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The Practical Modelling of Discontinuous Rock Masses with Finite Element Analysis

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ABSTRACT: The mechanical response of rock masses to loading and excavation, especially in low stress environments, is significantly affected by discontinuities. This paper examines the practical modelling of rock mass problems with explicit representation of discontinuities using special joint elements in the Finite Element Method. The paper discusses why it is possible to use this approach for routine, practical engineering analysis today. Through a few examples, it also presents the merits of the approach such as the ability to capture a range of mechanisms and scale effects due to discontinuities.

1. INTRODUCTION

The idea of using the continuum-based finite element method (FEM) to model blocky rock mass behaviour has been around since the 1960s. The first joint or interface element for simulating the behaviour of discontinuities was proposed then [1]. Over the years however, discontinuum-based numerical approaches such as the Discrete Element Method (DEM) and Discontinuous Deformation Analysis (DDA) surpassed the FEM as the tools of choice for modelling blocky rock masses.

This paper briefly discusses the effects discontinuities (the terms discontinuities and joints will be used interchangeable throughout the paper) have on rock mass response to excavation and mechanical loading. It will argue that the Shear Strength Reduction (SSR) method, combined with modern computing advances, has made it possible to apply the FEM to the practical and routine engineering of structures in discontinuous rock masses as originally envisaged.

Through a few examples, the paper will show how the FEM captures a spectrum of discontinuous rock mass behaviours ranging from individual block movements to continuum-like mechanisms, and combined modes. One of the examples looks at step-path (en-echelon) failure (that combines slip along joints with shearing through intact rock) determined by the FEM. The paper will outline two unique advantages of the FEM compared to pure discontinuum approaches.

2. RESPONSE OF DISCONTINUOUS ROCK MASSES TO EXCAVATION AND MECHANICAL LOADING

The influence of discontinuities on the mechanical response of rock masses to loadings and excavation has long been recognized [1, 2, 3, 4]. In some situations (especially in low stress environments such as are encountered in slopes and near surface excavations) discontinuities exert greater influence on behaviour than do intact rock properties.

Discontinuities can cause the distribution of stresses and displacements induced in a rock mass to differ significantly from those predicted by classical elastic or elasto-plastic theories for homogeneous continua. The strength, deformation modulus, and stress-strain responses to loading of rock masses can be all affected by discontinuities in non-linear and anisotropic fashion. As well, discontinuities can make it very difficult to predict the strength and deformation characteristics of rock masses.

The deformations of discontinuities contribute greatly to the behaviour of discontinuous rock masses under excavation. Discontinuities generally exhibit brittle (strain softening) behaviour; they typically have residual strength that is much lower than peak strength [2]. This leads to the development of progressive failure mechanisms. When the strength of a segment of a joint is reached, the material in this stressed zone yields and deforms considerably, while the strength drops to the lower, residual value. These developments cause stresses to be redistributed to adjacent rock areas and joint segments. The new distribution of stresses may then cause other material and joint segments to similarly fail. The failure continues to propagate until an equilibrium state is attained or complete collapse results.

The modes in which structures and excavations in rock masses fail are another way in which joints influence rock mass response. Because these failure modes change with the strength and deformation properties of both intact rock and joints, and the distribution of joints [2, 5], it can be difficult to anticipate or predict the manner in which failure can occur.

3. REALISTIC NUMERICAL MODELLING OF DISCONTINUOUS ROCK MASS PROBLEMS

One approach of incorporating the influence of joints on rock mass strength in numerical analysis is through modelling of rock masses as continua with reduced deformation and strength properties [6]. Methods based on this approach, although useful, are not able to model deformation mechanisms involving movements such as: separation, slip and rotations of blocks [7]. Unfavourably dipping joints, for example, can create unstable conditions in rock slopes or tunnels that may be difficult to model with continuum methods. As will be demonstrated in the examples section of this paper, another weakness of continuum-based theories is that in constant stress environments the stability of an excavation is independent of opening size. In reality this is not the case due to the effects of discontinuities.

The realistic modelling therefore of the mechanical behaviour of joints is a prerequisite for the successful numerical modelling of discontinuous rock. Such models better capture the broad range of behaviours caused by interactions between the different moduli and strengths of intact rock, joints and support.

To model the wide-ranging behaviours of discontinuous rock masses, special numerical techniques such as the DEM [3, 8] and DDA [9] were developed. (Widely-used codes for such analysis include UDEC [10]). These methods explicitly model a rock mass as an aggregate of discontinuities and intact material. The DEM and DDA have essentially been the de facto methods for simulating discontinuous rock masses.

Through the development of special elements – joint elements (sometimes also known as interface elements) [1, 11, 12] – the continuum-based Finite Element Method (FEM) can also be applied to the modelling of discontinuous rock masses. These elements can have either zero thickness or thin, finite thickness. They can assume linear elastic behaviour or plastic response when stresses exceed the strengths of discontinuities.

