# RS2

2D finite element program for stress analysis and support design around excavations in soil and rock

## Consolidation Verification Manual

# 1 One Dimensional Compression of a Finite Layer

## 1.1 Problem Description

### 1.1.1 Uniform Mesh

This problem analyzes the one-dimensional consolidation of an elastic layer of soil, under drained conditions with a permeable surface and impermeable base, with the application of a constant pressure over a dimensionless time period. The parameters for the soil model are outlined in Table 1-1.

**Table 1.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E) | 200 kPa |
| Poisson's ratio (ν) | 0.3 |
| Friction angle | 0º |
| Cohesion | 10.5 kPa |
| Permeability (κ) | 0.01 m/s |
| Thickness (H) | 10 m |
| Coefficient of consolidation ($c_v$) | 0.2744 $m^2/s$ |

The problem uses a uniform six-noded triangular mesh with horizontal and vertical supports as shown in Figure 1-1.

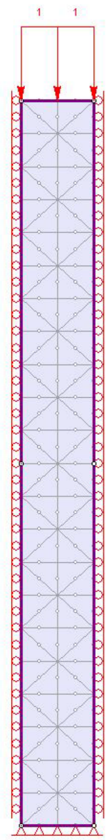



**Figure 1.1: One dimensional compression of a finite layer with uniform mesh as modeled in RS2**

Additionally, evident in Figure 1-2, a ramp load is imposed over the time period $t_0$ where the rate load is described as a dimensional time factor, $T_{v0}$, which in this problem is 0.0001.

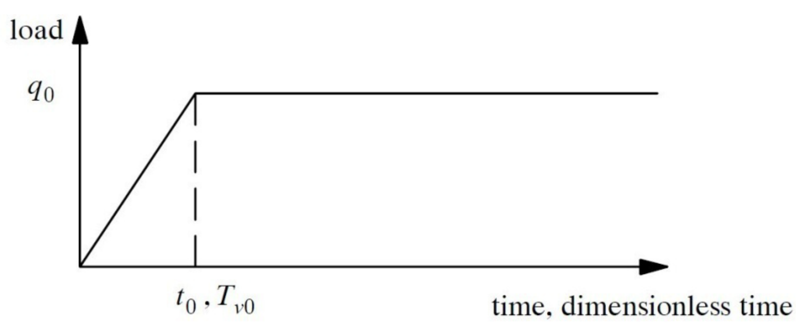



**Figure 1.2: Load vs. Time**

The dimensionless time of consolidation is determined by the equation

$$T_v = \frac{c_v t}{H^2}$$

where $c_v$ is the coefficient of consolidation described by the following equation, with *k, E,* and *v* being the drained permeability, the drained Young's modulus, and the drained Poisson's ration of the soil, respectively.

$$c_v = \frac{kE(1-v)}{\gamma_w(1+v)(1-2v)}$$

The degree of consolidation is defined by the equation

$$U = \frac{S_c}{S_{c,max}}$$

where $S_c$ is the settlement of the soil layer at each time step, measured from H = 0m.

### 1.1.2 Graded Mesh

This problem analyzes the effect of mesh refinement, specifically the application of a graded mesh, on pore pressure of a one dimensional consolidation of the soil with the same parameters described in Example 1.1.1. The graded mesh for this example is shown in Figure 1-3.

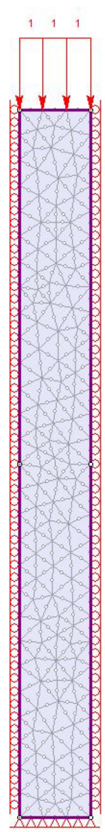



**Figure 1.3: One dimensional compression of a finite layer with graded mesh as modeled in RS2**

## 1.2 Analytical Solution

According to Terzaghi [1], to determine the rate and degree of consolidation, several assumptions are made: the coefficient of permeability and volume compressibility remains the same at every point in the layer, and for a consolidated compressed layer with uniform thickness the amount of water which leaves the layer per unit time exceeds the amount which enters, which is equivalent to the change in volume.

Based on these assumptions, and using the void equation and Darcy's law, the differential equation for consolidation under linear drainage is

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 y}{\partial z^2}$$

where $c_v$ is the coefficient of consolidation.

Furthermore, the percentage of settlement is described as a function of the dimensionless time of consolidation, or the time factor, with the equation

$$U\% = f(T_v)$$

where the settlement percentage is the same for the consolidation of every layer under specific loading and drainage conditions, to determine the relation between the time factor and degree of consolidation.

Although the solution does not include the secondary time effect, where the solution approaches a horizontal asymptote, it can be used to determine upper and lower limit values for the rate of settlement. Additionally the analytical solution used for this example is specific for the boundary conditions of a permeable surface and impermeable base.

## 1.3 Results

### 1.3.1 Uniform Mesh

The finite element results for the consolidation of the elastic soil with RS2 are shown in Figure 1-4. It can be see that the results are in accordance with the analytical solution derived by Terzaghi; an example of the settlement for $T_v$=1 is shown in Figure 1-5.

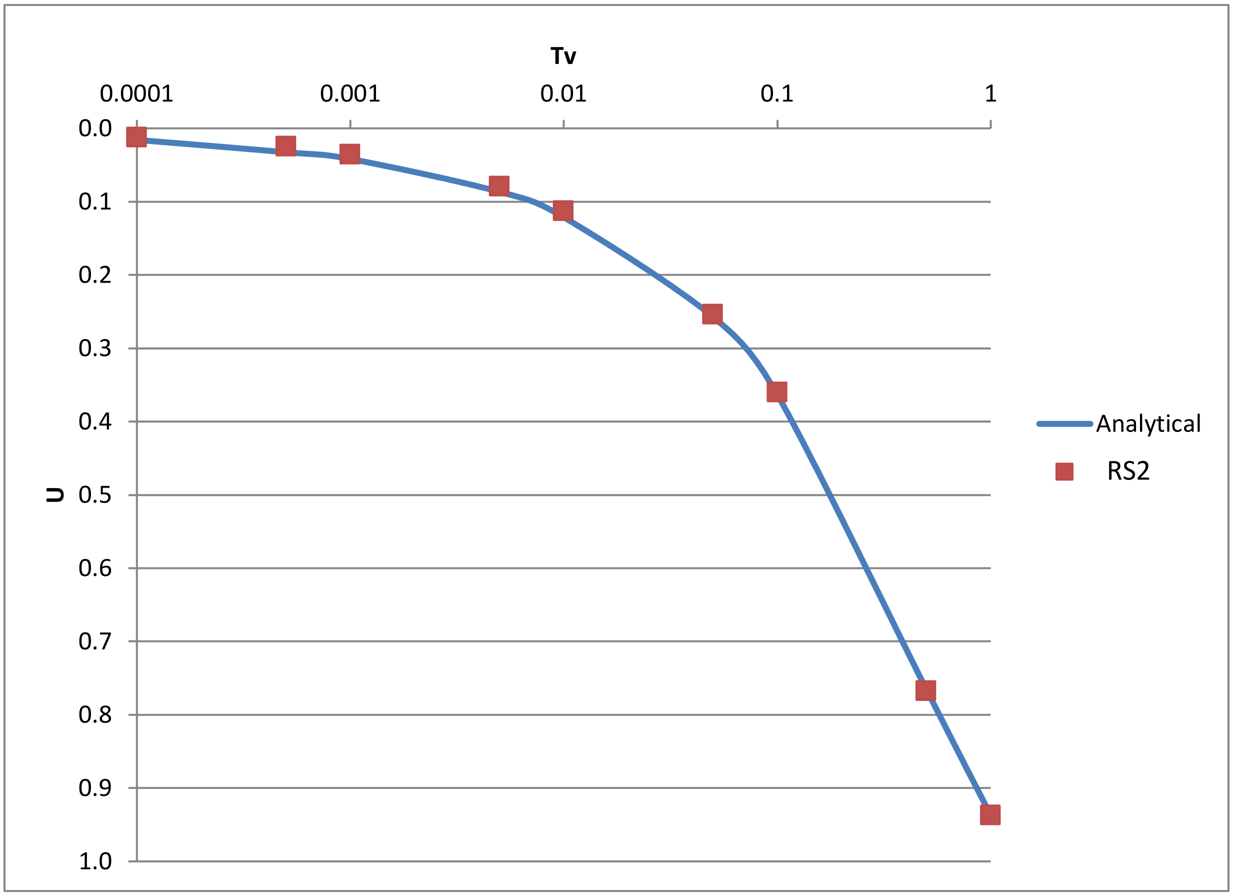



**Figure 1.4: Degree of consolidation versus time factor**

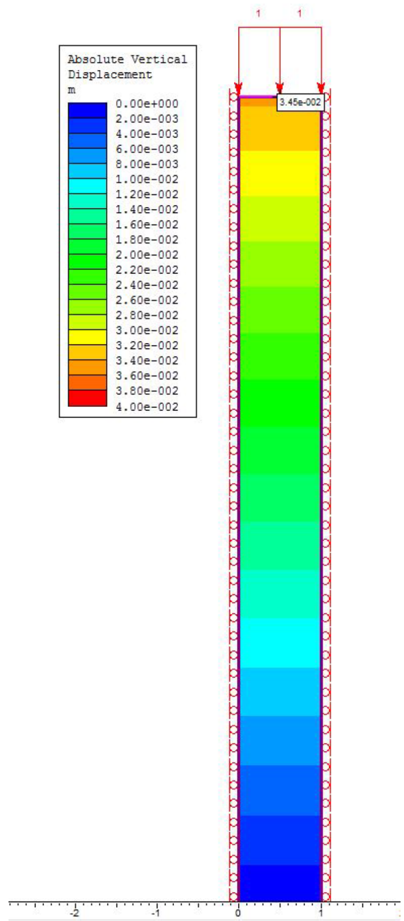



**Figure 1.5: Absolute vertical displacement contour for last load stage as modeled in RS2**

### 1.3.2 Graded Mesh

Figure 1-6, reveals that the finite element results with RS2 for five different time steps correspond accordingly with the analytical solution derived by Terzaghi. Figure 1-7 shows the contour and query locations for the $T_v = 1$ pore pressure.

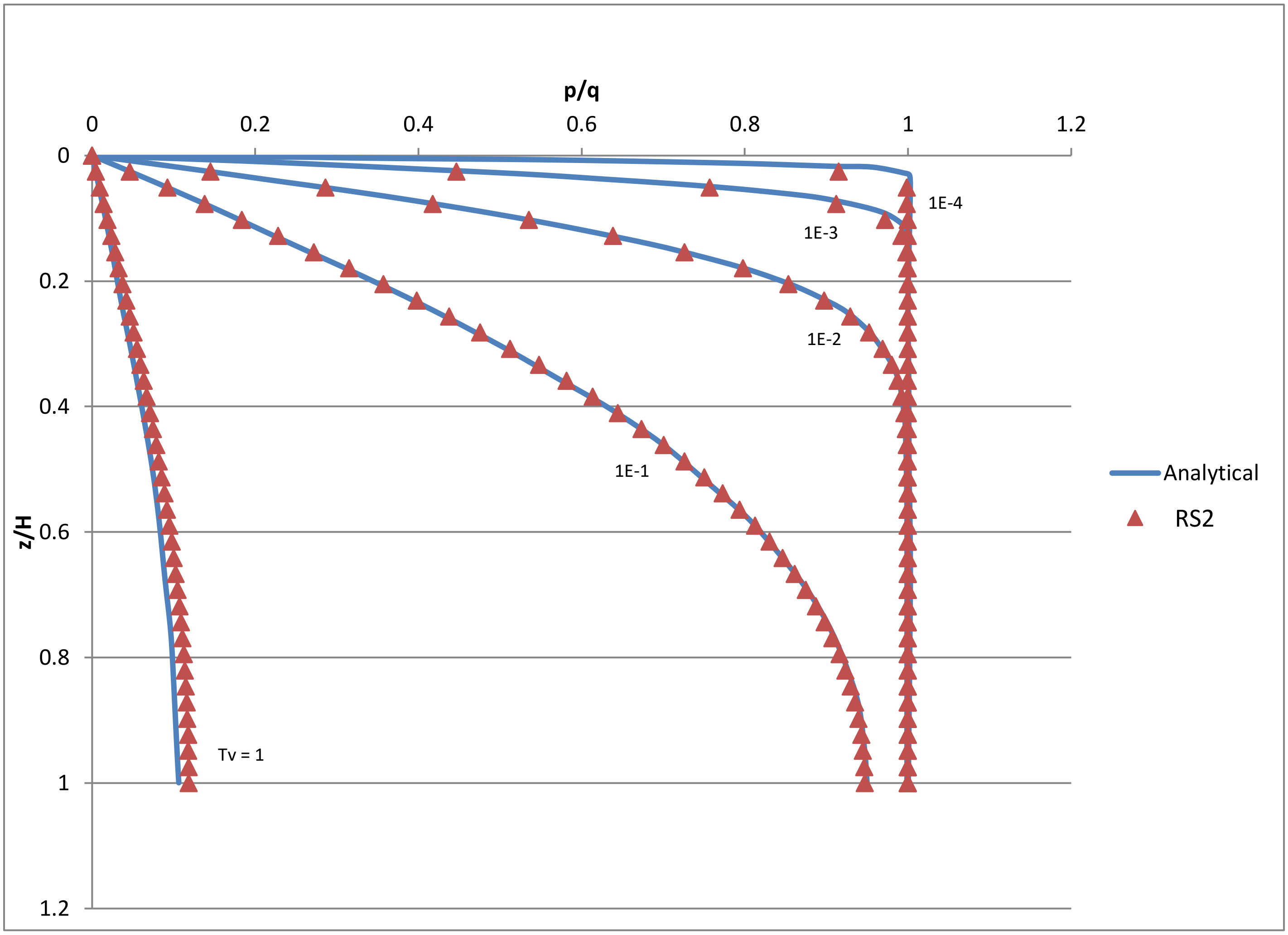



**Figure 1.6: Pore pressure versus ratio of depth using graded mesh**

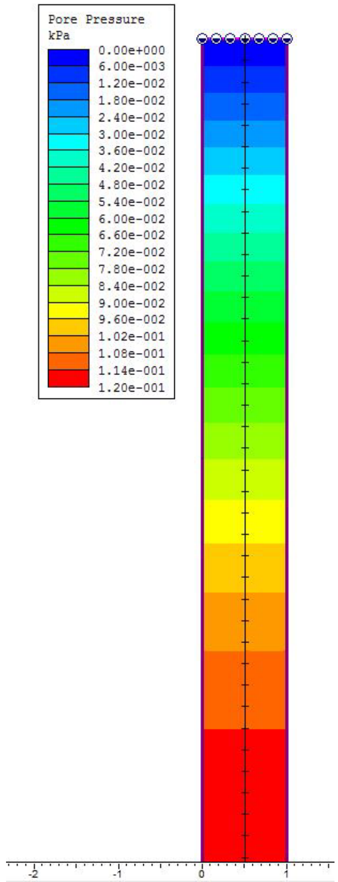



**Figure 1.7: Pore pressure contour for $T_v$ = 1 as modeled in RS2**

## 1.4 References

1. Terzaghi, K. 'Die Berechnung der Durchlassigkeitsziffer des Tones aus demVerlauf der hydrodynamischen Spannungsersceinungen', Originally published in 1923 and reprinted in *From Theory to Practice in Soil Mechanics*, John Wiley and Sons, New York, 133-146, 1960.

## 1.5 Data Files

The input files **consolidation#001_01.fez** and **consolidation#001_02.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 2 Consolidation of Finite Layer Compressed Between Rigid Plates

## 2.1 Problem Description

### 2.1.1 Uniform Mesh

Analysis of the consolidation of an elastic drained soil compressed between two smooth, impermeable plates, with the two ends open to flow. The parameters for this problem are outlined in Table 2-1, and the parameters for the composite liner with a joint are outlined in Table 2-2 and Table 2-3 respectively.

**Table 2.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young’s modulus (E) | 200 kPa |
| Poisson’s ratio (ν) | 0.3 |
| Friction angle | 0º |
| Cohesion | 10.5 kPa |
| Permeability (κ) | 0.0001 m/s |
| Half-width (a) | 1.25 m |
| Thickness | 1 m |
| Coefficient of consolidation ($c_v$) | 0.002744 $m^2/s$ |

**Table 2.2: Elastic Composite Liner Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's Modulus | $3x10^9$ kPa |
| Poisson's Ratio | 0.2 |
| Thickness | 20 m |

**Table 2.3: Joint Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Normal Stiffness | $1x10^5$ kPa/m |
| Shear Stiffness | 0 kPa/m |

The dimensionless time factor for this example is described by the equation

$$T_v = \frac{c_v t}{3a^2}$$

where $a$ is the half-width of the layer.

Additionally the soil is vertically supported with a constant pressure applied to the top plate, as shown in Figure 2-1.

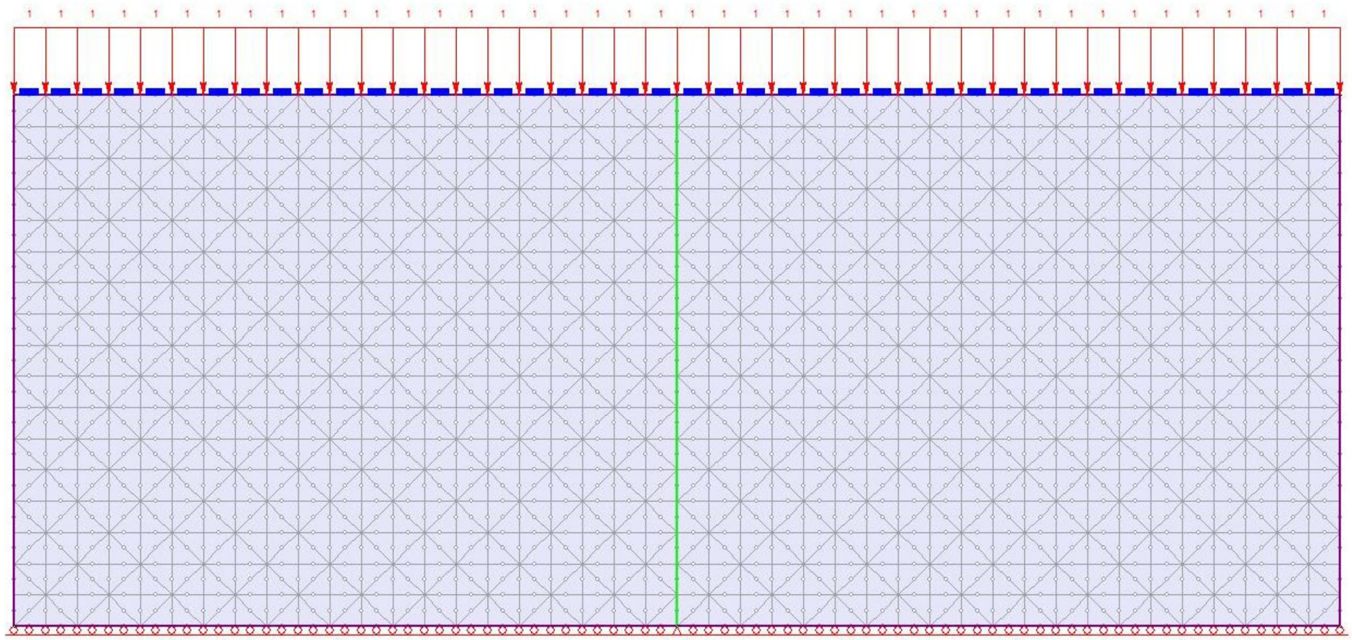


**Figure 2.1: Finite layer compressed between two rigid plates as modeled in RS2**

### 2.1.2 Graded Mesh

This problem analyzes the effect of meshing on the pore pressure of a one dimensional consolidation of the soil model in Example 2.1.1. All model parameters are the same as those described in Example 2.1.1, except the application of a graded mapped-mesh containing 60 external nodes, as shown in Figure 2-2.

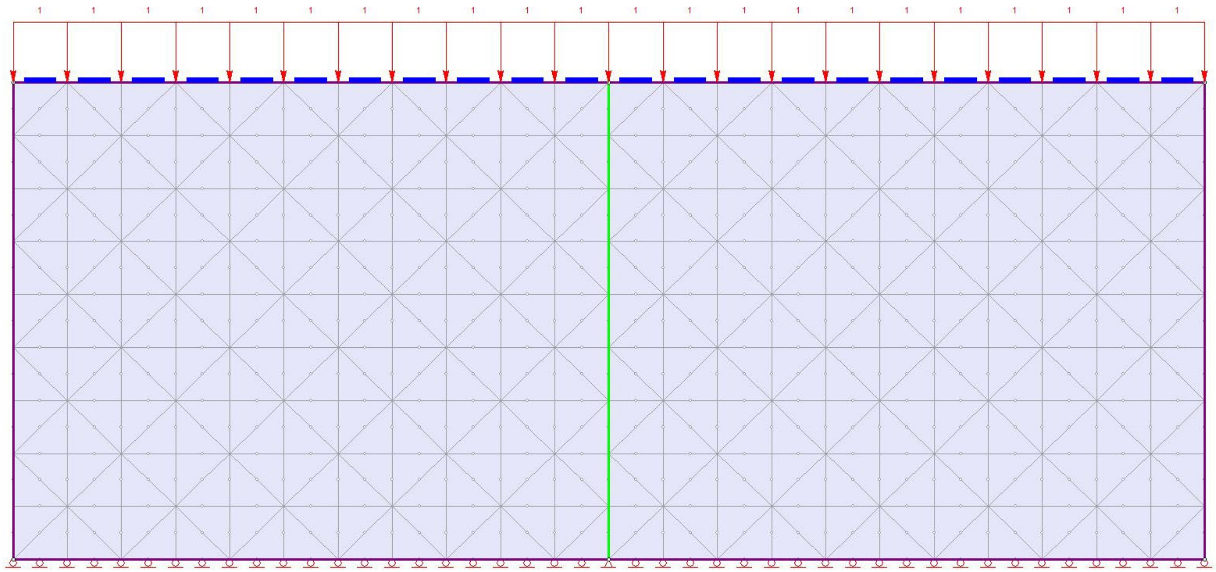


**Figure 2.2: Finite layer compressed between two rigid plates with graded mesh as modeled in RS2**

## 2.2 Analytical Solution

According to Mandel [1] the problem of an isotropic, elastic, and drained layer in the plane strain state under an axial load, $P_0H(t)$, between two smooth, rigid, impermeable plates involves the following components:

| | |
|---|---|
| Displacement | $u_x = u_x(x,t)$ <br> $u_z = u_z(z,t)$ |
| Strain | $\varepsilon_x = \varepsilon_x(x,t)$ <br> $\varepsilon_z = \varepsilon_z(t)$ |
| Stress | $\sigma_x = \sigma_x(x,t)$ <br> $\sigma_z = \sigma_z(x,t)$ |
| Pore fluid pressure | $p = p(x,t)$ |

with the following boundary conditions

$$x = \pm a$$

$$\sigma_x = 0$$

$$p = 0$$

where $x$ is the horizontal distance from centre of layer, $\sigma_x$ is the horizontal stress applied, and $p$ is the pore fluid pressure, and the following loading conditions

$$2\int_0^a \sigma_z(x,t)dx = -\,P_0H(t)$$

where $\sigma_z$ is the vertical stress applied and $H(t)$ is the heaviside unit step function.

Additionally, the equations for fluid-saturated, isotropic, poroelastic materials satisfying an irrotational condition has the following strain-displacement relations:

$$\sigma_{ij} = 2G\left(\varepsilon_{ij} + \frac{v}{1-2v}\varepsilon_{ll}\delta_{ij}\right) - \alpha p\delta_{ij}$$

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\right) = 0$$

where $\sigma_{ij}$, $\varepsilon_{ij}$, $p$, and $u_i$ are the total stress, average strain, pore fluid pressure, and average strain, respectively; $G$, $v$, and $\alpha$ are the shear modulus, drained Poisson's ratio, and Biot's coefficient of effective stress described by the equation

$$\alpha = \frac{3(v_u - v)}{B(1+v_u)(1-2v)}$$

where $B$ and $v_u$ are Skempton's pore pressure coefficient and undrained Poisson's ratio.

