Bolt Support Models

Introduction

Bolt models have been implemented in various numerical methods such as the Finite Element Method (FEM) (Goodman et al., 1968), the Boundary Element Method (BEM) (Crotty & Wardle, 1985) and block methods (Cundall, 1971).

This document outlines the background theories of the bolt support models used in *Phase*^{2}. Five different bolt models are available:

1. End Anchored
2. Fully Bonded
3. Plain Strand Cable
4. Swellex / Split Set
5. Tiebacks.

The bolts pass through the elements in the mesh, and are modeled by one or a series of one-dimensional elements.

End Anchored Bolt

The End Anchored rock bolt is represented by a one-dimensional deformable element (Figure 1).

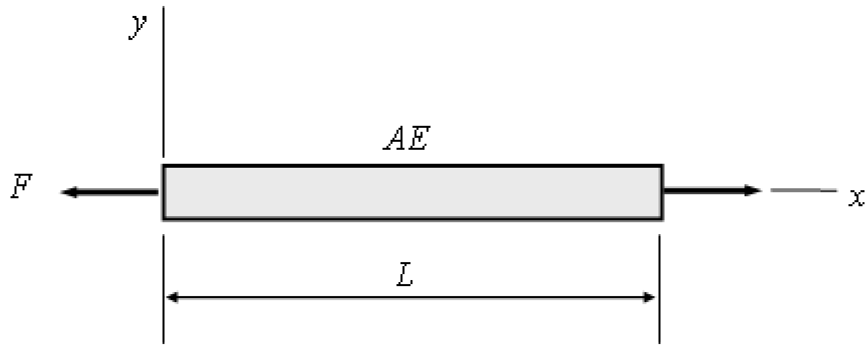

Figure 1: End-anchored bolt model

An End Anchored bolt in *Phase2* behaves as a single element. Interaction with the finite element mesh is through the endpoints only. The axial force, F is calculated from the axial displacement by:

$$F = K_b \Delta u \tag{1}$$

where K_b is the bolt stiffness which equals EA/L , Δu denotes the relative displacement between the two anchorage points which is $\Delta u = u_1 - u_2$. Failure of an End Anchored rock bolt occurs due to tensile yielding of the bolt material. Therefore, bolt failure is controlled by the yield strength (F_{yield}). An End Anchored bolt may also be assigned a residual capacity after failure. However in most cases the residual capacity of an End Anchored bolt would be equal to zero.

Fully Bonded bolt

Fully bonded bolts in *Phase2* are divided into bolt elements according to where the bolts cross the finite element mesh. These bolt elements act independently of each other. Neighbouring fully bonded bolt elements do not influence each other directly, but only indirectly through their effect on the rock mass.

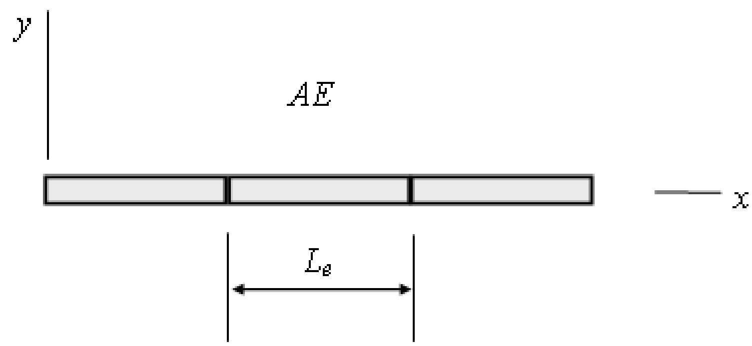

Figure 2: Fully bonded bolt model

The axial force along the bolt is determined from the elongation of the bolt element. If the length of a bolt element L_e, is increased by Δu_e then the induced force in the bolt is given by:

$$F_e = \frac{AE}{L_e} \Delta u_e \tag{2}$$

If the axial force exceeds the yield strength (F_{yield}) of the bolt material then the bolt force is set to the residual capacity F_{res} (Figure 3).

