INVESTIGATING THE PERFORMANCE OF THE SHEAR STRENGTH REDUCTION (SSR) METHOD ON THE ANALYSIS OF REINFORCED SLOPES

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ABSTRACT

This paper studies the performance of the Finite Element-based Shear Strength Reduction (SSR) method on the analysis of reinforced slopes. For two simplified reinforced slope examples, it compares SSR factor of safety and reinforcement loads to those of conventional limit equilibrium methods. The comparisons indicated good performance of the SSR technique. Although more detailed studies are required to assess the method’s results on a wider range of slope and reinforcement types, the outcomes of the tests in the paper demonstrate the usefulness of the SSR method as a complement to limit equilibrium analysis. The SSR can help uncover stiffness interactions and behaviour which may be missed when only limit equilibrium analysis is performed.

RÉSUMÉ

Cet article étudie l'exécution de la méthode Élément-basée finie de la réduction de résistance au cisaillement (SSR) sur l'analyse des pentes renforcées. Pour deux exemples renforcés simplifiés de pente, il compare le facteur de SSR des charges de sûreté et de renfort à ceux des méthodes conventionnelles d'équilibre de limite. Les comparaisons ont indiqué la bonne exécution de la technique de SSR. Bien que des études plus détaillées soient exigées pour évaluer les résultats de la méthode sur un éventail de types de pente et de renfort, les résultats des essais dans le papier démontrent l'utilité de la méthode de SSR comme complément à l'analyse d'équilibre de limite. Le SSR peut aider à découvrir les interactions et le comportement de rigidité qui peuvent être manqués quand seulement l'analyse d'équilibre de limite est exécutée.

1. INTRODUCTION

Computing power and resources available to the geotechnical engineer today, combined with low costs, have made slope stability analysis with the Finite Element Method (FEM) a viable alternative to traditional limit equilibrium methods. The Shear Strength Reduction (SSR) technique [Dawson et al, 1999, Griffith and Lane, 1999, Hammah et al, 2004, 2005a and 2005b] enables the FEM to be used to calculate factors of safety for slopes.

One of the most powerful and attractive features of SSR analysis commonly cited is the method’s ability to predict stresses and deformations of support elements, such as piles, anchors and geotextiles, at failure. Reinforced slopes such as mechanically stabilized earth walls, anchored walls, soil-nailed walls, and reinforced embankments, constitute a major segment of the geotechnical engineering industry. As such a proven ability to calculate reinforcement loads and deformations will offer geotechnical engineers a very powerful tool for improving and optimizing the design of reinforced slopes.

Some studies [Han and Leshchinsky, 2004, Leshchinsky and Han, 2004] have been done that apply the SSR method (using FLAC, which is based on the finite difference method) to the design of multitiered mechanically stabilized earth (MSE) walls. These studies compare the SSR’s performance to limit equilibrium results for these walls. The primary focus of this paper is to verify the performance of the SSR method (based on the FEM) in calculating the factor of safety for reinforced slopes and reinforcement loads at failure. Two reinforced slope examples will be considered – one involving a simple slope reinforced with a single bolt, and the other involving multiple layers of reinforcement.

Factor of safety values and reinforcement forces for the SSR method will be compared against limit equilibrium results. The paper will explain why there generally is a difference between reinforcement forces reported by the SSR and those specified in limit equilibrium analysis.

2. DIFFERENCES BETWEEN LIMIT EQUILIBRIUM AND FINITE ELEMENT REINFORCEMENT LOADS

Limit equilibrium analysis makes several simplifying assumptions to make slope stability problems tractable. These include:

1. *A priori* assumptions on the shapes and locations of failure surfaces
2. Assumption that a sliding mass moves as a rigid block, with movement occurring only along the failure surface

3. Assumption that at failure shear is immediately exceeded along the entire length of the failure surface, and
4. Various assumptions on interslice forces.

These methods do not consider stress-strain relationships, and consequently do not compute displacements. They include the influence of reinforcement in the following manner:

1. Assume a specific distribution of loads along reinforcement elements. For the simplest bolt type – end-anchored bolts – a constant load is assumed to act along the entire reinforcement length. For other bolt types, reinforcement forces vary in different bolt segments.
2. The magnitude of bolt force included in stability equations is selected based on the location along a bolt at which a failure surface intersects. As a result, bolt forces reported at the end of a limit-equilibrium analysis are exactly the same as the prescribed (input) forces.

The FEM provides a more complete solution by considering slope boundary conditions and constitutive (stress-strain) laws of behaviour, enforcing strain compatibility, and checking for the satisfaction of complete equilibrium. An FEM analysis (that converges to a solution), calculates bolt forces and displacements that satisfy all constraints and conditions.

