10- Mohr-Coulomb with Cap Material Model

This model is an extension to the elasto-perfect-plastic Mohr Coulomb model. The addition is a cap yield surface that closes the elastic domain in stress space $(p - q)$ on the hydrostatic (p) axis. With the help of cap the densification/compaction of material can be simulated.

This model could be an alternative to Cam Clay model in a sense that both models can simulate the compaction of material and limit its shear strength. The advantage for the Mohr Coulomb with cap model is that it will not have the issues that the Cam Clay model has when the stress state is above the critical state line on the dry side of the yield surface (e.g. the softening behavior and high ratios of shear to normal stresses).

10.1. Yield surface and plastic potential functions

The model can utilize up to three yield surfaces that includes Mohr Coulomb (shear), cap (compaction/densification) and tension cut off. The yield surfaces and hardening characteristics of this model are illustrated in Figure 10.1. Figures 10.2 and 10.3 show the 3D view of the yield surfaces with vertical and elliptical cap respectively. Based on the formulations of this model it is apt to say that this model has three different mechanisms, i.e. deviatoric, volumetric and tension cutoff.

The formulations of these three mechanisms, definition of yield surfaces and their corresponding plastic potential and hardening law are presented below.

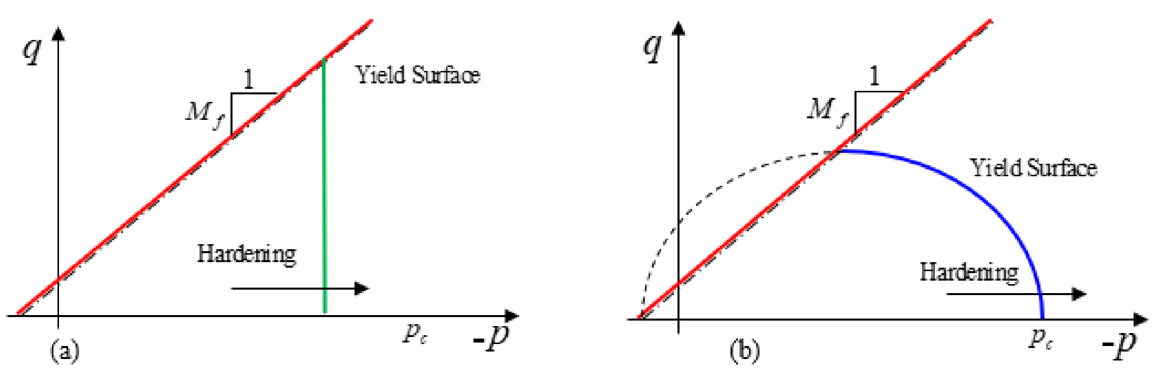

Figure 10.1- The yield surfaces of the Mohr Coulomb with Cap model; a) Mohr Coulomb yield surface (red) and the vertical cap (green); b) Mohr Coulomb yield surface (red) and elliptical cap (blue)