Due to the fundamental continuum analysis condition of displacement compatibility at element nodes, FEM programs do not allow the detachment of individual blocks [13]. Nevertheless they are very useful for determining the onset of instability (collapse mechanisms) or large movements that cause block detachments.

Hybrid numerical methods such as those that combine FEM and DEM are another category of powerful tools for modelling discontinuous rock masses. ELFEN [14] is one such program. A most recent approach for modelling jointed rock mass behaviour is the synthetic rock mass approach [15].

4. FRACTURE NETWORKS

The generation of a network (system) of discontinuities, which is representative of field conditions, is one of the most important steps in modelling a jointed rock masses. Characterization of discontinuity networks in order to generate them is not a trivial step though. First, adequate description of the discontinuities in a geological domain is difficult, because of the three-dimensional nature of discontinuities and their partial exposure in outcrops or excavations [16, 17]. Because only parts of discontinuities are visible on exposed faces, thorough description of exposed samples is not possible. At the same time, indirect methods of measurement are not very accurate [17].

The geometric attributes (orientation, length, spacing, persistence, etc.) of the discontinuities in a set are not deterministic values but random [18]. However they exhibit patterns that may be approximated with some statistical distributions and, as a result, discontinuities in a network are best described as a group with statistical characteristics in space [17, 19].

Fracture network models are useful tools for generating systems of discontinuities for blocky rock mass modelling [17]. Approaches exist for characterizing the fractures in a rock mass and simulating them according to some fracture network model. Some of these models are implemented and described by Dershowitz et al [20] in a program known as FracMan. With the computing power available today, it has become possible to generate networks of discontinuities numerically and to simulate the mechanical behaviour of each member of a network.

5. THE SHEAR STRENGTH REDUCTION METHOD AND PRACTICAL BLOCKY ROCK MASS MODELLING WITH THE FEM

Although the idea of modelling rock masses with the FEM and joint elements has been around since the 1960s, it is only recently however that the method is being applied to routine rock engineering analysis. In addition to widespread availability of powerful desktop and laptop computers, the development of the shear strength reduction (SSR) method of slope stability analysis with the FEM has contributed greatly to the ability to use the FEM for practical blocky rock mass [21]. Quite a few publications have shown that the SSR method is a powerful alternative to conventional limit equilibrium methods of slope stability analysis, and have highlighted its several advantages (including the ability to solve a broader range of problems) [22 – 28].

5.1. Non-Traditional Use of the SSR Method

Because the SSR method is able to reveal the formation and progress of failure mechanisms, it can be readily applied to non-slope problems. For example, for the stability analysis of a tunnel, when the contour patterns of displacement or maximum shear strain are arranged in order (e.g. from the lowest strength reduction factor to the highest), they show the sequence in which blocks and joints move and deform. This often gives a good picture of how the failure mechanism is formed and how it propagates from zone to zone.

5.2. A Note on Joint Stiffness and Strength Parameters

The shear and normal stiffness parameters of discontinuities, as mentioned earlier, as well their strength parameters, influence the behaviour of rock masses. Unfortunately though, information on these parameters is not easily obtained [4]. In many practical situations however, knowledge of the ratios of these parameters for the different types of discontinuities is adequate enough to generate reasonable answers. Goodman et al [1] provide a good discussion on the factors that most affect joint parameters. The ideas they express provide a good framework for making reasonable guesses at joint stiffness and strength parameters based on geological observations.

An example is the description of how the filling material in joints influences stiffness and strength. Goodman et al [1] describe that clay infill generally leads to low normal and shear discontinuity stiffness, and also leads to low strength, except when joint surfaces in strong rock interlock extensively and therefore require shearing of asperities during failure. Another example they give is the role of joint cementation by quartz, calcite or epidote. Such cementation gives rise to joint properties that are as good as or even stronger than those of intact rock material.

5.3. Advantages of the FEM in the Modelling of Discontinuous Rock Masses

In the authors' opinion, the FEM enjoys two unique advantages over the DEM and DDA:

1. It facilitates integrated analysis in which the different components (such as foundations, rock, joints and support elements) of a rock engineering system interact with each other. The FEM can help establish the interactions between these different components, and
2. It can handle cases in which fractures intersect in a manner such that discrete blocks may not necessarily be formed, i.e. cases in which joints may terminate within intact rock and not only at intersections with other joints. One of the examples in the paper will show how the method can handle rock slope failures involving step-path mechanisms.

