Probabilistic analyses in *Phase²* 8.0

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"Everything must be made as simple as possible. But not simpler."

— Albert Einstein

Introduction

In order to develop a reliable design approach, one must use statistical methods to deal with the variability of the input parameters. However tools usually used in geomechanics, like stress analyses (e.g. Finite Element Analyses, FEM), are in essence deterministic (a single set of input parameters leads to a single answer). Also, these tools are often computing time intensive and are not well-suited for the multiple runs needed for systematic sensitivity analyses or statistical simulations (e.g. Monte-Carlo). That's the reason why the Center for Excellence in Mining Innovation (CEMI) recently contracted RocScience Inc. to introduce an alternate method, the Rosenblueth point-estimate method (PEM, Rosenblueth, 1975), a simple, computing efficient probabilistic method, into their FEM software *Phase²* version 8.0. This paper presents the approach and discusses its applicability.

Uncertainty, variability and heterogeneity

When considering statistical distributions of input parameters in geomechanics problems, three different concepts must be considered: uncertainty, variability and heterogeneities. These three concepts must be treated separately as they have various impacts on the rock mass behaviour and, therefore, different approaches must be used to tackle them.

Uncertainties arise from the difficulty in measuring key geomechanical properties like rock stresses, rock modulus or rock strength. Any of these measurements involves some error due to the sampling process, sample preparation or sensitivity and calibration of the measuring devices. This uncertainty is usually evaluated and reduced by acquiring repeated measurements during the development of a project (Fig. 1). Considering a given design criterion, the probability of failure is given by the gray areas on Fig. 1 which, when combined with the consequence of failure, allows the computation of the risk of a given design (taking the standard definition of risk being probability of occurrence times consequence) and therefore, an evaluation of whether this risk is acceptable.

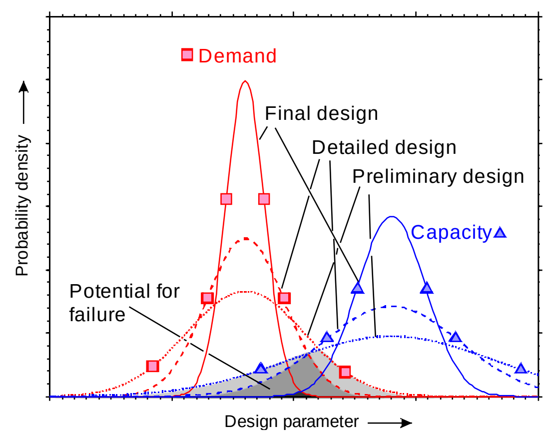

Fig. 1 Illustration of the uncertainty reduction during the development of a project until the potential for failure is minimised to an acceptable level (after Hoek, 1992)

The PEM/FEM method presented in this paper is particularly suited to handle this kind of situation, i.e., it allows one to track how uncertainties in the input parameters are propagated through the analyses and produce uncertainty in the design parameters. It allows the engineer to not limit the design to a single deterministic analysis with the most probable parameters (the mode of the distribution of Fig. 1), but to evaluate the reliability of the design by considering the dispersion of the design parameters.

Variability is an inherent property of natural materials and rocks or rock masses are no exception. It arises from the various formation and transformation processes of rock and rock masses which have a local influence on their mechanical parameters and characteristics. Due to this variability, rock mass properties will vary, for example, within a rock unit along the trace of a tunnel. Thus, a failure mechanism will affect more or less severely various locations along this tunnel. Here again the PEM/FEM approach presented in this paper is well-suited and will, for example, allow the engineer to anticipate what percentage of a tunnel section will be affected by a failure mechanism for a given severity level. It will also allow for an evaluation of the range of severity of a given failure mechanism that should be anticipated and thereby permit the inclusion of flexibility in the design to handle the less probable but potentially more severe situation. Having an estimate of the distribution of the severity of a potential failure mechanism will also permit the optimisation of the support systems and allow a better estimation of the cost and thus the economical risk of a project.