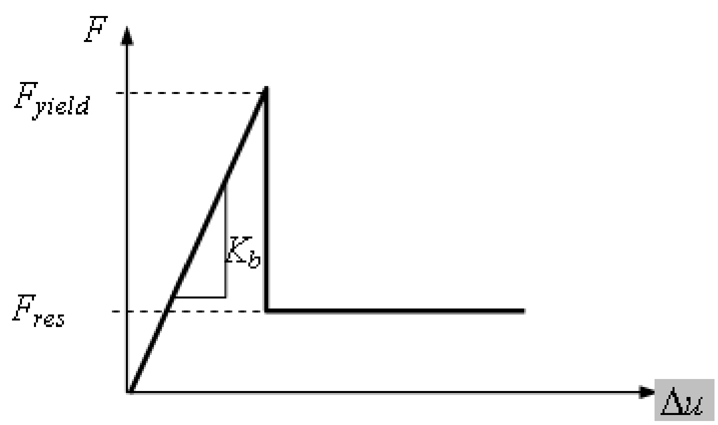

Figure 3: Fully bonded bolt failure criteria

Plain Strand Cable Bolt

For the Plain Strand Cable bolt model in *Phase2*, the entire bolt behaves as a single element (i.e. the behaviour of each segment of the bolt has a direct effect on adjacent segments). This is in contrast to the Fully Bonded bolt model, where bolt elements on the same bolt act independently of each other. The stiffness of the grout, and the strength and stiffness of the bolt/grout interface is taken into account. The failure mechanism of the bolt is by tensile rupture of the cable.

Failure of the cable/grout interface also occurs, but it is not a failure mechanism as such, since this interface is always assumed to be in a plastic state as the rock moves. The amount of relative slip at this interface, and the stiffness of the interface, determines how much shear force is generated at the cable.

For information about the development of this model, see the following references (Moosavi, 1997, Moosavi et al. 1996, Hyett et. al. 1996, Hyett et. al. 1995).

Shear Bolt (Swellex / Split Sets)

Swellex / Split Set (shear bolts) consider the shear force due to relative movement between bolts and the rock mass. The equilibrium equation of a fully grouted rock bolt, Figure 4, may be written as (Farmer, 1975 and Hyett et al., 1996):

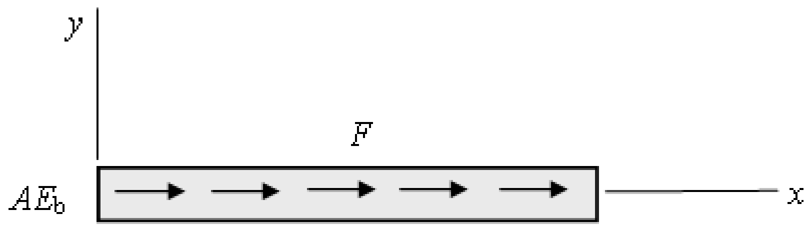

Figure 4: Elastic bar

$$AE_b \frac{d^2 u_x}{dx^2} + F_s = 0 \tag{3}$$

where F_s is the shear force per unit length and A is the cross-sectional area of the bolt and E_b is the modulus of elasticity for the bolt. The shear force is assumed to be a linear function of the relative movement between the rock and the bolt and is presented as:

$$F_s = k\left(u_r - u_x\right) \tag{4}$$

Usually, k is the shear stiffness of the bolt-grout interface measured directly in laboratory pull-out tests. Substitute equation (4) in (3), then the weak form can be expressed as:

$$\delta\Pi = \int (AE_b \frac{d^2 u_x}{dx^2} - ku_x + ku_r)\, \delta u\, dx \tag{5}$$

$$\begin{aligned} &= \int \left\{ AE_b \left[\frac{d}{dx}\left(\frac{du_x}{dx}\delta u\right) - \frac{du_x}{dx}\frac{d\delta u}{dx} \right] - (ku_x - ku_r)\delta u \right\} dx \\ &= AE_b \delta u \frac{du_x}{dx}\Big|_0^L \quad - \int \left(AE_b \frac{du_x}{dx}\frac{d\delta u}{dx} + ku_x \delta u \right) dx \quad + \int \left(ku_r \delta u \right) dx \end{aligned} \tag{6}$$

Let us consider the generic element shown in Figure 5.

