CPillar

Crown Pillar Stability Analysis

Theory Manual

Factor of Safety Calculations – Elastic Analysis (Rectangular Pillar)

Table of Contents

Elastic Analysis (Rectangular Pillar)

1. Introduction

This paper documents the calculations used in *CPillar* to determine the shear failure and elastic buckling factors of safety for surface or underground crown pillars, and laminated roof beds. This involves the following series of steps:

1. Determine the crown pillar geometry
2. Determine the horizontal and vertical stress state of the soil/rock
3. Determine the normal stresses on the abutments
4. Compute the driving forces due to surcharge and self-weight
5. Compute the moments and stresses due to bending
6. Compute the resisting forces due to abutment shear strength
7. Calculate the safety factor(s)

2. Crown Pillar Geometry

2.1. Rectangular Pillar

A rectangular pillar is defined by its length (x), width (y), and thickness (z). A thickness of overburden (t_o) can also be added above the pillar. The height of water (h_w) can be specified to any height above the base of the pillar.

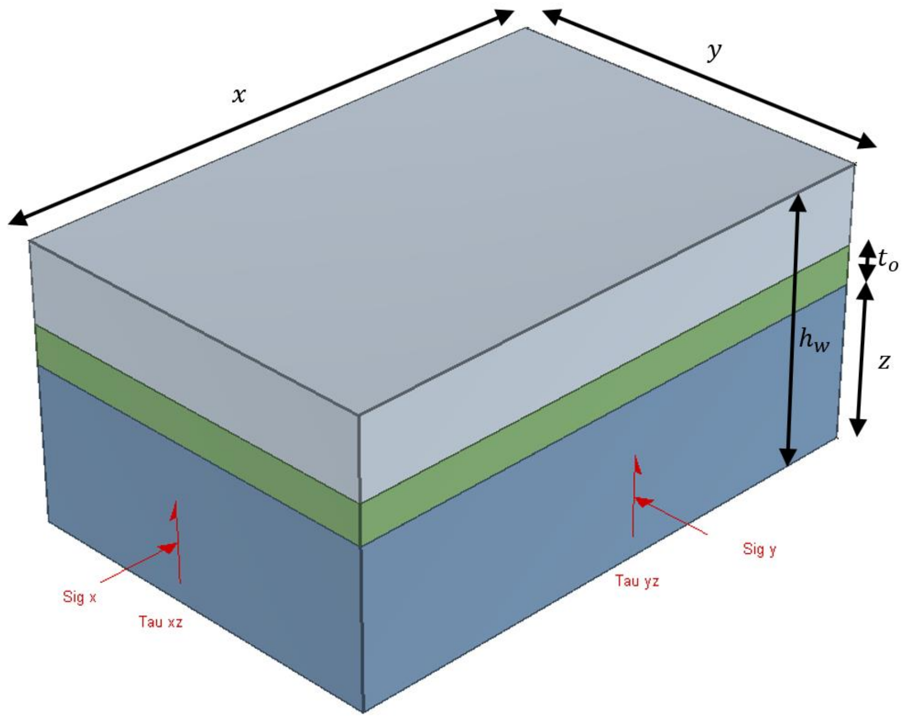

Figure 2-1: Rectangular Pillar Geometry

Where:

x	is the pillar length
y	is the pillar width
z	is the pillar height
t_o	is the thickness of overburden above the pillar
h_w	is the height of water from the base of the pillar
γ_r	is the rock unit weight
γ_o	is the overburden unit weight
γ_w	is the water unit weight

3. Stresses in the Soil

3.1. Water Pressure

In *CPillar*, the water height is specified from the bottom of the pillar. Water pressure is taken into account if the pillar is specified as **Permeable** (RIGID and ELASTIC options only).

Impermeable Pillar

If the pillar is modelled as impermeable, then any water height less than the combined pillar and overburden thickness will have no impact on the effective vertical and horizontal stress computations. Porewater pressure, μ = 0.

If water height is greater than the combined pillar and overburden thickness, then the weight of the free water will have an effect as an extra deadload on the pillar.

Permeable Pillar

If the pillar is modelled as permeable, then the effect of water pressure on vertical and lateral effective stress will be taken into account when computing stresses from GRAVITY.