6. EXAMPLES

Three examples were analyzed in order to investigate the application of the FEM to problems in discontinuous rock masses. Each of the first two examples involves two rock mass cases: a homogeneous rock mass with no joints, and a second case in which the rock mass has two joint sets. This helps assess the joint effects such on the stability of excavations and distribution of stresses.

The third example is designed to illustrate the versatility of the FEM by analyzing a step-path failure mechanism. This is done without use of any special assumptions or treatment; the mechanism is uncovered from straightforward SSR analysis.

The FEM computational tool used to carry out the analysis is a version of the Rocscience program Phase2 [29] with an automatic generator of discontinuity networks and joint sets.

6.1. Example 1 – Stability of a Cut in Continuous and Discontinuous Rock Masses

This example explores the stability of a rock cut that slopes at one horizontal to two vertical (1H: 2V). Different heights (15m, 30m, 45m and 60m) of the cut are considered. The basic geometry of a slope height of 60m is shown on Figure 1. The intact rock and joint properties used in the example are described in Table 1.

The results of the analyses of slope heights of 15m, 30m, 45m and 60m, respectively, for the cases of homogeneous and discontinuous rock masses are given in Table 2.

The results show that for near-surface excavations size effects on stability exist even for conventional elasto-plastic assumptions on material behaviour. This is due to the fact that in such cases the magnitudes of the stresses driving failure are directly related to excavation or slope size. As the height of the slope increases the factor of safety reduces. The results also show that presence of the joints reduced the factor of safety for the slopes.

Table 1: Properties of Intact Rock and Joints

Material	**Properties**
Intact Rock	Unit weight = 0.027 MN/m3 Young's Modulus = 20000 MPa Poisson's ratio = 0.3 Tensile strength = 0 MPa Cohesion = 1 MPa Friction angle = 30 degrees Dilation angle = 0 degrees
Joints	Dip (of Joint Set 1) = 0 degrees Dip (of Joint Set 2) = 45 degrees Spacing = 3m Normal stiffness = 100000 MPa/m Shear stiffness = 10000 MPa/m Tensile strength = 0 MPa Cohesion = 0.5 MPa Friction angle = 20 degrees

Table 2: Factors of Safety for the Different Slope Cases

Slope Height	Factor of Safety for Homogeneous, Unjointed Rock Mass	Factor of Safety for Discontinuous Rock Mass
15m	11.01	6.66
30m	5.6	3.5
45m	3.93	3.48
60m	3.12	1.86

Of also great interest in this example is the analysis of the critical failure mechanisms for the different slope cases. The failure mechanism of the slopes can be identified from the contours of maximum shear strain from SSR finite element analysis. We will examine of these contours (as shown on Figures 2 and 3) for the smallest (15m) and largest (60m) slope heights. Figure 2a depicts a rotational-type failure for the case of the homogeneous 15m slope. The failure mechanism for the corresponding 15m cut in the discontinuous rock mass (Figure 2b) involves block movements (sliding) relative to each other. In the 60m slope height cases (Figures 3a and 3b), the contours of maximum shear strain and total displacement for the discontinuous rock mass slope indicate a failure mechanism with greater shearing through intact rock in the upper right zones of the slope.

Another case of the 60m slope was analyzed in which the joints in the rock mass were more closely spaced (spacing of 1.25m). As seen on Figure 3c, the failure surface for this case has acquired a much more rotational character. Its factor of safety was 1.78.

These results indicate the range of mechanisms the FEM is able to capture. The results are also consistent with rock mass behaviour widely noted in rock slope stability that the larger (the term 'large' in this case refers to the ratio of slope height to joint spacing) a slope, the closer the failure mechanism gets to the rotational-type failures common to soils.

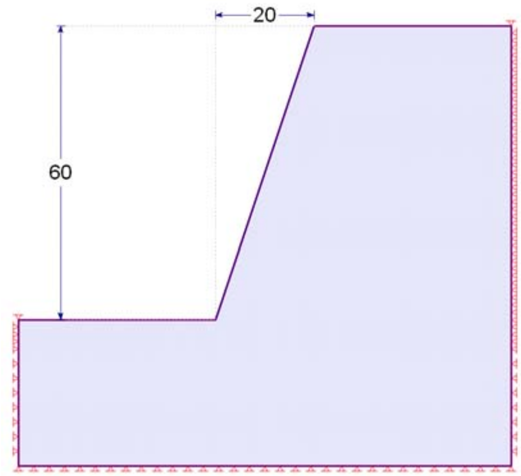

Figure 1a: Basic geometry of 60 m rock cut in homogeneous rock mass.