The diffusion equation of the pore fluid satisfying the irrotational condition is given by the following equation

$$\frac{\partial p}{\partial t} = c\Delta^2 p - \frac{\alpha}{S}\frac{d}{dt}g(t)$$

with the storage coefficient $S$ and diffusivity coefficient $c$ both given by

$$S = \frac{\alpha^2(1-2v)^2(1-v)}{B(v_u - v)(1-v)}, \qquad c = \frac{\kappa}{S}$$

where $\kappa$ is the permeability coefficient, $g(t)$ is the auxiliary function of $t$, with the following relations between volumetric strain and fluid pressure

$$\varepsilon_{ll} = \frac{\eta}{G} p + g(t), \qquad \eta = \frac{\alpha(1-2v)}{2(1-v)}$$

Finally, the 2D Mandel solution (in solved in Laplace space) has the following final equations

$$\frac{\tilde{p}(\chi,\xi)}{(P_0/2a)(a^2/c)} = \frac{B}{3}(1+v_u)(1-v)\frac{1}{\xi D(\xi)}\left\{1-\frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\},$$

$$\frac{\tilde{\sigma}_z(\chi,\xi)}{(P_0/2a)(a^2/c)} = -\frac{1}{\xi D(\xi)}\left\{(1-v)-(v_u-v)\frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\},$$

$$\frac{\tilde{\sigma}_x(\chi,\xi)}{(P_0/2a)(a^2/c)} = 0,$$

$$\frac{2G\tilde{\varepsilon}_z(\xi)}{(P_0/2a)(a^2/c)} = -(1-v)(1-v_u)\frac{1}{\xi D(\xi)},$$

$$\frac{2G\tilde{\varepsilon}_x(\chi,\xi)}{(P_0/2a)(a^2/c)} = \frac{1}{\xi D(\xi)}\left\{v_u(1-v)-(v_u-v)\frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\}.$$

## 2.3 Results

### 2.3.1 Uniform Mesh

As can be seen in **Figure 2-3**, the finite element data produced from RS2 is very similar to the analytical results derived by Mandel. The pore pressure values were measured at the centre of the soil layer, as shown in Figure 2-4.

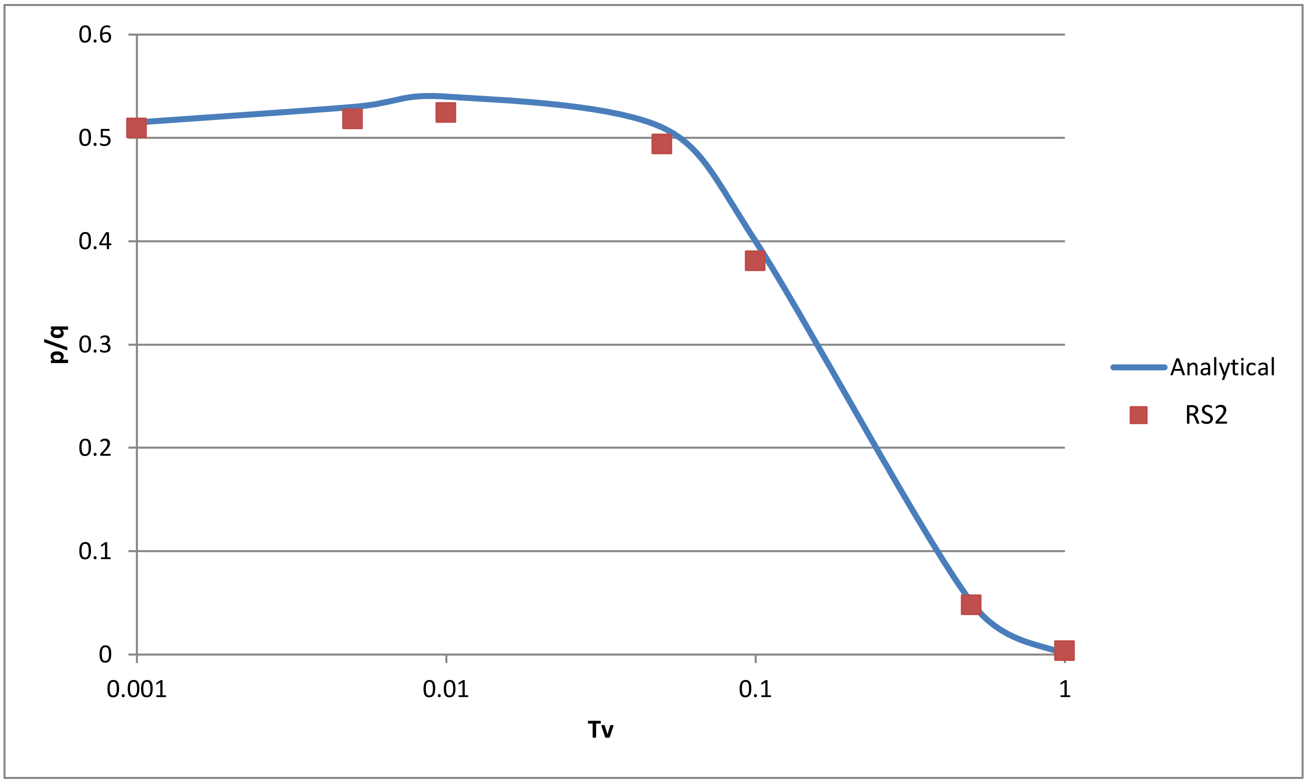



**Figure 2.3: Pore pressure versus time factor**

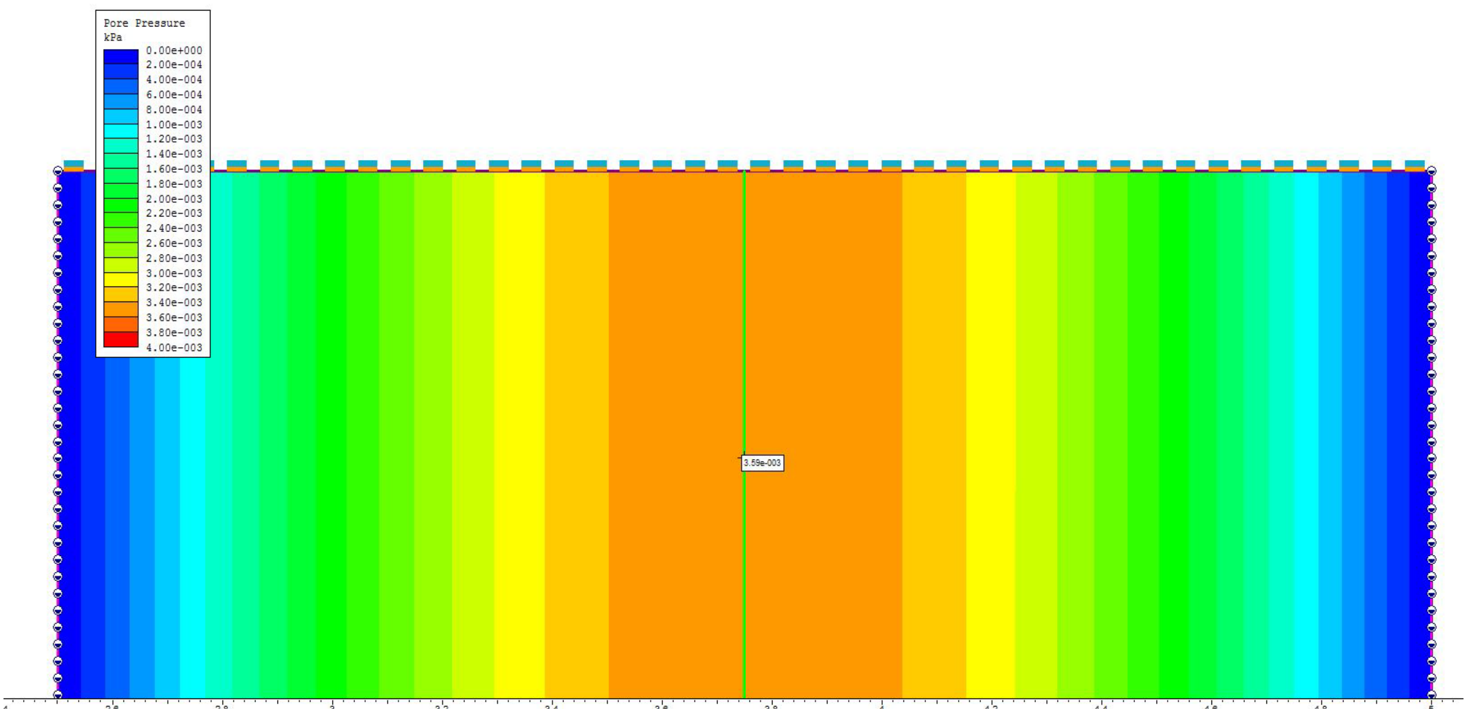



**Figure 2.4: Pore pressure contour for $T_v = 1$ in RS2**

### 2.3.2 Graded Mesh

As can be seen in Figure 2-5, the data computed using RS2 based on a graded mesh is in good agreement with the analytical solution developed by Mandel [1].

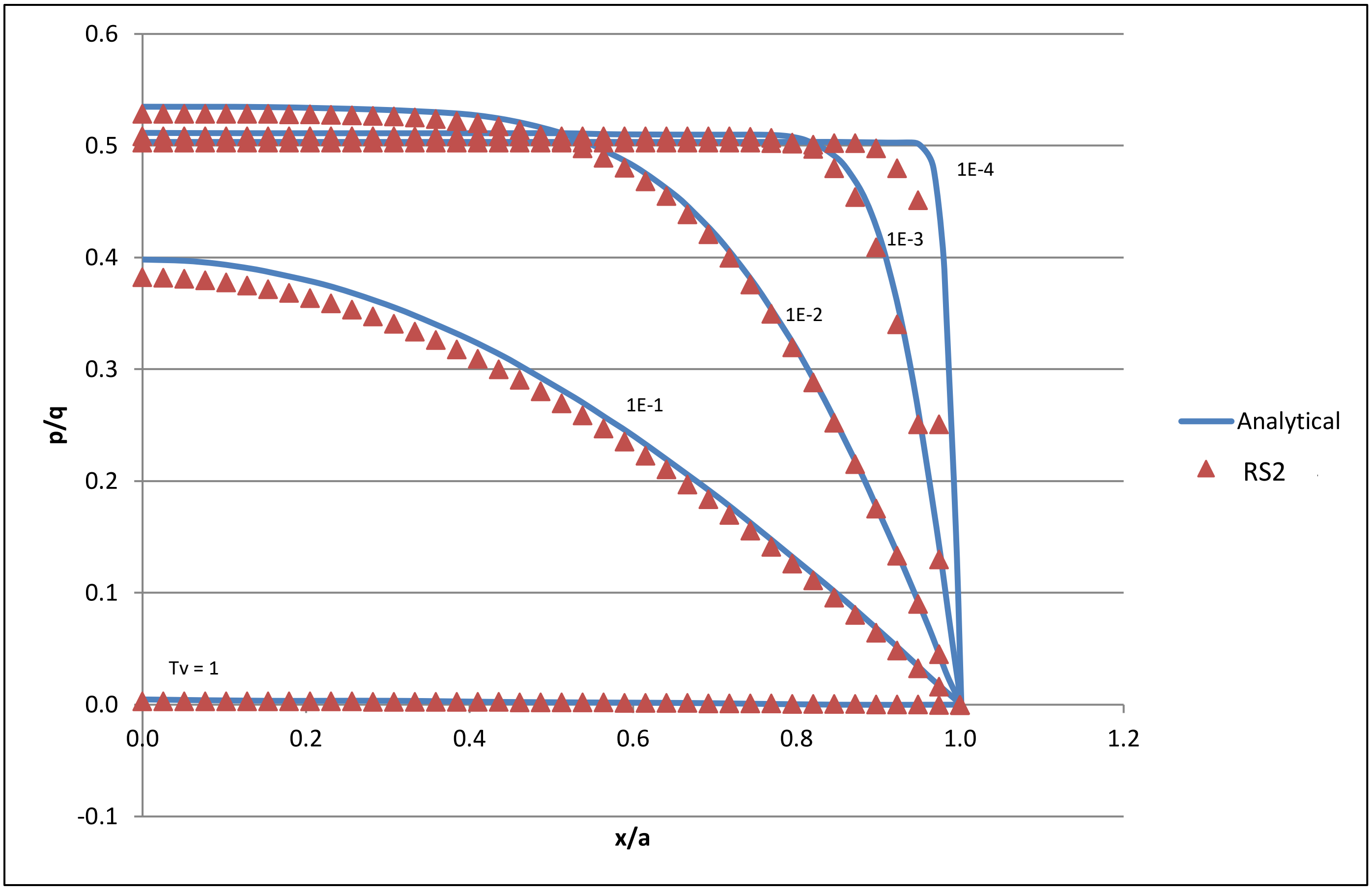



**Figure 2.5: Pore pressure versus distance ratio from centre using graded mesh**

The pore pressures values along the centre of the soil layer, as shown in Figure 2-6, were analyzed at each time step.

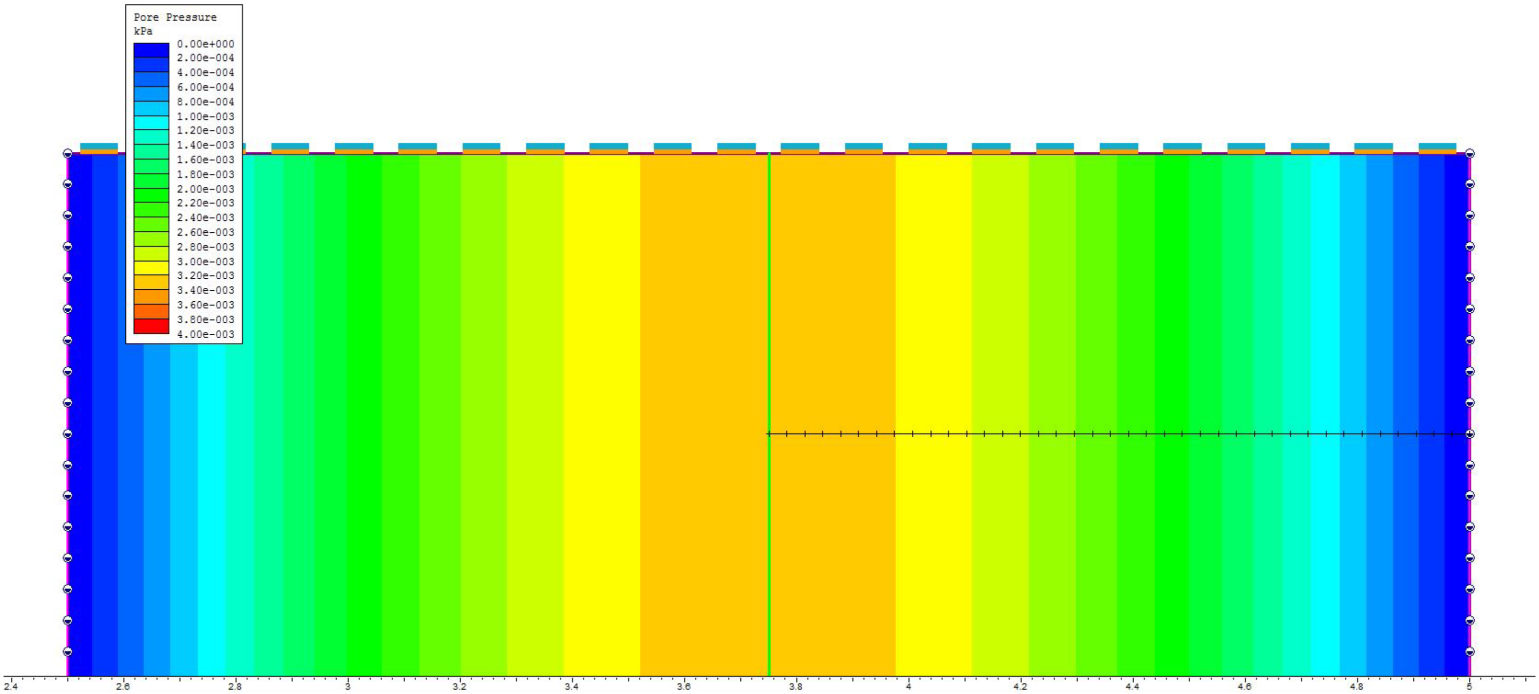



**Figure 2.6: Pore pressure contour plot at $T_v$ = 1 in RS2**

## 2.4 References

1. Mandel, J. 'Consolidation des sols', *Geotechnique*, **III**, 287-299, 1953.

## 2.5 Data Files

The input files **consolidation#002_01.fez** and **consolidation#002_02.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 3 Flexible Strip Footing on Finite Layer

## 3.1 Problem Description

This problem analyzes the one dimensional consolidation of a flexible strip footing on a porous elastic soil layer with the model parameters described in Table 3-1.

**Table 3.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus | 200 kPa |
| Poisson's ratio (ν) | 0 |
| Cohesion | 10.5 kPa |
| Friction angle | 0º |
| Permeability (κ) | 0.0001 m/s |
| Thickness (H) | 5 m |
| Width | 10 m |
| Coefficient of consolidation ($c_v$) | 0.002039 $m^2/s$ |

The dimensionless time consolidation factor is defined by the equation

$$T_v = \frac{c_v t}{H^2}$$

and the degree of consolidation defined by the equation

$$U = \frac{S_c}{S_{c,max}}$$

where $S_c$ is the settlement of the soil layer. Additionally, the top surface of the model is permeable with a constant pressure applied over the 1m long footing strip, shown in Figure 3-1.

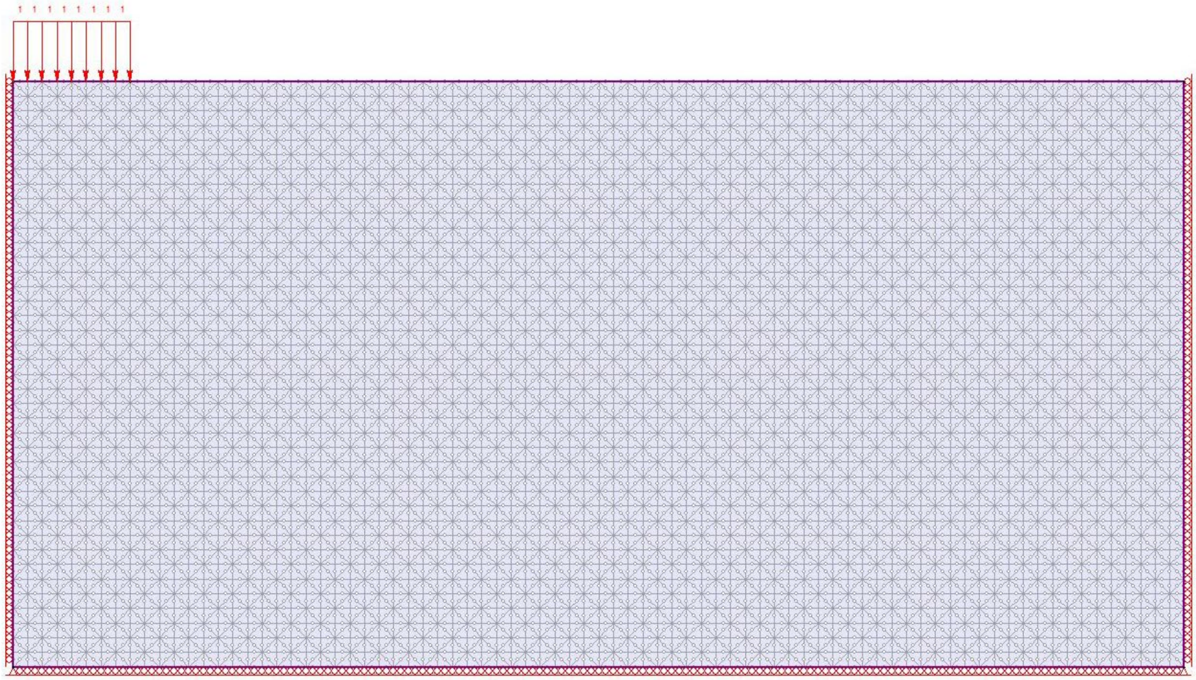


**Figure 3.1: Flexible strip footing on finite layer as modeled in RS2**

## 3.2 Analytical Solution

Determined by Booker [1], the equation for a uniformly loaded strip developed through Laplace and Fourier transformations is

$$w = \frac{1}{4}\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[p_0 + \sum_{n=t}^{\infty} p_n e^{Sn(\alpha,\beta)t}\right] e^{(i\alpha x + i\beta y)}\, d\alpha\, d\beta$$

which, when evaluated gives the following solutions

$$z = z_1 \begin{cases} \sigma_{zz} = q & -a \le x \le a \\ \sigma_{zz} = 0 & elsewhere \end{cases}$$

$$q^F = q\frac{sin\alpha a}{\alpha}\delta(\beta)$$

Where $q$ is the uniform pressure applied, $z$ is the layer depth, $a$ is the footing width, $\alpha$ and $\beta$are coordinate points, $\delta(\beta)$ is the Dirac delta function, and $\sigma_{zz}$ is the vertical stress given by the equation

$$\sigma_{zz} = \sigma + \lambda e_v - 2G\frac{dw}{dz}$$

where $\lambda$ and $G$ are Lame's parameters for the soil skeleton, $e_v$ is the void ratio, and $w$ is the soil deformation.

### 3.3 Results

As can be seen in Figure 3-2, the results produced from RS2 correspond well with the analytical solution derived by Booker.

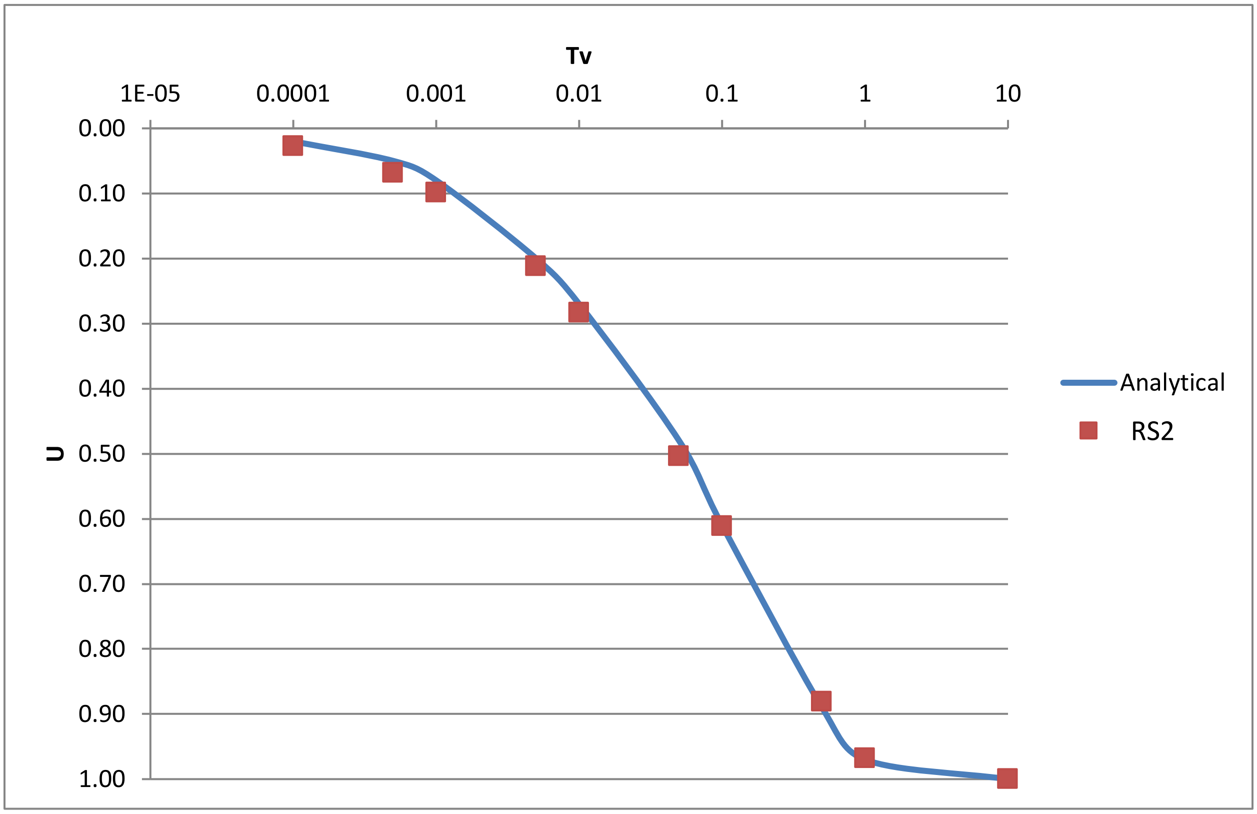



**Figure 3.2: Degree of consolidation versus time factor for elastic strip footing**

The settlement is measured for each time step at the centre of the footing, at H = 0m, as shown in Figure 3-3. The RS2 solution was analyzed using the initial stage (1 s) as the reference stage.