The average porewater pressure (located mid-height of the pillar) is computed as follows:

If $h_w < z$: **(1)**

$$\mu = \frac{1}{2}\gamma_w h_w \left(\frac{h_w}{z}\right)$$

If $h_w \geq z$:

$$\mu = \gamma_w \left(h_w - \frac{1}{2}z\right)$$

Where:

μ is the porewater pressure
γ_w is the water unit weight
h_w is the water height
z is the pillar height

3.2. Horizontal Soil Pressure

In *CPillar*, lateral stresses can be specified as either CONSTANT or GRAVITY.

Constant Stress Type

Constant lateral stresses are entered directly into *CPillar* as σ_x and σ_y. These horizontal stresses act normal to the abutments.

Gravity Stress Type

Gravity lateral stresses are computed based on:

- Locked-in stress at the top surface of the pillar,
- Horizontal-to-vertical stress ratio constant,

- Rock unit weight,
- Pillar height, and
- Porewater pressure (if the pillar is permeable).

The average horizontal stresses (located mid-height of the pillar) is computed as follows:

$$\sigma'_x = \sigma_{lockedin} + \frac{1}{2} K_x \gamma_r z - \mu \qquad \textbf{(2)}$$

$$\sigma'_y = \sigma_{lockedin} + \frac{1}{2} K_y \gamma_r z - \mu$$

Where:

σ'_x and σ'_y	are the lateral effective soil stresses
$\sigma_{lockedin}$	is the constant locked in soil stress due to soil loading history
K_x and K_y	are the horizontal-to-vertical stress ratio constants
γ_r	is the rock unit weight
z	is the pillar height
μ	is the porewater pressure

4. Dead Load

The driving forces responsible for the destabilization of the crown pillar is attributed by the deadload of the entire system. The total dead load is computed by summing all the rock, overburden, and water weights.

$$q = W_r + W_o + W_w \tag{3}$$

Where:

q	is the total dead load (force per area)
W_r	is the weight of rock (i.e. pillar) (force per area)
W_o	is the weight of overburden (force per area)
W_w	is the weight of free water (force per area)

4.1. Self-Weight

The self-weight of the pillar is calculated as follows:

$$W_r = \gamma_r z \tag{4}$$

Where:

W_r	is the weight of rock (i.e. pillar)
γ_r	is the unit weight of rock
z	is the height of pillar

4.2. Overburden Weight

The weight of the overburden is calculated as follows:

$$W_o = \gamma_o t_o \tag{5}$$

Where:

W_o	is the weight of overburden
γ_o	is the unit weight of overburden
t_o	is the thickness of overburden

4.3. Water Weight

The weight of the free water is calculated as follows:

If $h_w \leq z + t_o$: $$W_w = 0 \tag{6}$$

If $h_w > z + t_o$: $$W_w = \gamma_w [h_w - (z + t_o)]$$

Where:

W_w	is the weight of free water
γ_w	is the unit weight of water
h_w	is the height of water
z	is the height of pillar
t_o	is the thickness of overburden

5. Plate Bending

In *CPillar*, the bending of the pillar is calculated using the equations in “Theory of Plates and Shells” by Timoshenko and Woinowsky-Krieger (1987). The pillar is idealized as a plate of uniform thickness, z, with dimensions a and b. The plate is subject to a uniformly distributed area load, q (total dead load). The boundary conditions are assumed to be clamped on all edges (Figure 5-1).

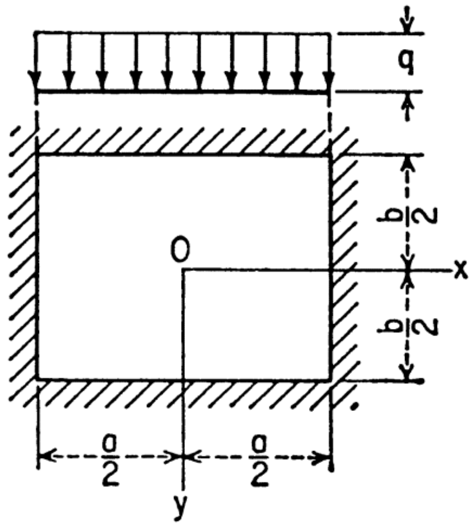

Figure 5-1: Rectangular Plate with All Edges Built In, Uniform Load q (Timoshenko and Woinowsky-Krieger, 1987)

The solution involves the superposition of deflection solutions for a simply supported rectangular plate, and a rectangular plate subject to moments distributed along the edges (Timoshenko and Woinowsky-Krieger, 1987).

