Undrained Behaviour

RS2 offers an undrained effective stress analysis for soil layers in combination with effective strength parameters. RS2 uses the undrained effective stress analysis combined with effective strength parameters (φ' and c') to obtain the undrained shear strength of a material. Pore pressure development is critical in identifying the correct effective stress path leading to a realistic failure value of undrained shear strength (c_u or s_u).

1. Undrained Effective Stress Calculation

When the undrained behaviour is selected for the material, special elements developed in RS2 will be automatically selected to account for the numerical stability when dealing with incompressible material.

Pore pressure in a soil body can usually be attributed to the presence of water and contributes to the total stress level within the soil. Terzaghi's principle (see also 1.5) states that total stresses (σ) are divided into effective stresses (σ'), pore pressure (p), and pore water pressure (ρ_w). Note that water does not sustain shear stress, therefore total shear stress is equal to effective shear stress.

$$\underline{\sigma} = \underline{\sigma'} + mp \tag{1.1a}$$

Where,

$$m = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} and\ p = \alpha S_e p_w \tag{1.1b}$$

$$\sigma_{xx} = \sigma'_{xx} + \alpha S_e p_w \tag{1.1c}$$

$$\sigma_{yy} = \sigma'_{yy} + \alpha S_e p_w \tag{1.1d}$$

$$\sigma_{zz} = \sigma'_{zz} + \alpha S_e p_w \tag{1.1e}$$

$$\sigma_{xy} = \sigma'_{xy} \tag{1.1f}$$

$$\sigma_{yz} = \sigma'_{yz} \tag{1.1g}$$

$$\sigma_{zx} = \sigma'_{zx} \tag{1.1h}$$

Where α = Biot's pore pressure coefficient and Se = effective degree of saturation. Considering incompressible gains, α = 1. Additional details about compressible grains and compressible solid material (α < 1) are provided in the Biot's coefficient section below.

Pore pressure is the product $\alpha S_e P_w$ and is termed p in RS2. An additional distinction must be made between steady state pore stress (p_{steady}) and excess pore stress (p_{excess}).

$$p_w = p_{steady} + p_{excess} \tag{1.2}$$

Steady state pore pressures (p_{steady}) are determined based on phreatic levels or by a groundwater flow calculation; this is considered input data. During plastic calculations, excess pore pressures are generated for undrained material behaviour or during a consolidation analysis. The following equations can be used to describe undrained material behaviour and related excess pore pressure calculations:

The time derivative of the steady state component is equal to zero, therefore:

$$\delta\rho w = \delta\rho excess \tag{1.3}$$

Inverting Hooke's law gives the following:

$$\begin{bmatrix} \delta\varepsilon^e xx \\ \delta\varepsilon^e yy \\ \delta\varepsilon^e zz \\ \gamma\varepsilon^e xy \\ \gamma\varepsilon^e yz \\ \gamma\varepsilon^e zx \end{bmatrix} = \frac{1}{E\prime} \begin{bmatrix} 1 & -v' & -v' & 0 & 0 & 0 \\ -v' & 1 & -v' & 0 & 0 & 0 \\ -v' & -v' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2v' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2v' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2v' \end{bmatrix} \begin{bmatrix} \delta\sigma'_{xx} \\ \delta\sigma'_{yy} \\ \delta\sigma'_{zz} \\ \delta\sigma'_{xy} \\ \delta\sigma'_{yz} \\ \delta\sigma'_{zx} \end{bmatrix} \tag{1.4}$$

Substituting in Equation 1.1 gives the following:

$$\begin{bmatrix} \delta\varepsilon^e xx \\ \delta\varepsilon^e yy \\ \delta\varepsilon^e zz \\ \gamma\varepsilon^e xy \\ \gamma\varepsilon^e yz \\ \gamma\varepsilon^e zx \end{bmatrix} = \frac{1}{E\prime} \begin{bmatrix} 1 & -v' & -v' & 0 & 0 & 0 \\ -v' & 1 & -v' & 0 & 0 & 0 \\ -v' & -v' & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2v' & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2v' & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2v' \end{bmatrix} \begin{bmatrix} \delta\sigma'_{xx} - \alpha\delta p_w \\ \delta\sigma'_{yy} - \alpha\delta p_w \\ \delta\sigma'_{zz} - \alpha\delta p_w \\ \delta\sigma'_{xy} \\ \delta\sigma'_{yz} \\ \delta\sigma'_{zx} \end{bmatrix} \tag{1.5}$$

In cases with slightly compressible water, excess pore pressure can be defined using the following:

$$\delta p_{excess} = \frac{\alpha\delta p_v}{nC_w + (\alpha - n)C_s} \tag{1.6a}$$

$$C_w = \frac{1}{K_w} \tag{1.6b}$$

$$C_s = \frac{1}{K_s} \tag{1.6c}$$

$K_w$represents the bulk modulus of the water, K_s represents the bulk modulus of the solid material, C_w represents the compressibility of the water, $C_s$is the compressibility of the solid material, and *n* is the porosity of the soil.

$$n = \frac{e_0}{1 + e_0} \tag{1.7}$$

Where e_0 is the initial void ratio, as defined in the soil properties.

