Modelling composite sections (e.g., steel sets and shotcrete) with FLAC and Phase2

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The problem is represented in Figure 1a. We want to model a composite section using beam or liner elements in FLAC or Phase2. We consider a stretch of 1 meter of composite section. The section is composed of regularly spaced elements '1' (e.g., steel sets) and regularly spaced elements '2' (e.g., shotcrete). There are n elements '1' and '2' in a meter of section —this is equivalent to saying that the spacing between elements is $s = 1.0/n$. Each element has a Young's modulus E, a cross-sectional area A and a moment of inertia I. So each element '1' is characterized by parameters E_1, A_1 and I_1 and each element '2' is characterized by parameters E_2, A_2 and I_2.

The elements '1' and '2' in the composite section are considered to be rigidly attached to each other, so that the elements will deform uniformly in the axial direction, if a thrust N is imposed on the composite section, and the elements will rotate uniformly, if a bending moment M is imposed on the composite section —see lower sketch in Figure 1a.

To model the problem in FLAC or Phase2 we smear the geometrical and mechanical properties of the elements in the composite section into an equivalent rectangular element of width 1 meter, and height h_{eq} —see Figure 1b. Furthermore, we consider that the equivalent rectangular section has a Young's modulus E_{eq}. We compute the values of h_{eq} and E_{eq} as follows:

$$h_{eq} = 2\frac{\sqrt{3C_A C_I}}{C_A} \tag{1}$$

$$E_{eq} = \frac{\sqrt{3}}{6}\frac{{C_A}^2}{\sqrt{C_A C_I}} \tag{2}$$

where

$$C_A = n\,(A_1E_1 + A_2E_2) \tag{3}$$

$$C_I = n\,(I_1E_1 + I_2E_2) \tag{4}$$

By solving the problem in terms of the equivalent section (Figure 1b), we get from FLAC or Phase2 the *total* values of thrust N and bending moment M. We need to re-distribute then these values into each element '1' and '2' in the section (see Figure 2). This is done as follows.

The thrust N that acts in each element '1' and '2', is, respectively

$$N_1 = \frac{N}{n}\frac{A_1E_1}{A_1E_1 + A_2E_2} \tag{5}$$

$$N_2 = \frac{N}{n}\frac{A_2E_2}{A_1E_1 + A_2E_2} \tag{6}$$

The moment M that acts in each element '1' and '2', is, respectively

$$M_1 = \frac{M}{n} \frac{I_1 E_1}{I_1 E_1 + I_2 E_2} \tag{7}$$

$$M_2 = \frac{M}{n} \frac{I_2 E_2}{I_1 E_1 + I_2 E_2} \tag{8}$$

Normally, each element in the composite section will have to be be verified/designed to sustain the stresses induced by the values of thrust (N_1 for element '1' and N_2 for element '2') and bending moment (M_1 for element '1' and M_2 for element '2') using classical equations of strength of materials.
Note also that cases in which elements '2' do not exist can be accounted for with these expressions —this case would correspond, for example, to the case in which regularly spaced steel sets are installed without shotcrete. To consider this situation, we simply make $E_2 = 0$ in the equations above.

Demonstration of equations 1 through 8

We analyze first the behavior of the elements '1' and '2' in the axial direction. Under the application of thrust, the axial strain that these elements undergo is

$$\varepsilon_1 = \frac{N_1}{A_1 E_1} \tag{9}$$

$$\varepsilon_2 = \frac{N_2}{A_2 E_2} \tag{10}$$

Equilibrium and compatibility of axial deformation conditions can be written as follows,

$$N = n(N_1 + N_2) \tag{11}$$

$$\varepsilon_1 = \varepsilon_2 \tag{12}$$

Solving for N_1 and N_2 from the set of equations (9) through (12), we get the equations (5) and (6).
Considering the equivalent section in Figure 1b, a similar relationship as in equations (9) and (10) can be constructed using the total trust N, i.e.,

$$\varepsilon = \frac{N}{A_{eq} E_{eq}} \tag{13}$$

Then, in view that the axial deformation ε of the equivalent section must also be equal to the deformation of each individual element (equations 9 and 10), and that the relationship between the axial force in each element and the total axial force in the equivalent section is given by equations (5) and (6), one gets,

$$A_{eq} E_{eq} = n\,(A_1 E_1 + A_2 E_2) \tag{14}$$

A similar analysis as done above for axial deformation induced by thrust, can be done for rotation induced by bending moment, using the following relations

$$\theta_1 = \frac{M_1}{I_1 E_1} \tag{15}$$

$$\theta_2 = \frac{M_2}{I_2 E_2} \tag{16}$$

$$\theta = \frac{M}{I E} \tag{17}$$

Then, one gets,

$$I_{eq} E_{eq} = n \left(I_1 E_1 + I_2 E_2 \right) \tag{18}$$

For the rectangular equivalent section represented in Figure 1b, the cross-sectional area A_{eq} and the moment of inertia I_{eq} are,

$$A_{eq} = h_{eq} \times 1.0\,\mathrm{m} \tag{19}$$

$$I_{eq} = \frac{h_{eq}^3}{12} \times 1.0\,\mathrm{m} \tag{20}$$

The expressions for h_{eq} and E_{eq} given by equations (1) and (2) are the ones that satisfy both, the conditions given by equations (14) and (18), with cross-sectional area and moment of inertia computed as in equations (19) and (20).