Deflection

The deflection of this system is symmetric about each x-axis and y-axis of interest. The maximum deflection occurs in the middle of the plate ($x = 0, y = 0$).

The deflection of a simply supported rectangular plate can be represented by the following series expansion:

For $-\frac{a}{2} \le x \le \frac{a}{2}$ and $-\frac{b}{2} \le y \le \frac{b}{2}$: **(7)**

$$w = \frac{4qa^4}{\pi^5 D} \sum_{m=1,3,5,\ldots} \frac{(-1)^{(m-1)/2}}{m^5} \cos\frac{m\pi x}{a}\left(1 - \frac{\alpha_m \tanh\alpha_m + 2}{2\cosh\alpha_m}\cosh\frac{m\pi y}{a} + \frac{1}{2\cosh\alpha_m}\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)$$

with

$$\alpha_m = \frac{m\pi b}{2a}$$

Where:

w	is the vertical deflection
q	is the uniform load magnitude (areal load)
D	is the flexural rigidity $D = \frac{Eh^3}{12(1-\nu^2)}$
a	is the pillar length
b	is the pillar width

The deflection caused by a moment distributed along the edges of a rectangular plate can be represented by the following series expansions:

For $-\frac{a}{2} \leq x \leq \frac{a}{2}$ and $-\frac{b}{2} \leq y \leq \frac{b}{2}$: **(8)**

$$w = \frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\ldots} \frac{\sin\frac{m\pi x}{a}}{m^2 \cosh\alpha_m} E_m \left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{a} - \frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)$$

with

$$\alpha_m = \frac{m\pi b}{2a}$$

From Equation (17), **(9)**

Sub $x + \frac{a}{2}$ for x:

$$w_1 = -\frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\ldots} E_m \frac{(-1)^{\frac{m-1}{2}}}{m^2 \cosh\alpha_m} \cos\frac{m\pi x}{a}\left(\frac{m\pi y}{a}\sinh\frac{m\pi y}{a} - \alpha_m \tanh\alpha_m \cosh\frac{m\pi y}{a}\right)$$

Sub $y + \frac{b}{2}$ for y:

$$w_2 = \frac{a^2}{2\pi^2 D} \sum_{m=1,3,5,\ldots} \frac{\sin\frac{m\pi x}{a}}{m^2 \cosh\alpha_m} E_m \left(\alpha_m \tanh\alpha_m \cosh\frac{m\pi\left(y+\frac{b}{2}\right)}{a} - \frac{m\pi\left(y+\frac{b}{2}\right)}{a}\sinh\frac{m\pi\left(y+\frac{b}{2}\right)}{a}\right)$$

Rotation

Likewise, the rotation must also take into account the effects from the simple supports and the moment along the edges.

The rotation from the simply supported condition:

From Equation (7), **(10)**

Rotation at edge $y = \frac{b}{2}$:

$$\left(\frac{\partial w}{\partial y}\right)_{y=b/2} = \frac{2qa^3}{\pi^4 D} \sum_{m=1,3,5,\ldots} \frac{(-1)^{(m-1)/2}}{m^4} \cos\frac{m\pi x}{a}\left(\frac{\alpha_m}{\cosh^2\alpha_m} - \tanh\alpha_m\right)$$

Rotation at edge $x = \frac{a}{2}$:

$$\left(\frac{\partial w}{\partial x}\right)_{x=a/2} = \frac{2qb^3}{\pi^4 D} \sum_{m=1,3,5,\dots} \frac{(-1)^{(m-1)/2}}{m^4} \cos\frac{m\pi y}{b} \left(\frac{\beta_m}{\cosh^2\beta_m} - \tanh\beta_m\right)$$

The rotation from the distributed moment along the edges:

From Equation (9), **(11)**

Rotation along edge $y = \frac{b}{2}$:

$$\left(\frac{\partial w_1}{\partial y}\right)_{y=\frac{b}{2}} = -\frac{a}{2\pi D} \sum_{m=1,3,5,\dots} E_m \frac{(-1)^{\frac{m-1}{2}}}{m} \cos\frac{m\pi x}{a} \left(\tanh\alpha_m + \frac{\alpha_m}{\cosh^2\alpha_m}\right)$$

$$\left(\frac{\partial w_1}{\partial x}\right)_{x=a/2} = -\frac{1}{4D} \sum_{m=1,3,5,\dots} E_m \frac{1}{\cosh^2\alpha_m} \left(b\sinh\alpha_m \cosh\frac{m\pi y}{a} - 2y\cosh\alpha_m \sinh\frac{m\pi y}{a}\right)$$

$$= -\frac{4b^2}{\pi^2 Da} \sum_{m=1,3,5,\dots} \frac{E_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{(i-1)/2}}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2}\right)^2} \cos\frac{i\pi y}{b}$$

