RS2 THEORY

Soil Behaviors in Unsaturated Zones

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It is well known that soil in unsaturated zone has strength increase as the suction increases, with proof that vertical cut in clays has been stabilized without any support. Unsaturated soil mechanics has been studied by much research in both shear strength and consolidation and results in many formulas.

RS2 provides various way to account for effective stresses has been implemented. Each of the four ways is described in detail below.

Single Effective Stress

This approach is available for all failure criteria models in RS2 except for the Barcelona model. Effective stress in unsaturated zone can be calculated by (Bishop, 1959)

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \tag{1}$$

where σ' is the effective stress, σ is total stress, u_a is the pore-air pressure, u_w is the pore water pressure, and χ is the coefficient.

In general, pressure is implied as an absolute value, but many soil mechanics problems use relative pressure and u_a is often assumed to be equal to the atmospheric pressure. If this is the case, then the effective stress becomes:

$$\sigma' = \sigma + I\chi p^w \tag{2}$$

where p^w is the suction, and I is the unit matrix.

In RS2, χ can be calculated by nine different methods:

a. Based on Bishop (1959), χ is calculated as:

$$\chi = S_r \tag{3}$$

Where S_r is the degree of saturation, $S_r = \frac{\theta}{n}$, θ is the water content, n is the soil porosity.

b. The user can define a table of χ vs. matric suction values.

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c. The user can define a table of χ vs. degree of saturation (S_r) values.

d. The user can define a table of χ vs. effective degree of saturation (S_e) values. The effective degree of saturation can be calculated as: $S_e = \frac{S_r - S_{re}}{S_{sat} - S_{re}}$, where S_{re} is the residual degree of saturation and S_{sat} is the maximum degree of saturation.

e. Based on Gudehus (1995), χ is calculated as:

$$\chi = S_r(2 - S_r) \tag{4}$$

Where S_r is the degree of saturation.

f. Based on Khalili (2004), χ is calculated as:

$$\chi = \begin{cases} \left(\frac{s}{s_e}\right)^{-0.55} & if\ s > s_e \\ 1 & if\ s \leq s_e \end{cases} \tag{5}$$

Where s is the matric suction, $s = u_a - u_w$, and s_e is the air entry suction.

g. Based on Bolzon (1996), χ is calculated as:

$$\chi = S_e \tag{6}$$

Where S_e is the effective degree of saturation.

h. Based on Aitchison (1961), χ is calculated as:

$$\chi = \begin{cases} 1 & if\ S_r = 1 \\ ({}^{\alpha}/_{s})_{s_e} & if\ S_r < 1 \end{cases} \tag{7}$$

Where α is a unitless material parameter, s is the matric suction, $s = u_a - u_w$, s_e is the air entry suction, and S_r is the degree of saturation.

i. Based on Kohgo et al (1993), the effective stress is defined otherwise in the form:

$$\sigma' = \sigma - u_{eq} \tag{8}$$

Where u_{eq} is the equivalent pore pressure. This pressure is aimed at averaging the effects of all fluid pressures within the pores. It is also designed to recover Terzaghi's effective stress on saturated states. Consequently, authors had to express the equivalent pore pressure in terms of air entry suction value (s_e), a critical suction (s_c), and a material parameter (a_e):

$$u_{eq} = u_a - s \quad if\ s \leq s_e \tag{9}$$

$$u_{eq} = u_a - \left(s_e + \frac{s_c - s_e}{(s - s_e) + a_e}(s - s_e)\right) \quad if\ s > s_e \tag{10}$$

This formulation is equivalent to using Bishop's method with

$$\chi = a_e (s_c - s_e)/(s - s_e + a_e)^2 \quad (11)$$

Note that in RS2, the **Use effective stress analysis** option must be selected to enable the Single Effective Stress method for unsaturated materials. See the topic for details in modelling.

Independent Stress Variables Constitutive Model

The effects of suction were accounted using state variables of $(\sigma, u_a \; and \; u_w)$. Any of two variables can be used to define the stress state based on the multiphase continuum mechanics. Thus, the possible combinations are:

(1) $(\sigma - u_a)$ and $(u_a - u_w)$

(2) $(\sigma - u_w)$ and $(u_a - u_w)$

(3) $(\sigma - u_a)$ and $(\sigma - u_w)$

Note that in the approach, sigma is the total stress for the unsaturated part. When constitutive model that used this approach was selected, the correspondent single effective stress approach will be automatically selected. There are several shortcomings of this approach:

- The use of total stress in constitutive model cause difficulties to develop.
- Transition between saturate and unsaturated are not natural and sometimes are not smooth.
- More parameters need to be determined compared with effective stress approach for the hydraulic components.

Because of the shortcomings, the approach is **not** implemented in RS2.

