VERIFICATION PROBLEM #1

1.1 Introduction
This verification is from FLAC example 1.4.4 Dynamic Pore-Pressure Generation:

Itasca Consulting Group Inc. (2011). FLAC Dynamic Analysis Version 7.0 (pp. 105-120).

1.2 Background
Under rapid loading such as earthquake shaking, the pore pressure is increased and therefore the soil particles can readily move with respect to each other in saturated soil, causing a loss in the strength and stiffness of the soil. This phenomenon is defined as liquefaction. Particularly, when a saturated cohesionless soil is under rapid loading, the soil tends to densify, causing a reduce in the effective stress, which leads to liquefaction.

Although liquefaction is induced by the build up in pore pressure under rapid loading, the direct cause of liquefaction is indeed the reduce in effective stress due to the decrease in contact forces between soil particles (Dinesh et al. 2004). Under repeated shear cycle, the soil grains are forced to rearranged continuously, which then may be forced to move up against the adjacent soil particles, leading to dilation of the soil. Therefore, dilation is an important element in the liquefaction process.

Liquefaction is expected where induced stresses exceed the soil resistance. In standard practise, a liquefaction analysis is performed on soil based on a total stress analysis in the following three steps to access the potential for liquefaction of the soils, assuming the liquefiable soil remains undrained at the in-situ void ratio (Byrne and Wijewickreme 2006).

1. Triggering of liquefaction: The cyclic stress ratio (CSR) determined from numerical simulation is compared to the cyclic resistance ratio (CRR) derived from empirical curves and the factor of safety against triggering liquefaction is determined.
2. Flow Slide Assessment: After the triggering analysis, zones that are predicted to liquefy are assigned with post-liquefaction (undrained) strengths, which can be analyzed from penetration resistance using empirical charts. A standard limit-equilibrium analysis is then performed to determine the factor of safety against a flow slide.
3. Seismic Displacements: In this step, the displacement of the potential sliding block of soil is predicted using the Newmark approach. The potential sliding block of soil is simulated as a rigid mass resting on an inclined plane. An acceleration is applied at the base to determine the displacement of the block caused by shaking.

The main concerns of this three-step approach are that the three steps are considered as separate steps even though there might be interaction locally in some zones of the soil structure, changing the overall behaviour of the soil mass. Moreover, the changes in pore pressure is not considered by this assessment.

In order to evaluate the pore pressure redistribution, four methods: total-stress synthesized procedure, loosely coupled effective-stress procedure, fully coupled effective-stress procedure and fully coupled effective-stress bounding-surface procedures can be used, depending on the material models of the soils.

Total-stress synthesized procedure derived by Beaty and Byrne (2000) combines the above three steps into one single analysis, the assumption of the undrained behaviour of the soil holds true. This procedure uses a total stress approach to liquefaction analysis and relied on the adjustment of liquefied element properties at the instant of triggering liquefaction.

Loosely coupled effective-stress procedure uses the Seed cyclic stress approach by Seed and Idrisis (1971) to generate pore pressure from shear stress cycles. This coupled effective-stress constitutive model measures the cyclic stress ratio (CSR) of each shear stress cycle to compute the incremental excessive pore pressure. The model counts shear stress cycles by tracking the shear stress acting on horizontal planes and looking for stress reversal and it incorporates residual strength by using a two-segment failure envelope consisting of a residual cohesion and zero friction angle that is extended to meet with the traditional Mohr-Coulomb failure envelope. The Finn model in RS2 is currently using this approach.

Fully coupled effective-stress procedure focuses on predicting seismic response and liquefaction of cohesionless soils in plan strains. The elasto-plastic model is based on a hyperbolic relation between stress ratio and plastic shear strain similar to Duncan and Chang (1970)'s, which is applicable to the *Manzari and Dafalias* model in RS2.

