

Surface Reconstruction

Many features in *EX3* require interpolation to generate 3D surfaces from a set of data points, such as the geometry tool [Reconstruct Surface From Table](#) or the groundwater method Add Water by Location.

Interpolation Methods:

You can generate surfaces by using various interpolation methods on defined XYZ coordinates in the table. A grid is generated from the farthest x, y and z coordinates entered by the user. Within the grid, various interpolation methods can be used to generate a surface based on the entered points and the surface **does not** need to intersect every point entered by the user. The user may use any of the following Interpolation methods:

INVERSE DISTANCE

The Inverse Distance Interpolation method weights every grid point according to its distance from the sample point. This scheme is also known as the Shepard method (Shepard, 1968) and can be written in the form:

$$F(P) = \frac{\sum_{i=1}^N \frac{F_i}{\|P - P_i\|^2}}{\sum_{i=1}^N \frac{1}{\|P - P_i\|^2}}$$

where P is the location of the sample point, F(P) is the interpolated value at the sample point, P_i the location of the grid points, F_i are the grid point values, and ||P-P_i||² represents the distance from P to P_i. The main deficiencies of this method are: 1) the local extrema are typically located at the grid points and this results in poor shape properties, and 2) undue influence of grid points which are far away from the sample point.

THIN-PLATE SPLINE

The Thin-Plate Spline method utilizes the concept of an infinite thin elastic plate under tension, to determine a spline surface (a smooth 3-dimensional surface which fits through all of the data points). The spline surface is used to determine the sample value at any location (Franke, 1985).

CHUGH

The **Chugh** interpolation method is based on finding the nearest grid point in each of the four quadrants with origin centered at the point where the interpolation is required. A plane is then fit through each combination of three quadrant grid points, and an interpolation is performed for each plane. This results in four interpolations, which are then averaged to obtain the final interpolated value at the desired point (Chugh, 1981).

NOTE: if a grid point does not exist in each of the four quadrants around a sample point, then a secondary interpolation method must be used, for such sample locations. See the note below about the secondary interpolation method.

LOCAL THIN-PLATE SPLINE

The **Local Thin Plate Spline** method is an extension of the Thin Plate Spline interpolation technique, and is recommended for use with a large number of grid points (>200). The only difference between the methods is that instead of using all the grid points for the interpolation, the Local version takes a maximum of 10 closest points to the sample point and fits a spline surface through them. The local spline surface is then used to determine the sample value.

NOTE: if there are less than 10 data points then this method defaults to the non-local version.

LINEAR/TRIANGULATION (TIN TRIANGULATION)

TIN Triangulation (Triangulated Irregular Network) takes the grid points and triangulates them using the Delaunay triangulation method. To calculate the value at a sample point, the program first determines which triangle the point lies within. Once the triangle that contains the sample point is found, the interpolated value is calculated using linear interpolation. This is done by calculating the plane equation that fits through the 3 grid points at the triangle vertices, then solving for the value using the coordinates of the sample point and the plane equation.

NOTE: if a sample point lies outside the convex hull of the user-defined grid points, then a secondary interpolation method must be used for such sample locations. The convex hull is a convex polygon, with grid points for vertices, that encloses all of the grid points. See the note below about the secondary interpolation method.

GAUSSIAN

Gaussian or Normal distribution is a commonly used radial basis function (Fornberg & Piret) that follows the form:

$$\phi(r) = e^{-(\epsilon r)^2}$$

Where $\phi(r)$ is the influence of a data point to the interpolated point based on the distance between them denoted as r . ϵ is a shape parameter and is taken as the average distance between all data points. Similar to other radial basis functions such as

thin-plate spline, the interpolated point is obtained by summing the weighted $\phi(r)$ values of all data points. The weights for each $\phi(r)$ value are determined by the magnitude of the data point. In surface reconstruction, the magnitude of the data point is the elevation at the XY coordinate. In pore pressure interpolation, the weight would be pore pressure at the XYZ coordinate of the data point.

MULTI QUADRATIC

Multi Quadratic radial basis function (Fornberg & Piret) used in EX3 follows the form:

$$\phi(r) = \sqrt{1 + (\epsilon r)^2}$$

Where $\phi(r)$ is the influence of a data point to the interpolated point based on the distance between them denoted as r . ϵ is a shape parameter and is taken as the average distance between all data points. Similar to other radial basis functions such as thin-plate spline, the interpolated point is obtained by summing the weighted $\phi(r)$ values of all data points. The weights for each $\phi(r)$ value are determined by the magnitude of the data point. In surface reconstruction, the magnitude of the data point is the elevation at the XY coordinate. In pore pressure interpolation, the weight would be pore pressure at the XYZ coordinate of the data point.

POLYHARMONIC SPLINE

Polyharmonic spline radial basis function (Fornberg & Piret) used in EX3 follows the form:

$$\phi(r) = r^3$$

Where $\phi(r)$ is the influence of a data point to the interpolated point based on the distance between them denoted as r . Thin-Plate Spline is a special case of the polyharmonic spline functions. Similar to other radial basis functions such as thin-plate spline, the interpolated point is obtained by summing the weighted $\phi(r)$ values of all data points. The weights for each $\phi(r)$ value are determined by the magnitude of the data point. In surface reconstruction, the magnitude of the data point is the elevation at the XY coordinate. In pore pressure interpolation, the weight would be pore pressure at the XYZ coordinate of the data point.

COMPACTLY SUPPORTED

Compactly supported methods perform interpolation for a point in space using the closest data points within a specified radius. All data points outside of the specified radius have no influence on the interpolation for that specific point (Wendland, 1995).

The compactly-supported radial basis function implemented in EX3 to calculate the influence of a data point to the interpolation follows the form:

$$\phi(r) = \begin{cases} (1-r)^8 + (32r^3 + 25r^2 + 8r + 1) & r < 1 \\ 0 & \textit{otherwise} \end{cases}$$

Where $\phi(r)$ is the influence of a data point to the interpolated point based on the distance between them denoted as r . Similar to other radial basis functions such as thin-plate spline, the interpolated point is obtained by summing the weighted $\phi(r)$ values of all data points. The weights for each $\phi(r)$ value are determined by the magnitude of the data point. In surface reconstruction, the magnitude of the data point is the elevation at each XY coordinate.

Resolution:

The surface reconstruction resolution defines the number of sample points used in the interpolation method to generate the surface. Generally, the more sample points that are used the more detailed the generated surface will become. However, this is highly dependent on the available data points inputted by the user to conduct the interpolation. **Coarse** represents the lowest number of sample points and **Fine** is the highest.