Effective Stress Calculation in Undrained Materials

RS2 offers an undrained effective stress analysis for soil layers in combination with effective strength parameters. RS2 uses the undrained effective stress analysis combined with effective strength parameters (φ' and c') to obtain the undrained shear strength of a material. Pore pressure development is critical in identifying the correct effective stress path leading to a realistic failure value of undrained shear strength (c_u or s_u).

1. Undrained Effective Stress Calculation

When the undrained behaviour is selected for the material, special elements developed in RS2 will be automatically selected to account for the numerical stability when dealing with incompressible material.

Pore pressure in a soil body can usually be attributed to the presence of water and contributes to the total stress level within the soil. Terzaghi's principle states that total stresses (σ) are divided into effective stresses (σ'), pore pressure (p), and pore water pressure (ρ_w). Note that water does not sustain shear stress, therefore total shear stress is equal to effective shear stress.

$$\underline{\sigma} = \underline{\sigma'} + mp \tag{1.1a}$$

Where,

$$m = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; and \; p = \alpha S_e p_w \tag{1.1b}$$

$$\sigma_{xx} = \sigma'_{xx} + \alpha S_e p_w \tag{1.1c}$$

$$\sigma_{yy} = \sigma'_{yy} + \alpha S_e p_w \tag{1.1d}$$

$$\sigma_{zz} = \sigma'_{zz} + \alpha S_e p_w \tag{1.1e}$$

$$\sigma_{xy} = \sigma'_{xy} \tag{1.1f}$$

$$\sigma_{yz} = \sigma'_{yz} \tag{1.1g}$$

$$\sigma_{zx} = \sigma'_{zx} \tag{1.1h}$$

Where α = Biot's pore pressure coefficient and Se = effective degree of saturation. Considering incompressible gains, α = 1. Additional details about compressible grains and compressible solid material (α < 1) are provided in the Biot's coefficient section below.

Pore pressure is the product $\alpha S_e P_w$ and is termed p in RS2. The system of equations to be solved at each iteration for initial stiffness is:

$$\boldsymbol{K_{eq}} \Delta U^i = R^{i-1} \tag{1.2}$$

Where $\boldsymbol{K_{eq}}$ is the stiffness matrix, ΔU^i is the change in displacement for the current iteration, R^{i-1} is the residual force from the previous iteration, which is calculated as the difference between the external force and the internal force at the current iteration.

The stiffness matrix ($\boldsymbol{K_{eq}}$) formulation is:

$$\boldsymbol{K_{eq}} = \boldsymbol{K_{eff}} + \boldsymbol{K_u} \tag{1.3a}$$

$$\boldsymbol{K_{eff}} = \int \boldsymbol{B^T D_e B} dV \tag{1.3b}$$

where $\boldsymbol{D_e}$ is the elastic stress-strain matrix and $\boldsymbol{B}$ is the strain-displacement matrix.

$$\boldsymbol{K_u} = \boldsymbol{Q E^T F E \overline{Q}^T} \tag{1.3c}$$

$$\boldsymbol{Q} = \int \boldsymbol{B^T} \alpha \chi m \boldsymbol{N_p} \tag{1.3d}$$

$$\boldsymbol{E} = \int \boldsymbol{B_u}^{\boldsymbol{T}} \alpha S_w m \boldsymbol{N_p} \tag{1.3e}$$

$$\boldsymbol{B_u} = \boldsymbol{L N_u} \tag{1.3f}$$

$$\boldsymbol{F} = \int \boldsymbol{N_u}^{\boldsymbol{T}} \boldsymbol{K_f N_u} \tag{1.3g}$$

$$\boldsymbol{\overline{Q}} = \int \boldsymbol{B^T} \alpha S_w m \boldsymbol{N_p} \tag{1.3h}$$

where S_w is the degree of saturation, and χ is the coefficient.

If negative pore pressure cutoff and single effective was not selected:

$$\begin{cases} K_f = 0 \; for \; pore \; pressure \; \leq 0 \\ K_f = \dfrac{1}{(\alpha - n)K_s + \dfrac{n}{K_w}} \; for \; pore \; pressure > 0 \end{cases} \tag{1.4}$$

If single effective parameters are activated:

$$\begin{cases} K_f = \dfrac{1}{(\alpha - n)K_s + \dfrac{n}{K_w}} \; for\ pore\ pressure\ \geq 0 \\ K_f = \dfrac{1}{\dfrac{(\alpha - n)}{K_s} S_w \left(S_w + \dfrac{C_s}{n} p \right) + \dfrac{n S_w}{K_w} + C_s} \; for\ pore\ pressure < 0 \end{cases} \tag{1.5}$$

Where K_s is the bulk modulus of the solid material, $K_w = 2 * 10^6\ kN/m^2$, C_s is the compressibility of the solid material, p is the pore pressure, and n is the porosity of the soil.