Rotation along edge $x = \frac{a}{2}$:

$$\left(\frac{\partial w_2}{\partial y}\right)_{y=b/2} = -\frac{4a^2}{\pi^2 Db} \sum_{m=1,3,5,\dots} \frac{F_m}{m^3} \sum_{i=1,3,5,\dots} \frac{i(-1)^{(i-1)/2}}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2}\right)^2} \cos\frac{i\pi x}{a}$$

$$\left(\frac{\partial w_2}{\partial x}\right)_{x=a/2} = -\frac{b}{2\pi D} \sum_{m=1,3,5,\dots} F_m \frac{(-1)^{(m-1)/2}}{m} \cos\frac{m\pi y}{b} \left(\tanh\alpha_m + \frac{\beta_m}{\cosh^2\beta_m}\right)$$

Boundary Conditions

Since the edges are clamped, the rotation is restricted:

Edges $y = \pm\frac{b}{2}$: **(12)**

$$\left(\frac{\partial w}{\partial y}\right)_{y=\frac{b}{2}} + \left(\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial y}\right)_{y=\frac{b}{2}} = 0$$

Edges $x = \pm\frac{a}{2}$:

$$\left(\frac{\partial w}{\partial x}\right)_{x=\frac{a}{2}} + \left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial x}\right)_{x=\frac{a}{2}} = 0$$

System of Equations

From Equation (12), **(13)**

Edges $y = \pm\frac{b}{2}$:

$$\frac{2qa^3}{\pi^4 D}\sum_{i=1,3,5,\ldots}\frac{(-1)^{\frac{i-1}{2}}}{i^4}\cos\frac{i\pi x}{a}\left(\frac{\alpha_i}{\cosh^2\alpha_i}-\tanh\alpha_i\right)$$
$$-\frac{a}{2\pi D}\sum_{i=1,3,5,\ldots}E_i\frac{(-1)^{\frac{i-1}{2}}}{i}\cos\frac{i\pi x}{a}\left(\tanh\alpha_i+\frac{\alpha_i}{\cosh^2\alpha_i}\right)$$
$$-\frac{4a^2}{\pi^2 Db}\sum_{m=1,3,5,\ldots}\frac{F_m}{m^3}\sum_{i=1,3,5,\ldots}\frac{i(-1)^{\frac{i-1}{2}}}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}\cos\frac{i\pi x}{a}=0$$

therefore,

$$\frac{E_i}{i}\left(\tanh\alpha_i+\frac{\alpha_i}{\cosh^2\alpha_i}\right)+\frac{8ai}{\pi b}\sum_{m=1,3,5,\ldots}\frac{F_m}{m^3}\frac{1}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}=\frac{4qa^2}{\pi^3 i^4}\left(\frac{\alpha_i}{\cosh^2\alpha_i}-\tanh\alpha_i\right)$$

Edges $x=\pm\frac{a}{2}$:

$$\frac{2qb^3}{\pi^4 D}\sum_{m=1,3,5,\ldots}\frac{(-1)^{(m-1)/2}}{m^4}\cos\frac{m\pi y}{b}\left(\frac{\beta_m}{\cosh^2\beta_m}-\tanh\beta_m\right)$$
$$-\frac{4b^2}{\pi^2 Da}\sum_{m=1,3,5,\ldots}\frac{E_m}{m^3}\sum_{i=1,3,5,\ldots}\frac{i(-1)^{\frac{i-1}{2}}}{\left(\frac{b^2}{a^2}+\frac{i^2}{m^2}\right)^2}\cos\frac{i\pi y}{b}$$
$$-\frac{b}{2\pi D}\sum_{i=1,3,5,\ldots}F_i\frac{(-1)^{\frac{i-1}{2}}}{i}\cos\frac{i\pi y}{b}\left(\tanh\alpha_i+\frac{\beta_i}{\cosh^2\beta_i}\right)=0$$

therefore,

$$\frac{F_i}{i}\left(\tanh\beta_i+\frac{\beta_i}{\cosh^2\beta_i}\right)+\frac{8bi}{\pi a}\sum_{m=1,3,5,\ldots}\frac{E_m}{m^3}\frac{1}{\left(\frac{b^2}{a^2}+\frac{i^2}{m^2}\right)^2}=\frac{4qb^2}{\pi^3 i^4}\left(\frac{\beta_i}{\cosh^2\beta_i}-\tanh\beta_i\right)$$

Where:

E_i, E_m, F_i and F_m are coefficients (unknown)

Expanding to the n^{th} term of the series results in a $2n$ by $2n$ system of equations.

