Verification - Example 3: Stress variation around underground sphere cavity (Uniaxial stress field)

Problem Description

Example 3 is very similar to Example 2, which compares normalized radial and tangential and angular stresses with analytical solution. Unlike in Example 2 though, here only vertical stress is considered as the field stress, i.e.

σ_x=σ_x=0.0 and σ_z=30.0 MPa

For this example we assume the sphere cavity exists in an infinite domain.

Model Information

- The sphere radius is a = 1.0 m.
- A constant 30MPa in uniaxial direction for field stress is assumed.
- Host rock material property:
    - E = 2000 MPa, v=0.3

Analytical Solution

The σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ along the direction of θ=0 and ϕ=90 can be determined in the spherical coordinate by:

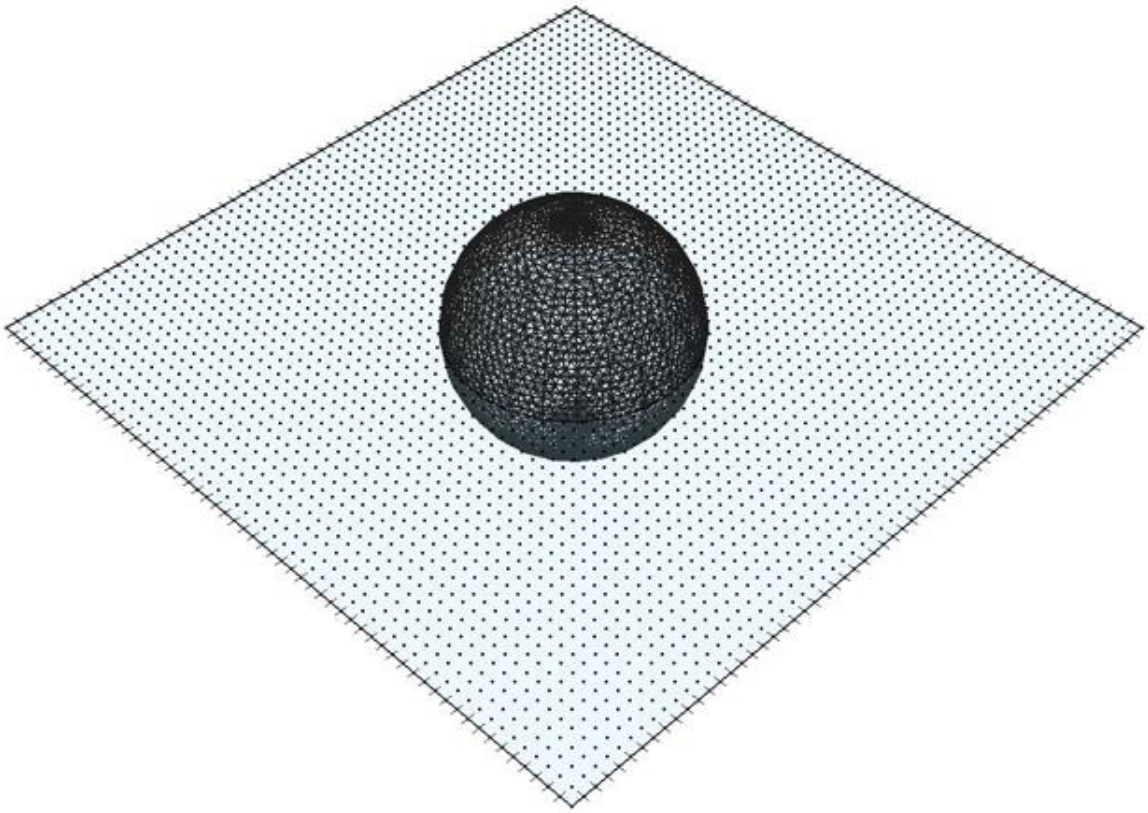
$$\sigma_{rr} = P\left[\frac{6\beta^3(1-\beta^2)(3cos^2\theta-2)}{7-5\nu} + (1-\beta^3)sin^2\theta\right]$$