The last one, fully coupled effective-stress bounding-surface procedure provides the capability to consider cyclic stress reversal in two and three dimensions. This constitutive model can reproduce the behaviour of soil under cyclic loading, including the reduce in shear modulus, the increase of hysteretic damping with cyclic shear strain amplitude, the shear and volumetric strain accumulation at a decreasing rate as the numbers of cycle increases, and the increase in liquefaction resistance with density. This model is applicable to the *Bounding Surface Plasticity* model in RS2.

1.2.1 Finn Model Formulation

Since the primary effect of liquefaction is the irrecoverable volume contraction in the soil grains, meaning a change in volumetric strain, when the soil is under a strain cycle with constant confining stress. If the voids are filled with fluid, the pore pressure and effective pressure stay constant if the volume is constant; however, when there is a volume contraction, the pore pressure increases and the effective pressure decreases.

This independency of the volumetric strain and cyclic shear-strain amplitude with respect to confining stress is noted by Martin et al. (1975) defined this mechanism as the following empirical equation:

$$\Delta\epsilon_{vd} = C_1(\gamma - C_2\epsilon_{vd}) + \frac{C_3\epsilon_{vd}^2}{\gamma + C_4\epsilon_{vd}} \tag{1.1}$$

Where $\Delta\epsilon_{vd}$ is the cyclic shear-strain amplitude,
γ is the engineering shear strain,
$C_1, C_2, C_3, and\ C_4$ are constants equal to 0.8, 0.79, 0.45 and 0.73 respectively.
It should be noted that the equation takes account of the accumulated irrecoverable volume strain ϵ_{vd} by decreasing the increment in volume strain as the volume strain is accumulated. Presumably, $\Delta\epsilon_{vd}$ should

be zero if γ is zero. Martins et al. also compute the change in pore pressure by assuming certain boundary conditions that were not clearly stated, which are taken care by RS2.
Another similar formula derived by Byrne (1991) also define this mechanism, in a simpler way:

$$\frac{\Delta\epsilon_{vd}}{\gamma} = C_1 exp\left(-C_2 \frac{\epsilon_{vd}}{\gamma C_1}\right) \tag{1.2}$$

Where $\Delta\epsilon_{vd}$ is the cyclic shear-strain amplitude,
γ is the engineering shear strain,
C_1, C_2 are constants, $C_1 = 7600(D_r)^2, C_2 = \frac{0.4}{C_1}$
Figure 1.1 illustrates the shear induced volumetric strain for constant amplitude of cyclic shear strain predicted by this formula. It can be shown from figure 1.1 that the formula predicts the volumetric strain to have an upward tendency with a decreasing rate of accumulation as the number of cycles grows.