The inverted form of Hooke's law may be written in terms of the total stress rates and the undrained parameters E_u and v_u:

$$\begin{bmatrix} \delta\varepsilon^e xx \\ \delta\varepsilon^e yy \\ \delta\varepsilon^e zz \\ \gamma\varepsilon^e xy \\ \gamma\varepsilon^e yz \\ \gamma\varepsilon^e zx \end{bmatrix} = \frac{1}{E_u} \begin{bmatrix} 1 & -v_u & -v_u & 0 & 0 & 0 \\ -v_u & 1 & -v_u & 0 & 0 & 0 \\ -v_u & -v_u & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2v_u & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+2v_u & 0 \\ 0 & 0 & 0 & 0 & 0 & 2+2v_u \end{bmatrix} \begin{bmatrix} \delta\sigma'_{xx} \\ \delta\sigma'_{yy} \\ \delta\sigma'_{zz} \\ \delta\sigma'_{xy} \\ \delta\sigma'_{yz} \\ \delta\sigma'_{zx} \end{bmatrix} \tag{1.8}$$

Where,

$$E_u = 2G(1 + v_u);\ v_u = \frac{3v' + \alpha B(1-2v')}{3 - \alpha B(1-2v')};\ B = \frac{\alpha}{\alpha + n(\frac{K'}{K_w} + \alpha - 1)} \tag{1.9}$$

where B represents Skempton's B-parameter.

In RS2 undrained models, the effective parameters *G* and *v'* are converted into their undrained counterparts E_u and V_u (see Equation 1.9 above).

To obtain realistic results, the bulk modulus of the water must be higher than the bulk modulus of the soil skeleton (K_w >> *n*K'). This can be achieved by requiring that v' ≤ 0.35.

2. Skempton B-Parameter

When using undrained drainage conditions, RS2 employs an implicit undrained bulk modulus (K_u) for the soil in its entirety (soil skeleton and water). The following equations are used to differentiate between total stress rates, effective stress rates, and rates of excess pore pressure:

Total stress: $\delta\sigma p = K_u \delta\varepsilon_v$ (2.1a)

Excess pore pressure: $\delta p_{excess} = B\delta p = \frac{\alpha\delta\varepsilon_v}{nC_w+(\alpha-n)C_s}$ (2.1b)

Effective stress: $\delta p' = (1-\alpha B)\delta p = K'\delta\varepsilon_v$ (2.1c)

Undrained effective stiffness parameters should be entered with material properties (define E' and v', not E_u and V_u). Hooke's law of elasticity is used to automatically calculate the undrained bulk modulus:

$$K_u = \frac{2G(1+v_u)}{3(1-2v_u)} \; where \; G = \frac{E'}{2(1+v')} \tag{2.2}$$

When the user manually inputs Skempton's B-parameter and Biot's pore pressure coefficient, v_u is calculated using the following equation:

$$v_u = \frac{3v' + \alpha B(1-2v')}{3-\alpha B(1-2v')} \tag{2.3}$$

The value of Skempton's B-parameter is calculated using the ratio of the bulk stiffnesses of the soil skeleton and the pore fluid:

$$B = \frac{\alpha}{\alpha + n(\frac{K'}{K_w} + \alpha - 1)} \tag{2.4}$$

Where $K_w = 2 * 10^6 \; kN/m^2$.

The following equation is used to calculate the rate of excess pore pressure using the (small) volumetric strain rate:

$$\delta p_{excess} = \frac{\alpha\delta\varepsilon_v}{nC_w + (\alpha-n)C_s} \tag{2.5}$$

The special undrained behaviour option in RS2 requires calculations that using effective stiffness parameters, with a distinction between effective stresses and excess pore pressures. The calculations may not completely address shear induced effective pore pressures.

This analysis requires effective soil parameters and is therefore very useful when these parameters are available. However, with soft soil projects, accurate effective soil parameter data may not be accessible. Therefore, in situ and laboratory tests may be performed to obtain undrained soil parameter data. In these cases, Hooke's law can be used to convert measured values of undrained Young's moduli into effective Young's moduli:

$$E' = \frac{2(1+v')}{3} E_u \tag{2.6}$$

This direct conversion from measured to effective values is not possible for advanced models. In these advanced cases, it is suggested to estimate the effective stress parameter from the measured parameter, then perform an undrained test to verify the resulting undrained stiffness (and adapt the estimated effective stiffness value if needed). The *Soil* test facility (Reference Manual) may be used in these circumstances.

3. Biot Pore Pressure Coefficient, α

Generally, the compressibility of the soil skeleton will be greater than that of the individual grains of soil in a mass; therefore, deformations of individual soil grains can be disregarded. In cases with deep soil layers at high pressures, the stiffness of the soil/rock matrix will approach that of the material of the soil/rock grains; in these cases, the compressibility of the solid material must be considered. This affects the division of total stress into effective stress and pore pressure. In cases with compressible solid material, Terzaghi's effective stress can be defined using the following equation:

$$\underline{\sigma'} = \underline{\sigma} - \alpha S_e \underline{m} p_w \tag{3.1}$$

Where α is Biot's pore pressure coefficient, S_e is the effective degree of saturation, $\underline{m}$ is a vector (with unity values for normal components and 0-values for shear components), and p_w is the pore water pressure. The following equation shows the definition of Biot's pore pressure coefficient:

$$\alpha = 1 - \frac{K'}{K_s} \tag{3.2}$$

Where K' is the effective bulk modulus for the soil matrix and K_s is the bulk modulus of the solid material. Note that for an incompressible solid material ($K_s = \infty$), Terzaghi's original stress definition holds true. Lower values of α indicate that for given values of total stress and pore water pressure, the resulting effective stress is higher than that of an incompressible solid material ($\alpha = 1$).

The value of Biot's pore pressure coefficient is automatically calculated by RS2, but the value may be changed manually by the user.