Initial Stiffness Method

Newton's method is a commonly used computational algorithm for elasto-plastic finite element analysis. One variation of this method is the initial stiffness method, in which a global stiffness matrix is assembled at the beginning of the analysis, and is kept constant for all subsequent calculation steps. This reduces the computational effort since the stiffness matrix is not being formed for each iteration, as is done in Newton's method. However, this method requires a greater number of iterations to converge; this increase in the number of iterations may outweigh the reduction in computation time from keeping the stiffness matrix constant.

The system of equations to be solved at each iteration is:

$$\Delta U^i = K^{-1} R^{i-1} \tag{1}$$

where K is the stiffness matrix, R^{i-1} is the residual force from the previous iteration and ΔU^i is the change in displacement for the current iteration. The residual force, R^{i-1}, is calculated as the difference between the external force and the internal force at the current iteration. The internal force at iteration *i* is commonly obtained by the relation:

$$F_{int}^i = \int B^T \sigma^i dV \tag{2}$$

where B is the strain-displacement matrix and σ^i is the stress tensor.

In the initial stiffness method, the global stiffness matrix is calculated using:

$$K_e = \int B^T D_e B dV \tag{3}$$

where D_e is the elastic stress-strain matrix and B is the strain-displacement matrix. The total displacement at the ith iteration is calculated as:

$$U^i = U^{i-1} + \Delta U^i \tag{4}$$

The total displacement is thus the summation of the displacement at the previous iteration and the displacement calculated using (1).

Accelerated Initial Stiffness Method

The accelerated initial stiffness method accelerates the convergence of the initial stiffness method by scaling the iterative displacements using an acceleration parameter, α, for every other iteration. Thus, for the accelerated initial stiffness method, rather than using (4) to calculate the displacement, it is calculated using:

$$U^{2i} = U^{2i-1} + \Delta U_e^{2i} \tag{5i}$$

$$U^{2i+1} = U^{2i} + \alpha^{2i} \Delta U_e^{2i+1} \tag{5ii}$$

where α is the acceleration parameter applied for every other iteration. Equations (5i) and (5ii) are alternated for each iteration such that the acceleration parameter is only applied for every other iteration.

The acceleration parameter is calculated as follows:

$$\alpha^{2i} = \begin{cases} \alpha^{2i-2} + \dfrac{\{\Delta U_e^{2i}\}^T \{\Delta U_e^{2i}\}}{\{\Delta U_e^{2i}\}^T \{\alpha^{2i-2}\Delta U_e^{2i-1}\}} \ for \ i > 1 \\ 1 \ for \ i = 1 \end{cases} \qquad (6)$$

In Phase2 9.0 the user may implement the accelerated initial stiffness method by selecting the option in the "Stress Analysis" tab of the Project Setting window. The user may also set maximum and minimum values for α, the acceleration parameter.
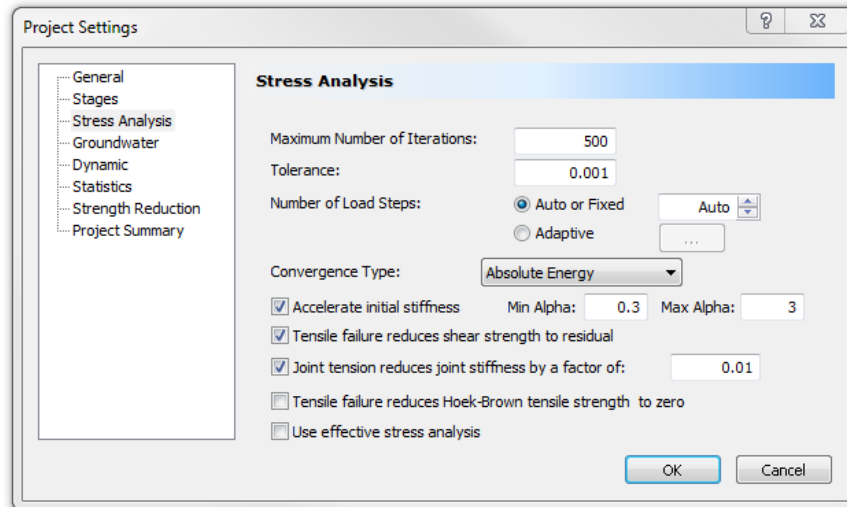
References

Dang, H.K. et al. (2014) "Evaluate the performance of an accelerated initial stiffness method in three dimensional finite element analysis." Accepted for publication in Computers and Geotechnics.