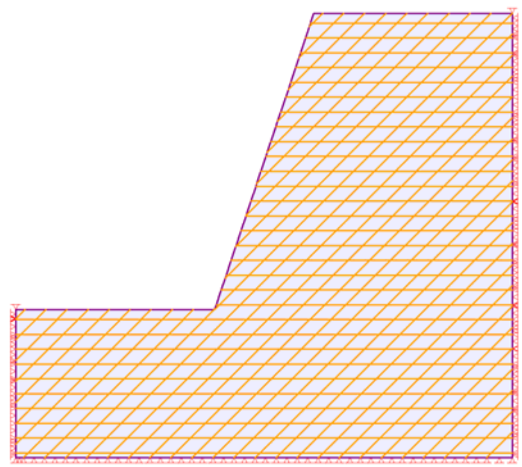

Figure 1b: Basic geometry of 60 m rock cut in discontinuous rock mass.

6.2. Example 2 – Distribution of Major Principal Stress around Circular Hole

This example examines the distribution of stresses around the perimeter of a circular hole in two rock mass types: a homogenous, unjointed rock mass, and a rock mass with two joint sets (one horizontal and the other vertical). For the jointed rock mass, the joints in each set had lengths of 10m, length persistence of 0.8, and spacing of 3m. The resulting jointing pattern is shown on Figure 3. Because of the presence of rock bridges (due to the fractional length persistence), some of the joints terminated in intact rock and created 'partial' blocks.

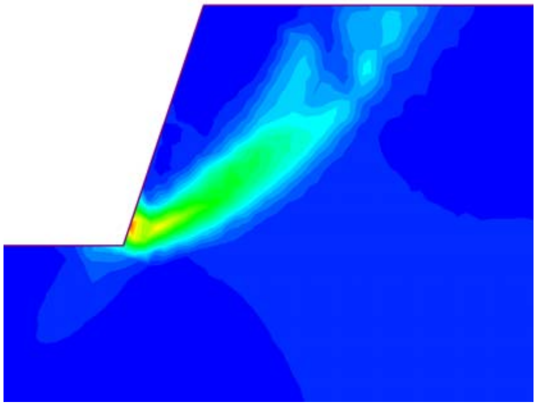

Figure 2a: Contours of maximum shear strain for 15 m cut in discontinuous rock mass. The contours indicate a rotational type failure.

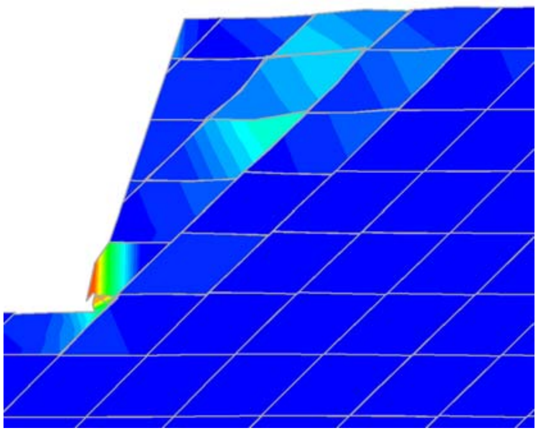

Figure 2b: Contours of maximum shear strain for 15 m cut in discontinuous rock mass. The contours indicate a failure mechanism involving block movements. (The block displacements shown on the figure are exaggerated just to make it easier to visualize the movements and deformations.)

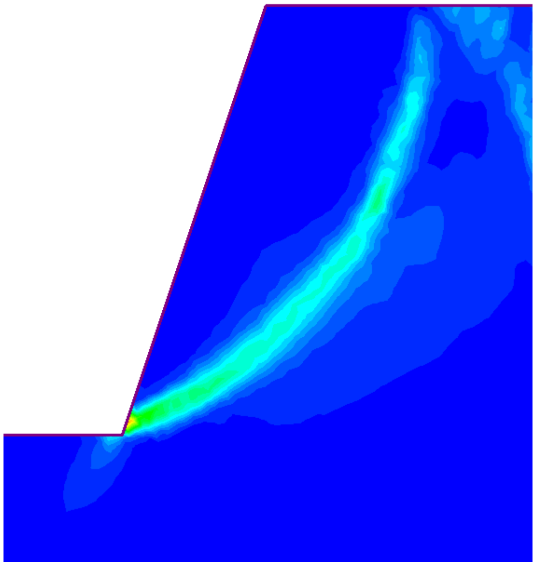

Figure 3a: Contours of maximum shear strain for 15 m cut in discontinuous rock mass. The contours indicate a rotational type failure.