Bending Moment

Moment along edges $y=\pm\frac{b}{2}$: **(14)**

$$(M_y)_{y=\pm b/2}=\sum_{m=1,3,5,\ldots}(-1)^{\frac{m-1}{2}}E_m\cos\frac{m\pi x}{a}$$

Moment along edges $x=\pm\frac{a}{2}$:

$$(M_x)_{x=\pm a/2}=\sum_{m=1,3,5,\ldots}(-1)^{\frac{m-1}{2}}F_m\cos\frac{m\pi y}{b}$$

Where:

M_y is the bending moment about the y-axis, per unit perpendicular to edge

M_x is the bending moment about the x-axis, per unit perpendicular to edge

To be conservative, the moments are taken at the mid-point along the edge (maximum edge moments).

Note: In *CPillar*, the series expansion is taken to 20 terms. 20 terms are sufficient since the functions series solutions converge rapidly.

5.1. Axial Stress Due to Bending

The tensile stresses due to the bending moments are computed as follows:

For a rectangular cross-section, **(15)**

Tensile (axial in x-direction) stress due to bending about the y-axis:

$$\sigma_{b_x} = \frac{6M_y}{z^2}$$

Tensile (axial in y-direction) stress due to bending about the x-axis:

$$\sigma_{b_y} = \frac{6M_x}{z^2}$$

Where:

σ_{b_x} and σ_{b_y} are the tensile stresses due to bending, per unit perpendicular to edge

z is the pillar height

6. Shear Strength

The resisting forces are provided by the shear strength along the abutments of the pillar.

6.1. Normal Stress

The normal stresses to the abutments are equal to the lateral (principal) effective soil stresses.

$$N_x = \sigma'_x \qquad \textbf{(16)}$$
$$N_y = \sigma'_y$$

Where:

N_x and N_y	are the normal stresses along the abutments
σ_x and σ_y	are the lateral soil stresses

6.2. Shear Strength

The following shear strength criteria are available in *CPillar* for defining the strength of the rock:

1. Mohr-Coulomb
2. Hoek-Brown
3. Generalized Hoek-Brown

Mohr Coulomb

$$\tau = c + N \cdot \tan\phi \qquad \textbf{(17)}$$

Where:

c	is the cohesion
N	is the normal stress along the abutments
ϕ	is the friction angle

Hoek-Brown

Note that this is a special case of the **Generalized Hoek-Brown** criterion, with the constant a = 0.5.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci}\left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s\right)^{0.5} \qquad \textbf{(18)}$$

(Hoek and Bray, 1981)

If $s = 0$:

$$\tau = 0$$

If $s \neq 0$:

$$\tau = \left(\frac{1}{\tan\phi_i} - \cos\phi_i\right)\frac{m_b \sigma_{ci}}{8}$$

with

$$\phi_i = \tan^{-1}\left(\frac{1}{\sqrt{4h\cos^2\theta - 1}}\right)$$
$$h = 1 + \frac{16(m_b * N + s * \sigma_{ci})}{3{m_b}^2\sigma_{ci}}$$

$$\theta = \frac{1}{3}\left(\frac{\pi}{2} + \tan^{-1}\frac{1}{\sqrt{h^3 - 1}}\right)$$

Where:

- m_b is a reduced value (for the rock mass) of the material constant *mi* (for the intact rock)
- s is a constant which depends upon the characteristics of the rock mass
- σ_{ci} is the uniaxial compressive strength (UCS) of the intact rock pieces
- σ_1' is the axial effective principal stress
- σ_3' is the confining effective principal stress

Generalized Hoek-Brown

Generalized Hoek-Brown (m_b, s, a):

$$\sigma_1' = \sigma_3' + \sigma_{ci}\left(m_b\frac{\sigma_3'}{\sigma_{ci}} + s\right)^a \tag{19}$$

(Hoek and Bray, 1981)

Check for tensile strength:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b}$$

If $N < \sigma_t$:

Generalized Hoek-Brown (GSI, mi, D):

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \tag{20}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \tag{21}$$

$$a = \frac{1}{2} + \frac{1}{6}\left[\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right)\right] \tag{22}$$

Where:

- m_b is a reduced value (for the rock mass) of the material constant *mi* (for the intact rock)
- s, a are constants which depend upon the characteristics of the rock mass
- σ_{ci} is the uniaxial compressive strength (UCS) of the intact rock pieces
- σ_1' is the axial effective principal stress
- σ_3' is the confining effective principal stress
- GSI is the Geological Strength Index
- m_i is a material constant for the intact rock
- D is a "disturbance factor" which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation (varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses)

7. Factor of Safety

7.1. Elastic Analysis

For an ELASTIC analysis, two failure modes are considered:

1. Shear, and
2. Elastic Buckling.

The pillar is treated as a clamped beam, with span and breadth equal to the shorter and longer of the x and y dimensions, respectively.

Shear Factor of Safety

Similar to RIGID analysis, the factor of safety of the pillar against vertical downward sliding is given by the ratio of the sum of the shear forces acting on the four sides of the pillar, to the total weight of the pillar, including overburden and free water.

In an ELASTIC analysis, the area on which the shear stresses act is lowered if bending stresses are high. If confining stresses are very low, the shear factor of safety will be about half that calculated from a RIGID analysis. As confining stress is increased, the correction factor for bending approaches 1. Therefore, at high confining stress, the shear factor of safety calculated from either a RIGID or ELASTIC analysis will be the same.

If the abutments are under tension, then apply a correction factor:

If $\sigma_x + \sigma_{t_x} < 0$: **(23)**

$$C_x = \frac{\left(\sigma_{b_x} - \sigma_x\right)}{2\sigma_{t_x}}$$
$$z_1 = C_x z$$

If $\sigma_y + \sigma_{t_y} < 0$:

$$C_y = \frac{\left(\sigma_{b_y} - \sigma_y\right)}{2\sigma_{t_y}}$$
$$z_2 = C_y z$$

Where:

C_x and C_y	are the correction factors for bending in the x and y directions (between 0.5 to 1.0)
σ_{b_x} and σ_{b_y}	are the tensile axial stresses due to bending
σ_x and σ_y	are the axial stresses due to lateral soil pressure
z_1 and z_2	are the corrected pillar heights

The correction factor effectively reduces the shear area in the factor of safety computations.

$$FS_{shear} = \frac{shear\ resistance}{dead\ load} = \frac{2\left(\frac{\tau_{xz} z_1}{x} + \frac{\tau_{yz} z_2}{y}\right)}{q} \quad (24)$$

Where:

FS_{shear}	is the shear factor of safety
τ_{xz} and τ_{yz}	are the shear strengths
z_1 and z_2	are the corrected pillar heights
x	is the pillar length
y	is the pillar width
q	is the total dead load

Elastic Buckling Factor of Safety

The horizontal confining stress is set equal to the stress along the breadth of the beam (the longer dimension).

Buckling or bending is assumed to occur in the longer dimension. If the x and y dimensions are equal, then bending direction is dictated by the principal horizontal stress magnitudes.

If $x = y$: **(25)**

$$\sigma_c = \max(\sigma_x, \sigma_y)$$
$$L_{span} = x = y$$

If $x > y$:

$$\sigma_c = \sigma_y$$
$$L_{span} = y$$

If $x < y$:

$$\sigma_c = \sigma_x$$
$$L_{span} = x$$

Where:

σ_{ci}	is the horizontal confining stress
L_{span}	is the span of the pillar
x	is the pillar length
y	is the pillar width
σ_x and σ_y	are the principal horizontal stresses

If the lateral stresses are very low, the elastic buckling safety factor will be very high.

$$FS_{buckling} = \frac{Euler\ buckling\ stress}{horizontal\ confining\ stress} = \frac{\left(\frac{\pi^2 E_{rm} z^2}{3{L_{span}}^2}\right)}{\sigma_c} \tag{26}$$

Where:

FS_{shear}	is the shear factor of safety
E_{rm}	is the rock modulus
z	is the pillar height
L	is the span of the pillar
σ_c	is the horizontal confining stress

There is a high degree of uncertainty in computing elastic buckling factor of safety for rock material. Therefore, span-to-depth ratios more than 3 are not recommended for an ELASTIC analysis.

8. References

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