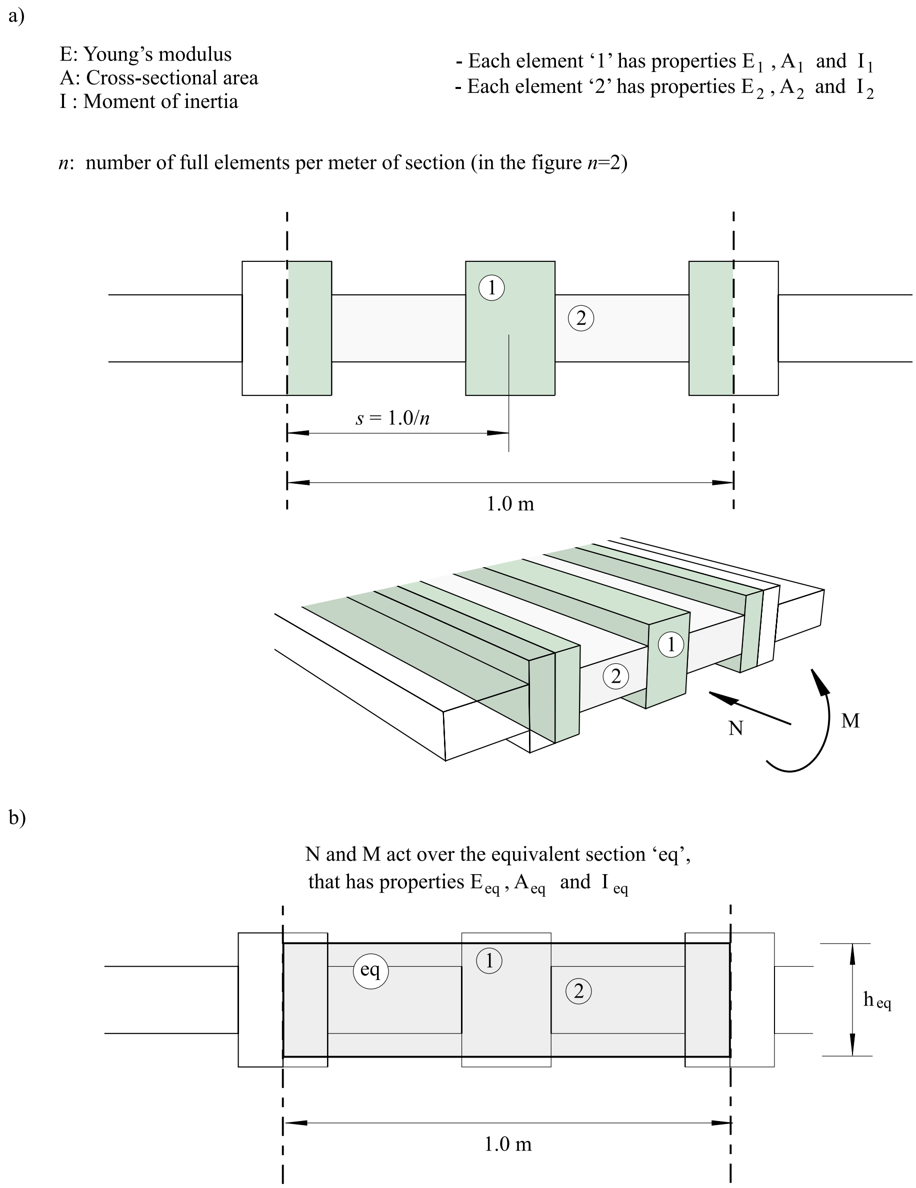
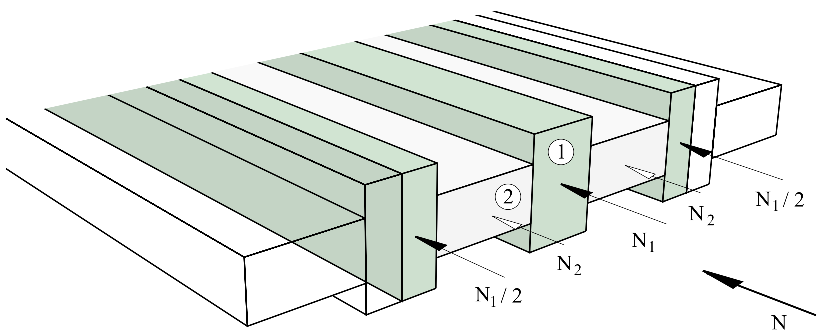


Figure 1: Problem statement.

a)



b)

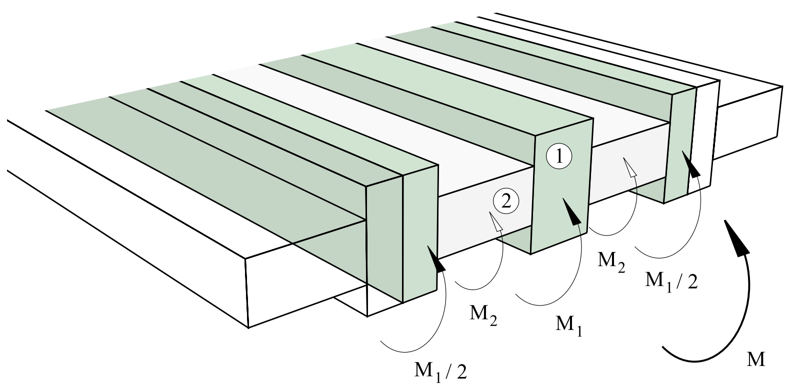


Figure 2: Distribution of thrust and bending moment to each section.