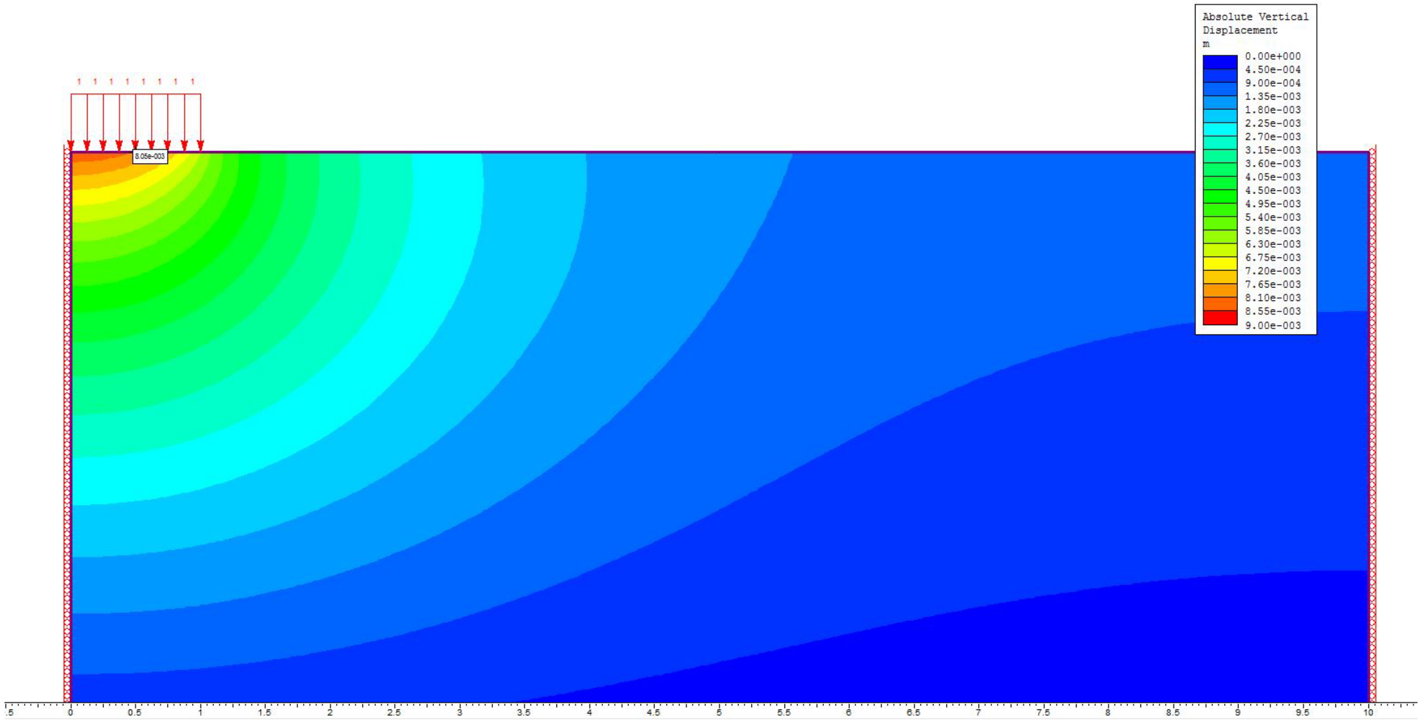



**Figure 3.3: Absolute vertical displacement contour in RS2**

## 3.4 References

1. Booker, J.R. 'The consolidation of a finite layer subject to surface loading', *International Journal for Solids and Structures*, **10**, 1053-1065, 1974.

## 3.5 Data Files

The input files **consolidation#003.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 4 Analysis of Consolidation of Thick Cylinder

## 4.1 Problem Description

### 4.1.1 Drained Conditions

This problem analyzes elastoplastic consolidation through the expansion of a thick cylinder under drained loading conditions. The cylinder has an inner radius $a$ and an outer radius $b$, subject to an internal pressure $q$, with zero external pressure. Additionally the model follows an axisymmetric finite element mesh and the parameters described in Table 4-1.

**Table 4.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E') | 200 kPa |
| Poisson's ratio (ν') | 0.0 |
| Friction angle (ϕ') | 30º |
| Cohesion (c') | 1 kPa |
| Dilitancy angle (ψ') | 0º |
| Permeability (κ) | 0.01 m/s |
| Internal radius ($a$) | 2 m |
| Outer radius ($b$) | 4 m |
| Coefficient of consolidation ($c_v$) | 0.2039 $m^2$/s |

Where the dimensionless time factor follows the equation

$$T_v = \frac{c_v t}{a^2}$$

and a load rate parameter with the equation

$$\omega = \frac{\Delta q / c'}{\Delta T_v}$$

Due to the cylinder having drained conditions, a slow loading rate of $\omega = 0.01$ was used to apply the internal pressure over 12 time steps. The model has a uniform six-noded triangular mesh, with approximately 30 mesh elements, shown in Figure 4-1.

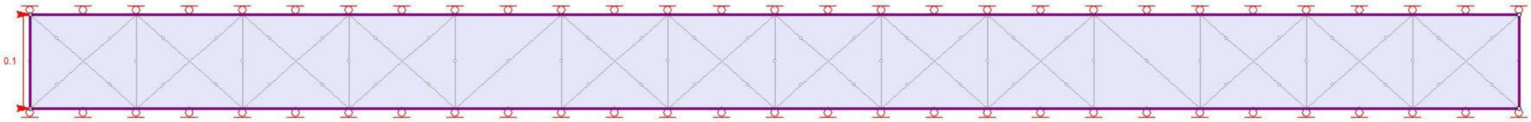



**Figure 4.1: Drained thick cylinder as modeled in RS2**

### 4.1.2 Undrained Conditions

This problem analyzes elastoplastic consolidation for a cylinder under undrained loading conditions, by either applying a faster loading rate of $\omega$ = 10,000 or applying undrained boundary conditions to the same soil model described in Example 4.1.1 with the model shown in Figure 4-1. The equations for the coefficient of consolidation, dimensionless time factor, and load rate also remain the same.

## 4.2 Analytical Solution

According to Yu [1], a cylinder with inner and outer radii boundaries, $a$ and $b$, and an internal pressure of $q$, with elastic-perfectly plastic material, will obey Hooke's law until yielding occurs, which is determined by the Mohr-Coulomb criterion.

Under the elastic stage, the radial and tangential stress, and the pore pressure are given by the respective equations:

$$\sigma_r = -p_0 + (p - p_0)\left[\frac{1}{\left(\frac{b}{a}\right)^2 - 1} - \frac{1}{\left(\frac{r}{a}\right)^2 - \left(\frac{r}{b}\right)^2}\right]$$

$$\sigma_\theta = -p_0 + (p - p_0)\left[\frac{1}{\left(\frac{b}{a}\right)^2 - 1} + \frac{1}{\left(\frac{r}{a}\right)^2 - \left(\frac{r}{b}\right)^2}\right]$$

$$u = \frac{p - p_0}{2G\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}\left[\frac{1 - 2v}{b^2} r + \frac{1}{r}\right]$$

with the yield equation:

$$\alpha\sigma_\theta - \sigma_r = Y$$

The elastic-plastic stage is analyzed by two separate regions which have the following respective equations:

**Plastic Region:** $a \leq r \leq p$

$$\sigma_r = \frac{Y}{\alpha - 1} + Ar^{-\frac{(\alpha-1)}{\alpha}}$$

$$\sigma_\theta = \frac{Y}{\alpha - 1} + \frac{A}{\alpha} r^{-\frac{(\alpha-1)}{\alpha}}$$

**Elastic Region:** $b \geq r \geq p$

$$\sigma_r = -p_0 + B\left(\frac{1}{b^2} - \frac{1}{r^2}\right)$$

$$\sigma_\theta = -p_0 + B\left(\frac{1}{b^2} + \frac{1}{r^2}\right)$$

and the equation for the internal pressure applied as:

$$p_f = \frac{Y + (\alpha - 1)p_0}{\alpha - 1}\left[\left(\frac{b}{a}\right)^{\frac{\alpha-1}{\alpha}} - 1\right] + p_0$$

In terms of displacement analysis, the displacement in the elastic zone has the following equation:

$$u = \frac{1+v}{M}\left[\frac{1-2v}{\alpha - 1 + (1+\alpha)\left(\frac{b}{p}\right)^2} r + \frac{1}{(\alpha - 1)\left(\frac{r}{b}\right)^2 + (1+\alpha)\left(\frac{b}{p}\right)^2} r\right]$$

and displacement in the plastic zone follows the plastic flow rule with the equation:

$$\beta\dot{\epsilon_r} + \dot{\epsilon_\theta} = \frac{1-v^2}{E}\left\{\left(\beta - \frac{v}{1-v}\right)\sigma^r + \left(1 - \frac{\beta v}{1-v}\right)\sigma_\theta + \left(\beta + 1 - \frac{v(1+\beta)}{1-v}\right)p_0\right\}$$

Additionally, according to Small [2] the undrained parameters for the thick cylinder model was determined from the drained parameters according to the following equations

$$E_u = \frac{3E'}{2(1+v')}$$

$$c_u = \frac{2c'\sqrt{N_\phi}}{1+N_\phi}$$

where

$$N_\phi = \frac{1 + \sin\phi}{1 - \sin\phi}$$

Finally, the drained collapse pressure of the cylinder given by the following equation

$$\frac{q}{c'} = 1.02$$

and the undrained collapse pressure by the following equation

$$\frac{q}{c'} = 1.2$$

or

$$\frac{q}{c_u} = 1.4$$

## 4.3 Results

### 4.3.1 Drained Conditions

Figure 4-2 shows the graph of pressure versus displacement ratios for both the RS2 and the analytical solution by Small [1], which have very similar results. Figure 4-3 shows results of the drained model. The displacement ratio uses the drained shear modulus which follows the equation

$$G' = \frac{E'}{2(1 + v')}$$

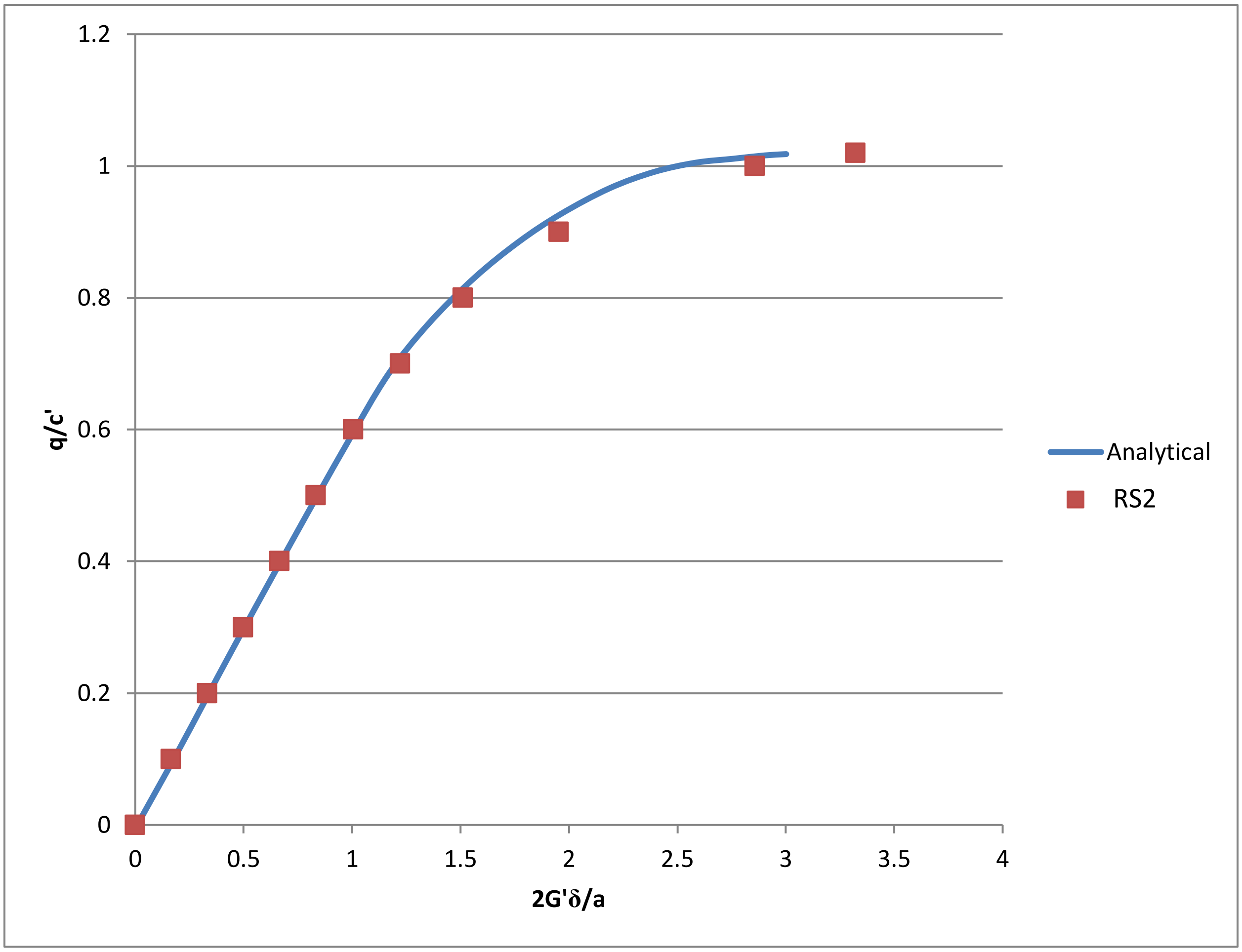



**Figure 4.2: Pressure versus displacement for drained loading**

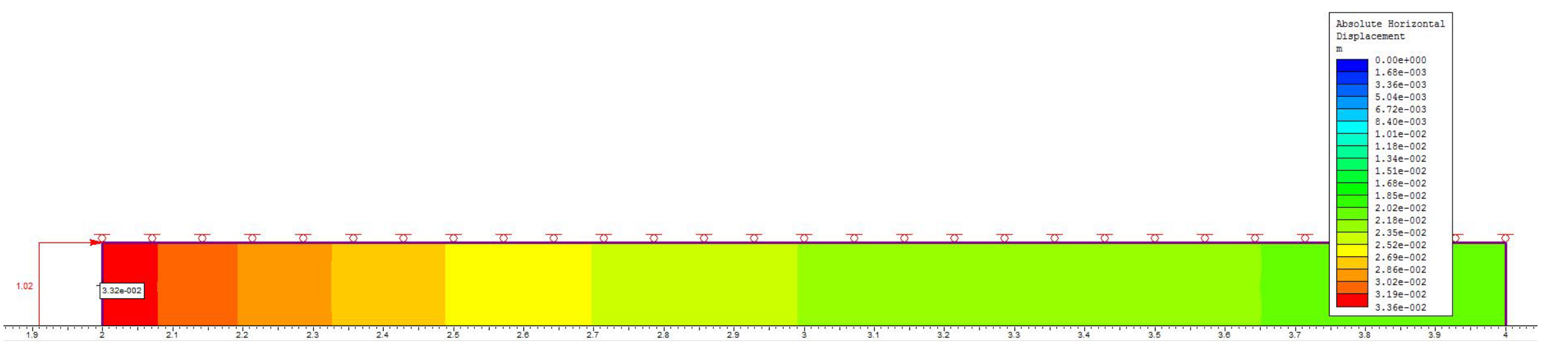



**Figure 4.3: Drained inner radius displacement contour at peak load as modeled in RS2**

### 4.3.2 Undrained Conditions

As can be seen in Figure 4-4, the results from RS2 by using undrained boundary conditions and by applying a rapid load rate are in accordance with the analytical solution by Hill [3]. In addition, a time step of 20 was used to determine the solution using a rapid load rate. The displacement contour of the inner radius of the model is shown in Figure 4-5.

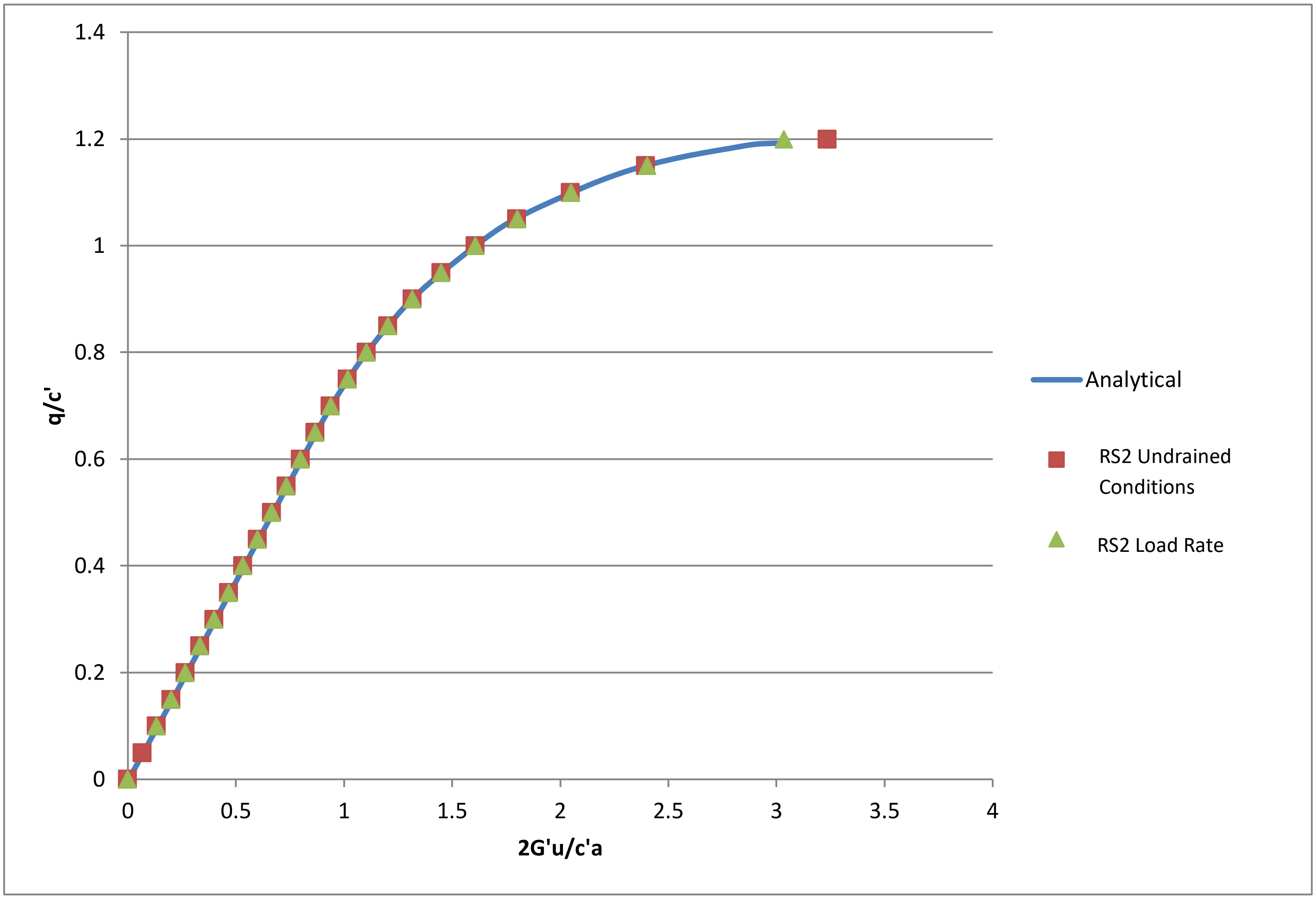



**Figure 4.4: Pressure versus displacement for undrained loading**

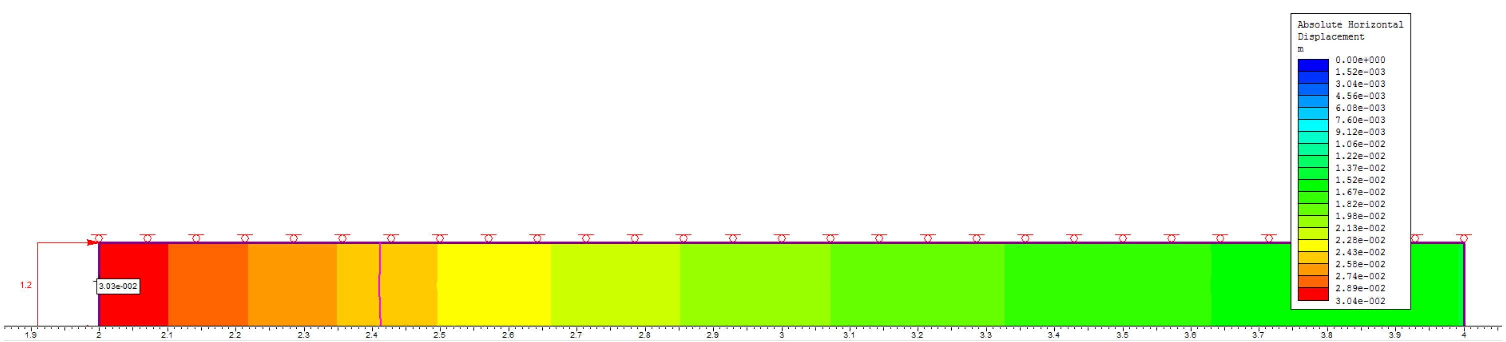



**Figure 4.5: Undrained (load rate) inner radius displacement contour at peak load as modeled in RS2**

## 4.4 References


1. Yu, H.S, 'Expansion of a thick cylinder of soil', *Computers and Geotechnics*, **14**, 21-41, 1992.

2. Small, J.C., *Elasto-plastic consolidation of Soils*, PhD thesis, University of Sydney, 1977.

3. Hill, R., *The Mathematical Theory of Plasticity*, Clarendon Press, Oxford, 1950.


## 4.5 Data Files

The input files **consolidation#004_01.fez**, and **consolidation#004_02 (undrained conditions).fez** and **consolidation#004_02 (load rate).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 5 Undrained Analysis of Strip Footing

## 5.1 Problem Description

This problem analyzes the consolidation of a smooth flexible strip footing to which a constant pressure is applied, under undrained conditions through the application of a rapid load rate of $\omega$ = 150. The model parameters of the soil model are outlined in Table 5-1.

**Table 5.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E') | 200 kPa |
| Poisson's ratio (ν') | 0.3 |
| Friction angle (ϕ') | 20º |
| Cohesion (c') | 1 kPa |
| Dilitancy angle (ψ') | 0º |
| Permeability (κ) | 0.01 m/s |
| Thickness | 8 m |
| Half-width | 16 m |
| Coefficient of consolidation ($c_v$) | 0.1960 $m^2$/s |

The load rate has the equation

$$\omega = \frac{\Delta q/c'}{\Delta T_{v2}}$$

and the dimensionless time factor has the equation

$$T_{v2} = \frac{c_{v2}t}{B^2}$$

where $c_{v2}$ is the two dimensional consolidation coefficient with the equation

$$c_{v2} = \frac{kE}{2\gamma_w(1+v')(1-2v')}$$

and $B$ is the half-width of the footing strip. Due to the simulation of undrained conditions, the top surface of the model is impermeable with a constant pressure applied over the footing strip, shown in Figure 5-1.

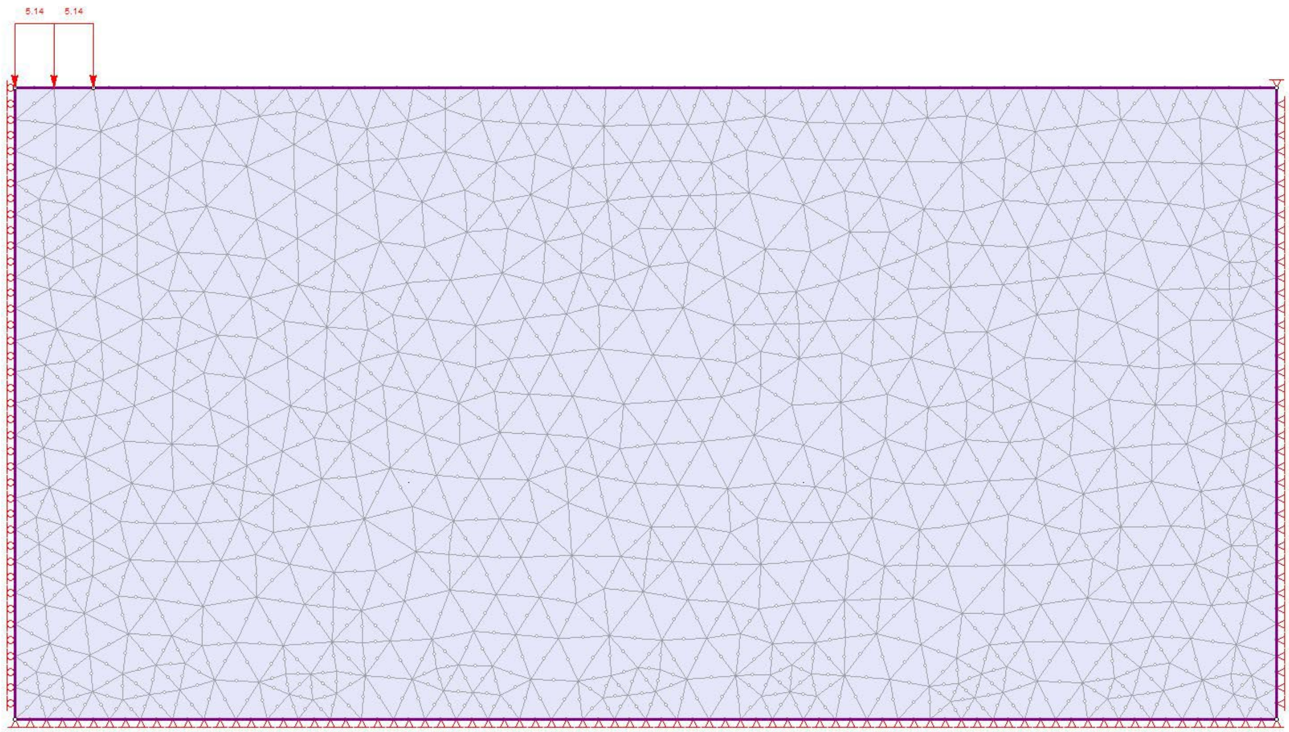



**Figure 5.1: Flexible strip footing on elastoplastic layer as modeled in RS2**

## 5.2 Analytical Solution

According to Small [1], the Biot consolidation formulation combined with the simple elastoplastic Mohr-Coulomb model requires a zero dilation angle, to prevent a change in volume and a gain in strength due to plastic shearing.