The final forces in bolts in FE slope analysis result from interactions between input parameters, including strength and deformation (stiffness) properties of slope materials and reinforcement. These parameters influence the manner in which stresses are redistributed and displacements accumulated.

The differences in reported reinforcement forces for limit equilibrium and FE analyses are most evident in slopes involving multiple reinforcement elements. In FE analysis the loads developed in reinforcement elements are generally proportional to the amount of deformation they experience. Slope stability analysis is such that very rarely do reinforcement experience the same amounts of straining. As a result, in multi-reinforcement elements models, the individual bolts generally experience different loads except when they all attain their ultimate capacity, fail, and loads are forced to residual capacity.

The SSR calculations in this paper are performed using the two-dimensional finite element program, Phase^2 [Phase^2, 2005]. Limit equilibrium models, equivalent to the finite element examples, are analyzed with Slide [Slide, 2004].

3. METHODOLOGY FOR COMPARING LIMIT EQUILIBRIUM AND SSR RESULTS FOR THE ANALYSIS OF REINFORCED SLOPES

To determine how well the SSR method analysis works on reinforced slopes, factors of safety and reinforcement loads computed by the method were compared to values given by limit equilibrium methods. Two of the most widely used limit equilibrium methods – Bishop's and Spencer's methods of slices – were used to analyze the slope examples.

Two slope examples were considered. The first example, shown on Fig. 1, involved a simple two-material slope reinforced with a single end-anchored bolt, the simplest type of reinforcement. This reinforcement model assumes a constant load along the entire length of reinforcement. The upper material had zero friction angle, cohesion of 500 psf, and Young's modulus of 1 x 10^6 psf. In order to restrict the failure mechanism to the upper slope material only, the lower material was assigned infinite strength (very high strength for finite element analysis).

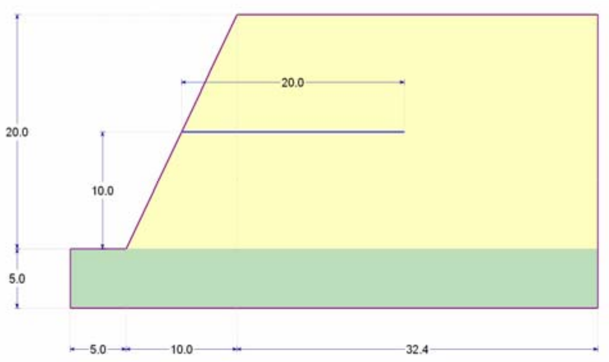

Fig. 1: Geometry of the reinforced slope in Example 1.

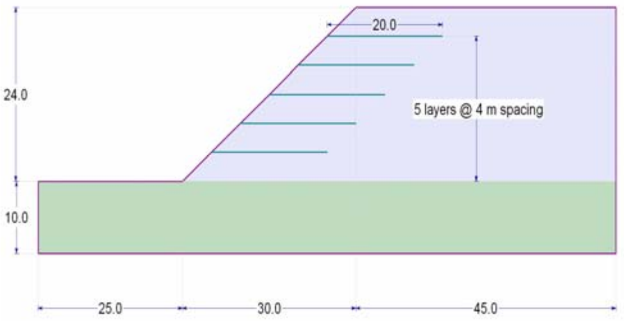

Fig. 2: Geometry of the reinforced slope in Example 2.

The SSR model for Example 1 had the following reinforcement properties:

- Diameter = 0.75 in
- Young's modulus (for steel) = 4177090000 psf, and
- Tensile capacity = 10,000 lbs.

A reinforcement force of 1144 lbs was specified for the limit equilibrium equivalent to this model. This was the force calculated for the reinforcement by the SSR method at failure of the slope, i.e. at its factor of safety. The reasoning this force was specified for the limit equilibrium analyses was that, if the SSR analysis was accurate, then this value should produce a limit equilibrium factor of safety very similar to the SSR result.

The second example (Fig. 2) analyzes a slope reinforced with multiple layers of geotextile. It was taken from page 135 of [Duncan & Wright, 2005]. The upper material has the following properties:

- Unit weight = 130 pcf
- Cohesion = 0, and
- Friction angle = 37^o

The geotextile has a tensile capacity of 800 lbs per lineal foot. In the finite element SSR analysis, it was assigned a tensile modulus of 13704.4 lb/ft, a representative value for geotextiles.

For the SSR models to be equivalent to their limit equilibrium counterparts, no slip is allowed between the geotextiles and adjacent slope material. In general, however, the FEM can easily incorporate slip at the interfaces between geotextiles and slope materials.

The presence of a single end-anchored bolt in the first example allows direct comparison of both factor of safety values and reinforcement loads, since the bolt force specified for limit equilibrium analysis can be selected such that it is equal to the bolt force at slope failure calculated by the SSR technique.