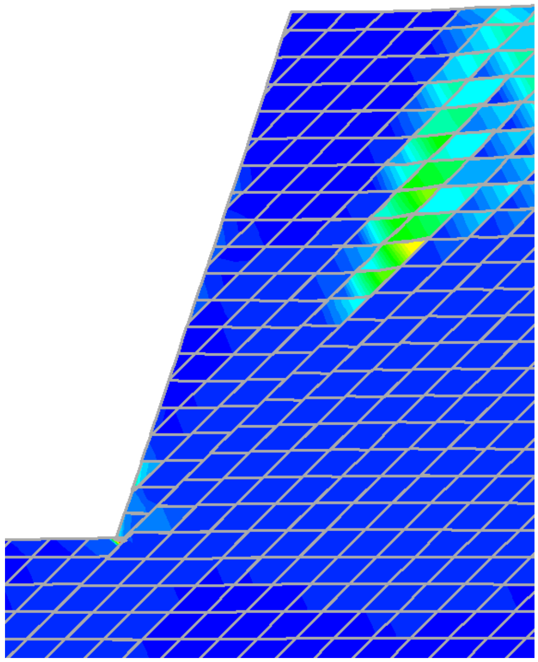

Figure 3b: Contours of maximum shear strain for 60 m cut in discontinuous rock mass. The contours indicate a failure mechanism involving block movements and some shearing through intact rock.

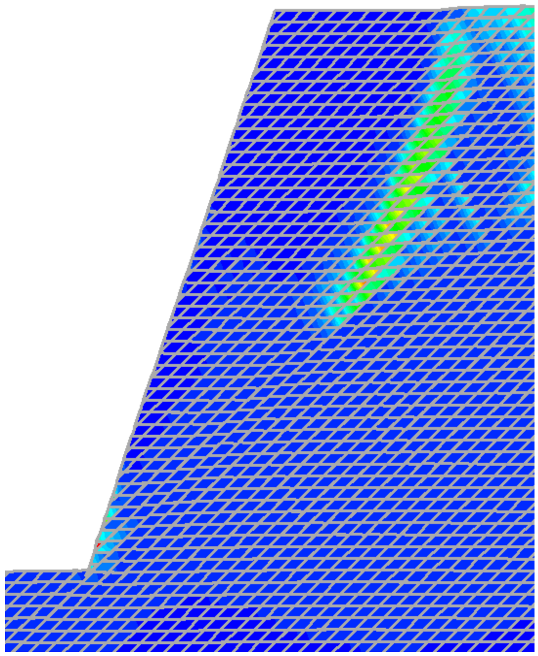

Figure 3c: Contours of maximum shear strain for 60 m cut in highly discontinuous rock mass (joints at 1.25m spacing). The failure mechanism has acquired a more rotational character.

A constant in situ stress field of 10 MPa in both the horizontal and vertical directions was assumed. The stresses around three different excavation sizes (1m, 5m and 10m diameters) were studied. Two cases on intact rock behaviour were considered – elastic, and elastic-perfectly plastic stress-strain behaviour. The properties assumed for the intact rock and the joints are provided in Table 3.

Table 3: Properties of Intact Rock and Joints

Material	Properties
Elastic Rock	Young's Modulus = 20000 MPa Poisson's ratio = 0.3
Plastic Rock	Young's Modulus = 20000 MPa Poisson's ratio = 0.3 Tensile strength = 0 MPa Cohesion = 1 MPa Friction angle = 30 degrees Dilation angle = 0 degrees
Joints	Normal stiffness = 100000 MPa/m Shear stiffness = 10000 MPa/m Tensile strength = 0 MPa Cohesion = 0.5 MPa Friction angle = 20 degrees

To help analyze the impact the joints had on both the elastic and elasto-plastic rock materials in the simplest manner, the mean, minimum and maximum values of major principal stress along the excavation perimeter were calculated. These results are in Table 4. They indicate the following:

1. For both the elastic and elasto-plastic unjointed rock, the distribution of stresses was practically independent of excavation size. (The small differences were more as a result of differences in meshes.)
2. The presence of joints reduced the mean major principal stress value for both the elastic and elasto-plastic rock.
3. The larger the excavation the smaller the scatter in results (minimum and maximum stress values) around the excavation perimeter. The reduction in the scatter of stress values with increasing excavation size can be interpreted as a trend towards the behaviour exhibited in homogeneous, unjointed rock.

Table 4: Major Principal Stress Results around the Excavation Perimeter

Case	Hole Diameter	Major Principal Stress (MPa)		
		Mean	Min	Max
Elastic homogeneous rock	1m	19.8	19.75	20.0
	5 m	19.3	18.6	20.3
	10 m	18.6	18	19.4
Jointed, elastic rock mass	1m	18.9	3.1	31.2
	5 m	14.5	3.4	28.2
	10 m	10.2	2.8	25.4
Elastic-perfectly plastic homogeneous rock	1m	6.9	6.6	7.3
	5 m	7.3	6.6	8.5
	10 m	7.2	6.6	7.9
Jointed, elasto-plastic rock mass	1m	7.8	3.2	9.8
	5 m	6.5	3.2	8.7
	10 m	5.6	2.8	7.7