Heterogeneities need to be treated separately as they will influence the severity of a failure mechanism and, more importantly, change the behaviour of the rocks or rock masses. For example, increasing modulus heterogeneities in a rock will promote the development of local tensile stress even in an overall compressive field, which will affect the failure mode, e.g. change from a shear mechanism to a tensile dominated mechanism like spalling (e.g. Diederichs, 2007). The PEM/FEM approach proposed in this paper does not simulate the effect of heterogeneities. Heterogeneities must be handled differently, either by the use of a classification system, coupled with equivalent homogeneous properties (Hoek and Brown, 1997) or by explicit modelling of the heterogeneities (e.g. Valley et al., 2010a).

The PEM approach

In the simplest case, when closed-form analytical solutions are available for an analysis and when the input parameters are independent and uncorrelated, the propagation of errors can be approximated by using a first order Taylor series. However, such an approach requires that it is possible to extract a partial derivative for the solution function. This is not always feasible and is obviously impossible when the solution to a problem is found by a numerical method like FEM. The point estimate method proposed by Rosenblueth (1975), allows one to propagate error even if no closed-form analytical solution is available. The principle of PEM is to compute solutions at various estimation points and to combine them with proper weighting in order to get an approximation of the distribution of the solution (see Fig. 2). The PEM implemented in *Phase²* 8.0 is the two-point estimate method for the first and second moment of uncorrelated variables. It needs 2^n evaluations of the solution, where n is the number of random variables. The distribution of the solution for $y = f(x_1, x_2, \ldots, x_n)$ is given by:

$$\bar{y} = \sum_{i=1}^{2^n} w f_i \tag{1}$$

$$\sigma_y = \sqrt{\sum_i^{2^n} w f_i^{\,2} - \left[\sum_i^{2^n} w f_i\right]^2} \tag{2}$$

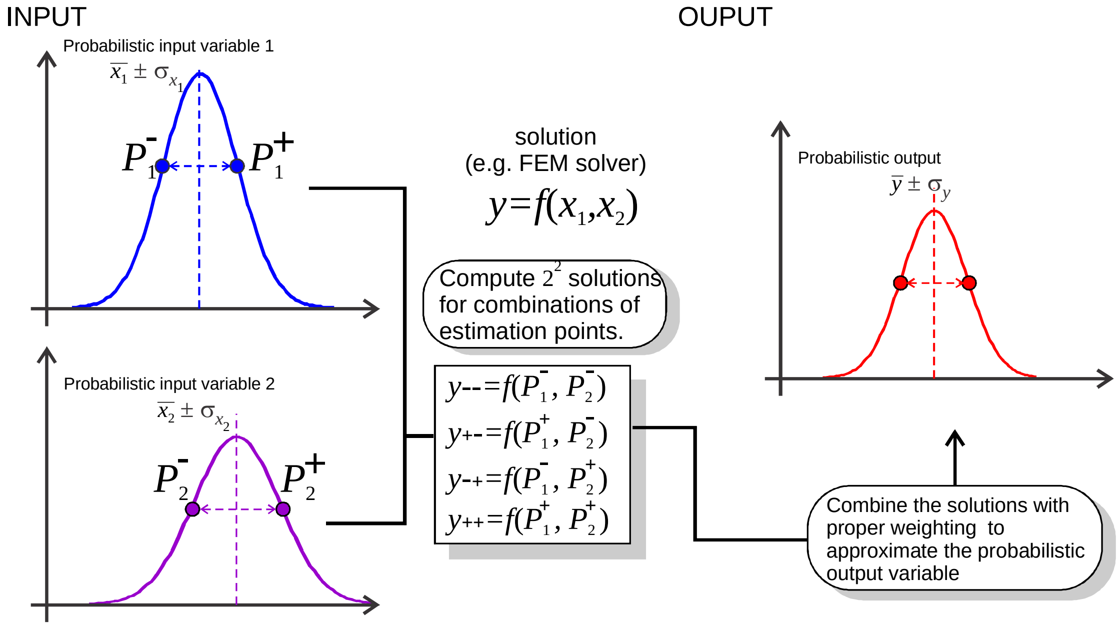

Fig. 2 Illustration of the computation principle of an approximation of the output probabilistic variable using the point estimate method. In this example, the case with only two probabilistic input variables is assumed.

where the weights w are given by $1/2^n$. f_i are successive evaluations of f at the 2^n possible combinations of the random variables at the point estimate locations, i.e. at $\bar{x}_n - \sigma_{x_n}$ and $\bar{x}_n + \sigma_{x_n}$. In the solution presented here, all input and variable and output variables are assumed to follow a normal distribution given by their mean $\bar{x}$ and standard deviation σ_x.