Two assumptions regarding the post-yield tensile strength of reinforcement were tested for the SSR models. The first assumption was that the reinforcement had zero strength once its peak (yield) strength was attained. Under the second assumption, the post-yield strength of reinforcement was deemed equal to the peak value, i.e. the reinforcement was assigned an elastic-perfectly plastic strength response.

The sensitivity of SSR factor of safety results to mesh size (or degree of refinement) was also tested for Example 2. Three different mesh sizes – 1000, 1500 and 2000 elements – were applied. The mesh refinement was mostly restricted to the slope region containing the layers of reinforcement. All the SSR models used 500 iterations and a tolerance of 0.001 as the thresholds for determining the non-convergence of Phase^2 FEM models.

It is important to note that the SSR method used in this paper does not divide reinforcement loads by the strength reduction factor (factor of safety) in an analysis. The strength reduction factor is applied only to the strengths of slope materials. Duncan and Wright [2005] prefer this approach

4. RESULTS

4.1 Example 1

The SSR and limit equilibrium factor of safety results for Example 1 are given in Table 1. Fig. 3 superimposes the Bishop circular failure surface (the Spencer circle is very similar) on the contours of maximum shear strain for the SSR analysis. (The non-circular Bishop surface also mimics this mechanism closely.) From the table and figure, it is clear that the failure mechanism predicted by the SSR method agrees well with the limit equilibrium results. As noted above the reinforcement force is the same for both the SSR and limit equilibrium method.

Table 1: Comparison of SSR and Limit Equilibrium Factors of Safety for Example 1

Method	**Factor of Safety**
Bishop (circular)	1.312
Spencer (circular)	1.367
Bishop (non-circular)	1.288
Spencer (non-circular)	1.491
SSR (elastic-perfectly plastic reinforcement strength)	1.300
SSR (zero post-failure reinforcement strength)	1.300

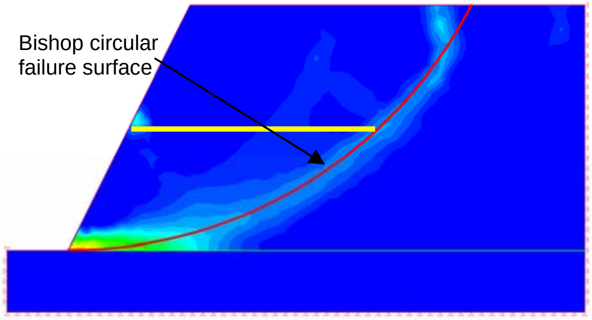

Fig. 3: The contours of maximum shear strain at failure computed by the SSR method with the Bishop failure circle superimposed. The two failure mechanism show very good agreement.

It is evident from Table 1 that for Example 1 the different assumptions on reinforcement post-failure strength did not affect the factor of safety results. Although it is not shown, the differences in the assumptions also did not affect the failure mechanism.

4.2 Example 2

The SSR and limit equilibrium factor of safety results for Example 2 with multiple layers of geotextile reinforcement are given in Table 2. The table also reports the factor of safety for the slope obtained by Duncan and Wright [2005]. In their limit equilibrium model they assume a constant variation of reinforcement force that decreases linearly to zero over the last four feet of embedded length.

Study of the factor of safety values produces two key insights. Firstly, the comparison shows that the assumption of elastic-perfectly plastic yield characteristics for reinforcement produces factor of safety values closest to the results of limit equilibrium analysis. For the example, the two different post-failure assumptions produced differences in failure mechanisms. Figs. 4 and 5 below show the contours of maximum shear strain for the elastic-perfectly plastic and zero post-failure strength, respectively. The contours shown are for the slope models that use the finest mesh in this study. Also superimposed on these figures is the Bishop critical circle (the non-circular surface is quite similar). It can be seen that the zero post-failure assumption (Fig. 4) leads to a failure mechanism, which closely mirrors the limit equilibrium critical slip surface. This failure surface passes through the reinforcement layers. Although the results are shown only for the most refined models, the same could be observed in the other models.

Table 2: Comparison of SSR and Limit Equilibrium Factors of Safety for Example 2

Method	**Factor of Safety**
Duncan & Wright [Ref]	1.610
Bishop (circular)	1.658
Spencer (circular)	1.652
Bishop (non-circular)	1.635
Spencer (non-circular)	1.645
SSR Base Mesh (elastic-perfectly plastic reinforcement strength)	1.670
SSR Intermediate Mesh (elastic-perfectly plastic reinforcement strength)	1.650
SSR Fine Mesh (elastic-perfectly plastic reinforcement strength)	1.600
SSR Base Mesh (zero post-failure reinforcement strength)	1.560
SSR Intermediate Mesh (zero post-failure reinforcement strength)	1.540
SSR Fine Mesh (zero post-failure reinforcement strength)	1.530