Biot's equations for soil consolidation include the following:

Equilibrium stresses

$$\frac{\partial \sigma_{ij}}{\partial x_i} - F_i = 0$$

Effective stresses in relation to Hooke's law

$$\sigma'_{ij} = \sigma_{ij} - \rho\delta_{ij} = -H_{ijkl}\epsilon_{kl}$$

Flow of water determined by Darcy's law

$$v_i = -\frac{k_{ij}}{\gamma_w}\frac{\partial p}{\partial x_j}$$

Incompressible pore water resulting in the rate of volume decreasing equaling the rate at which water is expelled

$$\frac{\partial v_i}{\partial x_i} = -\frac{\partial \theta}{\partial t}$$

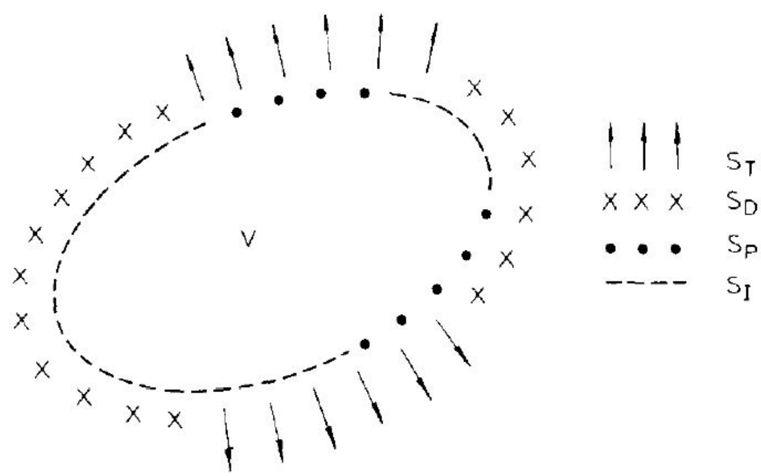
where

| | |
|---|---|
| $x_i$ | are coordinates |
| $\sigma_{ij}$ | are the components of the total stress tensor |
| $F_i$ | are the components of body force |
| $p$ | is the pore pressure |
| $\sigma'_{ij}$ | are components of effective stress |
| $v_i$ | are the components of superficial velocity vector |
| $\epsilon_{kl}$ | are components of the strain tensor with the equation |

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

| | |
|---|---|
| $\theta$ | is volume strain |
| $H_{ijkl}$ | are elastic coefficients in generalized Hooke's Law |
| $k_{ij}$ | are the coefficients of permeability in generalized Darcy's Law |
| $\gamma_w$ | unit weight of water |
| $t$ | time |

The equations are then integrated over the region V




with the following boundary conditions

$$\sigma_{ij} n_j = -T_i \quad \text{applied to } S_t$$
$$u_i = 0 \quad \text{applied to } S_D$$
$$p = 0 \quad \text{applied to } S_p$$
$$n_i v_i = 0 \quad \text{applied to } S_I$$

and the initial condition $\theta = 0$ when $t = 0^+$, where there are no instantaneous volume changes.

Additionally, according to Prandtl [2] the collapse pressure, the point to which the solution by Small [1] asymptotes towards, is given by the formula

$$\frac{q}{c_u} = 5.14$$

or

$$\frac{q}{c'} = 4.83$$

for simulated undrained and undrained, respectively.

## 5.3 Results

As can be seen in Figure 5-2, using undrained boundary conditions and using a rapid load rate for the soil model in RS2 produces results that correspond very well with the analytical results by Small [1]. In addition, a time step of 20 was used to determine the solution using a rapid load rate. Figure 5-3 displays the centre-displacement contour of the footing for the final load stage, under undrained boundary conditions.

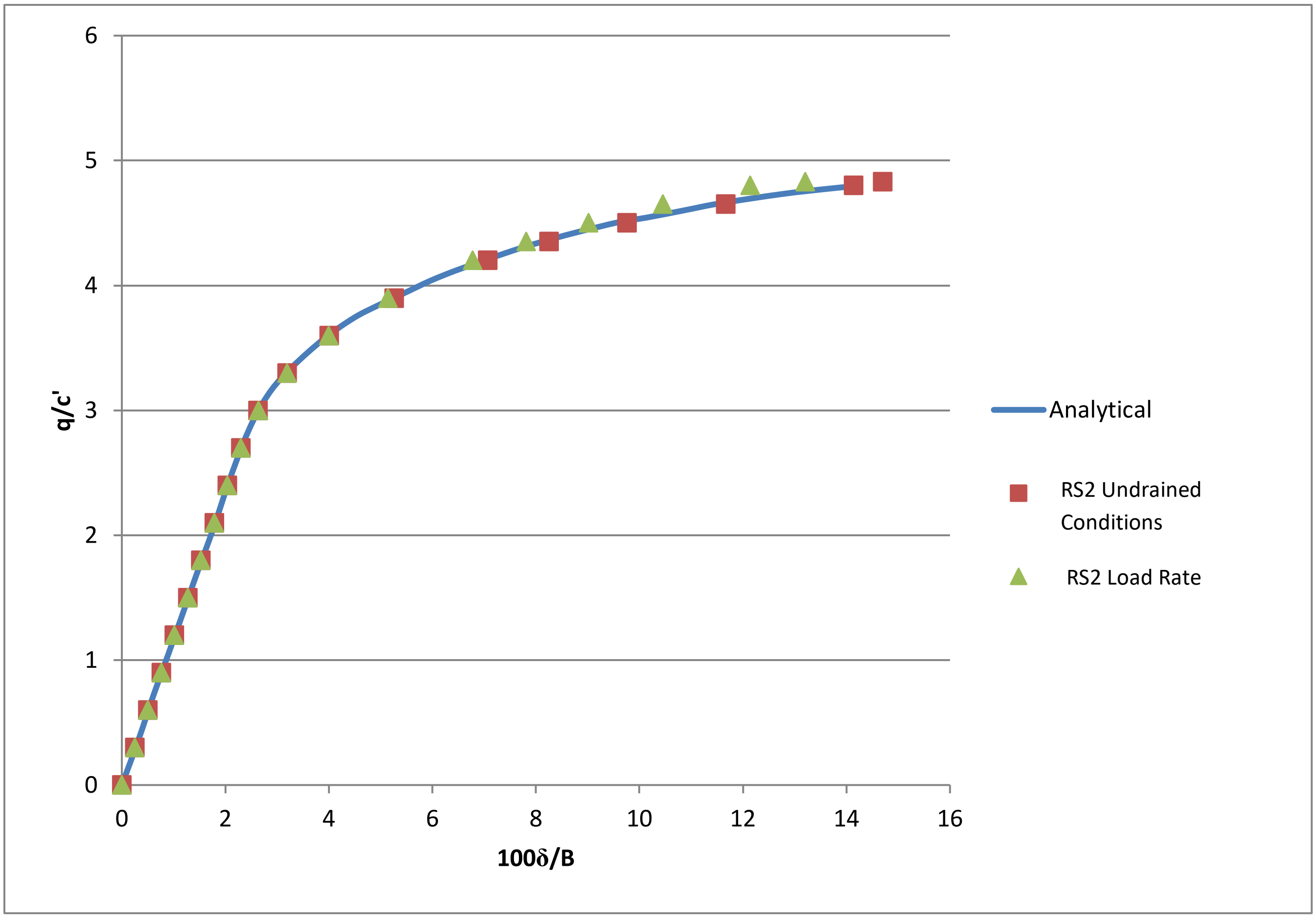



**Figure 5.2: Pressure versus displacement for undrained loading**

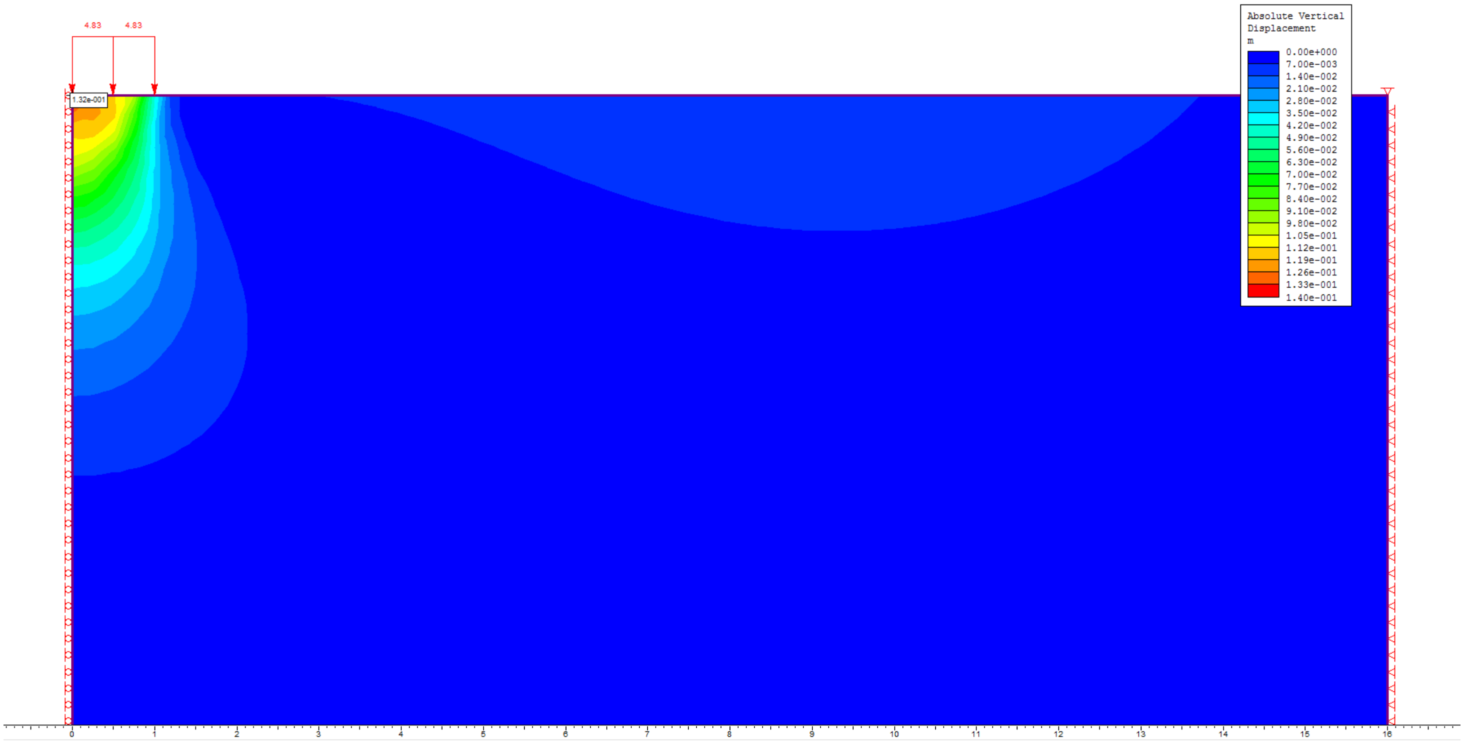



**Figure 5.3: Absolute vertical displacement contour as modeled in RS2**

## 5.4 References


1. Small, J.C., *Elasto-plastic consolidation of Soils*, PhD thesis, University of Sydney, 1977.

2. Prandtl, L., 'Spannungsverteilung in plastischen Koerpern', in *Proceedings of the 1st International Congress on Applied Mechanics*, Delft, 43-54, 1924.


## 5.5 Data Files

The input files **consolidation#005 (undrained conditions).fez** and **consolidation#005 (load rate).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 6 Strip Footing with Associated and Non-associated Flow Rules

## 6.1 Problem Description

### 6.1.1 Associated Flow Rule

This problem analyzes the behavior of a smooth flexible strip footing on an elastoplastic soil layer, under drained conditions. The dimensions, mesh quality, and equations for the time factor and coefficient of consolidation are the same as those in Example **Error! Reference source not found.**, but differs in the following model parameters outlined in Table 6-1.

**Table 6.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E') | 200 kPa |
| Poisson's ratio (ν') | 0.3 |
| Friction angle (ϕ') | 20º |
| Cohesion (c') | 1 kPa |
| Permeability (κ) | 0.01 m/s |
| Dilatancy angle (ψ') | 20º |
| Coefficient of consolidation ($c_{v2}$) | 0.1960 $m^2/s$ |

The model is shown in Figure 6-1, where the surface boundary is drained.

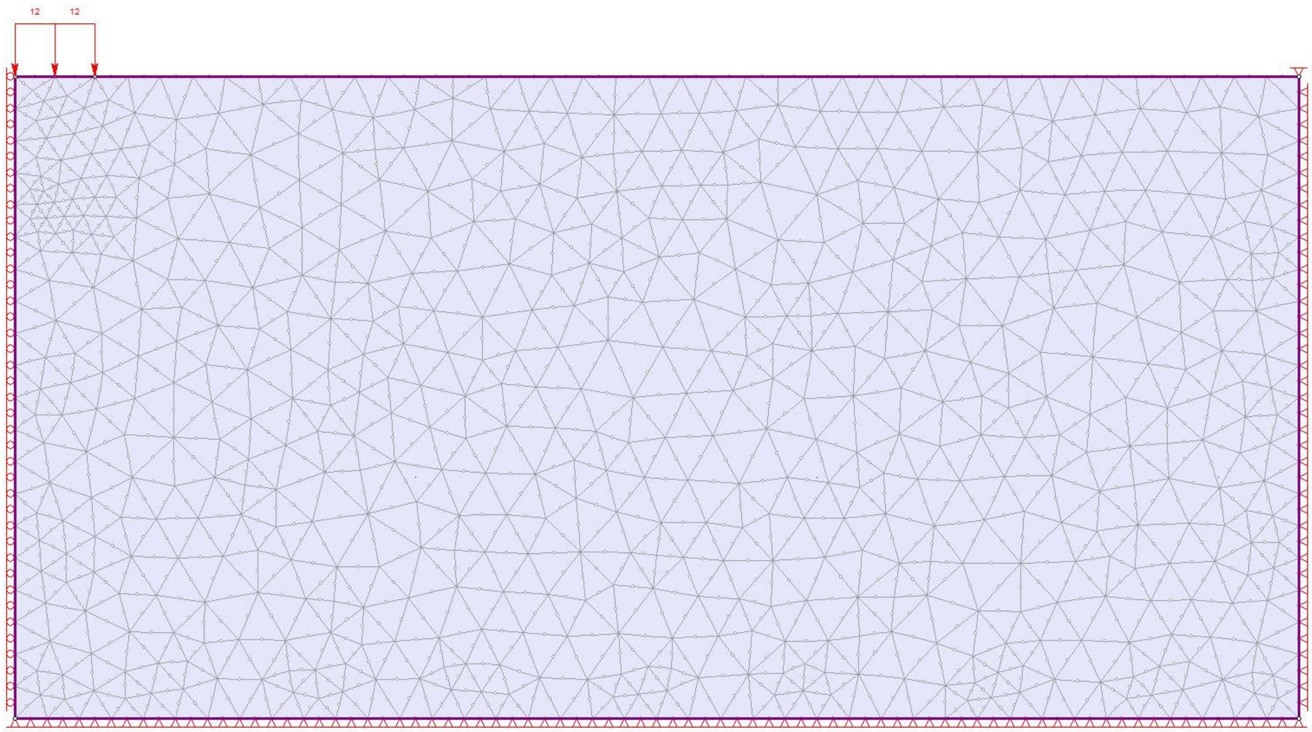



**Figure 6.1: Strip footing on elastoplastic layer under drained conditions as modeled in RS2**

Additionally, a ramp load is imposed until $T_{v0}$ = 0.01 at which it is held constant, displayed in Figure 6-2. In this problem, three different load values were applied, $q_0/c'$ = 5, $q_0/c'$ = 10, and $q_0/c'$ = 15.

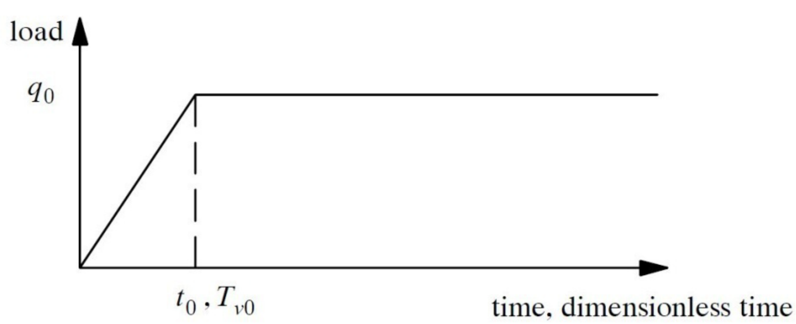



**Figure 6.2: Load vs. Time**

### 6.1.2 Non-associated Flow Rule

This problem uses the consolidation formula to predict the drained collapse pressure under a slow loading rate. The mesh quality and dimensions remain the same as that of the previous problem, with a different dilatancy angle of 0º. The soil model is shown in Figure 6-3 where the surface boundary is drained.

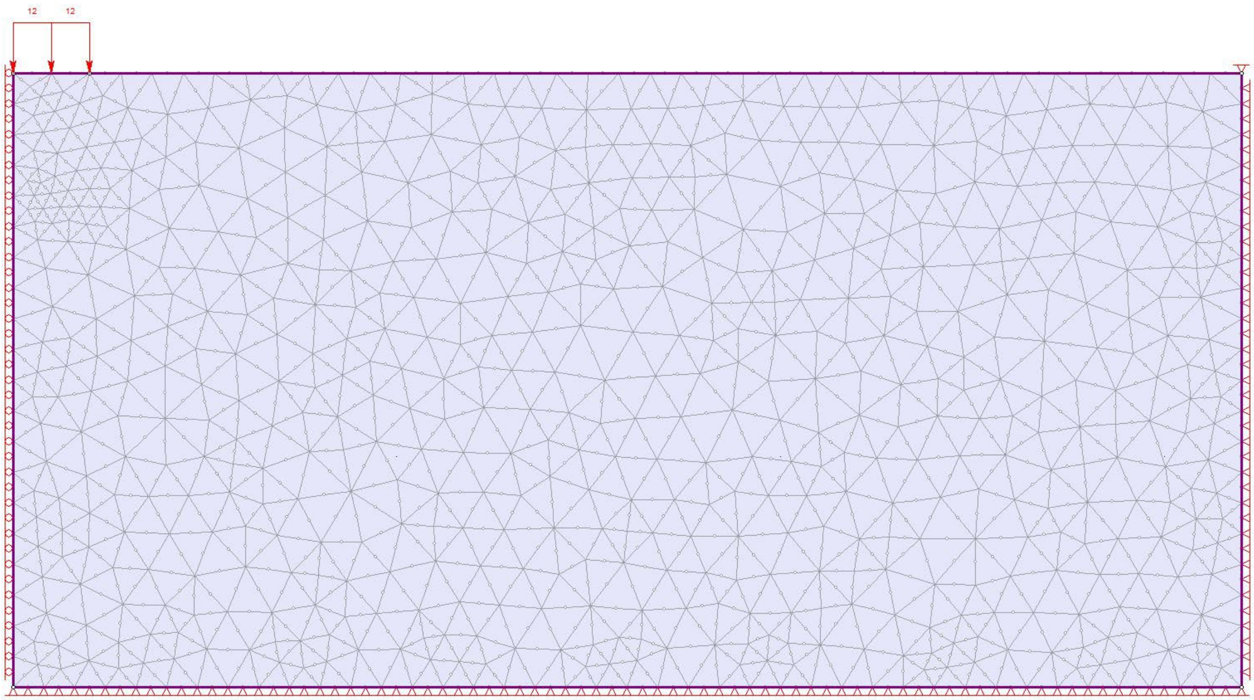



**Figure 6.3:** **Strip footing on elastoplastic layer under drained conditions as modeled in RS2**

According to Prandtl, the collapse pressure is given by the equation

$$\frac{q}{c'} = 14.83$$

Additionally, a slow load rate of $\omega$ = 0.015 was used for this example due to the drained conditions.

## 6.2 Analytical Solution

According to Manoharan and Dasgupta [1] regarding the finite element analysis of elastoplastic consolidation, is based on Biot's consolidation theory and has the following formulas:

**Displacement and pore pressure vector:**

$$\{\mathrm{u}\} = [N_u]\{\mathrm{u}_n\}$$

$$p = [N_p]\{\mathrm{p}_n\}$$

where $\{\mathrm{u}_n\}$ is the nodal displacement vector, $\{\mathrm{p}_n\}$ is the nodal pore pressure vector, and $[N_u]$ and $[N_p]$ are the shape functions.

**Strain in terms of nodal displacement:**

$$\{\epsilon\} = [B_u]\{\mathrm{u}_n\}$$

where $[B_u]$ is the strain-displacement matrix with the following equation

$$[B_u]^T = [N_u]^T \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{bmatrix}$$

**Pore pressure derivatives:**

$$\left[B_p\right] = \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{Bmatrix} [N_p]$$

## 6.3 Results

### 6.3.1 Associated Flow Rule

As can be seen in Figure 6-4, the RS2 solutions are in accordance with the analytical solutions for each load value, with the RS2 consolidation measured from $T_{v0} = 0.01$ under a constant load value. An example of the displacement contours for each load value show in Figure 6-5, Figure 6-6, and Figure 6-7.

