

CPillar

Factor of Safety Calculations – Rigid Analysis (Rectangular Pillar)

Theory Manual

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1. Introduction

This paper documents the calculations used in *CPillar* to determine the shear failure factor of safety for surface or underground crown pillars, and laminated roof beds. This involves the following series of steps:

1. Determine the crown pillar geometry
2. Determine the horizontal and vertical stress state of the soil/rock
3. Determine the normal stresses on the abutments
4. Compute the driving forces due to surcharge and self-weight
5. Compute the resisting forces due to abutment shear strength
6. Calculate the safety factor

2.Crown Pillar Geometry

2.1. Rectangular Pillar

A rectangular pillar is defined by its length (x), width (y), and thickness (z). A thickness of overburden (t_o) can also be added above the pillar. The height of water (h_w) can be specified to any height above the base of the pillar.

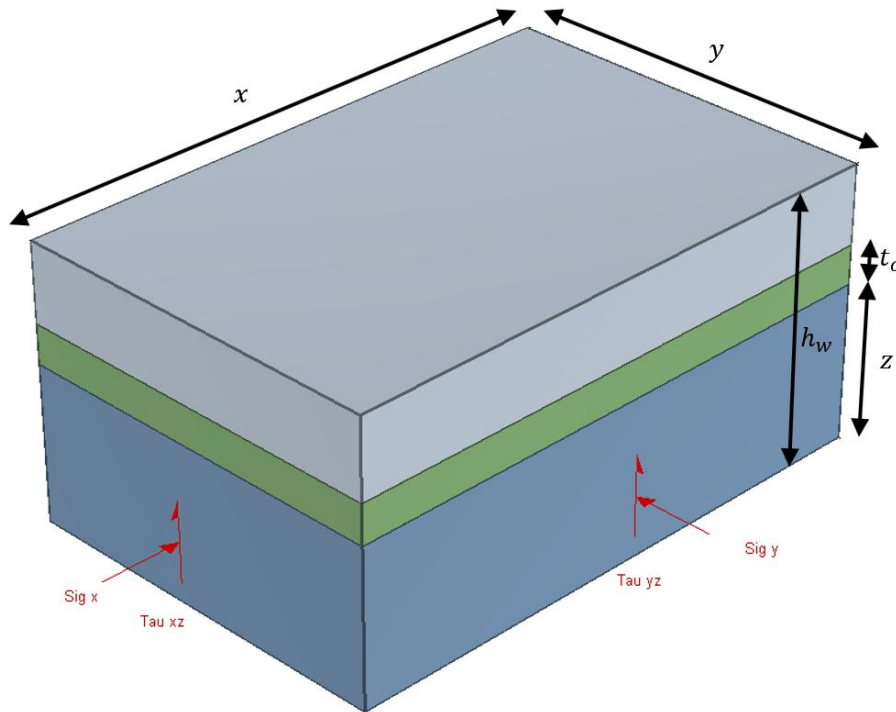


Figure 2.1-1: Rectangular Pillar Geometry

Where:

- x is the pillar length
- y is the pillar width
- z is the pillar height
- t_o is the thickness of overburden above the pillar
- h_w is the height of water from the base of the pillar
- γ_r is the rock unit weight
- γ_o is the overburden unit weight
- γ_w is the water unit weight

3. Stresses in the Soil

3.1. Water Pressure

In *CPillar*, the water height is specified from the bottom of the pillar. Water pressure is taken into account if the pillar is specified as **Permeable** (RIGID and ELASTIC options only).

3.1.1. Impermeable Pillar

If the pillar is modelled as impermeable, then any water height less than the combined pillar and overburden thickness will have no impact on the effective vertical and horizontal stress computations. Porewater pressure, $\mu = 0$.

If water height is greater than the combined pillar and overburden thickness, then the weight of the free water will have an effect as an extra deadload on the pillar.

3.1.2. Permeable Pillar

If the pillar is modelled as permeable, then the effect of water pressure on vertical and lateral effective stress will be taken into account when computing stresses from GRAVITY.

The average porewater pressure (located mid-height of the pillar) is computed as follows:

If $h_w < z$: (1a)

$$\mu = \frac{1}{2} \gamma_w h_w \left(\frac{h_w}{z} \right)$$

If $h_w \geq z$: (1b)

$$\mu = \gamma_w \left(h_w - \frac{1}{2} z \right)$$

Where:

μ is the porewater pressure

γ_w is the water unit weight

h_w is the water height

z is the pillar height

3.2. Horizontal Soil Pressure

In *CPillar*, lateral stresses can be specified as either CONSTANT or GRAVITY.

3.2.1. Constant Stress Type

Constant lateral stresses are entered directly into *CPillar* as σ_x and σ_y . These horizontal stresses act normal to the abutments.

3.2.2. Gravity Stress Type

Gravity lateral stresses are computed based on:

- Locked-in stress at the top surface of the pillar,
- Horizontal-to-vertical stress ratio constant,
- Rock unit weight,
- Pillar height, and
- Porewater pressure (if the pillar is permeable).