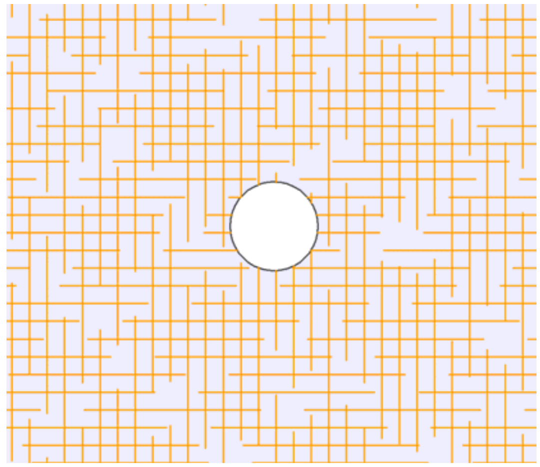

Figure 3: Jointing pattern for Example 2. Notice the termination of some joints in intact rock.

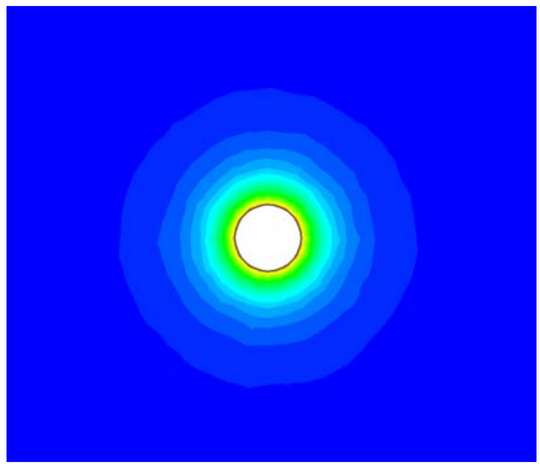

Figure 4a: Contours of major principal stress for the elastic-perfectly plastic, unjointed rock.

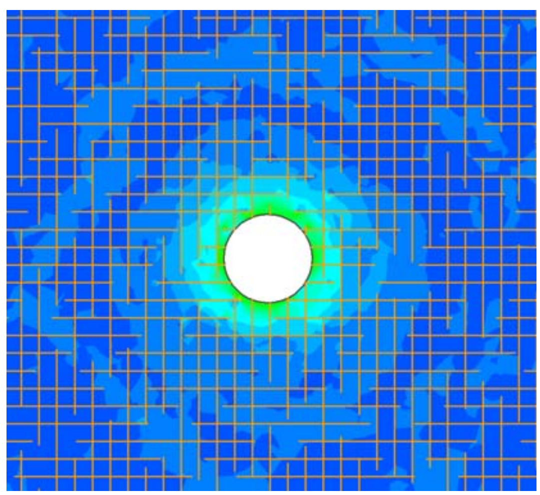

Figure 4b: Contours of major principal stress for the elastic-perfectly plastic, jointed rock mass.

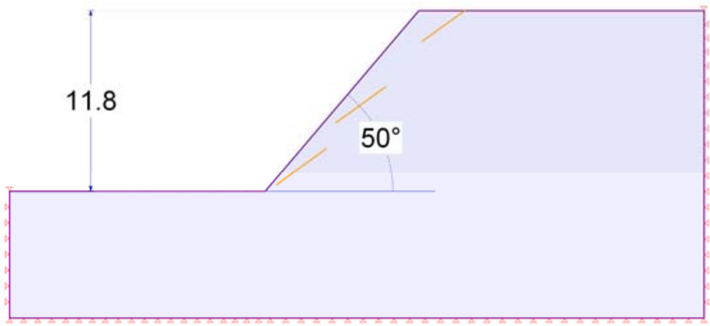

Figure 5: Geometry of slope for Example 3.

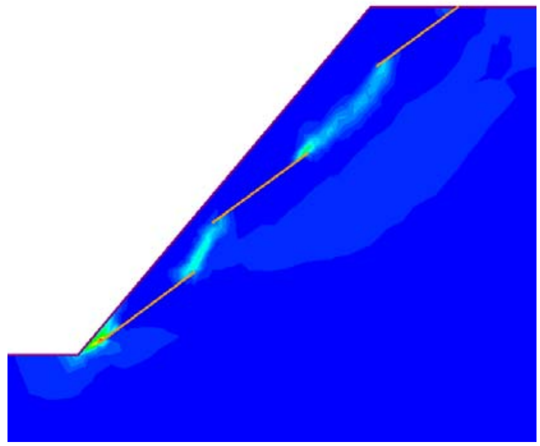

Figure 6: Step-path failure mechanism (as depicted by contours of maximum shear strain) for Example 3 predicted by SSR analysis.