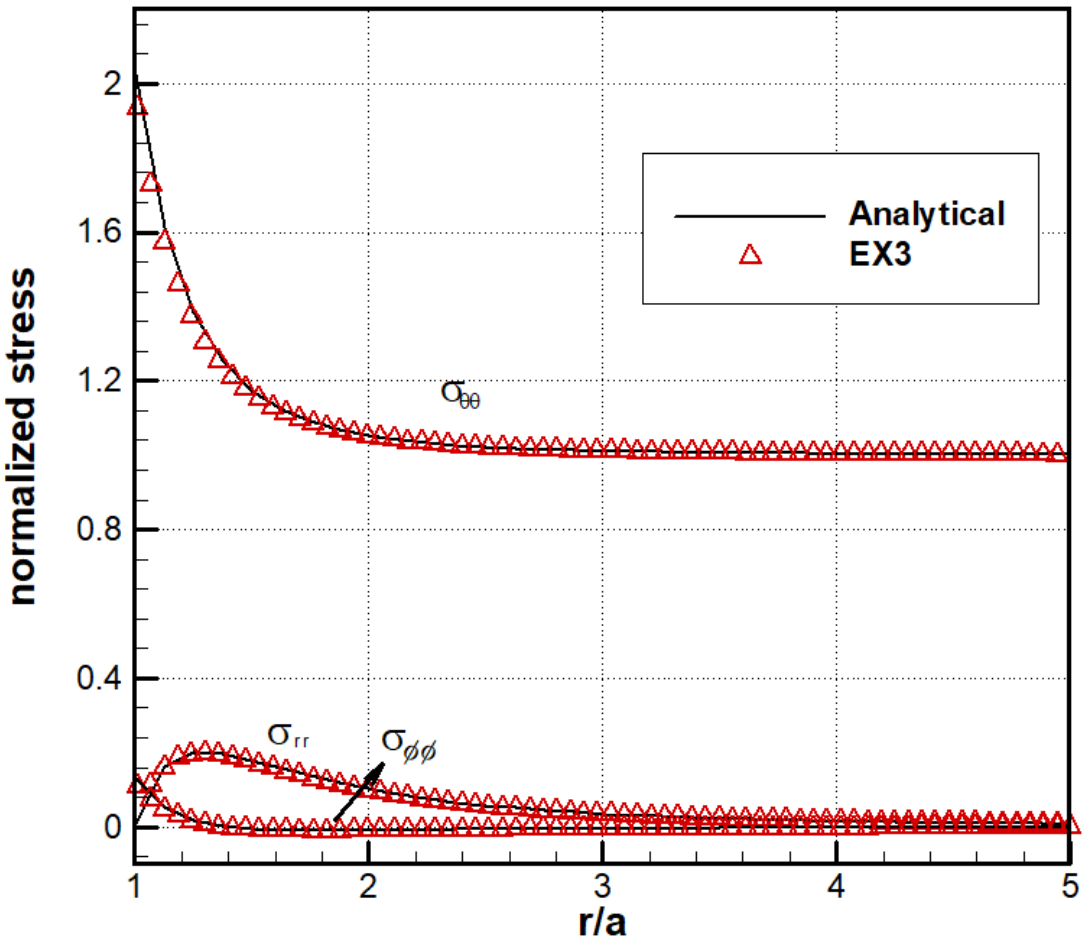
$$\sigma_{\theta\theta} = P\left[\frac{3\beta^3[(3+7\beta^2)cos^2\theta-4(1+\beta^2)]}{2(7-5\nu)} + (1-\beta^3)cos^2\theta + \frac{3\beta^3}{2}\right]$$

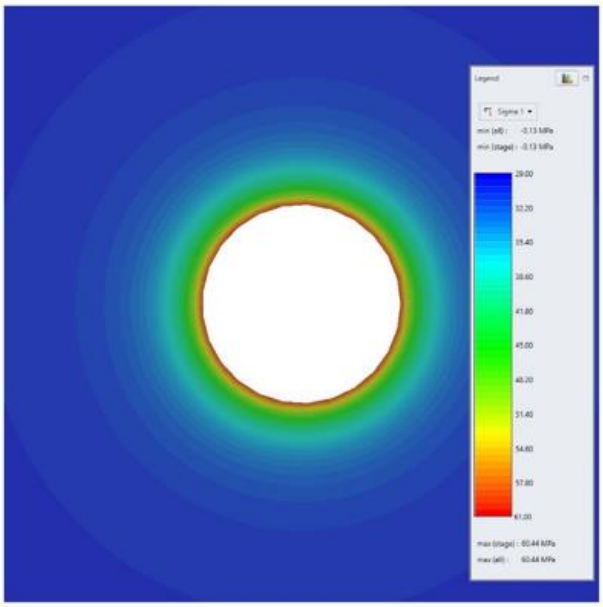
$$\sigma_{\phi\phi} = P\left[\frac{3\beta^3[(9+5\beta^2)cos^2\theta-4(1+\beta^2)]}{2(7-5\nu)} - \frac{3\beta^3}{2}cos^2\theta\right]$$