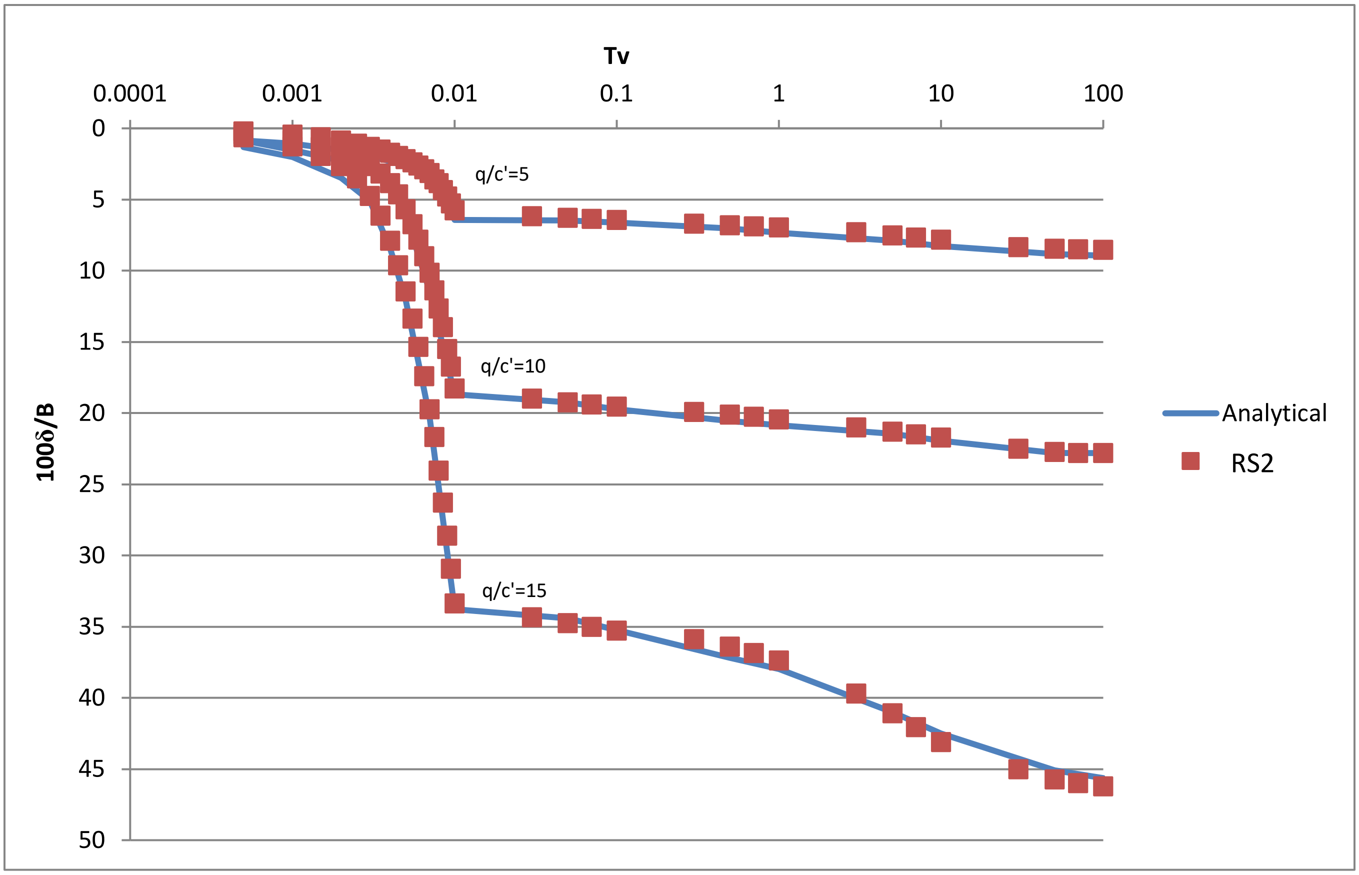



**Figure 6.4: Settlement versus time factor for elastoplastic strip footing**


**Figure 6.5: Absolute vertical displacement contour for q/c'= 5 as modeled in RS2**


**Figure 6.6: Absolute vertical displacement contour for q/c'= 10 as modeled in RS2**

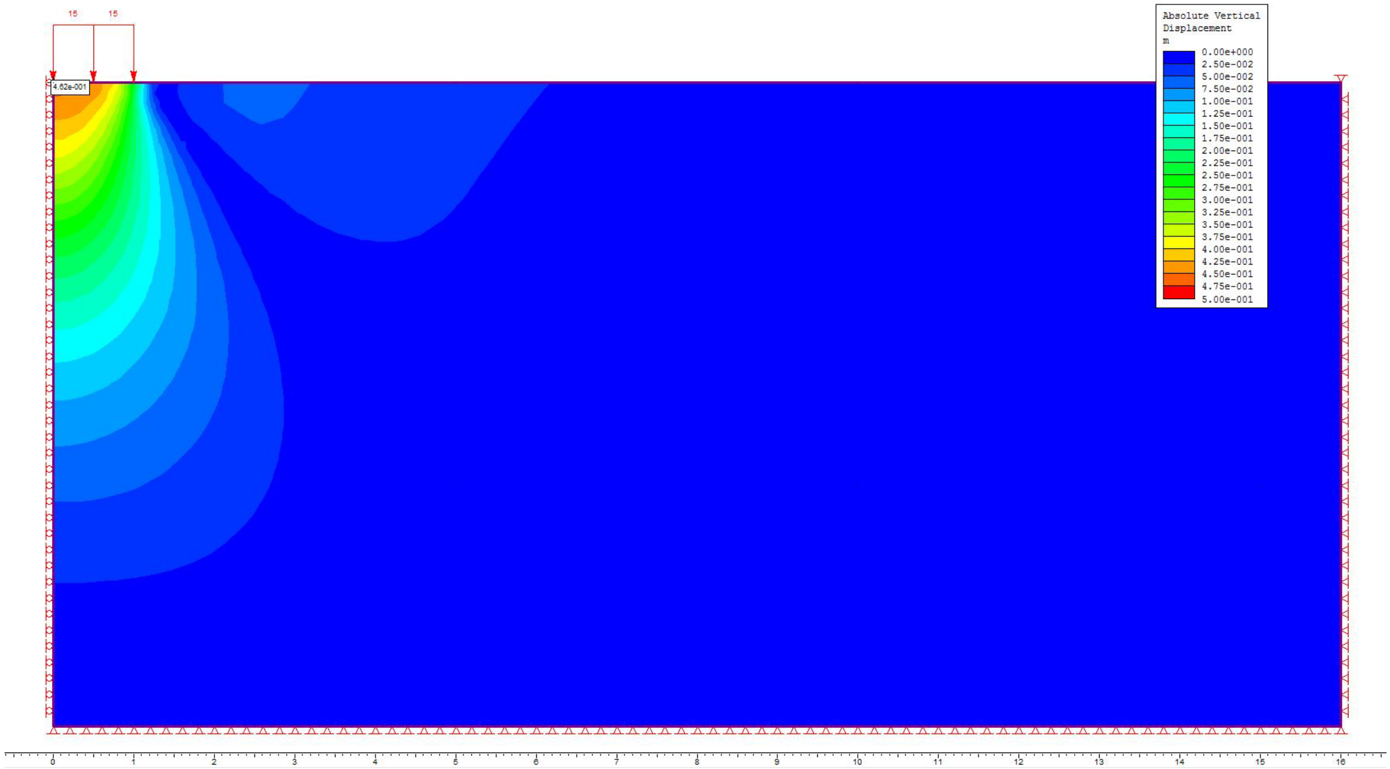



**Figure 6.7: Absolute vertical displacement contour for q/c'= 15 as modeled in RS2**

### 6.3.2 Non-associated Flow Rule

The solution determined from RS2 corresponds well with the analytical solution by Manoharan and Dasgupta, evident in Figure 6-8. An example of the contour for the displacement at the centre of the footing used for the RS2 solution is shown in Figure 6-9.

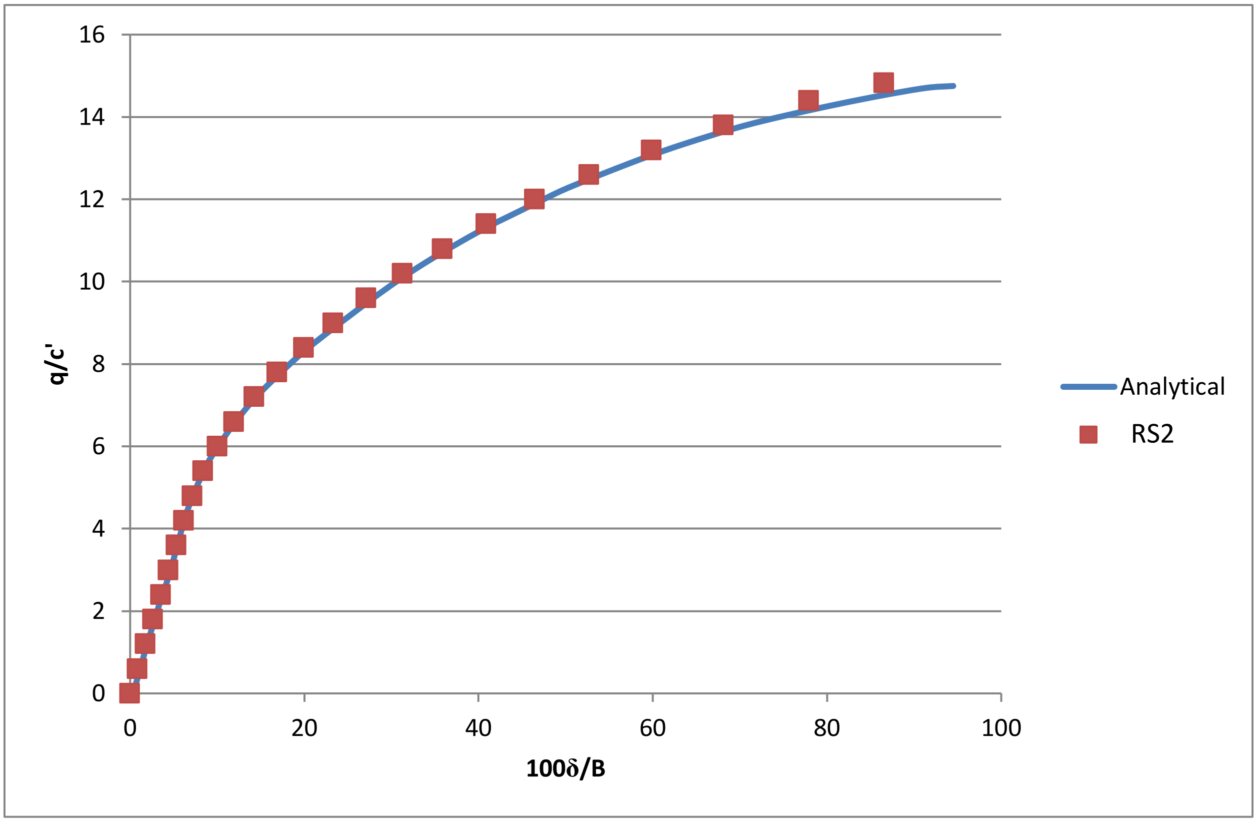



**Figure 6.8: Pressure versus displacement for flexible strip footing with varying loading rates**


**Figure 6.9: Absolute vertical displacement contour as modeled in RS2**

## 6.4 References


1. Manoharan, N. and Dasgupta, S.P., 'Consolidation analysis of elastoplastic soil', *Computers and Structures*, **54**, 1005-1021, 1995.


## 6.5 Data Files

The input files **consolidation#006_01 (q=5).fez** to **consolidation#006_01 (q=15).fez**, and **consolidation#006_02.fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 7 Uncoupled One-Dimensional Consolidation

## 7.1 Problem Description

This problem analyzes the correctness of the uncoupled consolidation approach using the equivalent value of mv to account for the effect of the solid deformation on the water flow. Model 1 using coupled approach and model 2 used the uncoupled approach.

The excess pore pressure dissipation of a one-dimensional consolidation under drained conditions with a permeable surface and impermeable base, with the application of a constant pressure over a dimensionless time period will be compared between the 2 approaches. The parameters for the soil models are outlined in Table 7.1.

**Table 7.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E) | 1000 kPa |
| Poisson's ratio (ν) | 0 |
| Strength criteria | Elastic |
| Permeability (κ) | 0.001 m/d |
| Thickness (H) | 1 m |
| Compressibility index ($m_v$) (for uncoupled) | 1D Elastic Consolidation |
| Coefficient of consolidation ($c_v$) | 0.1019 $m^2/d$ |

Both models use a graded eight-noded quadrilateral mesh with horizontal and vertical supports as shown in Figure 7.1.

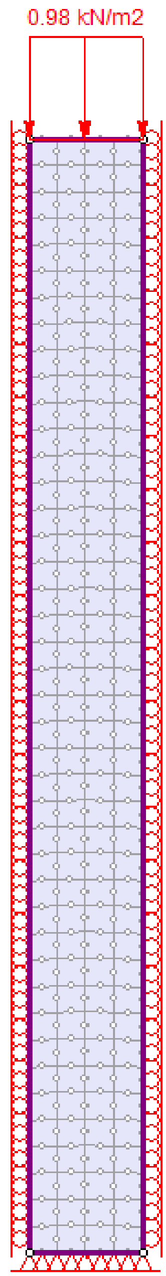



**Figure 7.1: Model mesh and load applied**

The dimensionless time of consolidation is determined by the equation

$$T_v = \frac{c_v t}{H^2}$$

where $c_v$ is the coefficient of consolidation described by the following equation, with k, E, and v being the drained permeability, the drained Young's modulus, and the drained Poisson's ratio of the soil, respectively.

$$C_v = \frac{kE(1-v)}{\gamma_w(1+v)(1-2v)}$$

The degree of consolidation is defined by the equation.

$$U = \frac{S_c}{S_{c,max}}$$

where Sc is the settlement of the soil layer at each time step, measured from H = 0m.

## 7.2 Results

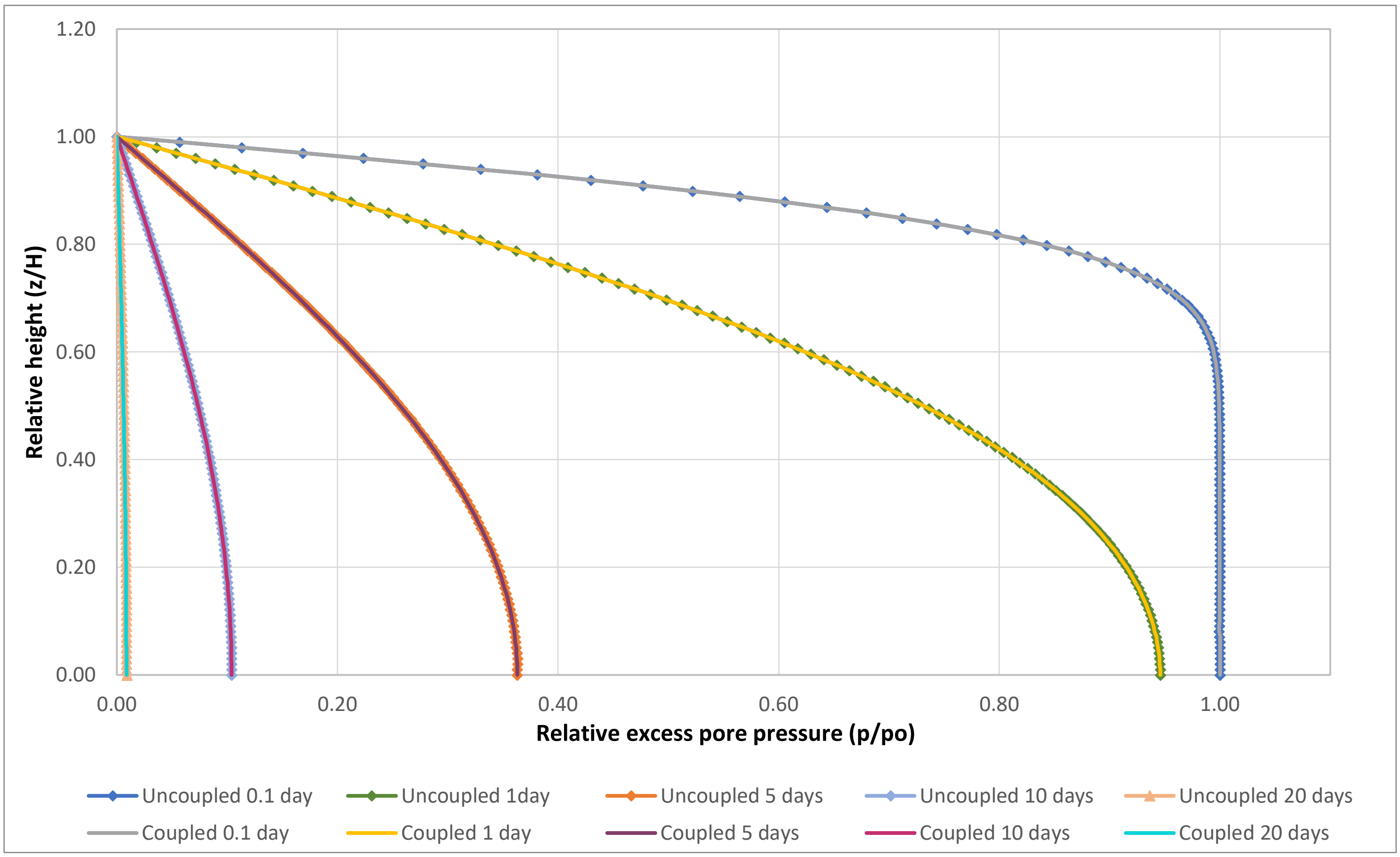



**Figure 7.2: Dissipation of Excess Pore Pressure**

From Figure 7.2 above, it can be seen that the excess pore pressure dissipation between the 2 approaches is very similar. The correctness of the uncoupled algorithm in 1D consolidation was verified.

## 7.3 References

1. Terzaghi, K. *'Die Berechnung der Durchlassigkeitsziffer des Tones aus demVerlauf der hydrodynamischen Spannungsersceinungen'*, Originally published in 1923 and reprinted in From Theory to Practice in Soil Mechanics, John Wiley and Sons, New York, 133-146, 1960.

## 7.4 Data Files

The input files **consolidation#007_01(coupled).fez** and **consolidation#007_02(uncoupled).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 8 Uncoupled Two-Dimensional Consolidation

## 8.1 Problem Description

This problem analyzes the correctness of the uncoupled consolidation approach using the equivalent value of $m_v$ to account for the effect of the solid deformation on the water flow. Model 1 using coupled approach and model 2 used the uncoupled approach.

The excess pore pressure dissipation of a two-dimensional consolidation under drained conditions with a permeable surface and impermeable base, with the application of a constant pressure over a dimensionless time period will be compared between the 2 approaches. The parameters for the soil models are outlined in Table 8.1.

**Table 8.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E) | 200 kPa |
| Poisson's ratio (ν) | 0.3 |
| Strength criteria | Elastic |
| Permeability (κ) | 0.01 m/s |
| Thickness (H) | 8 m |
| Compressibility index ($m_v$) (for uncoupled) | 2D Elastic Condolidation |
| Coefficient of consolidation ($c_v$) | 0.2744 $m^2/s$ |

Both models use a graded six-noded triangular mesh with horizontal and vertical supports as shown in Figure 8.1.

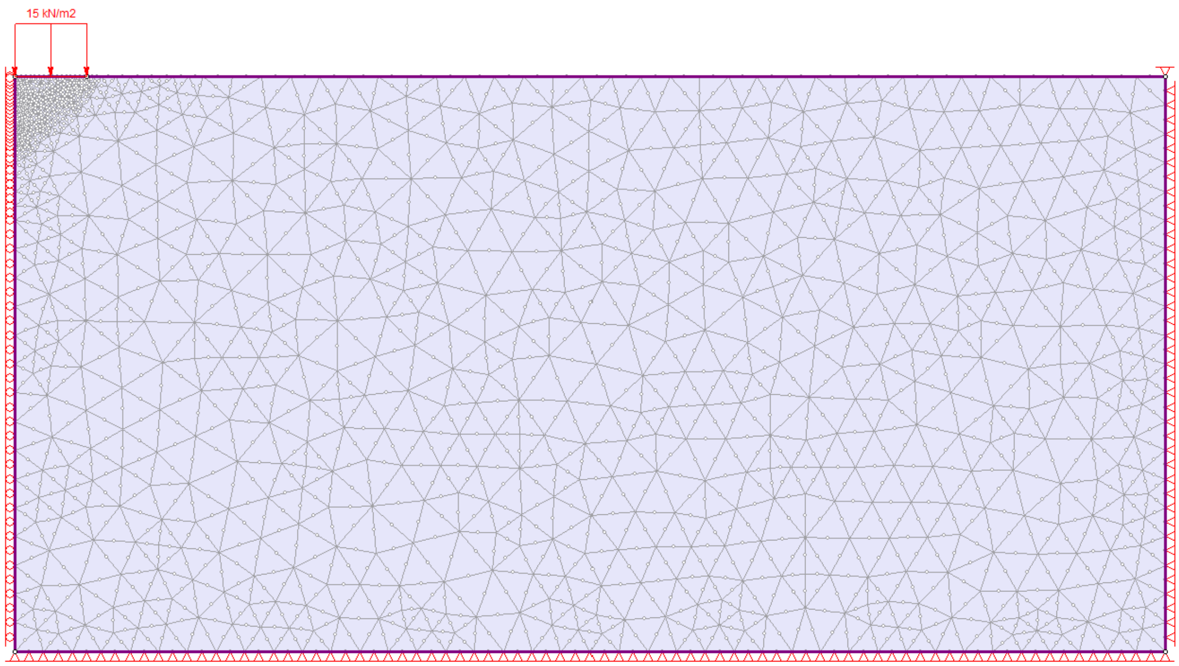



**Figure 8.1: Model mesh and load applied**

The dimensionless time of consolidation is determined by the equation

$$T_v = \frac{c_v t}{H^2}$$

where $c_v$ is the coefficient of consolidation described by the following equation, with k, E, and v being the drained permeability, the drained Young's modulus, and the drained Poisson's ratio of the soil, respectively.

$$C_v = \frac{kE(1-v)}{\gamma_w(1+v)(1-2v)}$$

The degree of consolidation is defined by the equation.

$$U = \frac{S_c}{S_{c,max}}$$

where $S_c$ is the settlement of the soil layer at each time step, measured from H = 0m.

## 8.2 Results

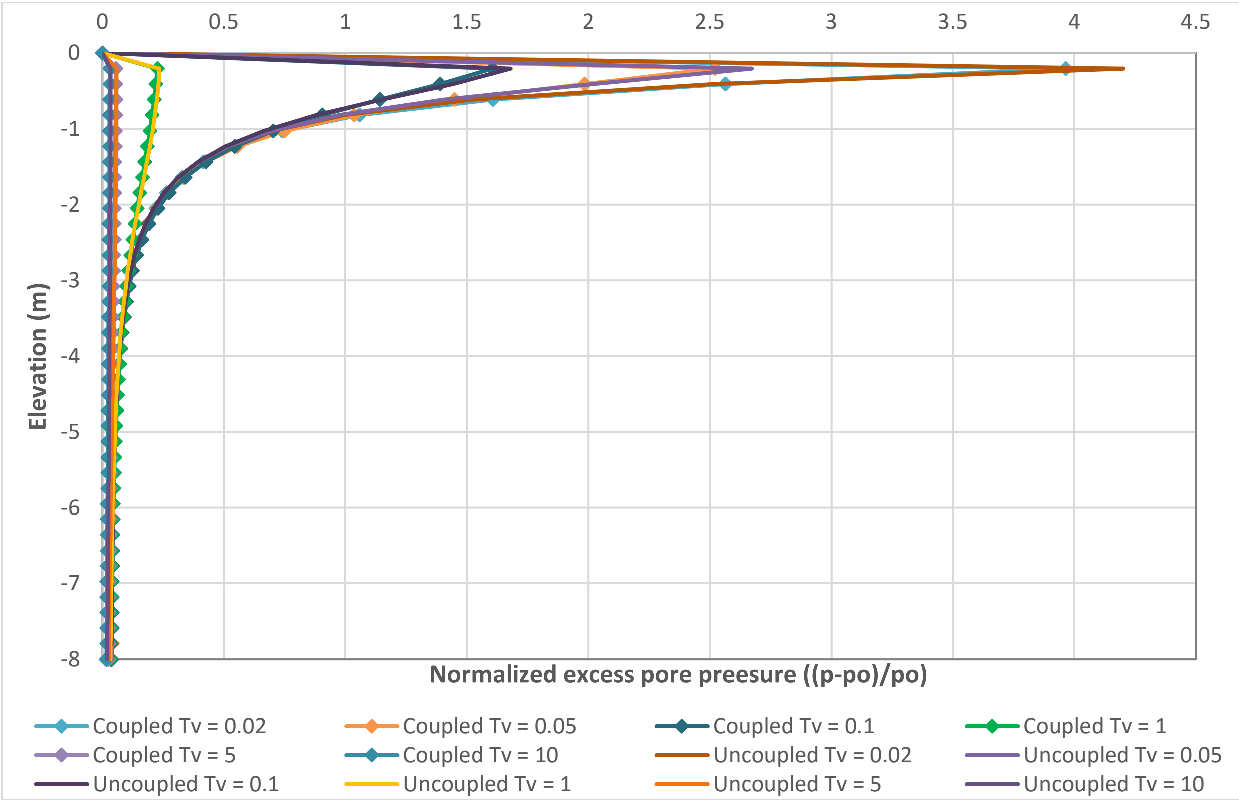



**Figure 8.2: Excess pore pressure dissipation**

From Figure 8.2 above, it can be seen that the excess pore pressure dissipation between the 2 approaches are very similar. The correctness of the uncoupled algorithm in 2D consolidation was verified.

## 8.3 References

1. Terzaghi, K. *'Die Berechnung der Durchlassigkeitsziffer des Tones aus demVerlauf der hydrodynamischen Spannungsersceinungen'*, Originally published in 1923 and reprinted in From Theory to Practice in Soil Mechanics, John Wiley and Sons, New York, 133-146, 1960.

## 8.4 Data Files

The input files **consolidation#008_01(coupled).fez** and **consolidation#008_02(uncoupled).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 9 Effective Stress Modelling in Unsaturated and Saturated Zones

## 9.1 Problem Description

This problem analyzes nine different methods in accounting for the unsaturated behavior of soil with single effective stress. See the RS2 Theory Manual – Soil Behaviors in Unsaturated Zones for more information.

The effective stress calculation is based on the formula in Equation (9-1) below (Bishop, 1959):

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \tag{9-1}$$

where $\sigma'$ is the effective stress, σ is total stress, $u_a$ is the pore-air pressure and $u_w$ is the pore water pressure, and $\chi$ is the coefficient obtained from the followed approach for each model.

However, in the RS2 models and many soil mechanics problems, $u_a$ is assumed to be equal to the atmospheric pressure. Hence, the effective stress formula shown in Equation (9-2) is implied in the models.

$$\sigma' = \sigma + I\chi p^w \tag{9-2}$$

where $p^w$ is the suction and $I$ is the unit matrix.