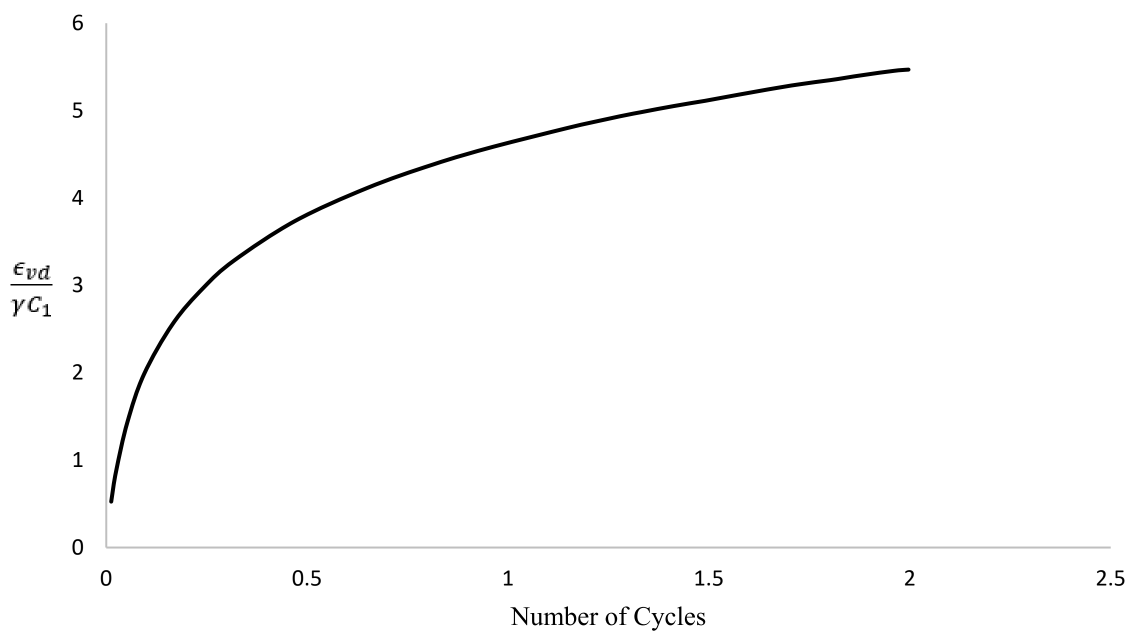

Figure 1.1 – Byrne Formula Graph

The incremental volumetric behaviour of the Byrne model can be expressed as equation (1.3):

$$\Delta\sigma_m + \alpha\Delta p = K(\Delta\epsilon + \Delta\epsilon_{vd}) \tag{1.3}$$

Where $\sigma_m = \sigma_{ii}/3$ is the mean stress,
P is pore pressure,
α is Biot coefficient (=1 for soil),
K is the drained bulk modulus of the soil,
ϵ is the volumetric strain.
For undrained conditions, the change in pore pressure is proportional to the change in volumetric strain as:

$$\Delta p = -\alpha M \Delta \epsilon \tag{1.4}$$

Where M is Biot modulus. After substitution of Equation (1.4) into (1.3), and solving for $\Delta\epsilon$, the following equation can be obtained:

$$\Delta \epsilon = \frac{\Delta \sigma_m - K \Delta \epsilon_{vd}}{K + \alpha^2 M} \tag{1.5}$$

If Equation (1.5) predicts no change in volume, then use $\Delta\epsilon = 0$ in Equation (1.3) gives us:

$$\Delta \sigma_m + \alpha \Delta p = K \Delta \epsilon_{vd} \tag{1.6}$$

Equation (1.6) predicts a decrease in magnitude of effective stress with cyclic shear strain which is produced by an increase of shear induced compaction. Under conditions of constant stress, $\Delta\sigma_m = 0$, an increase in pore pressure can be observed:

$$\Delta p = K \Delta \epsilon_{vd} \tag{1.7}$$

The increase in pore pressure is proportional to the drained bulk modulus of the soil. While under free stress conditions, the pore pressure will remain unchanged ($\Delta p = 0$), and the magnitude of the total stress will decrease according to:

$$\Delta \sigma_m = K \Delta \epsilon_{vd} \tag{1.8}$$

Please note that in both situations, the drained bulk modulus, K, is essential in determining the magnitude of the cyclic loading impact on effective stress. Therefore, the Byrne model captures the important physics of liquefaction.

1.3 Problem Description

A shaking table model consist of a box of sand is simulated in RS2. Periodic motion is applied at the base, on the two sides of the box and diminishes to zero at the top. Gravity is the only vertical loading in this case. The stresses and pore pressure are computed using the Martin et al. (1975) and the Byrne (1991) formulas. The α, β and time step are adjusted in RS2 to match the dynamic analysis damping parameters defined in FLAC. Figure 1.1 indicates the model geometry in RS2 and the numbers and locations of three different time queries. Figures 1.3a and 1.3b illustrates that the predicted pore pressure at three different time queries using either Martin or Byrne formulas has similar trends, and that the pore pressure results from RS2 and FLAC are almost identical.