Example of application

In order to illustrate how to use the PEM with FEM let's look at the following example: the stress distribution around a circular opening has to be evaluated, but the estimations of the far field stresses (S_1 and S_3) are uncertain. Let's assume that this uncertainty can be captured by a normal distribution, i.e., a mean and a standard deviation (see modelling properties given in Table 1). In order to evaluate the uncertainty associated with some design parameter (let's assume, for example, the maximum principal stress at the excavation boundary), four (2^2, because there are two random variables, S_1 and S_3) models (Fig. 3a to d) must be run, assuming the following combinations of input for the far field stresses: [S_1=25 MPa; S_3=13 MPa], [S_1=25 MPa; S_3=17 MPa], [S_1=35 MPa; S_3=13 MPa] and, [S_1=35 MPa; S_3=17 MPa]. These combinations consist of all possible combinations of the mean ± one standard deviation. The outputs of these models must then be combined using Equation (1) and Equation (2) in order to obtain the mean and standard deviation of the output design criteria (Fig. 3e and f).

It is interesting to see that even in this simple case of elastic stresses around a circular opening, the pattern of uncertainty (see Fig. 3f) is quite complex and not intuitive. The highest uncertainty is located where S_1 is maximum while an area of low uncertainty arises in the S_1 far field direction at about one tunnel radius (darker area on Fig. 3f).

Table 1 Input parameters for modelling

Input parameter	Mean	Standard deviation
Max. far field principal stress S_1 (horizontal)	30 MPa	5 MPa
Min. far field principal stress S_3 (vertical)	15 MPa	2 MPa
Out of plane stress S_z	10 MPa	- [a]
Young modulus E	20 GPa	- [a]
Poisson ratio v	0.25	- [a]

[a] – These variables are not considered as random variables and thus no standard deviation is defined for them.