In RS2, the χ coefficient can be calculated as:

1. Bishop (1959)

   $$\chi = S_r \tag{9-3}$$

   where $S_r$ is the degree of saturation, calculated as $S_r = \frac{\theta}{n}$, where $\theta$ is the water content and $n$ is the soil porosity.
2. Tabular values with respect to $p^w$
3. Tabular values with respect to $S_r$
4. Tabular values with respect to $S_e$
   where $S_e$ is effective degree of saturation and calculated as $S_e = \frac{S_r - S_{re}}{S_{sat} - S_{re}}$, where $S_{re}$ and $S_{sat}$ is the residual degree of saturation and maximum degree of saturation.
5. Gudehus (1995)

   $$\chi = S_r(2 - S_r) \tag{9-4}$$

   where $S_r$ is the degree of saturation

6. Khalili (2004)

$$\chi = \begin{cases} \left(\frac{s}{s_e}\right)^{-0.55} & if\ s > s_e \\ 1 & if\ s \leq s_e \end{cases} \tag{9-5}$$

where $s$ is the matric suction, $s = u_a - u_w$, and $s_e$ is the air entry suction.

7. Bolzon (1996)

$$\chi = S_e \tag{9-6}$$

where $S_e$ is the effective degree of saturation.

8. Aitchison (1960)

$$\chi = \begin{cases} 1 & if\ S_r = 1 \\ (\alpha/s)s_e & if\ S_r < 1 \end{cases} \tag{9-7}$$

Where $\alpha$ is a unitless material parameter, $s$ is the matric suction, $s = u_a - u_w$, $s_e$ is the air entry suction, and $S_r$ is the degree of saturation.

9. Kohgo (1993)

$$\sigma' = \sigma - u_{eq} \tag{9-8}$$

where $u_{eq}$ is called the equivalent pore pressure. This pressure is aimed at averaging the effects of all fluid pressures within the pores. It is also designed to recover Terzaghi's effective stress on saturated states. Consequently, authors had to express the equivalent pore pressure in terms of air entry suction value ($s_e$), a critical suction ($s_c$), and a material parameter ($a_e$):

$$u_{eq} = u_a - s \quad if\ s \leq s_e \tag{9-9}$$

$$u_{eq} = u_a - \left(s_e + \frac{s_c - s_e}{(s - s_e) + a_e}(s - s_e)\right) \quad if\ s > s_e \tag{9-10}$$

This formulation is equivalent to using Bishop's method with

$$\chi = a_e\,(s_c - s_e)/(s - s_e + a_e)^2 \tag{9-11}$$

## 9.2 Model Information

The problem uses a uniform six-noded triangular meshed soil column with horizontal and vertical supports as shown in Figure 9.1 below.

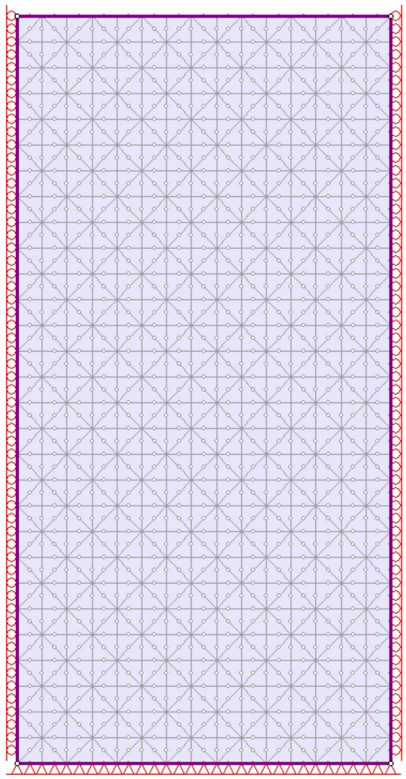


**Figure 9.1: Soil column modelled in RS2**

Nine models were created in RS2 covering each of the approaches, with each model number corresponding to the number of its approach. Table 9.1 below shows the soil parameters applied in all the models used to run the tests.

**Table 9.1: Model Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (E) | 10000 kPa |
| Poisson's ratio (ν) | 0.2 |
| Porosity (n) | 0.36 |
| Initial Pore Water Pressure | -10 kPa |
| Strength criteria | Elastic |
| Van Genuchten Alpha (1/m) | 2.24 |
| Van Genuchten n | 2.286 |

All the models use the same Van Genuchten function represented in the water retention curve illustrated in Figure 9.2 below:

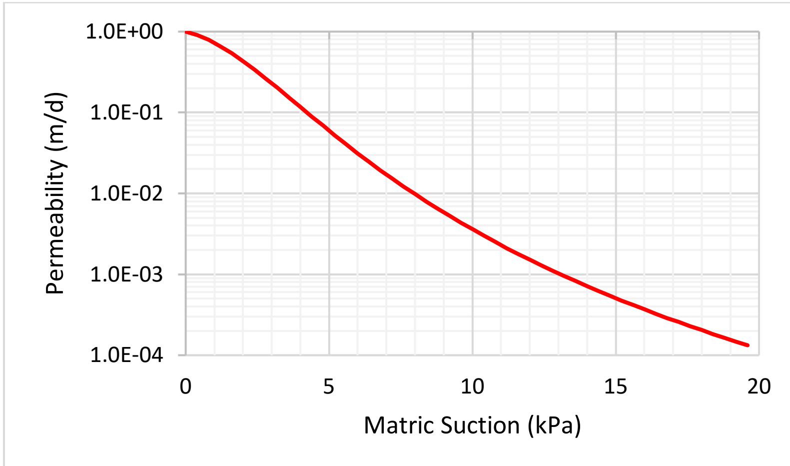



**Figure 9.2: Van Genuchten Function of Models**

In the unsaturated zones, each model had a different set of soil parameters. These parameters are listed in Table 9.2 to Table 9.5, which correspond to approaches 4, 6, 8, and 9, respectively:

**Table 9.2: Unsaturated Soil Parameters for the Model considering the Tabular Values with Respect to the Effective Degree of Saturation**

| ***Parameter*** | ***Value*** |
|---|---|
| Residual Water Content, wcr | 0 |
| Peak Water Content, wc | 0.5 |
| Porosity, n | 0.5 |

**Table 9.3: Unsaturated Soil Parameters for Khalili's Model**

| ***Parameter*** | ***Value*** |
|---|---|
| Air Entry Suction, se | 5 kPa |

**Table 9.4: Unsaturated Soil Parameters for Aitchison's Model**

| ***Parameter*** | ***Value*** |
|---|---|
| Air Entry Suction, se | 10 kPa |
| Alpha Parameter, α | 0.32 |

**Table 9.5: Unsaturated Soil Parameters for Khogo's Model**

| ***Parameter*** | ***Value*** |
|---|---|
| Air Entry Suction, se | 5 kPa |
| Critical Suction, sc | 118 kPa |
| Material Parameter, ae | 98 |

The graphs representing the tabulated values for approaches 2, 3, and 4, are shown on Figure 9.3, Figure 9.4, and Figure 9.5, respectively:

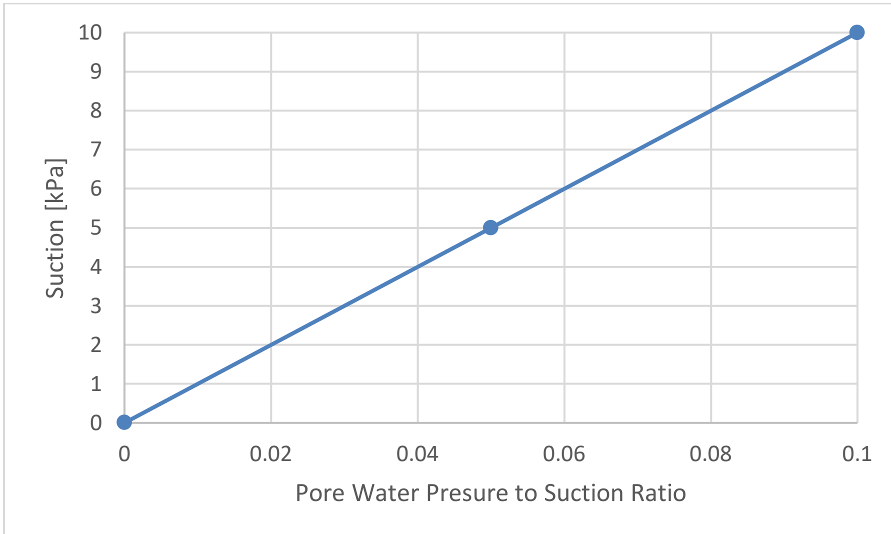



**Figure 9.3: Suction versus Pore Water Pressure to Suction Ratio for the Tabulated Values with Respect to $p^w$**

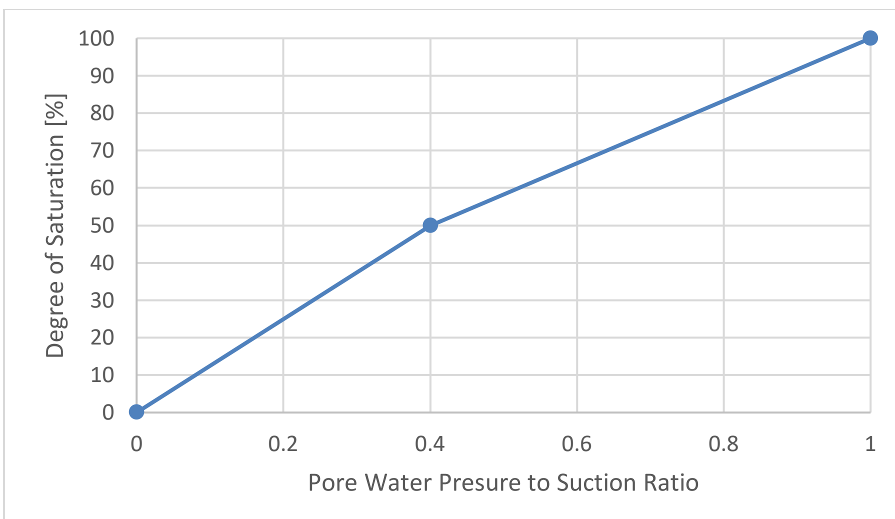



**Figure 9.4: Degree of Saturation versus Pore Water Pressure to Suction Ratio for the Tabulated Values with Respect to $S_r$**

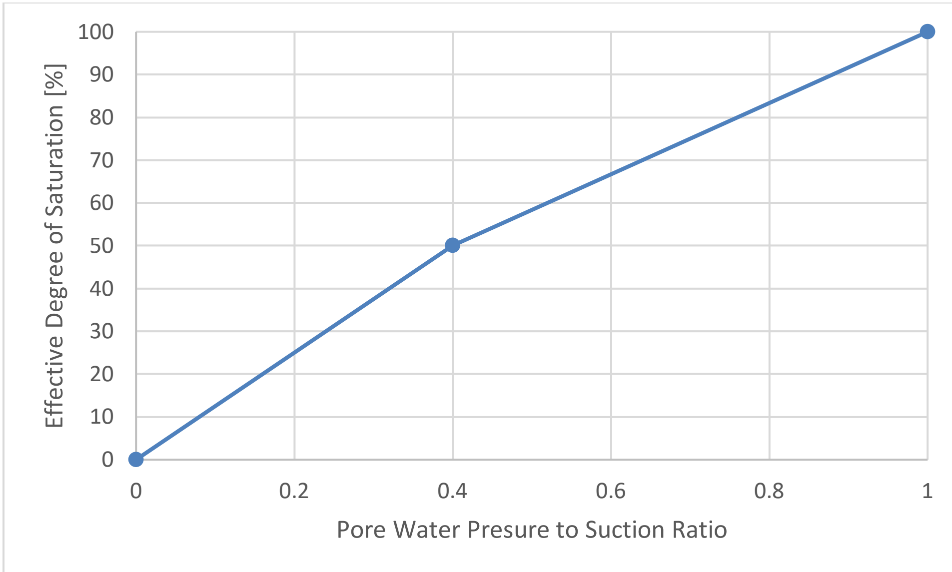



**Figure 9.5: Effective Degree of Saturation versus Pore Water Pressure to Suction Ratio for the Tabulated Values with Respect to $S_e$**

## 9.3 Results

Figure 9.6 to Figure 9.14 show the results of approaches 1 to 9 mentioned above in their respective order. Each figure compares the RS2 results for the approach with the analytical result utilizing the $\chi$ coefficient.

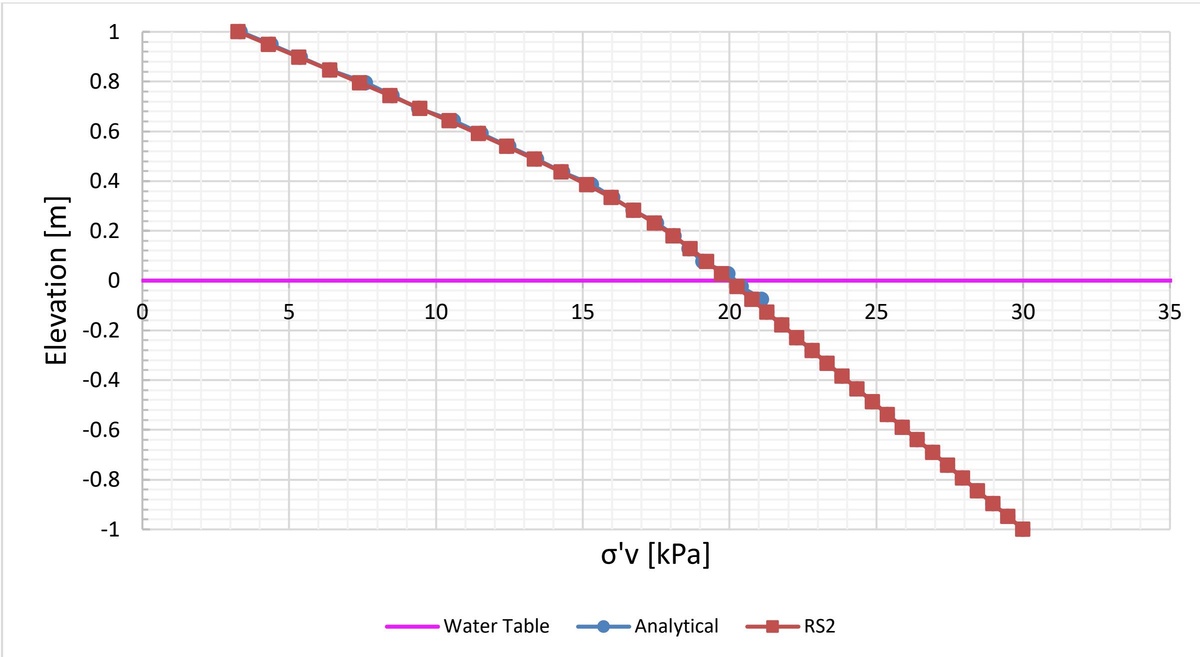



**Figure 9.6: Effective Stress (kPa) versus Depth (m) with Bishop's approach**

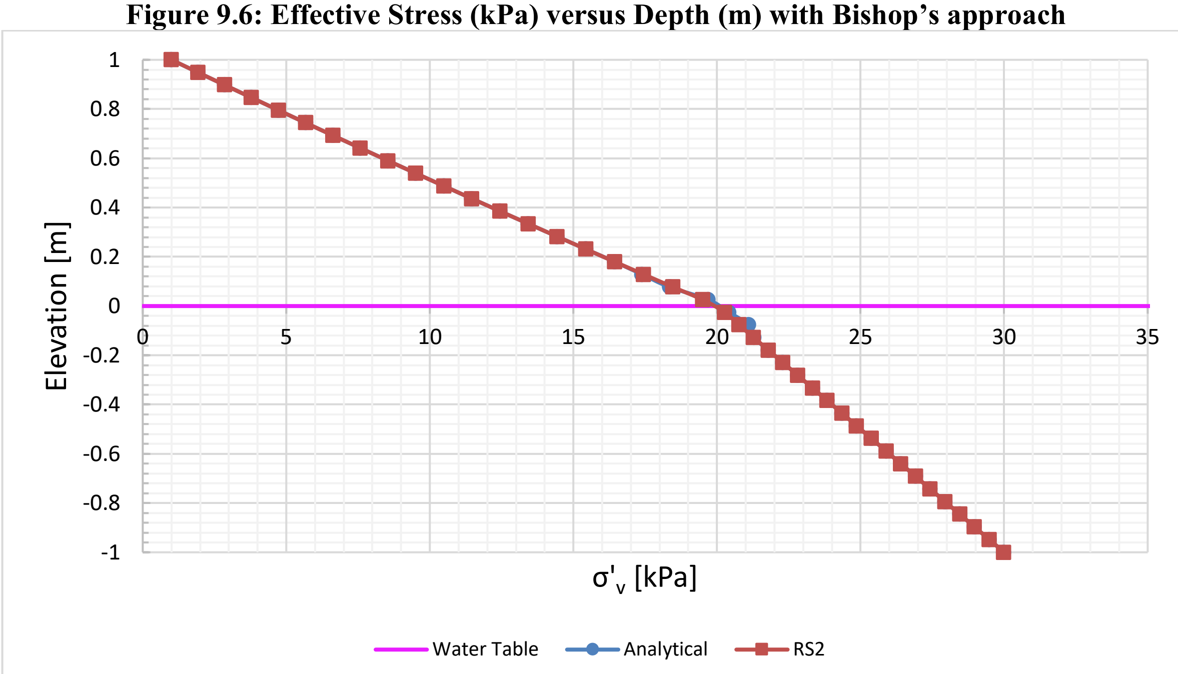



**Figure 9.7: Effective Stress (kPa) versus Depth (m) considering the tabular values with respect to suction**

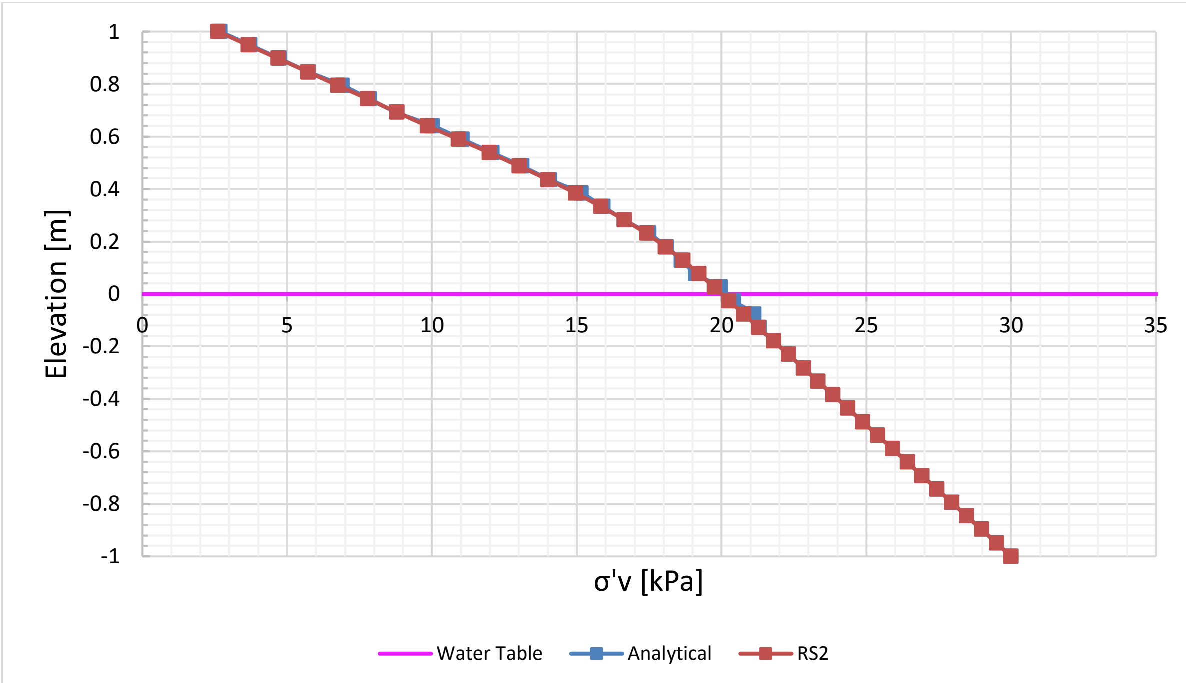



**Figure 9.8: Effective Stress (kPa) versus Depth (m) considering the tabular values with respect to the degree of saturation**

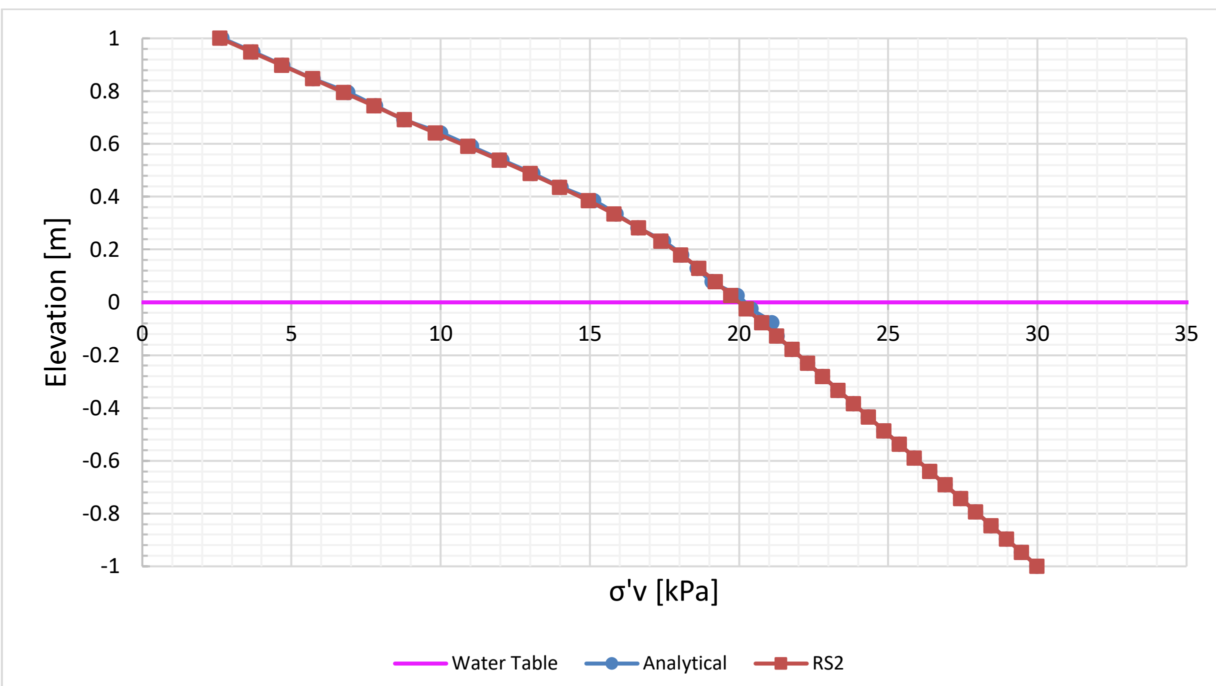



**Figure 9.9: Effective Stress (kPa) versus Depth (m) considering the tabular values with respect to the effective degree of saturation**

