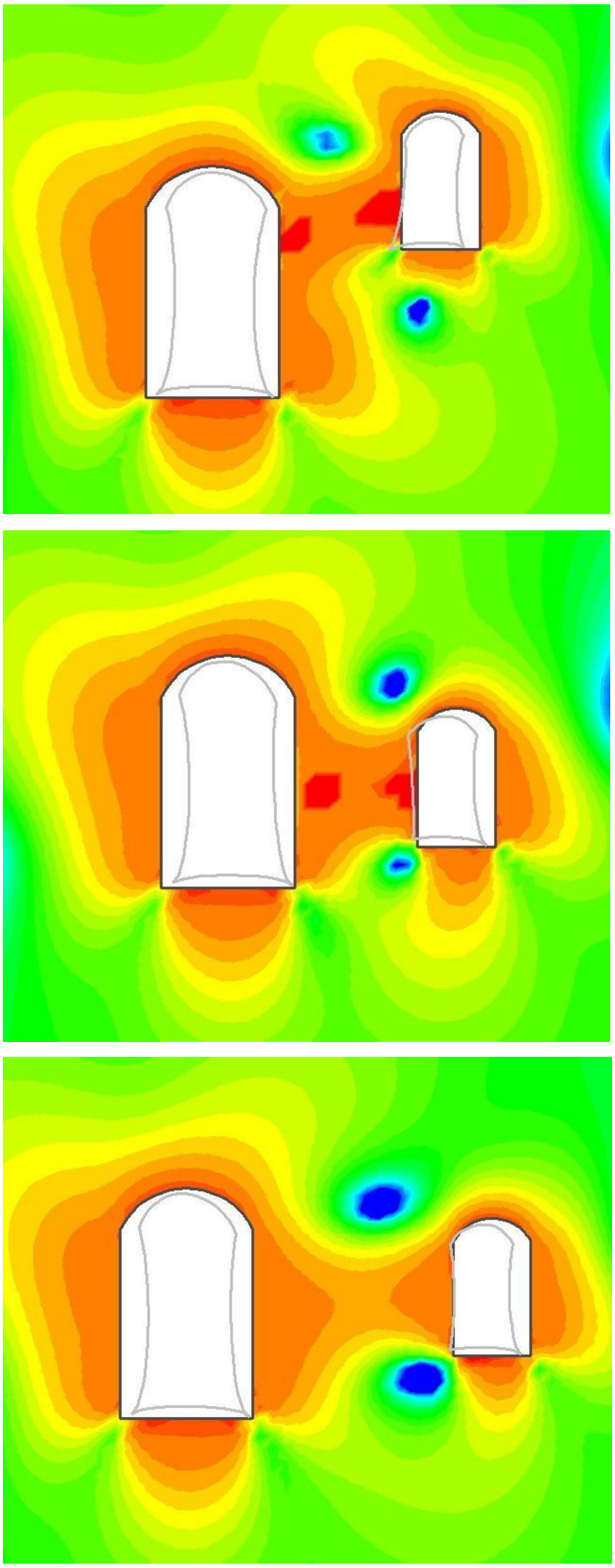
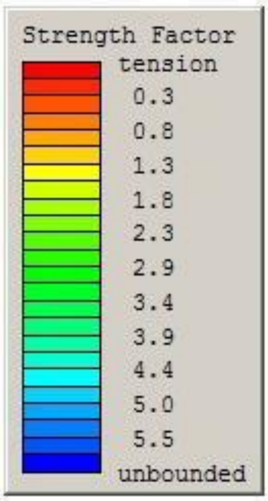


Figure 11: Cutaway sketch of the layout of an underground powerhouse cavern and a parallel transformer gallery.





In situ stresses:

Major principal stress σ_1 = 10 MPa
Minor principal stress σ_3 = 7 MPa
Intermediate stress σ_2 = 9 MPa
Inclination of major principal stress to the horizontal axis = 15º

Rock mass properties:

Friction angle ϕ = 35º
Cohesion c = 1 MPa
Tensile strength = zero
Deformation modulus E = 4600 MPa

Figure 12: Comparison of three underground powerhouse and transformer gallery layouts, using EXAMINE2D. The contours are for the Strength Factor defined by the ratio of rock mass strength to the induced stress at each point. The deformed boundary profile (exaggerated) is shown inside each excavation.