Unsaturated Shear Strength

This approach based on Mohr-Coulomb criterion has been widely used by introducing additional friction angle for unsaturated zone. In RS2, it can only be applied to the Mohr-Coulomb model. Two methods are introduced in this approach: by Fredlund et al (1978), or by Vanapalli et al (1996).

a. According to Fredlund et al (1978), the shear strength is written as follow,

$$\tau = c' + (\sigma - u_a) \tan \Phi' + (u_a - u_w) \tan \Phi^b \quad (12)$$

Where c' is the effective saturated cohesion, σ is the total stress, $(\sigma - u_a)$ is the net normal stress on the plane of failure at failure, $(u_a - u_w) = s$ is the matric suction, u_a is the pore-air pressure, u_w is the pore water pressure, Φ' is the effective saturated angle of friction, and Φ^b is the angle of friction accounting for matric suction contribution to shear strength.

In order to account for the approach correctly in RS2, the Maximum negative PWP for unsaturated strength option needs to be ON with an input value of 0, to avoid double counting of suction effects. See the link for more information with the option.

b. Another form of the shear strength was proposed by Vanapalli et al (1996), where it can be calculated in terms of either the degree of saturation or the water content.

with degree of saturation,

$$\tau = c' + (\sigma - u_a)\tan\Phi' + (u_a - u_w)\left[(\tan\Phi')\left(\frac{S_r - S_{re}}{100 - S_{re}}\right)\right] \tag{13}$$

where S_{re} is the residual degree of saturation and S_r is degree of saturation.

with water content,

$$\tau = c' + (\sigma - u_a)\tan\Phi' + (u_a - u_w)\left[(\tan\Phi')\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)\right] \tag{14}$$

where θ is the water content, θ_r is the residual water content, and θ_s is the saturated water content.

Constitutive Models that account for Unsaturated Behaviors

For the effect of the suction on the mechanical behaviors, alternative to the single effective stress approach, it can also be considered using the constitutive model that account for the suction and/or degree of saturation (i.e., Barcelona Basic).

In such cases, if undergoes steady state analysis or (and) no consolidation, the coefficient of permeability (kw) governs unsaturated behavior; if consolidation is included in the transient analysis, unsaturated behaviour will be considered differently, depending on the consolidation type (coupled or uncoupled).

a. For coupled consolidation, please refer to the Coupled Consolidation document for governing equilibrium equations, where Equation 15 in the document will be applied here.

b. For uncoupled consolidation, instead of the compressibility matrix S (in coupled consolidation), the coefficient of water volume change m_2^w (or symbol mv) is used to account for the deformation of solid influences on fluid. Please refer to the Transient Fluid Flow section of the Groundwater Seepage document for m_2^w calculations in both saturated and unsaturated soils.

References

Aitchison GD. (1961). Relationships of moisture stress and effective stress functions in unsaturated soils. Pore Pressure and Suction in Soils. Butterworths: London.

Bishop, A. W. (1959). The principle of effective stress.Tecknish Ukeblad 106, 859-863.

Blight, G. E. (1967). Effective stress evaluation for unsaturated soils. J. Soil Mech. Found. Engng Div.,ASCE 93, No. SM2, 125-148.

Bolzon, G., Schrefler, B. A., & Zienkiewicz, O. C. (1996). Elastoplastic soil constitutive laws generalized to partially saturated states. Géotechnique, 46(2), 279–289.

Fredlund, D. G., Morgenstern, N. R. & Widger, R. A. (1978). The shear strength of unsaturated soils. Can. Geotech. J. 15, No. 3, 313-321.

Gudehus. G. (1995) A comprehensive concept for non-saturated granular bodies, in Proceedings of the 1st International Conference on Unsaturated Soils. Paris, Balkema, Rotterdam.

Khalili, N.; Geiser, F.; Blight, G. E. (2004). Effective Stress in Unsaturated Soils: Review with New Evidence. International Journal of Geomechanics, 4(2), 115–126. doi:10.1061/(ASCE)1532-3641(2004)4:2(115).

Kohgo Y, Nakano M, Miyazaki T. Theoretical aspects of constitutive modelling for unsaturated soils. Soils and Foundations 1993; 33(4):49–63.

Nuth, M., & Laloui, L. (2008). Effective stress concept in unsaturated soils: Clarification and validation of a unified framework. International Journal for Numerical and Analytical Methods in Geomechanics, 32(7), 771–801.

Pereira J-M, Wong H, Dubujet P, Dangla P. Adaptation of existing behaviour models to unsaturated states: application to CJS model. International Journal for Numerical and Analytical Methods in Geomechanics 2005; 29(11):1127–1155.

Tamagnini R. An extended cam-clay model for unsaturated soils with hydraulic hysteresis. Geotechnique 2004; 54(3):223–228.

Vanapalli, S. K., Fredlund, D. G., Pufahl, D. E., & Clifton, A. W. (1996). Model for the prediction of shear strength with respect to soil suction. Canadian Geotechnical Journal, 33(3), 379–392.