2. Skempton B-Parameter

When using undrained drainage conditions, the following equations are used to differentiate between total stress rates, effective stress rates, and rates of excess pore pressure:

Total stress: $\delta\sigma_m = (K_f + K')\delta\varepsilon_v$ (2.1a)

Excess pore pressure: $\delta p_{excess} = K_f \delta\varepsilon_v$ (2.1b)

Effective stress: $\delta\sigma'_m = K'\delta\varepsilon_v$ (2.1c)

The value of Skempton's B-parameter is calculated using the ratio of the bulk stiffnesses of the soil skeleton and the pore fluid:

$$B = \frac{\alpha}{\alpha + n(\frac{K'}{K_w} + \alpha - 1)}$$

Where K' is the effective bulk modulus for the soil matrix and $K_w = 2 * 10^6\ kN/m^2$.

The special undrained behaviour option in RS2 requires calculations that using effective stiffness parameters, with a distinction between effective stresses and excess pore pressures. The calculations may not completely address shear induced effective pore pressures.

This analysis requires effective soil parameters and is therefore very useful when these parameters are available. However, with soft soil projects, accurate effective soil parameter data may not be accessible. Therefore, in situ and laboratory tests may be performed to obtain undrained soil parameter data. In these cases, Hooke's law can be used to convert measured values of undrained Young's moduli into effective Young's moduli:

$$E' = \frac{2(1 + v')}{3} E_u \tag{2.3}$$

This direct conversion from measured to effective values is not possible for advanced models. In these advanced cases, it is suggested to estimate the effective stress parameter from the measured parameter, then perform an undrained test to verify the resulting undrained stiffness (and adapt the estimated effective stiffness value if needed). The *Soil* test facility (Reference Manual) may be used in these circumstances.

3. Biot Pore Pressure Coefficient, α

Generally, the compressibility of the soil skeleton will be greater than that of the individual grains of soil in a mass; therefore, deformations of individual soil grains can be disregarded. In cases with deep soil layers at high pressures, the stiffness of the soil/rock matrix will approach that of the material of the soil/rock grains; in these cases, the compressibility of the solid material must be considered. This affects the division of total stress into effective stress and pore pressure. In cases with compressible solid material, Terzaghi's effective stress can be defined using the following equation:

$$\underline{\sigma'} = \underline{\sigma} - \alpha S_e \underline{m} p_w \tag{3.1}$$

Where α is Biot's pore pressure coefficient, S_e is the effective degree of saturation, $\underline{m}$ is a vector (with unity values for normal components and 0-values for shear components), and p_w is the pore water pressure. The following equation shows the definition of Biot's pore pressure coefficient:

$$\alpha = 1 - \frac{K'}{K_s} \tag{3.2}$$

Where K' is the effective bulk modulus for the soil matrix and K_s is the bulk modulus of the solid material. Note that for an incompressible solid material ($K_s = \infty$), Terzaghi's original stress definition holds true. Lower values of α indicate that for given values of total stress and pore water pressure, the resulting effective stress is higher than that of an incompressible solid material ($\alpha = 1$).

For most soil types, the value of α will be around 1. However, the α values for hard rocks may vary. Some typical values of α for rocks can be found in Table 3.1 below (Luo et al, 2015).

Table 3.1. Biot's Coefficient Values for Rocks

Rock type	Rock Porosity ϕ (%)	α (-)	Location	References
Siltstone	0.7	0.172	Altensalzwedel	Hou et al. (2009)
Claystone	2.5	0.358	Altensalzwedel	Hou et al. (2009)
Siltstone	1.1	0.210	Altensalzwedel	Hou et al. (2009)
Sandstone	7.0	0.645	Altensalzwedel	Hou et al. (2009)
Anhydrite	0.1	0.082	Altensalzwedel	Hou et al. (2009)
RRS	15.3	0.78	North German Basin	Trautwein (2005)
RRS	18.3	0.90	North German Basin	Trautwein (2005)
RRS	9.8	0.61	North German Basin	Trautwein (2005)
RRS	4.7	0.64	North German Basin	Trautwein (2005)
RRS: rotliegend reservoir sandstones				

The value of Biot's pore pressure coefficient is automatically calculated by RS2, but the value may be changed manually by the user.

References

Hou, M.Z., Yoon, J., Khan, I.U., and Wundram, L. (2009) *"Determination of the 3D primary stresses and hydromechanical properties of severalstrata in the Salzwedel reservoir area & multi-layer caprock-reservoir 2D-model for the Altensalzwedel test field for THMCcoupled numerical modelling"*. Report to the BMBF project03G0704Q, Milestones I & II of the Subproject PI.2 in the TV-III of the CLEAN-Project.

Luo, X., Were, P., Liu, J., and Hou, Z. (2015). *"Estimation of Biot's effective stress coefficient from well logs"*. Environmental Earth Sciences, 73(11), 7019-7028.

Trautwein, U. (2005). *“Poroelastische Verformung und petrophysikalische Eigenschaften von Rotliegend Sandsteinen”*. Doctor dissertation, Tehcnische Universität Berlin.