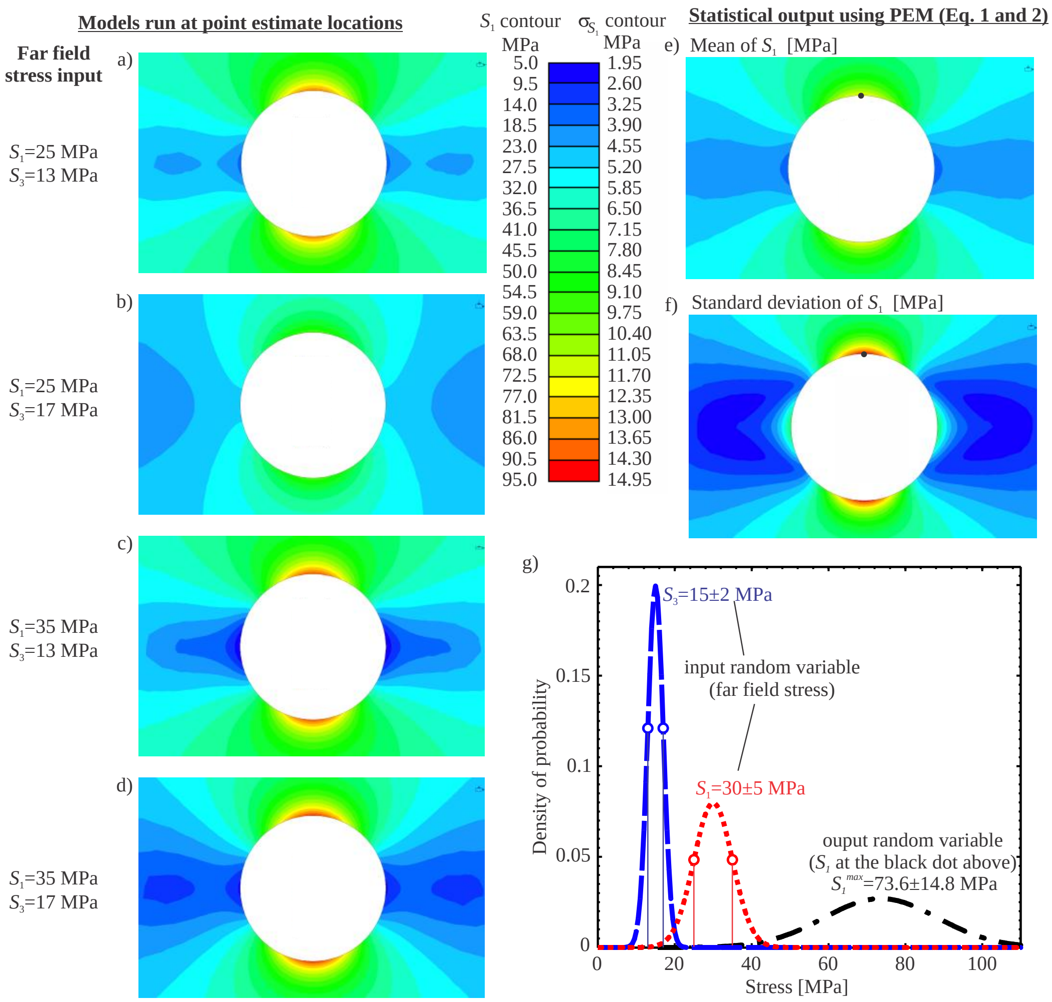

Figure 3 Example of PEM/FEM computation with parameters given in Table 1. a), b), c) and d) evaluation of the maximum principal stress S_1 (FEM elastic models) at the four combinations of the estimation points; e) and f) probabilistic output (mean and standard deviation) for S_1 obtained by combining the FEM results in the left with the PEM (Equations (1) and (2)); g) Probability density functions of the input (S_1 and S_3, dashed line and dotted line) and the output of the analyses where principal stress is maximum (S_1^{max}) (see black dot for location on e and f).

The example presented on Fig. 3 was selected for didactic purposes and is very simple. The implementation of the PEM in *Phase²* 8.0 permits inclusion of all the complexity that *Phase²* typically allows, including complex geometry, excavation stages, plasticity, etc. The *Phase²* 8.0 interface facilitates the interpretation of the probabilistic output by offering the appropriate visualisation tools, including standard deviation contouring, coefficient of variation contouring, and line plots with error bars.

When increasing the complexity of the model, one must however be aware of the limitations of the PEM. Particularly, in complex models, when multiple behaviours occur concomitantly, the actual output distribution can significantly differ from a normal distribution and thus the PEM may have difficulty in capturing it accurately. This may happen, for example, when looking at a location close to the fringe of a plasticity front where both mechanisms affect the output distribution. These effects were studied in detail by comparing PEM and Monte-Carlo output (Valley et al., 2010b) and the results are presented schematically on Fig. 4. When all combinations of estimation points, as well as the major part of the input distributions, generate the same mechanism (Fig. 4a), the PEM approximation of the output distribution is accurate. However, when mixed behaviour modes occur (Fig. 4b and c), the PEM output can be inaccurate and cannot capture the presence of tails in the output distribution (Fig. 4b) or the overall output distribution shape (Fig. 4c).