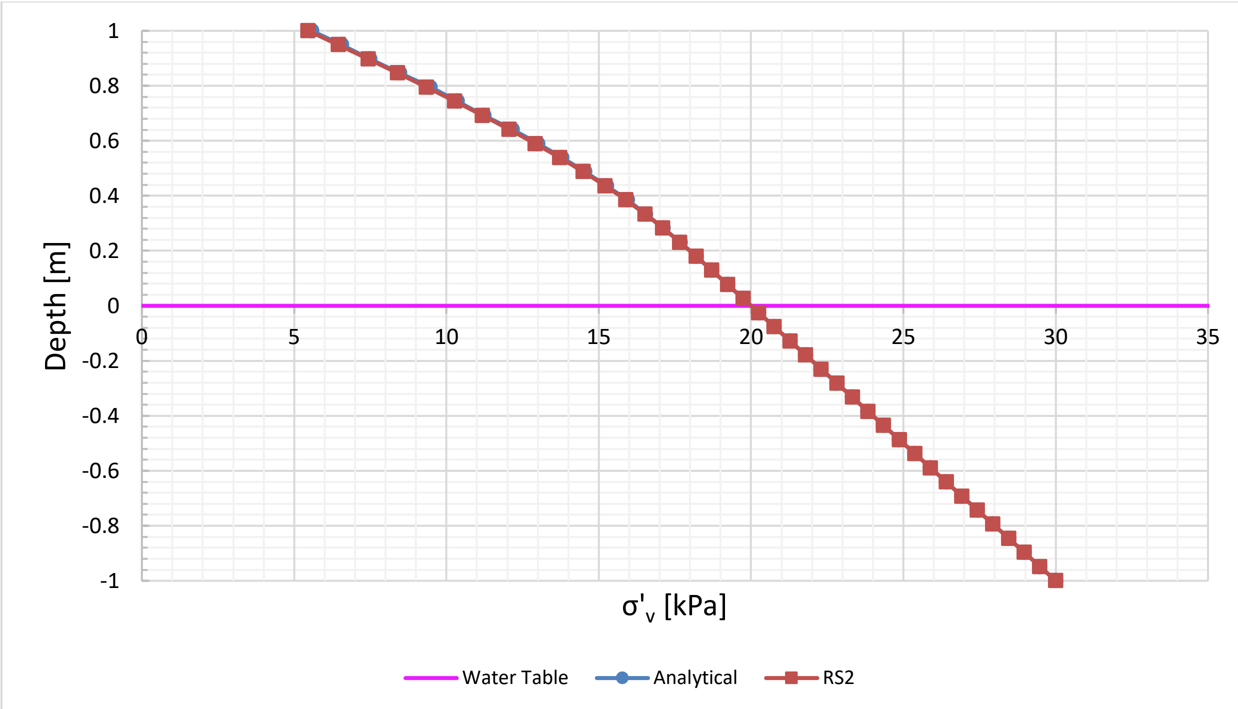



**Figure 9.10: Effective Stress (kPa) versus Depth (m) with Gudehus's approach**

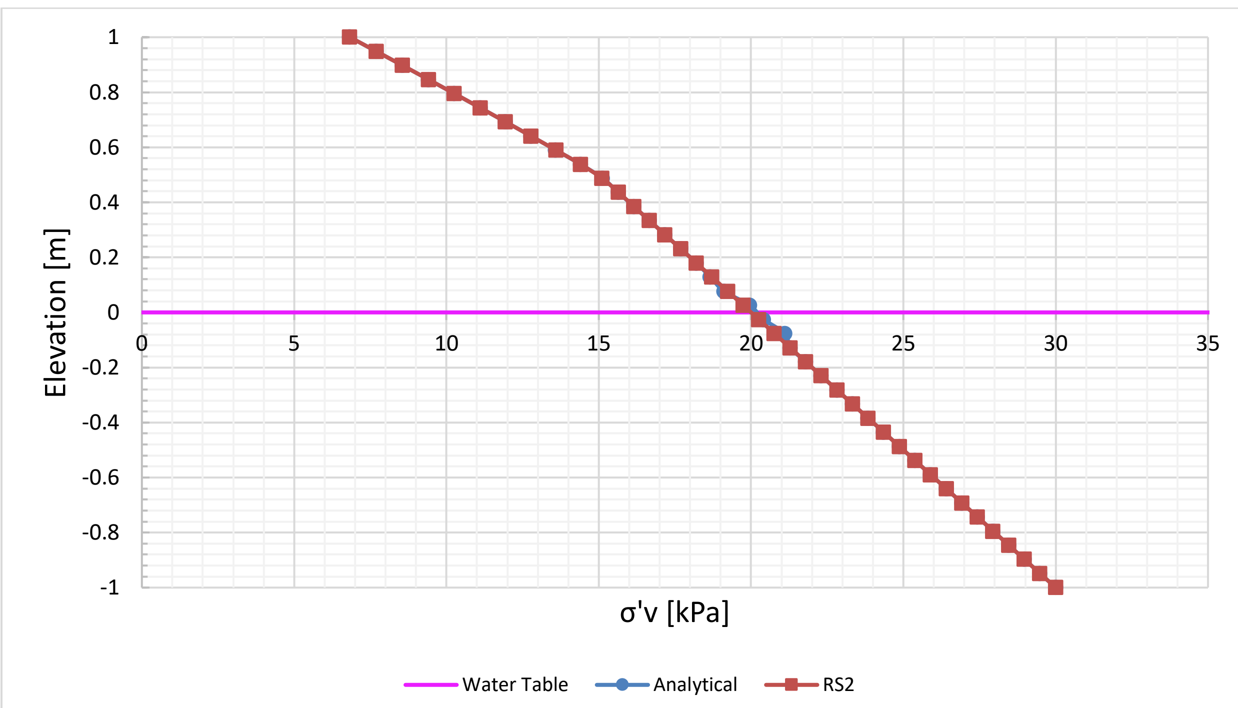



**Figure 9.11: Effective Stress (kPa) versus Depth (m) with Khalili's approach**

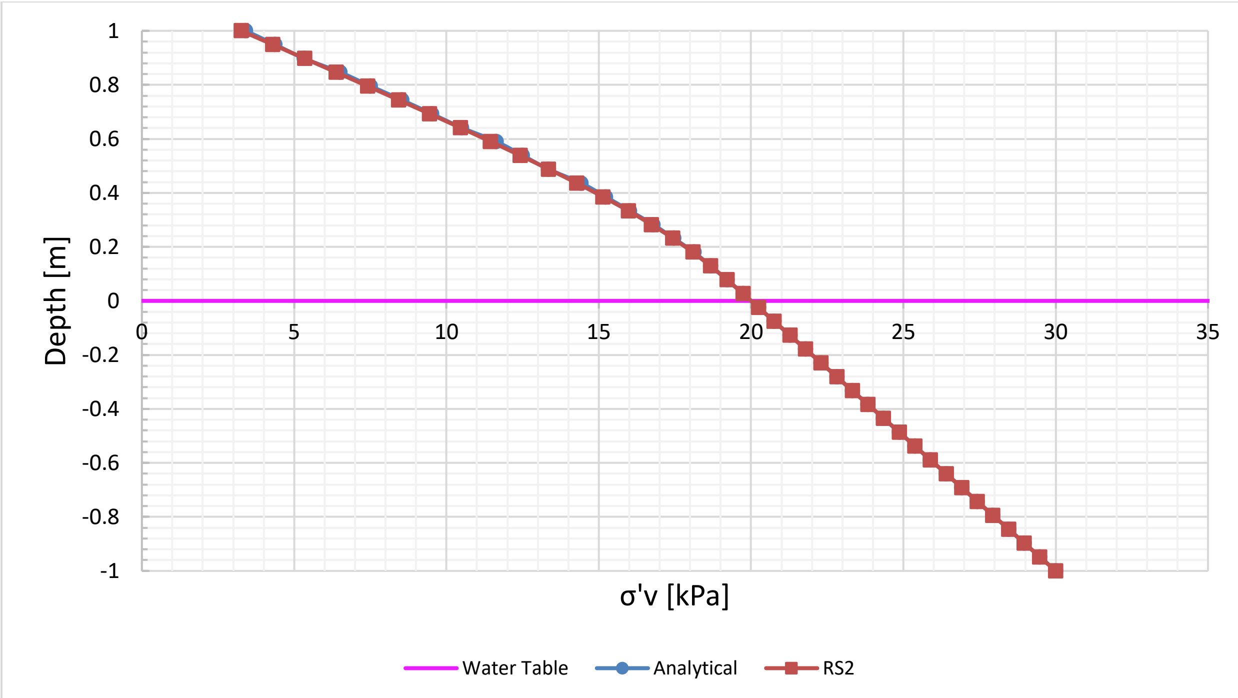



**Figure 9.12: Effective Stress (kPa) versus Depth (m) with Bolzon's approach**

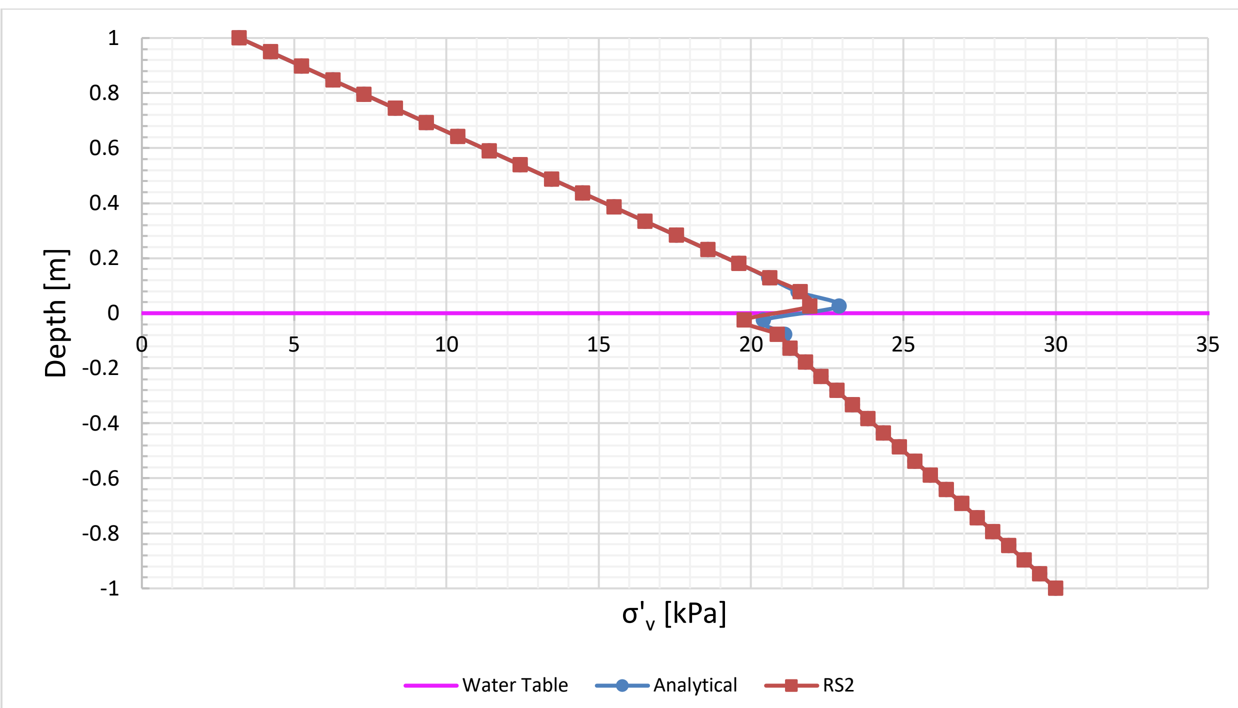



**Figure 9.13: Effective Stress (kPa) versus Depth (m) with Aitchison's approach**

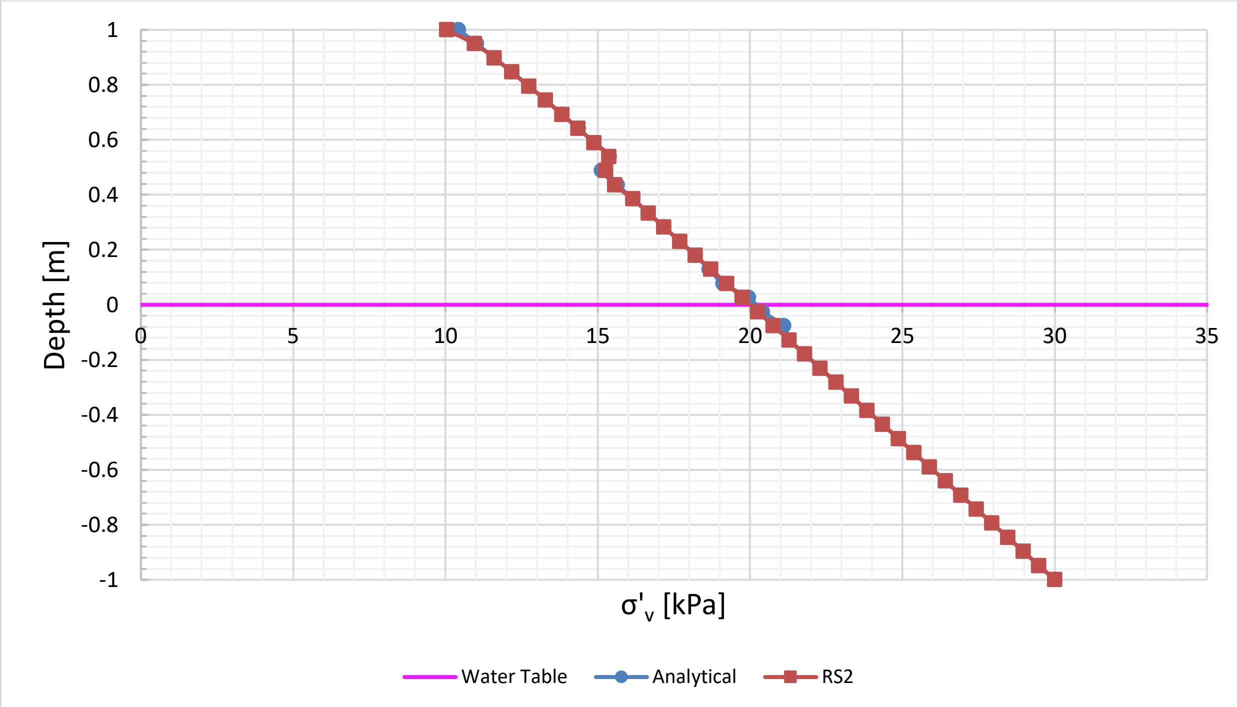



**Figure 9.14: Effective Stress (kPa) versus Depth (m) with Kohgo's approach**

All the results from RS2 are compiled in Figure 9.15 below. Note that for the "None" data set, unsaturated zone is not included in the model. Overall, all the effective stresses converged in the saturated zone below the water table with very minimal differences. However, above the water table, all the approaches showed similar trends of a linear relationship between the effective stress and the depth, with the main differences being the initial effective stress at the ground surface besides for the Aitchison approach.

It should be noted that Aitchison's approach was the only one that had greater effective stress magnitudes above the water table, especially at the depth of 0.92m down to a depth of around 1.80m.

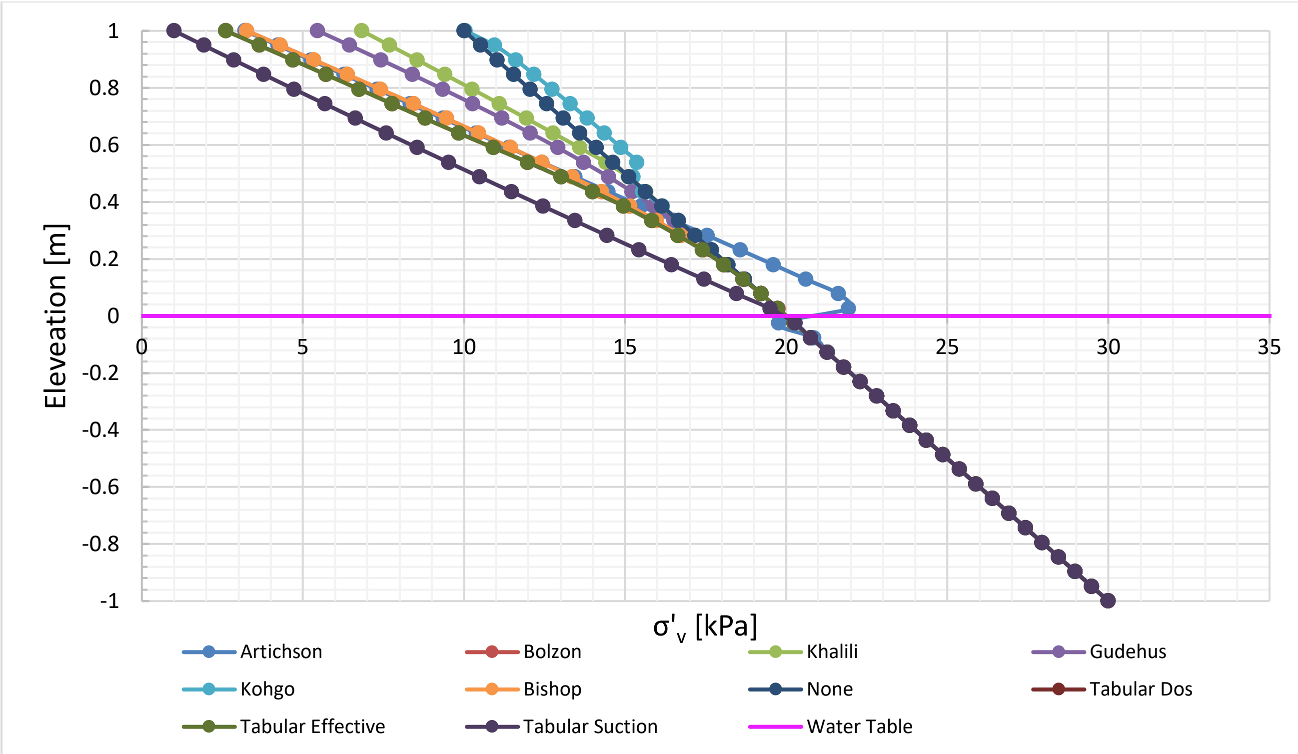



**Figure 9.15: Depth (m) versus Effective Stress (kPa) with RS2 Computations**

## 9.4 References


1. Aitchison, G. D. (1960). *Relationships of moisture stress and effective stress functions in unsaturated soils.* London: Butterworths.

2. Bishop, A. W. (1959). *The principle of effective stress.* Tecknish Ukeblad 106, 859-863.

3. Bolzon, G., Schrefler, B., & Zienkiewicz, O. (1996). *Elastoplastic soil constitutive laws generalized to partially saturated states.* Géotechnique, 46(2), 279–289.

4. Gudehus, G. (1995). *A comprehensive concept for non-saturated granular bodies, in Proceedings of the 1st International Conference on Unsaturated Soils.* Paris, Balkem, Rotterdam.

5. Khalili, N., Geiser, F., & Blight, G. E. (2004). *Effective Stress in Unsaturated Soils: Review with New Evidence.* International Journal of Geomechanics, 4(2), 115–126. doi:10.1061/(ASCE)1532-3641(2004)4:2(115).

6. Kohgo, Y., Nakano, M., & Miyazaki, T. (1993). *Theoretical aspects of constitutive modelling for unsaturated soils.* 33(4), 49-63. doi:33(4):49–63.

## 9.5 Data Files

The input files include:

1. **Consolidation#009_01(Bishop).fez**
2. **Consolidation#009_02(Suction).fez**
3. **Consolidation#009_03(Dos)fez**
4. **Consolidation#009_04(Eff_dos).fez**
5. **Consolidation#009_05(Gudehus).fez**
6. **Consolidation#009_06(Khalili).fez**
7. **Consolidation#009_07(Bolzon).fez**
8. **Consolidation#009_08(Aitchison).fez**
9. **Consolidation#009_09(Kohgo).fez**
10. **Consolidation#009_10(none).fez**

They can be downloaded from the RS2 Online Help page for Verification Manuals.

# 10 Porous Medium Consolidation with Liakpolous's Experiment

## 10.1 Problem Description

This problem analyzes the consolidation in porous media. Both uncoupled and coupled consolidations are considered. The problem is based on an experiment in Section 5.7 from "The Finite Element Method in the Static and Dynamic Deformation and Consolidation in Porous Media" (Lewis & Schrefler, 1998). As attributed to Liakpolous's experiment, a 1m x 0.1 m column of Del Monte sand is modelled. The RS2 results are compared with the benchmark experiment results.

## 10.2 Model Description and Set-up

Figure 10.1 illustrates the geometry of the prepared model on RS2, along with its use of a uniform four-noded quadrilateral mesh set-up.

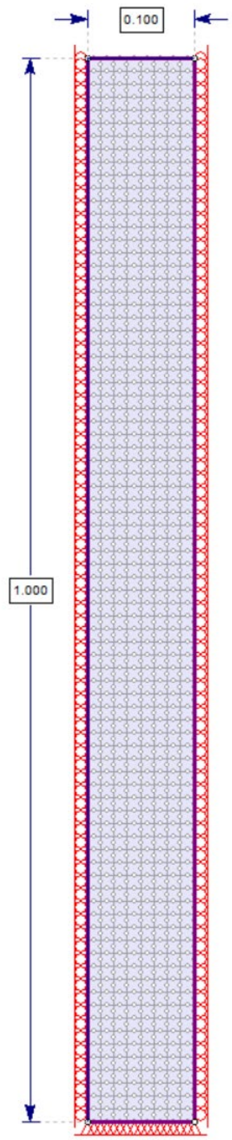



**Figure 10.1: Model Geometry and Mesh**

The soil parameters utilized in the model are shown in Table 10.1, which were based on the parameters listed in experiment (Lewis & Schrefler, 1998, p. 168).

**Table 10.1: Sand Parameters on RS2 Models**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (kPa) | 1300 |
| Poisson's ratio (ν) (-) | 0.4 |
| Peak Friction angle (ϕ) (º) | 35 |
| Peak Cohesion (c) (kPa) | 10.5 |
| Peak Tensile Strength (kPa) | 0 |
| Unit Weight (kN/m3) | 19.612 |

| | |
|---|---|
| Porosity (-) | 0.2975 |
| Failure Criterion | Mohr-Coulomb |
| Material Type | Elastic |
| Material Behavior | Drained |
| Mv Model | 1D Elastic Consolidation |

The permeability function used in the analysis is shown in Figure 10.2 below. Data can be found in the model file.

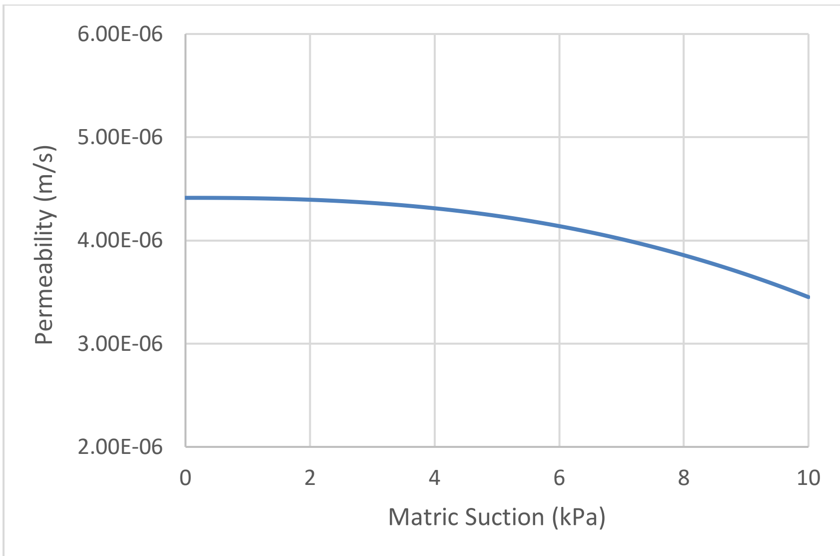



**Figure 10.2: Permeability function for the soil material**

The analyses were run on two separate models, one in uncoupled consolidation, and the other one in coupled consolidation. The readings of the pore pressure, vertical displacement, and degree of saturation were recorded at the 5, 10-, 20-, 30-, and 60-minute marks, following the testing in Liakpolous's experiment.

## 10.3 Results

The computed RS2 results were obtained, and they were compared to the results obtained from the experiment (Lewis & Schrefler, 1998, p. 170-172). Figure 10.3 to Figure 10.5 illustrate the data comparisons of pore pressure, vertical displacement, and degree of saturation, respectively.