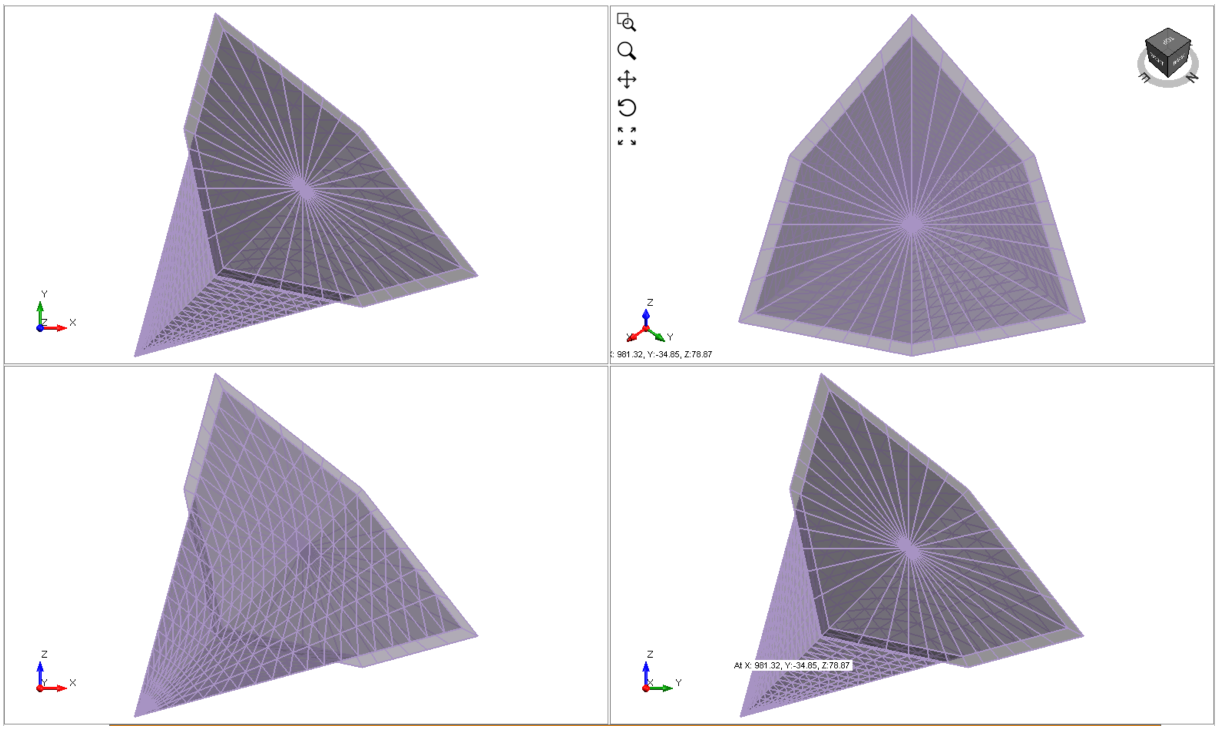

Figure 10.2- Yield surface of Mohr-Coulomb with vertical cap model in 3D stress space

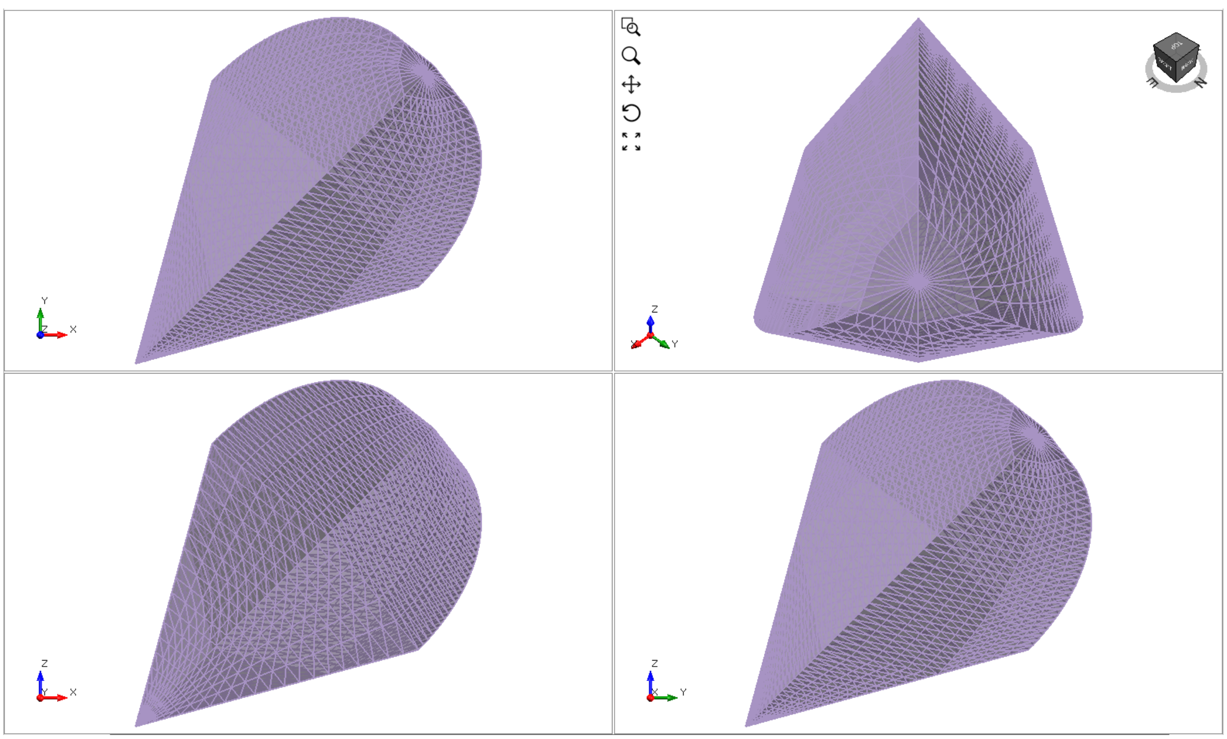

Figure 10.3- Yield surface of Mohr-Coulomb with elliptical cap model in 3D stress space