The average horizontal stresses (located mid-height of the pillar) is computed as follows:

$$\sigma'_x = \sigma_{lockedin} + \frac{1}{2}K_x\gamma_r z - \mu \quad (2a)$$

$$\sigma'_y = \sigma_{lockedin} + \frac{1}{2}K_y\gamma_r z - \mu \quad (3b)$$

Where:

σ'_x and σ'_y	are the lateral effective soil stresses
$\sigma_{lockedin}$	is the constant locked in soil stress due to soil loading history
K_x and K_y	are the horizontal-to-vertical stress ratio constants
γ_r	is the rock unit weight
z	is the pillar height
μ	is the porewater pressure

4. Dead Load

The driving forces responsible for the destabilization of the crown pillar is attributed by the deadload of the entire system. The total deadload is computed by summing all the rock, overburden, and water weights.

$$q = W_r + W_o + W_w \quad (4)$$

Where:

- q is the total dead load (force per area)
- W_r is the weight of rock (i.e. pillar) (force per area)
- W_o is the weight of overburden (force per area)
- W_w is the weight of free water (force per area)

4.1. Self-Weight

The self-weight of the pillar is calculated as follows:

$$W_r = \gamma_r z \quad (5)$$

Where:

- W_r is the weight of rock (i.e. pillar)
- γ_r is the unit weight of rock
- z is the height of pillar

4.2. Overburden Weight

The weight of the overburden is calculated as follows:

$$W_o = \gamma_o t_o \quad (6)$$

Where:

- W_o is the weight of overburden
- γ_o is the unit weight of overburden
- t_o is the thickness of overburden

4.3. Water Weight

The weight of the free water is calculated as follows:

$$\text{If } h_w \leq z + t_o: \quad W_w = 0 \quad (7a)$$

$$\text{If } h_w > z + t_o: \quad W_w = \gamma_w [h_w - (z + t_o)] \quad (6b)$$

Where:

W_w is the weight of free water

γ_w is the unit weight of water

h_w is the height of water

z is the height of pillar

t_o is the thickness of overburden

5. Shear Strength

The resisting forces are provided by the shear strength along the abutments of the pillar.

5.1. Normal Stress

The normal stresses to the abutments are equal to the lateral (principal) effective soil stresses.

$$N_x = \sigma'_x \quad (8a)$$

$$N_y = \sigma'_y \quad (7b)$$

Where:

N_x and N_y are the normal stresses along the abutments

σ_x and σ_y are the lateral soil stresses

5.2. Shear Strength

The following shear strength criteria are available in *CPillar* for defining the strength of the rock:

1. Mohr-Coulomb
2. Hoek-Brown
3. Generalized Hoek-Brown

5.2.1. Mohr Coulomb

$$\tau = c + N \cdot \tan \phi \quad (9)$$

Where:

c is the cohesion

N is the normal stress along the abutments

ϕ is the friction angle

5.2.2. Hoek-Brown

Note that this is a special case of the **Generalized Hoek-Brown** criterion, with the constant $a = 0.5$.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^{0.5} \quad (10)$$

(Hoek and Bray, 1981)

If $s = 0$:

$$\tau = 0$$

If $s \neq 0$:

$$\tau = \left(\frac{1}{\tan \phi_i} - \cos \phi_i \right) \frac{m_b \sigma_{ci}}{8}$$

with

$$\phi_i = \tan^{-1} \left(\frac{1}{\sqrt{4h \cos^2 \theta - 1}} \right)$$

$$h = 1 + \frac{16(m_b * N + s * \sigma_{ci})}{3m_b^2 \sigma_{ci}}$$

$$\theta = \frac{1}{3} \left(\frac{\pi}{2} + \tan^{-1} \frac{1}{\sqrt{h^3 - 1}} \right)$$

Where:

m_b is a reduced value (for the rock mass) of the material constant m_i (for the intact rock)

s is a constant which depends upon the characteristics of the rock mass

σ_{ci} is the uniaxial compressive strength (UCS) of the intact rock pieces

σ'_1 is the axial effective principal stress

σ'_3 is the confining effective principal stress

5.2.3. Generalized Hoek-Brown

Generalized Hoek-Brown (m_b, s, a):

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (11)$$

(Hoek and Bray, 1981)

Check for tensile strength:

$$\sigma_t = -\frac{s\sigma_{ci}}{m_b}$$

If $N < \sigma_t$:

Generalized Hoek-Brown (GSI, m_i, D):

$$m_b = m_i \exp \left(\frac{GSI - 100}{28 - 14D} \right) \quad (12)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (13)$$

$$a = \frac{1}{2} + \frac{1}{6} \left[\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right) \right] \quad (14)$$

Where:

- m_b is a reduced value (for the rock mass) of the material constant m_i (for the intact rock)
- s, a are constants which depend upon the characteristics of the rock mass
- σ_{ci} is the uniaxial compressive strength (UCS) of the intact rock pieces
- σ'_1 is the axial effective principal stress
- σ'_3 is the confining effective principal stress
- GSI is the Geological Strength Index
- m_i is a material constant for the intact rock
- D is a "disturbance factor" which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation (varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses)

6. Factor of Safety

6.1. Rigid Analysis

For a RIGID analysis, the only failure mode considered is shear. The crown pillar is idealized as a rigid block which fails by plug-sliding along the abutments. The factor of safety of the pillar against vertical downward sliding is given by the ratio of the sum of the shear forces acting on the four sides of the pillar, to the total weight of the pillar, including overburden and free water, if any.

6.1.1. Shear Factor of Safety

$$FS_{shear} = \frac{\text{shear resistance}}{\text{dead load}} = \frac{2 \left(\frac{\tau_{xz}z}{x} + \frac{\tau_{yz}z}{y} \right)}{q} \quad (15)$$

Where:

- FS_{shear} is the shear factor of safety
- τ_{xz} and τ_{yz} are the shear strengths
- z is the pillar height
- x is the pillar length
- y is the pillar width
- q is the total dead load (force per area)

7. References

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