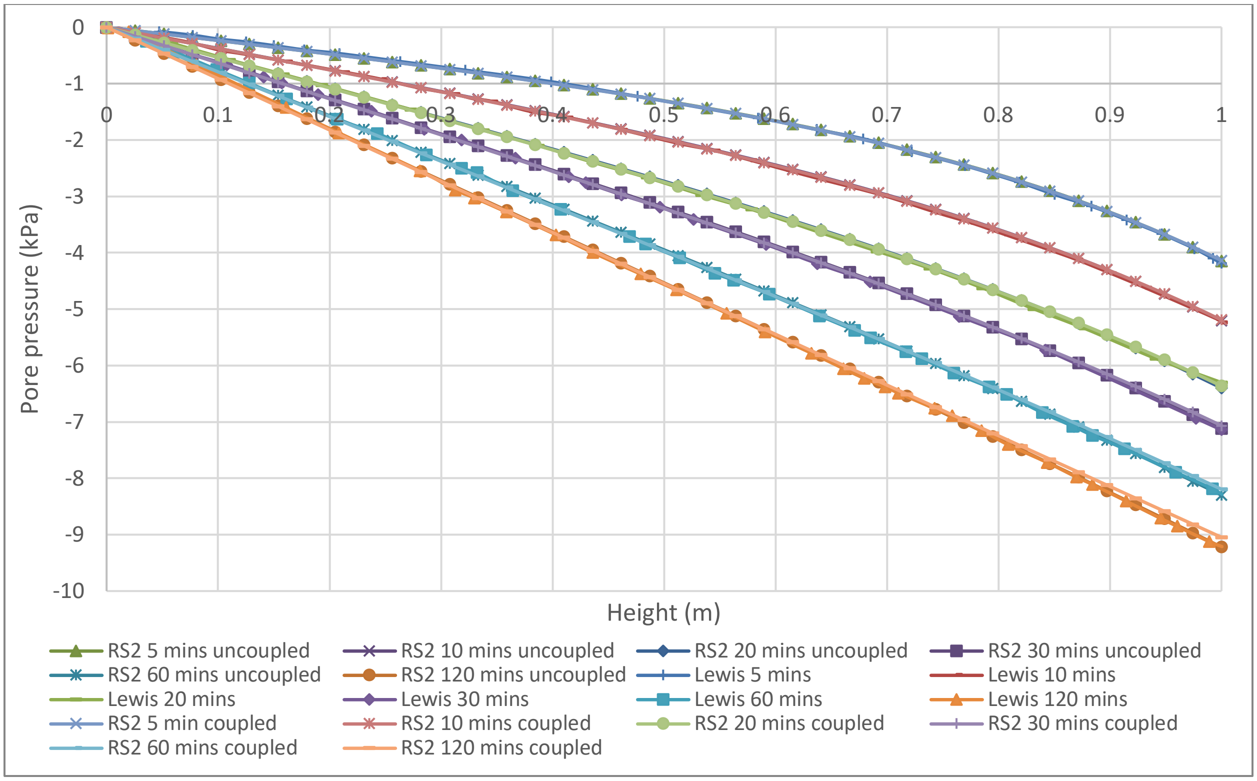



**Figure 10.3: Pore Pressure Graphs**

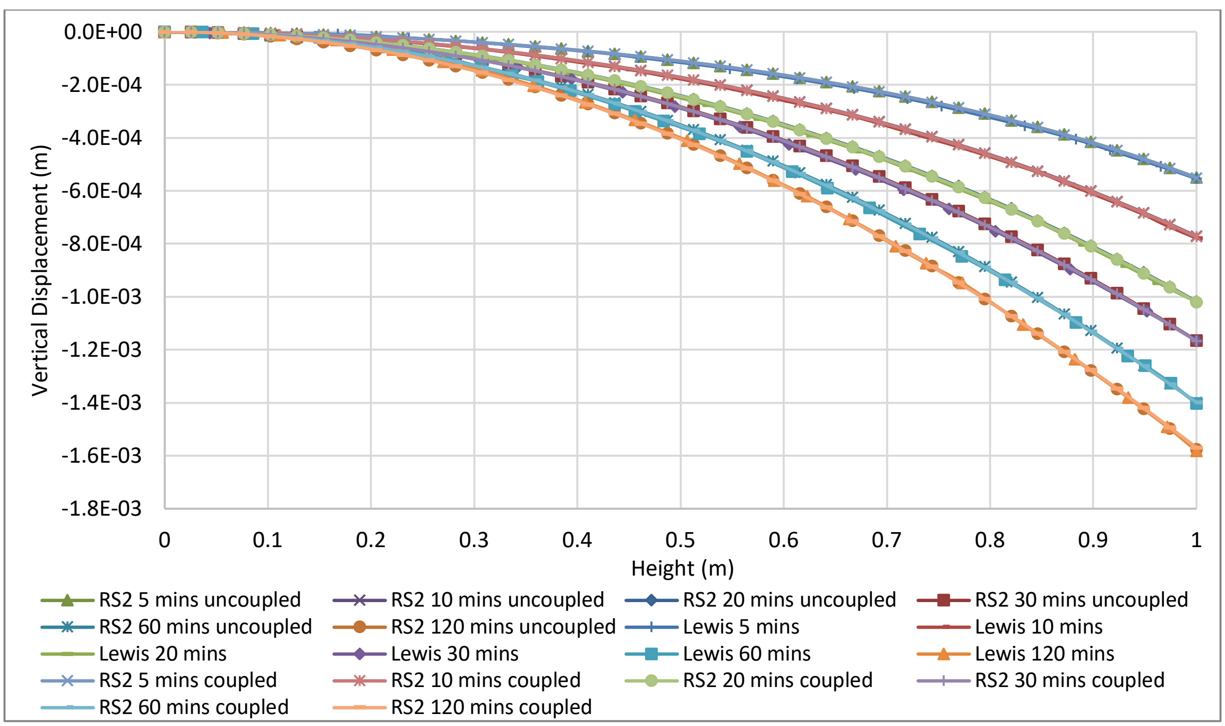



**Figure 10.4: Vertical Displacement Graphs**

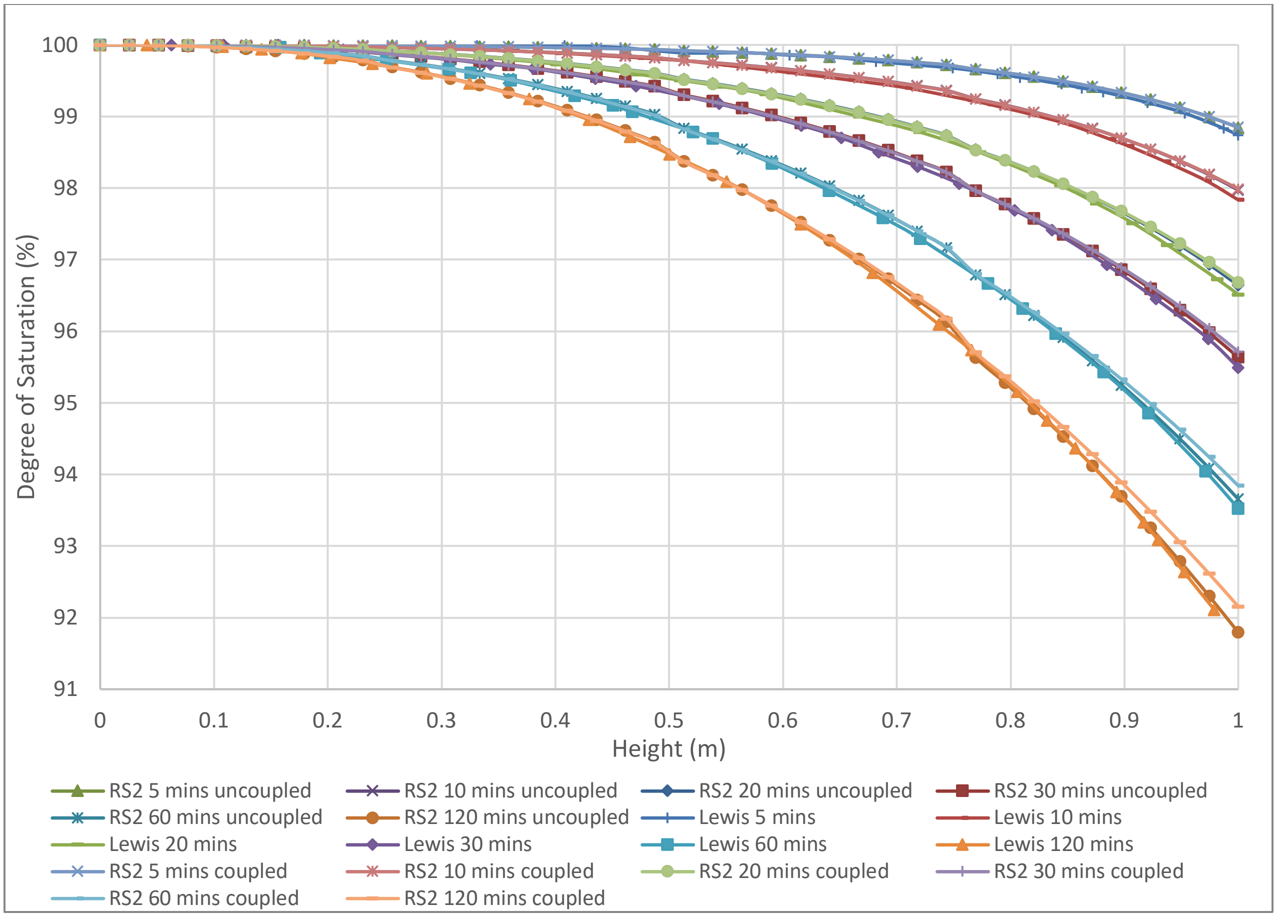



**Figure 10.5: Degree of Saturation Graphs**

## 10.4 Conclusions

Overall, the RS2 results were almost identical or very close to the numerical results within an acceptable margin. It demonstrates that both uncoupled and coupled consolidations could be successfully implemented to model the behavior of a porous medium.

## 10.5 References


Lewis, R. W., & Schrefler, B. (1998). *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media (2nd ed.)*. John Wiley & Sons Ltd.


## 10.6 Data Files

The input files **consolidation#010_01(uncoupled).fez** and **consolidation#010_02(coupled).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.

# 11 Unit Weights under Different Soil Conditions

## 11.1 Problem Description

This problem analyzes the use of unit weight under three different soil conditions. In RS2, you can either apply a uniform unit weight to the material, or account for moisture content in unit weight, which utilizes dry ($\gamma_d$), moist ($\gamma_{moist}$), and saturated ($\gamma_{sat}$) unit weights, and potentially the degree of saturated, to calculate unit weight for various soil conditions.

In this problem, the first scenario employs uniform unit weight for the soil, the second scenario addresses moist soil, and the last one considers unsaturated soil. The second and third scenarios accounted for the moisture content in unit weight. Furthermore, only the third scenario uses the single effective stress calculation for unsaturated zone, thus incorporates the degree of saturation. For more information about the Unit Weight in RS2, see the linked topic.

## 11.2 Models Set-up and Mesh

The geometry of the models has been set up as 0.1m x 1.0m samples with a uniform four-noded quadrilateral mesh set-up as seen in Figure 11.1. The water table for all models is at the elevation of 0.5m.

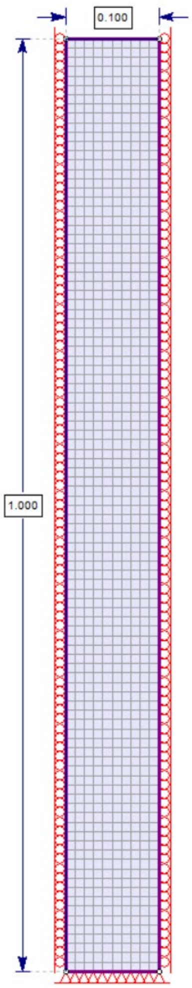



**Figure 11.1: Model Geometry and Mesh**

The soil parameters utilized in all models are shown in Table 11.1 below.

**Table 11.1: Sand Parameters on RS2 Models**

| ***Parameter*** | ***Value*** |
|---|---|
| Young's modulus (kPa) | 1300 |
| Poisson's ratio (ν) (-) | 0.4 |
| Peak Friction angle (ϕ) (º) | 35 |
| Peak Cohesion (c) (kPa) | 10.5 |
| Peak Tensile Strength (kPa) | 0 |
| Unit Weight (kN/m3) | 20 |
| Porosity (-) | 0.4 |
| Failure Criterion | Mohr-Coulomb |
| Material Type | Elastic |
| Material Behavior | Drained |

Three different models are analyzed as follows.

### 11.2.1 Uniform unit weight

The first model considers uniform unit weight for the soil. In the model, the "Account for moisture content in unit weight" option is unselected, and the unsaturated behavior is not included.

### 11.2.2 Moist Unit Weight

The second model considers soil above the ground level, and unsaturated component for calculating effective stress is not considered, thus the moist unit weight is utilized. In the model, the "Account for moisture content in unit weight" option is selected with input data as shown in Table 11.2 above. The unsaturated behavior is not included.

### 11.2.3 Unsaturated unit weight

The third model considers soil above ground level, and unsaturated component for calculating effective stress is considered. Thus, the unit weight will be calculated as:

$$\gamma = \gamma_d + n \cdot \mathrm{S} \cdot \gamma_w \tag{11-1}$$

Where $\gamma$ is the soil unit weight, $\gamma_d$ is the dry unit weight, $\gamma_w$ is the unit weight of water, n is the soil porosity, and S is the degree of saturation. Or:

$$\gamma = \gamma_d + \theta \cdot \gamma_w \tag{11-2}$$

Where $\theta$ is the volumetric water content of the sample.

It should be noted that when the unit weight yielded from the equation, $\gamma$, is larger than the saturated unit weight input, $\gamma_{sat}$, then $\gamma$ will be set to $\gamma_{sat}$.

In the model, the "Account for moisture content in unit weight" option is selected with input data as shown in Table 11.2 below. The unsaturated effective stress is calculated using the Bishop (1959) method.

Thus, the unit weight will be attained by Equation (11-1) or (11-2) as stated above.

**Table 11.2: Account for Moisture Content in Unit Weight Parameters**

| ***Parameter*** | ***Value*** |
|---|---|
| Dry Unit Weight (kN/m3) | 16 |
| Moist Unit Weight (kN/m3) | 18 |
| Saturated Unit Weight (kN/m3) | 20 |

## 11.3 Results

All three RS2 models produce results that correspond very well with the analytical results. For each model, the results for both vertical effective stress and vertical total stress are compared. Figure 11.2, Figure 11.3, and Figure 11.4 display those results for the uniform unit weight (Model 1), moist unit weight using the formulation (Model 2), and unsaturated unit weight above ground level (Model 3) respectively.

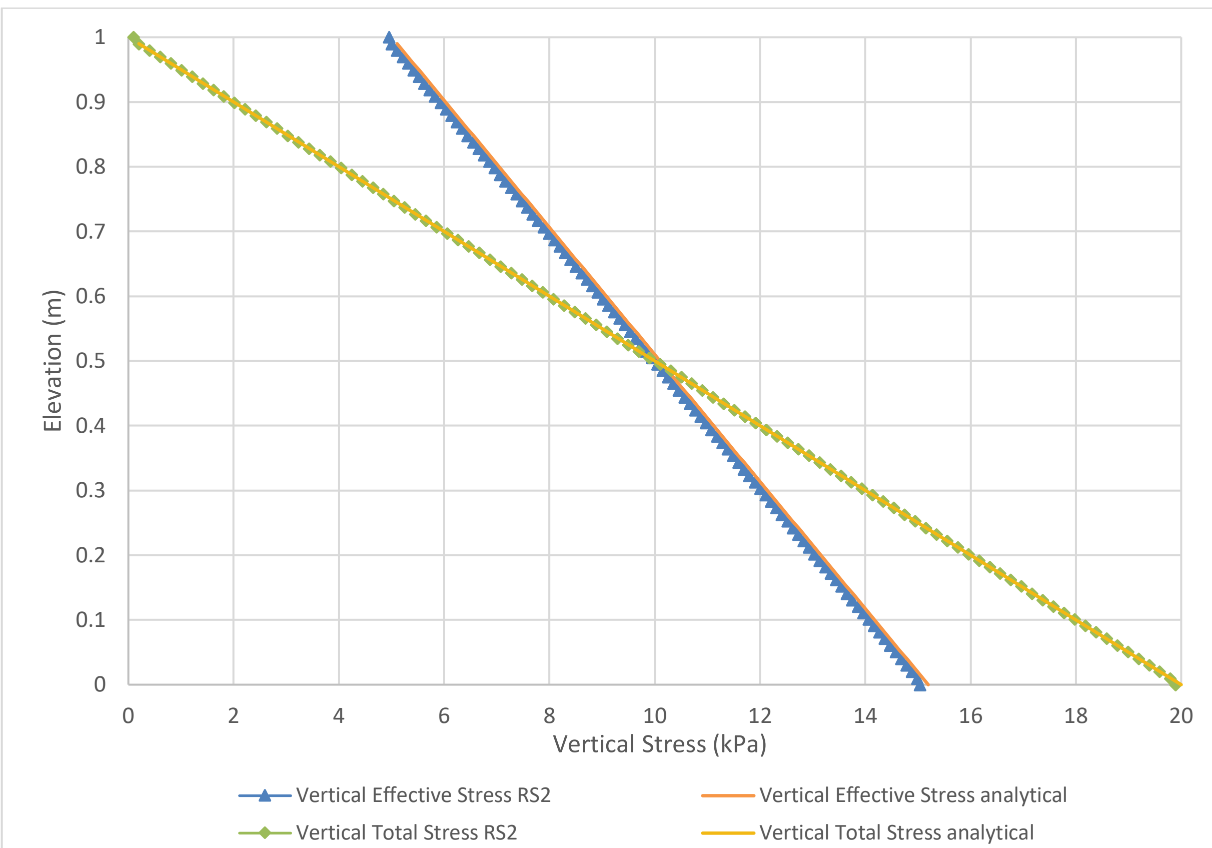



**Figure 11.2: Uniform Unit Weight - Vertical Stress Values**

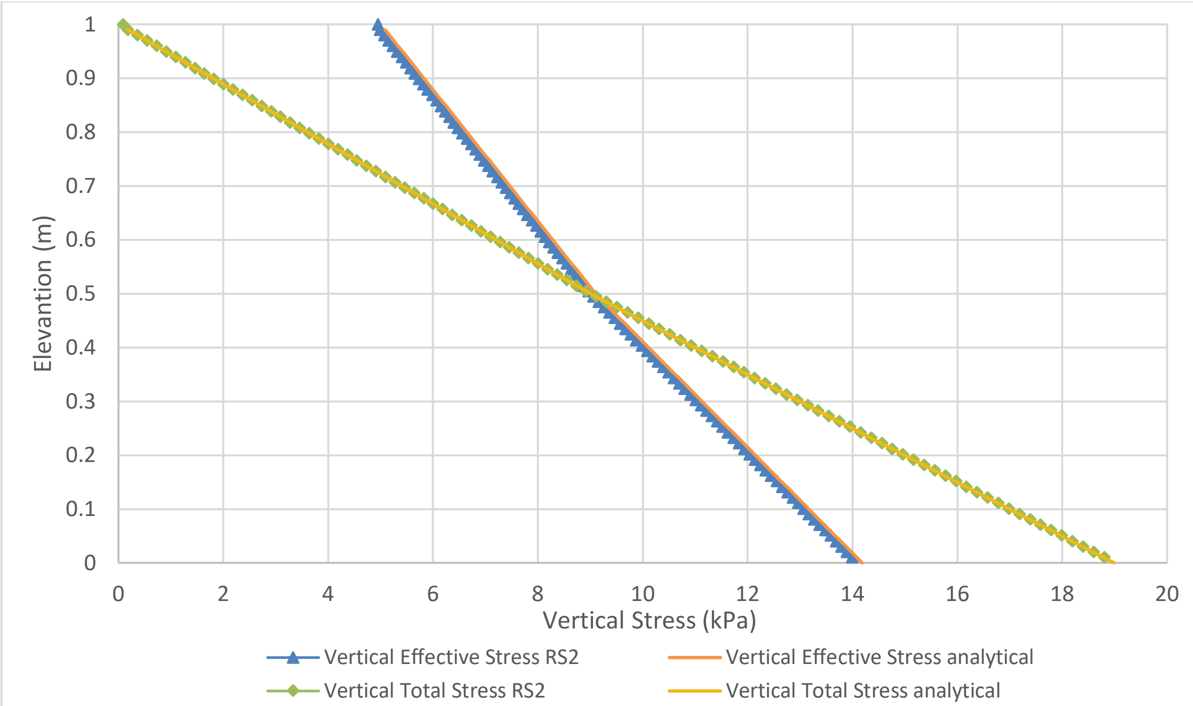



**Figure 11.3: Moist Unit Weight - Vertical Stress Values**

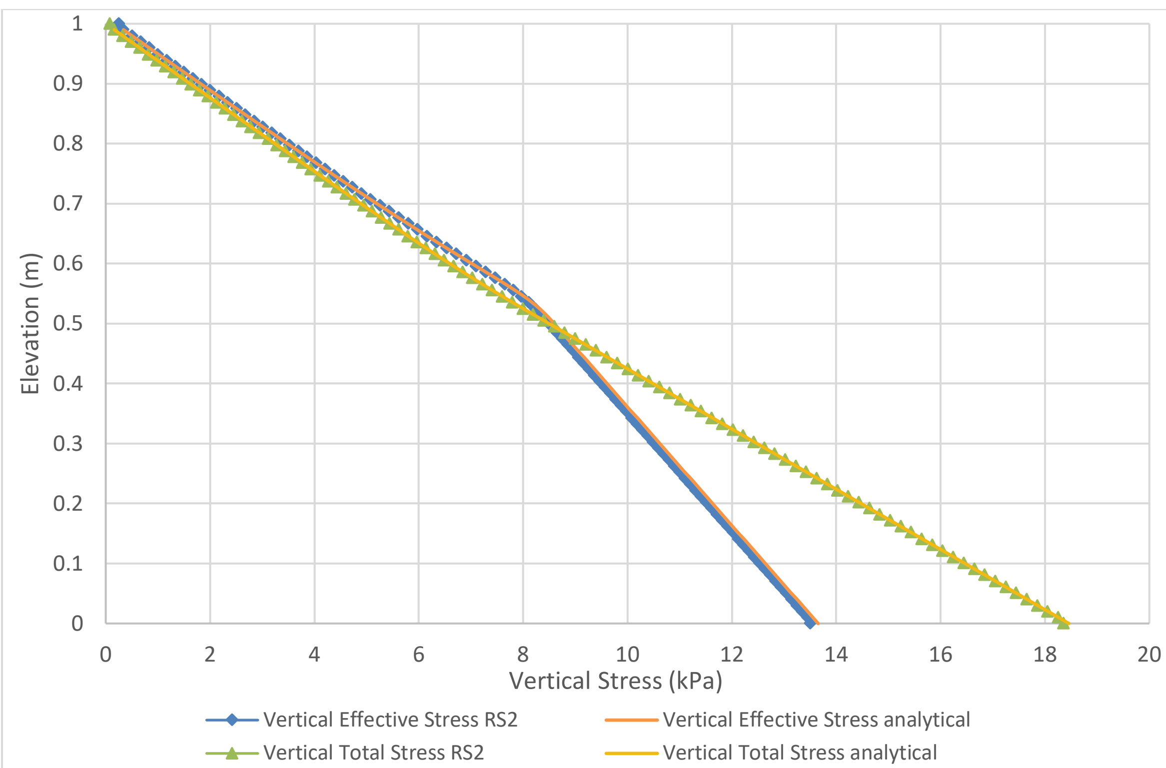



**Figure 11.4: Unsaturated Unit Weight Based on Saturation Degree - Vertical Stress Values**

Figure 11.5 below compares the effective vertical stress results for the three models.

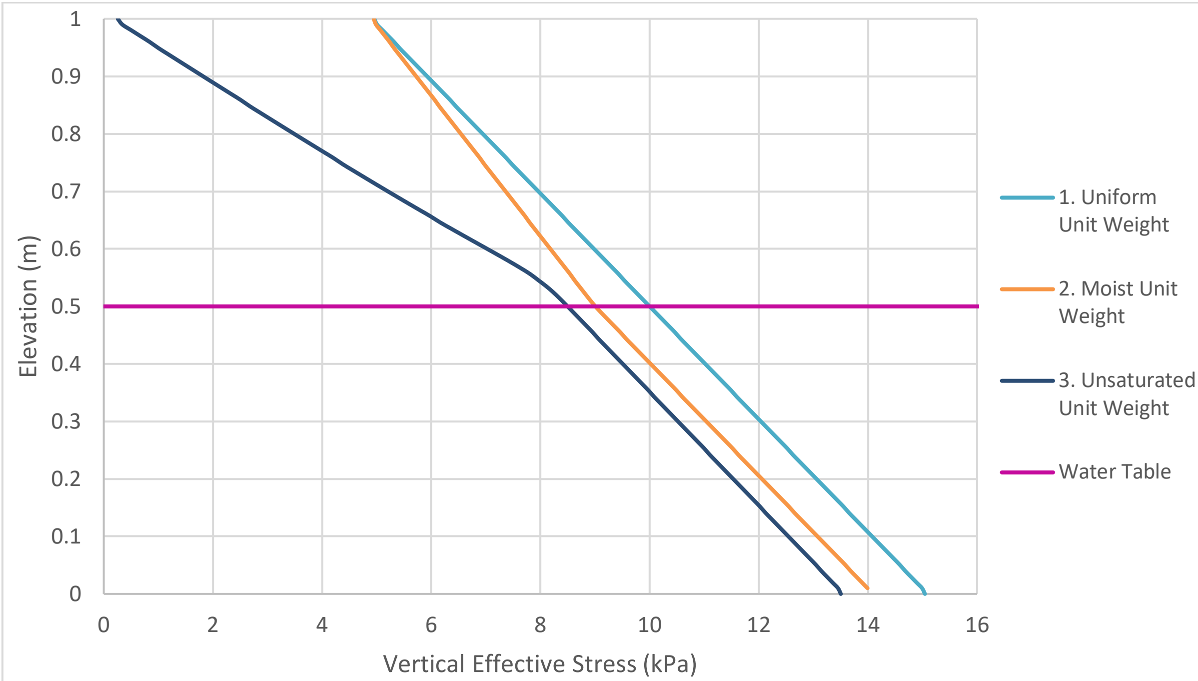



**Figure 11.5: Vertical Effective Stress Graph for All Models**

## 11.4 References


Bishop, A. W. (1959). *The principle of effective stress.* Tecknish Ukeblad 106, 859-863.


## 11.5 Data Files

The input files **consolidation#011_01(uniform).fez**, **consolidation#011_02(moist).fez** and **consolidation#011_03(unsaturated).fez** can be downloaded from the RS2 Online Help page for Verification Manuals.