The equation for the Mohr-Coulomb yield surface (e.g. Pietruszczak 2010). using the (p, q, θ) invariants is given by

$$F_s = q + M\left(p - \frac{c}{\tan\varphi}\right) = 0 \tag{10.1}$$

where

$$M = \frac{3\sin\varphi}{\sqrt{3}\cos\theta - \sin\theta\sin\varphi} \tag{10.2}$$

In above φ is the ultimate/failure friction angle and c is the cohesion.

This mechanism has no hardening/softening rule.

The plastic potential function of the model has a similar shape as the yield function with the dilation angle (ψ) replacing the friction angle.

$$Q_s = q + M_\psi p = const \quad , \quad M_\psi = \frac{3\sin\psi}{\sqrt{3}\cos\theta - \sin\theta\sin\psi} \tag{10.3}$$

There are two options for the yield surface of the volumetric mechanism or the cap, vertical and elliptical.

The yield surface of the vertical cap is defined as follows.

$$F_c = p + p_c = 0 \tag{10.4}$$

Where p_c is the location of the intersection of this yield surface with the p axis.

The elliptical cap is very similar to the yield surface of the modified Cam-Clay model with an offset to consider the cohesion.

$$F_c = \left(\frac{q}{M_f}\right)^2 + \left(p - \frac{c}{tan\varphi_f}\right)(p + p_c) = 0 \quad , \quad M_f = \frac{3\sin\varphi_f}{\sqrt{3}\cos\theta - \sin\theta\sin\varphi_f} \tag{10.5}$$

The flow rule is associated for this yield surface.

This mechanism is to incorporate the tensile strength of the material to this model. In this mechanism the minor principal stress is limited to the tensile strength of the material. The flow rule is associated and the mechanism has no hardening.

$$F_T = \sigma_1 - T = 0 \tag{10.6}$$

In above T is the tensile strength of the material.

10.2- Hardening Behavior

The hardening for these yield surfaces is considered for p_c and it is attributed to volumetric plastic strain. The built in function for the hardening follows the same hardening law as in the Modified Cam-Clay model:

$$(p_c)_{n+1} = (p_c)_n \exp\left(\frac{\Delta\varepsilon_v^p}{\lambda}\right) \tag{10.7}$$

where n is the step number, ε_v^p is the volumetric plastic strain, and λ is the difference between the slope of normal consolidation line and the swelling line.

Tabular hardening law which uses custom tabular piecewise linear values for p_c versus volumetric plastic strain $\left(\varepsilon_v^p, p_c\right)$ is also available for this model.

10.3 The Overconsolidation Ratio and Initial State

The current state of a soil can be described by its stress state (p, q), and yield stress, p_c (also known as preconsolidation pressure is a measure of the highest stress level the soil has ever experienced). The ratio of preconsolidation pressure to current pressure is known as the overconsolidation ratio (OCR).

The in-situ distribution of preconsolidation pressure for the cap can be generated using the OCR. An OCR value of 1 represents a normal consolidation state; a state in which the maximum stress level previously experienced by a material is not larger than the current stress level. OCR > 1 describes an overconsolidated state indicating that the maximum stress level experienced by the material is larger than the present stress level.

To compute models involving a cap, non-trivial initial effective stresses must be specified. RS^2 and RS^3 allow specification of gravity in-situ stresses or a constant stress field. The initial yield surfaces for all stress states must be specified by finding the corresponding p_c. This can be done by assigning the preconsolidation pressure directly or by specification of the OCR.

If a current stress state completely lies within a specified yield cap, the soil will initially respond elastically to loading. This implies that it is overconsolidated. If, however, the initial stress state is located on the cap, the soil will respond elasto-plastically upon loading, indicating that it is normally consolidated.

Since initial stress states that lie outside yield surfaces have no physical meaning, the programs will readjust the preconsolidation pressure to accommodate for the current level of stress

References

Pietruszczak, S. (2010). Fundamentals of Plasticity in Geomechanics. CRC Press.