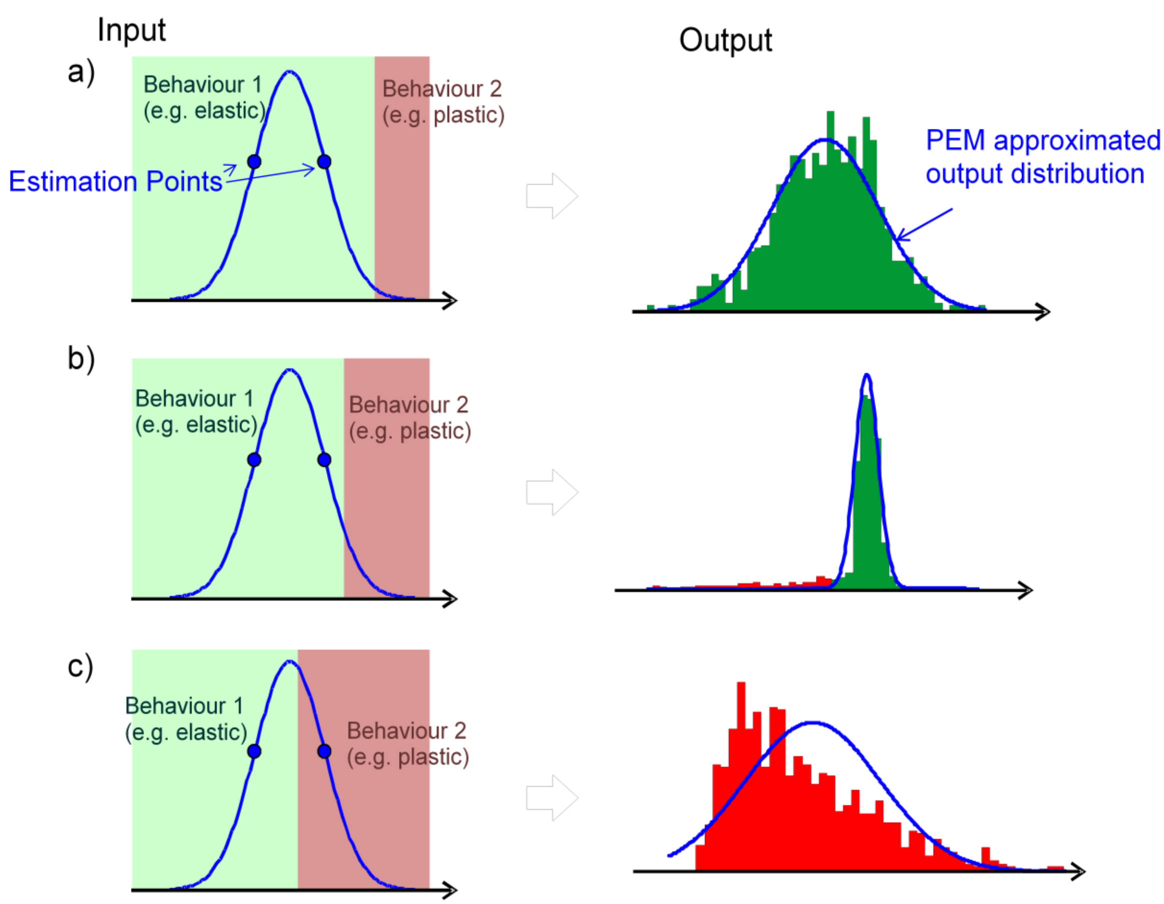

Fig. 4: Illustration of the effect of mixed behaviour on the accuracy of the PEM approximated output distribution compared to an actual output distribution evaluated using Monte-Carlo computations.

Conclusions

The PEM/FEM approach implemented in *Phase²* 8.0 presents an attractive method for handling the uncertainty and variability inherent in most geomechanical problems. The approximation using point estimates makes it computationally efficient and permits the performance of statistical analyses for problems for which other methods like Monte-Carlo simulation are not practical. However, its simplicity brings some limitations. The PEM approach, as presented here without correlation, is based on normal and uncorrelated distributions. When a modelled case differs from these assumptions the results can be inaccurate. Generally, the central tendency and some variability around it is well captured, but in many cases the tails may not be captured properly.

When modelling involves behaviour discontinuities, as for example when transitioning from elastic to plastic domains, the point estimate method shows further limitations and does not accurately capture the distribution of the design criteria. For this reason, it is recommended to test the effect of a limited number of random variables at a time. This will not only save computation time and allow deeper exploration of the possible outcomes but will also permit a better understanding and control over the potential bias introduced by the PEM/FEM approach. In addition to the outputs obtained using the proposed PEM/FEM approach, it is recommended to manually run some extreme cases of the targeted distributions in order to determine if it captures the tails of the output distribution properly. Never forget that a model must be as simple as possible, but not simpler.

In summary, when combined with an awareness of the assumptions and potential limitations, the PEM/FEM approach offers an attractive and very efficient way of considering uncertainty in FEM analyses. It should lead to a broader use of the probabilistic approach in the mining industry and a better assessment of the reliability level of the design of underground openings.

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