Here $\beta = \frac{a}{r}$

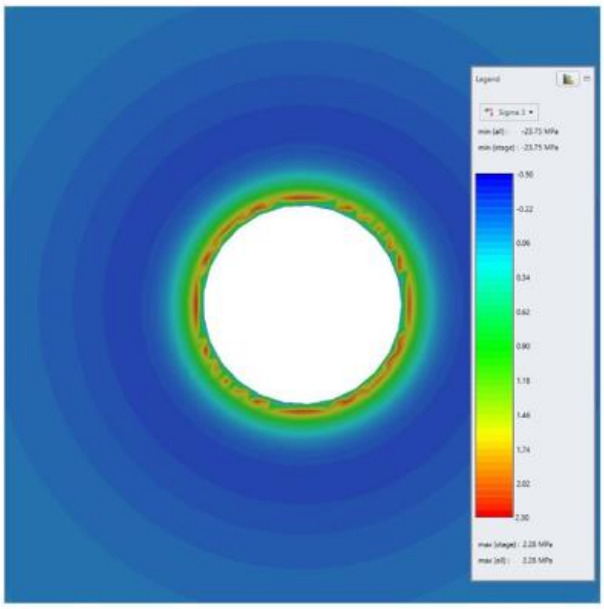
Results



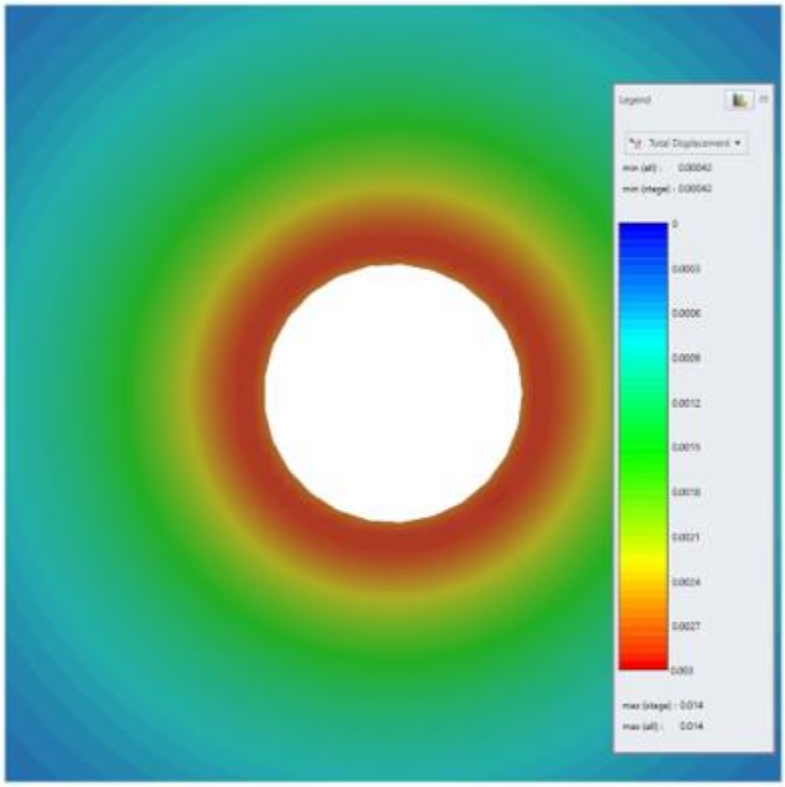

normalized stress
r/a
Analytical
EX3
σθθ
σrr
σφφ
2
1.6
1.2
0.8
0.4
0
1
2
3
4
5



Tangential Stress



Radial Stress



Total Displacement

References

Timoshenko, S.P. and Goodier, J.N. (1970) theory of Elasticity, McGrow Hill, New York.

Data Files

The data input file(s) and file for the finished model can be found in the EX3 installation folder.