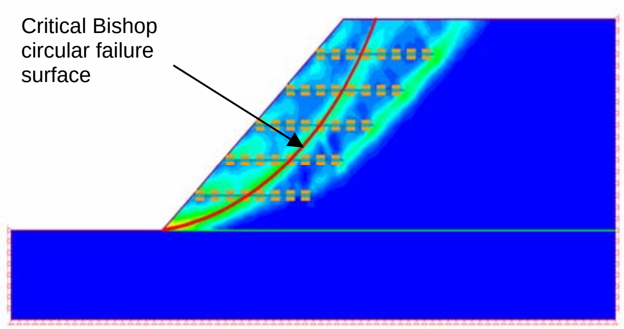

Fig. 4: Dominant failure mechanism as indicated by contours of maximum shear strain when reinforcement is assumed to have zero post-peak strength. Superimposed on the contours is the Bishop circular slip surface.

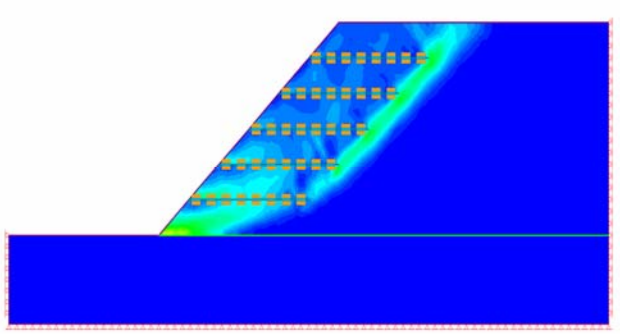

Fig. 5: Dominant failure mechanism as indicated by contours of maximum shear strain when reinforcement is assumed to have elastic-perfectly plastic strength profile.

A deeper mechanism just beyond the reinforced zone is also evident on Fig. 4. This mechanism involves movement of the entire reinforced zone as a block. As seen on Fig. 5 this second surface becomes the dominant mechanism when the reinforcement is assumed to have elastic-perfectly plastic strength. The presence of multiple failure mechanisms in a single SSR analysis has been noticed with other SSR models.

The study also shows that SSR factor of safety values, at least for this example, are quite insensitive to the degree of mesh refinement. This is a very important result; it means that users can be confident that they can begin their analyses with coarse models, and not miss any important mechanisms. At the latter stages of design or analysis, they can then use more refined meshes to better isolate the extents of failure mechanisms.

The load distributions along each of the five layers of reinforcement, starting with the uppermost, are shown on Fig. 6. The departures from the load distribution assumed in limit equilibrium analysis are evident on the images.

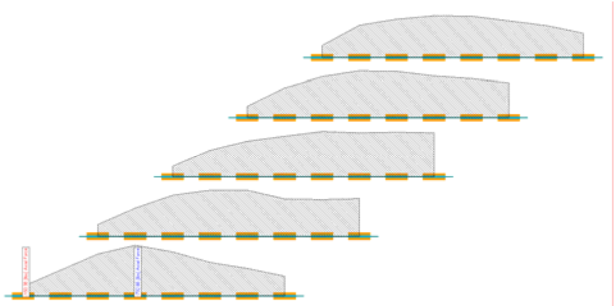

Fig. 6: The irregular distributions of forces along the layers of reinforcement, immediately before failure. These are very different from the limit equilibrium assumption of constant variation that decreases linearly over the last four feet of embedded length. (Reinforcement loads after slope failure may not be reliable due to the lack of equilibrium/non-convergence of FEM solutions.)

5. CONCLUDING REMARKS

Although the two simple reinforced slope examples analyzed in this paper are limited in scope, the results provide strong indication of the accuracy of the SSR method. In both examples the method produced factor of safety results similar to conventional limit equilibrium analysis. However, the SSR method offers the additional advantages of being able to predict more realistic force distributions along reinforcement, being able to accommodate more sophisticated yield behaviours of reinforcement, and estimating slope and reinforcement deformations at failure. It can also readily calculate reinforcement bending moments and shear stresses.

A perceived disadvantage of the SSR method of slope stability analysis is that it requires more parameters than limit equilibrium analysis. In the opinion of the authors, this should not be viewed as a disadvantage, but rather as an indication of greater flexibility. The additional parameters affect key aspects of slope behaviour such as failure mechanisms in ways that cannot be determined from limit equilibrium analysis. Consequently, the method should be viewed as a powerful complement to limit equilibrium analysis, since it can produce insights that may otherwise be missed.

Despite the promising results of the SSR method outlined in this paper, more comprehensive verification, similar to that for unreinforced slopes [Hammah et al], is needed to increase confidence in the method. A greater variety of reinforcement types should be analyzed, and the influence of parameters such as reinforcement Young's modulus studied into greater detail.

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