6.3. Example 3 – Step-Path Failure of Rock Slopes

Example 3 looks at the failure mechanism of a simple slope with three en-echelon joints. The geometry of the slope is shown on Figure 5. The intact slope rock had the following deformation and strength properties:

Young's modulus = 20000 MPa, Poisson's ratio = 0.3, tensile strength = 0 MPa, cohesion = 0 MPa, friction angle = 25 degrees, and dilation angle = 0 degrees.

The stress-strain behaviour of the rock was assumed to be elastic-perfectly plastic, i.e. the residual strength properties were taken to the same as the peak parameters given above.The joints had a dip of 36 degrees. They also had normal stiffness = 100000 MPa/m, shear stiffness = 10000 MPa/m, cohesion = 0 MPa, and friction angle = 35 degrees.

The failure mechanism predicted by finite element SSR analysis of the slope is shown on Figure 6 (the contours of maximum shear strain are shown). The SSR analysis predicted a step-path failure mechanism with a factor of safety of 1.26.

The critical mechanism combined slipping along discontinuity faces with shearing through intact rock (as evident on the contours of maximum shear strain).

7. DISCUSSIONS

The central argument of this paper is that due to the coming together of numerical modelling improvements such as the shear strength reduction method, automatic means for generating fracture networks, numerical formulations for joint behaviour, and the widespread availability of computing power, it is now possible to perform practical analysis of discontinuous rock masses with the FEM. A primary advantage of the FEM is versatility: it can model a broaden range of continuous and discontinuous rock mass behaviours without a priori assumptions on failure mechanisms. As a result, it is possible to examine different design ideas, and obtain meaningful results or make meaningful predictions with a single tool.

This does not in any way preclude use of other approaches such as the DEM and DDA though. There certainly are many situations, such as those involving large strains or complete separation of blocks, which would require DEM and DDA modelling tools.

In many cases of practical geotechnical engineering, the understanding of basic mechanisms of behaviour and their likely bounds, rather than precise quantitative details, is most important [30]. Burland [30] writes the following about the engineering of a solution for arresting the tilt of the Leaning Tower of Pisa: "It is true to say that the identification of the form of motion of the foundations of the tower is the single most important finding in the development of a strategy for stabilization. No amount of sophisticated analysis that did not capture the mechanism of leaning instability would have led to the adopted stabilization strategy." This is precisely the manner in which the authors believe FEM modelling of blocky rock masses should be used. Its worth to rock engineers lies in its ability to help identify overall behaviour and develop remedial measures.

REFERENCES

1. Goodman, R.E., Taylor, R.L. & T.L. Brekke. 1968. A model for the mechanics of jointed rock. *Journal of the Soil Mechanics and Foundations Division, ASCE*, 637-659.
2. Manfredini, G., Martinetti, S. & R. Ribacchi. 1975. Inadequacy of limiting equilibrium methods for rock slopes design. In Design Methods in Rock Mechanics, Proceedings of the 16th Symposium on Rock Mechanics, University of Minnesota, Minneapolis, American Society of Civil Engineers, 35-43.
3. Cundall, P., Voegele, M. & C. Fairhurst. 1975. Computerized design of rock slopes using interactive graphics for the input and output of geometrical data. In Design Methods in Rock Mechanics, Proceedings of the 16th Symposium on Rock Mechanics,