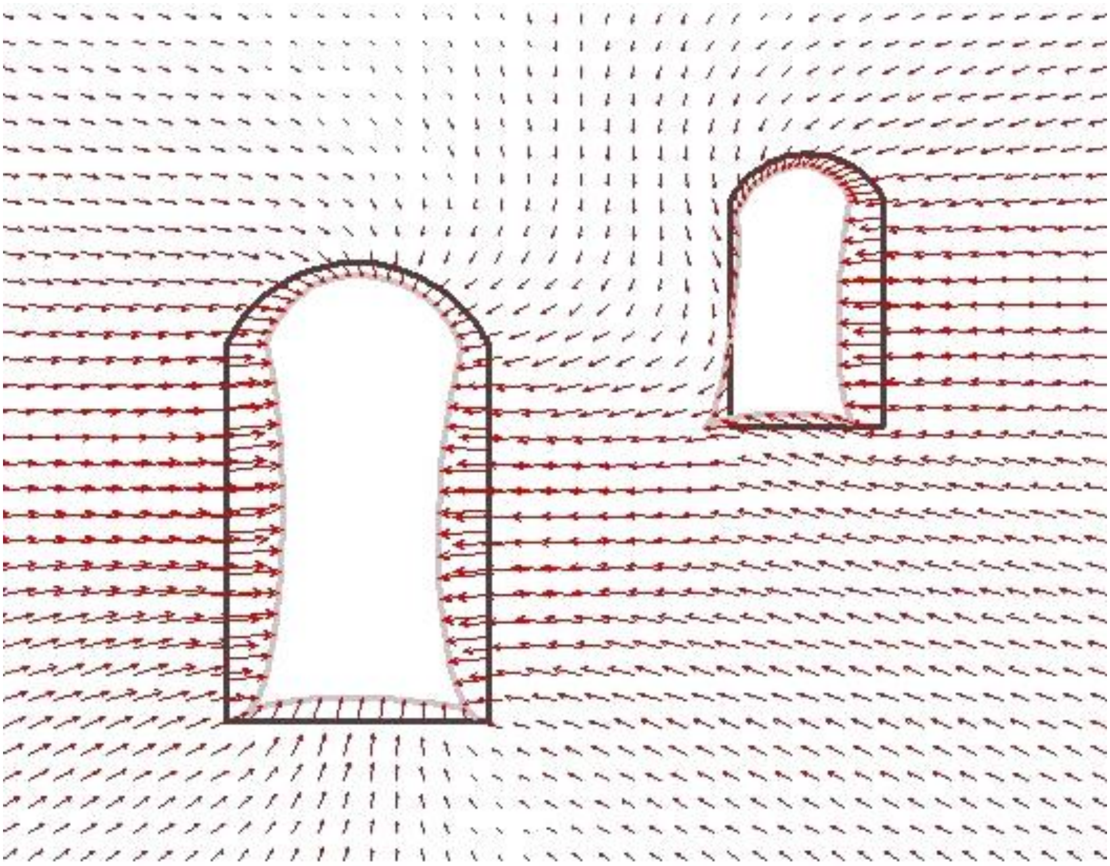


Figure 13: Displacement vectors and deformed excavation shapes for the underground powerhouse and transformer gallery.

A closer examination of the deformations induced in the rock mass by the excavation of the underground powerhouse and transformer gallery, in Figure 13, shows that the smaller of the two excavations is drawn towards the larger cavern and its profile is distorted in this process. This distortion can be reduced by relocating the transformer gallery and by increasing the spacing between the galleries as has been done in Figure 12.

Where the combination of rock mass strength and in situ stresses is likely to cause overstressing around the caverns and in the pillar, a good rule of thumb is that the distance between the two caverns should be approximately equal to the height of the larger cavern.

The interested reader is encouraged to download the program EXAMINE2D (free from www.rocscience.com) and to use it to explore the problem, such as those illustrated in Figures 10 and 12, for themselves.

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