1.3 Geometry and Properties

Table 1.1 - Material Properties

Analysis	ϕ′ (deg.)	E (kPa)	γ (kN/m^3)	C1	C2	C3	C4
Martin	35	491000	25.0	0.8	0.79	0.45	0.73
Byrne	35	491000	25.0	0.463234	0.431747	NA	NA

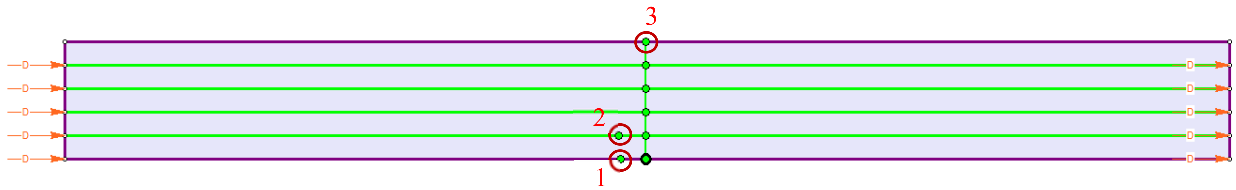

Figure 1.2 – RS2 Model Geometry

1.4 Results

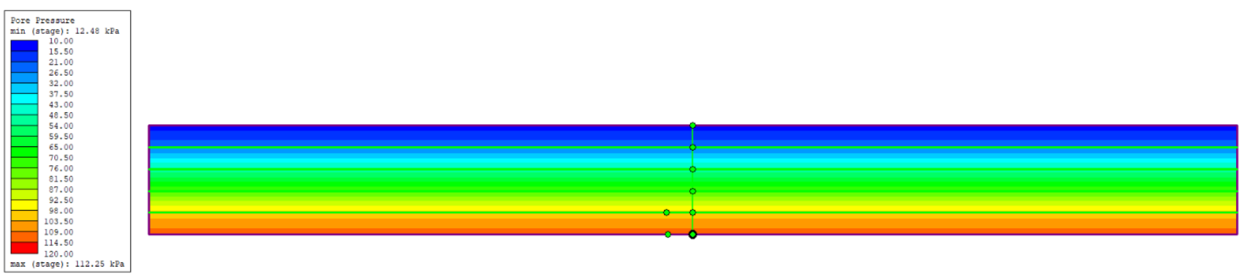