University of Minnesota, Minneapolis, American Society of Civil Engineers.
4. Bandis, S.C., Lumsden, A.C. & N.R. Barton. 1983. Fundamentals of rock joint deformation. International *Journal of Rock Mechanics, Mining Sciences & Geomechanics Abstracts*, vol. 20, no. 6, 249-268.
5. Stimpson, B. 1978. Failure of slopes containing discontinuous planar joints. In Proceedings of the 19th US Symposium on Rock Mechanics, Stateline, Nevada, 296-302.
6. Dershowitz, W.S., La Pointe, P.R. & T. Doe. 2000. Advances in discrete fracture network modeling. In Proceedings of the US EPA/NGWA Fractured Rock Conference - info.ngwa.org, 882-894.
7. Vargas, E. 1985. Continuum and discontinuum modelling of some blocky type foundation problems. In Proceedings of the International Symposium on Fundamentals of Rock Joints, Bjorkliden, Sweden, 543-553.
8. Lemos, J.V., Hart, R.D. & P. Cundall. 1985. A generalized distinct element program for modelling jointed rock mass. In Proceedings of the International Symposium on Fundamentals of Rock Joints, Bjorkliden, Sweden, 335-343.
9. Shi, G.-H. 1993. Block system modeling by Discontinuous Deformation Analysis. Computational Mechanics Publications, Southampton UK.
10. Itasca Consulting Group, Inc. 2004. UDEC - Universal Distinct Element Code, Version 4. Itasca: Minneapolis.
11. Ghaboussi, J., Wilson, E. L., & J. Isenberg. 1973. Finite element for rock joints and interfaces. *Journal of the Soil Mechanics and Foundations Division, ASCE*, vol. 99, no. M10, 833-848.
12. Beer, G. 1985. An isoparametric joint/interface element for finite flement analysis. *International Journal for Numerical Methods in Engineering*, vol. 21, 585-600.
13. Jing, L. 2003. A review of techniques, advances and outstanding issues in numerical modelling for rock mechanics and rock engineering. *International Journal of Rock Mechanics & Mining Sciences*, vol. 40, 283-353.
14. ELFEN. 2001. ELFEN 2D/3D numerical modelling package. Rockfield Software Ltd.: Swansea.
15. Mas Ivars, D., Deisman, N., Pierce, M. & C. Fairhurst. 2007. The synthetic rock mass approach – a step forward in the characterization of jointed rock masses. In The Second Half Century of Rock Mechanics, Proceedings of the 11th Congress of the International Society for Rock Mechanics, Lisbon, vol. 1, 485-490. L. Ribeiro e Sousa, C. Olalla, and N. Grossmann, Eds. London: Taylor & Francis Group.
16. Hudson, J. & S.D. Priest. 1979. Discontinuities and rock mass geometry. *International Journal of Rock Mechanics, Mining Sciences & Geomechanics Abstracts*, vol. 16, 339-362.
17. Dershowitz, W.S. & H.H. Einstein. 1988. Characterizing rock joint geometry with joint system models. *Rock Mechanics and Rock Engineeering*, vol. 21, 21-51.
18. Glynn, E.F., Veneziano, D. & H.H. Einstein. 1978. The probabilistic model for shearing resistance of jointed rock. In Proceedings of the 19th US Symposium on Rock Mechanics, Stateline, Nevada, 66-76.
19. Lee, J.S., D. Veneziano & H.H Einstein, 1990. Hierarchical fracture trace mode. In Proceedings of the 31st US Symposium on Rock Mechanics, Golden, Colorado, 261-269.
20. Dershowitz, W., Lee, G., Geier, J. & La Pointe P. 1995. FracMan - Interactive Discrete Feature Data Analysis, Geometric Modelling and Exploration Simulation). User documentation. Seattle, Golder Associates Inc.
21. Hammah, R.E., Yacoub, T.E., Corkum, B., Wibowo, F. & J.H. Curran. 2007. Analysis of blocky rock slopes with finite element shear strength reduction analysis. In Proceedings of the 1st Canada-U.S. Rock Mechanics Symposium, Vancouver, Canada, 329-334.
22. Dawson, E.M., Roth, W.H. & Drescher, A. 1999. Slope stability analysis by strength reduction. *Geotechnique*, vol. 49, no. 6, 835-840.
23. Griffiths, D.V., & Lane, P.A. 1999. Slope stability analysis by finite elements. *Geotechnique*, vol. 49, no. 3, 387-403.
24. Hammah, R.E., Yacoub, T.E., & Curran, J.H. 2006. Investigating the performance of the shear strength reduction (SSR) method on the analysis of reinforced slopes. In Proceedings of the 59th Canadian Geotechnical and 7th Joint IAH-CNC and CGS Groundwater Specialty Conferences – Sea to Sky Geotechnique 2006. Vancouver, Canada.
25. Hammah, R.E., Yacoub, T.E., Corkum, B. & Curran, J.H. 2005. The Shear Strength Reduction Method for the Generalized Hoek-Brown Criterion. In Proceedings of the 40th U.S. Symposium on Rock Mechanics, Alaska Rocks 2005, Anchorage, Alaska.
26. Hammah, R.E., Yacoub, T.E., Corkum, B. & Curran, J.H. 2005. A comparison of finite element slope stability analysis with conventional limit-equilibrium investigation. In Proceedings of the 58th Canadian Geotechnical and 6th Joint IAH-CNC and CGS Groundwater Specialty Conferences – GeoSask 2005. Saskatoon, Canada.
27. Hammah, R.E., Curran, J.H., Yacoub, T.E., & Corkum, B. 2004. Stability analysis of rock slopes using the finite element method. In Proceedings of the ISRM Regional Symposium EUROCK 2004 and the 53rd Geomechanics Colloquy, Salzburg, Austria.
28. Matsui, T. & San K.C. 1992. Finite element slope stability analysis by shear strength reduction technique. *Soils and Foundations*, vol. 32, no. 1, 59-70.
29. Rocscience Inc. 2005. Phase2 v6.0 – Two-dimensional finite element slope stability analysis.
30. Burland, J.B. 2007. Interaction between structural and geotechnical engineer. *The News of HSSMGE*, no. 9, 4-16.