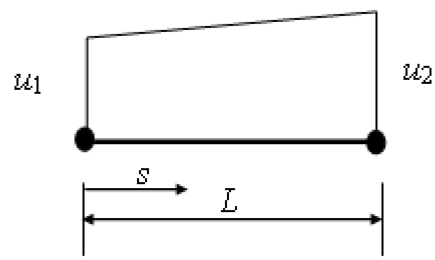

Figure 5: Linear displacement variation

The displacements u are to be linear in axial coordinates (Cook, 1981). The displacement field equals u_1 at one end and u_2 at the other. Then, the displacement at any point along the element can be given as:

$$u = \frac{L-s}{L} u_1 + \frac{s}{L} u_2 \qquad \text{or} \qquad u = \lfloor N \rfloor \{d\} \tag{7}$$

where $\lfloor N \rfloor = \left\lfloor \frac{L-s}{L} \quad \frac{s}{L} \right\rfloor$ and $\{d\} = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

for the two displacement fields, equation 7 can be written as:

$$u = \begin{Bmatrix} u_x \\ u_r \end{Bmatrix} = \begin{bmatrix} N_1 & N_2 & 0 & 0 \\ 0 & 0 & N_1 & N_2 \end{bmatrix} \begin{Bmatrix} u_{x1} \\ u_{x2} \\ u_{r1} \\ u_{r2} \end{Bmatrix} \tag{8}$$

Then equation (4) can be written as:

$$-\int \left(AE_b \frac{du_x}{dx} \frac{d\delta u}{dx} + ku_x \delta u \right) dx + \int (ku_r \delta u) dx = - \quad \begin{bmatrix} u_{x1} & u_{x2} & u_{r1} & u_{r2} \end{bmatrix} \begin{bmatrix} K_b & 0 \\ 0 & -K_r \end{bmatrix} \delta \begin{Bmatrix} u_{x1} \\ u_{x2} \\ u_{r1} \\ u_{r2} \end{Bmatrix} \tag{9}$$

and let us introduce the notation $\lfloor B \rfloor = \lfloor N_{,x} \rfloor$

then $u_{,x} = \frac{du}{dx} = \lfloor B \rfloor \{d\} = \left\lfloor -\frac{1}{L} \quad \frac{1}{L} \right\rfloor \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{10}$

Hence,

$$\left[K_b\right]=\int_0^L\left\{AE_b\begin{bmatrix}N_{1,x}N_{1,x} & N_{1,x}N_{2,x}\\ N_{2,x}N_{1,x} & N_{2,x}N_{2,x}\end{bmatrix}+k\begin{bmatrix}N_1N_1 & N_1N_2\\ N_2N_1 & N_2N_2\end{bmatrix}\right\}dx \tag{11}$$

$$\left[K_b\right]=\frac{AE_b}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+k\int_0^L\begin{bmatrix}\left(1-\frac{x}{L}\right)^2 & \left(1-\frac{x}{L}\right)\frac{x}{L}\\ \left(1-\frac{x}{L}\right)\frac{x}{L} & \left(\frac{x}{L}\right)^2\end{bmatrix}dx \tag{12}$$

$$\left[K_b\right]=\frac{AE_b}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{kL}{3}\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix} \tag{13}$$

and

$$\left[K_r\right]=k\begin{bmatrix}N_1N_1 & N_1N_2\\ N_2N_1 & N_2N_2\end{bmatrix}=\frac{kL}{3}\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix} \tag{14}$$

Equations (13) and (14) are used to assemble the stiffness for the bolts. *Phase2* uses bolts that are not necessarily connected to the element vertices, therefore a mapping procedure is carried out to transfer the effect to the element vertices. This procedure is done for each bolt segment by mapping the stiffness by the shape function depends on the intersected side of the elements.

Tiebacks

The Tieback bolt model in *Phase2* allows you to model grouted tieback support. Bolts may be pre-tensioned and grouted with a user-defined bonded length. Tieback bolts consist of a bonded length in series with an unbonded length, and consider the shear resistance of the bonded length.

In terms of its implementation in the *Phase2* analysis engine, a tieback uses the same formulation as the Swellex/Split Set bolt model, with allowance for an unbonded length.

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