Figure 1.3 – Example Pore pressure Results in RS 2 using Byrne formula

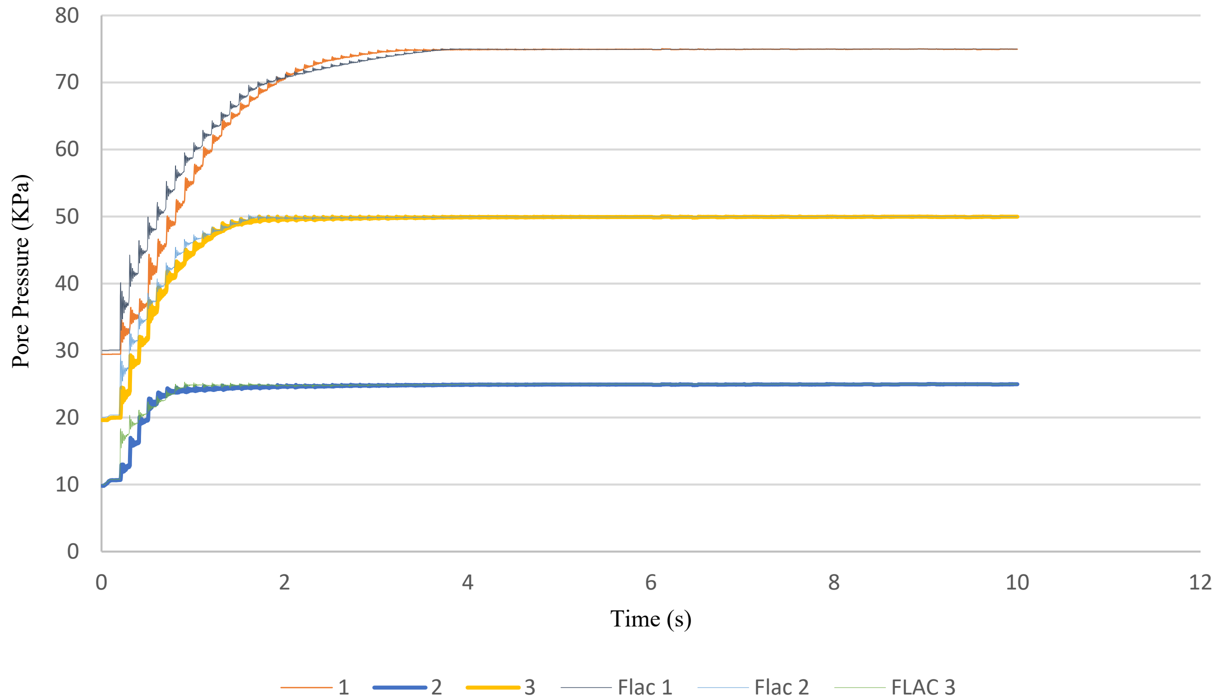

Figure 1.4a – Pore pressure vs Time at different time queries using Martin formula

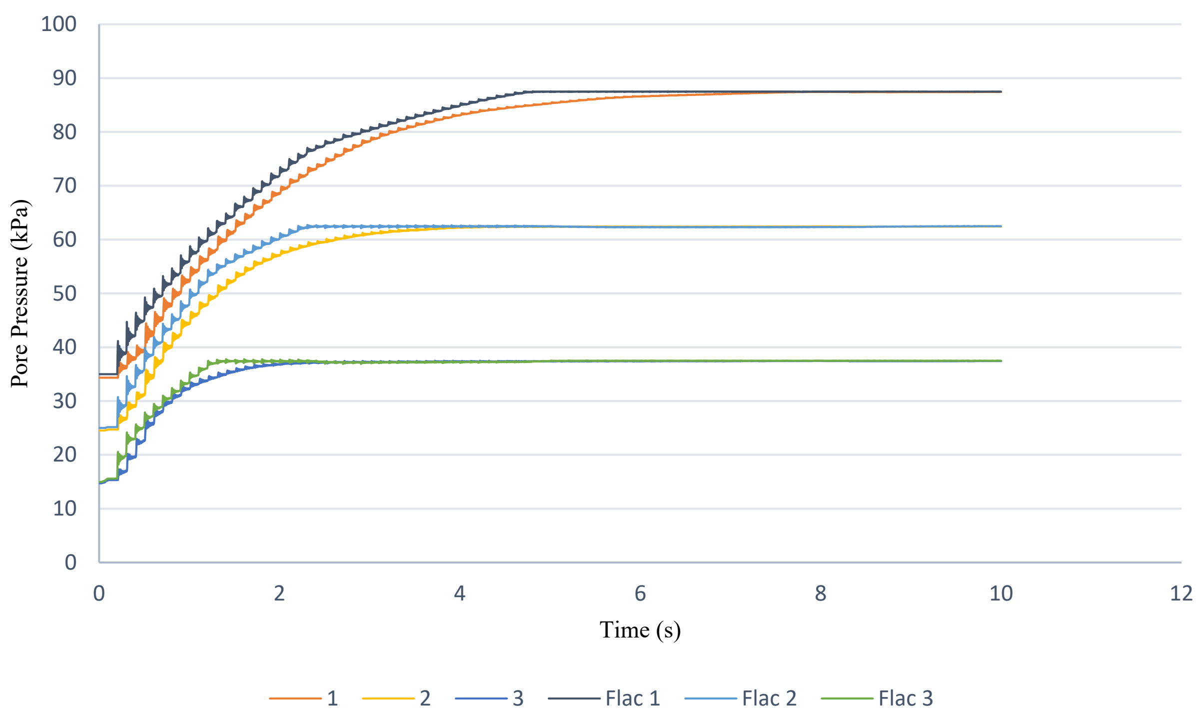

Figure 1.4b – Pore pressure vs Time at different time queries using Byrne formula

Reference

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