16- Hardening Soil Model with Small Strain Stiffness - PLAXIS

This model is the Hardening Soil Model with Small Strain Stiffness as presented in PLAXIS. The model is developed using the user-defined material model option in RS^2 and RS^3.

This model is an extension of the Hardening Soil Model with the major difference that the elastic properties are when the strains are very small. The base for this extension is illustrated in Figure 16.1.

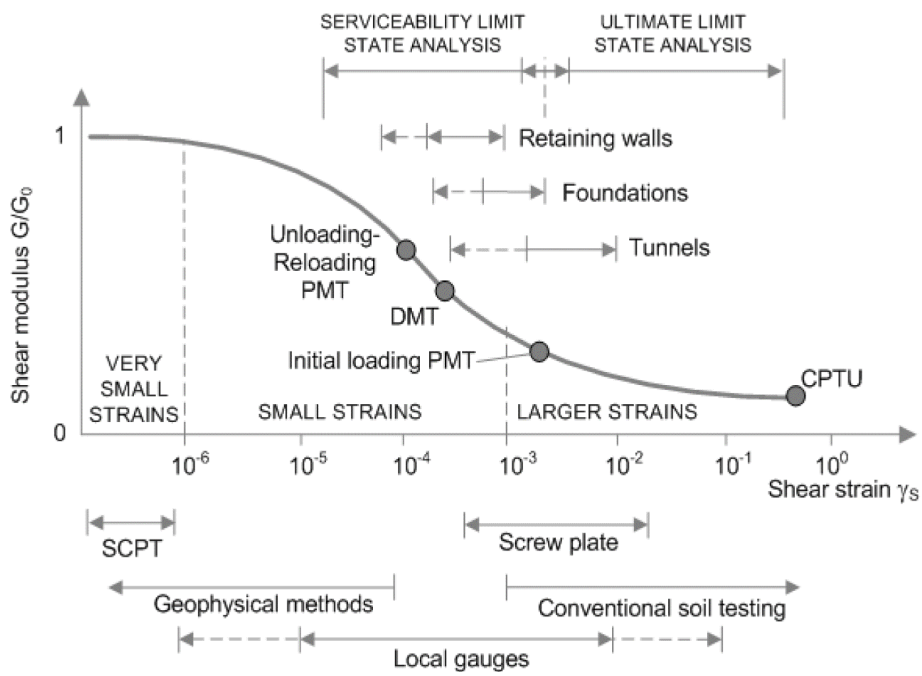

Figure 16.1- Characteristic stiffness-strain curve for soils

The S shaped characteristic stiffness-strain shows that the elastic modulus of soil is dependent on the level of shear strain, and major part of this curve is actually ignored when the material properties are evaluated from the conventional soil tests. In this graph, G_0 is the maximum/initial shear modulus and G is the shear modulus at a certain level of shear strain γ. Based on Figure 16.1, the elastic modulus that should be used in a numerical analysis should account for its dependency on the strain level.

16.1- Small Strain Stiffness

The S shape characteristic curve that is commonly used in soil dynamics is based on the Hardin and Drnevich (1972) equation

$$\frac{G}{G_0} = \frac{1}{1+\left|\frac{\gamma}{\gamma_r}\right|} \quad , \quad \gamma_r = \frac{\tau_{max}}{G_0} \qquad (16.1)$$

where τ_{max} is the shear stress at failure. This relationship covers the small and large range of strain up to failure. The use of smaller range for strain has been proposed for example by Santos and Correia (2001). They suggested to use $\gamma_r = \gamma_{0.7}$ at which the shear modulus is reduced to 70% of its initial value.

$$\frac{G}{G_0} = \frac{1}{1+a\left|\frac{\gamma}{\gamma_{0.7}}\right|} \quad , \quad a = 0.385 \qquad (16.2)$$

On top of the decay function that is applied to the shear modulus in 16.2, it is assumed that the elastic modulus is dependent on the level of stress similar to the relationships that was presented for Hardening Soil model. So equation 16.2 is used in combination of equation 16.3.

$$G_0 = G_0^{ref} \left(\frac{c \cos\varphi + \sigma_1 \sin\varphi}{c \cos\varphi + p_{ref} \sin\varphi} \right)^m \qquad (16.3)$$

where G_0^{ref} is the reference shear modulus at reference pressure.

Note that the cutoff for the decay function in 16.2 is the value of E_{ur}, meaning that the elastic modulus predicted for the numerical analysis cannot be less than this limit.

To be able to capture the hysteretic behavior of the material in loading-unloading-reloading stress paths the proposed approach by Masing (1926) is adopted in this model. In this method the initial shear modulus in unloading path is the same as the initial modulus in the initial loading path, and the shape of unloading and reloading path is similar to the initial loading but twice its size.

To obtain such a behavior in reloading conditions the Hardening Soil Model with Small Strain Stiffness uses the following modification

$$\gamma_{0.7-reloading} = 2\gamma_{0.7-virgin\ loading} \qquad (16.4)$$

16.2- History Dependent Shear Strain

The shear strain used in the decay function for the shear modulus is actually dependent on the loading history. Once the direction of loading is changed the stiffness regains a maximum recoverable value in the order of its initial value at that stress level. The shear strain used in the decay function is in general

$$\gamma = \frac{3}{2}\varepsilon_q \tag{16.5}$$

To make it history dependent Benz (2006) has proposed a modification

$$\gamma_{hist} = \sqrt{3}\frac{\|\underline{\dot{e}}_{km}\underline{H}^*_{ml}\|}{\|\underline{\dot{e}}_{kl}\|} \tag{16.6}$$

The $\|\dots\|$ denotes the Hilbert-Schmidt norm, $\|a\| = \sqrt{a_{ij}a_{ij}}$. The formulation to calculate the history dependent shear strain, γ_{hist}, is as follows.

The deviatoric strain history is stored in a symmetric tensor H_{kl}. In an increment of load the increment of deviatoric strain is $\dot{e}_{kl}$ and its eigen values and eigen vectors can be found by solving

$$\left(\dot{e}_{kl} - \lambda^{(i)}\delta_{kl}\right)V_l^{(i)} = 0 \tag{16.7}$$

where pairs of (λ, V_l) are the eigen value and eigen vectors, and δ_{kl} is the Kronecker delta.

$$\delta_{ij} = \begin{cases} 0 & , \ i \neq j \\ 1 & , \ i = j \end{cases} \tag{16.8}$$

The set of orthogonal eigen vectors forms a coordinate base V_{lm} in which the increment of deviatoric strain is a diagonal tensor with diagonal element being the eigen values. To identify a change in loading direction the deviatoric strain history and the increment of deviatoric strain are compared in this base.

$$\underline{\dot{e}}_{kl} = V_{km}\dot{e}_{mn}V_{lm} \tag{16.9}$$

$$\underline{H}_{kl} = V_{km}H_{mn}V_{lm} \tag{16.10}$$

In above $\underline{\dot{e}}_{kl}$ is the transformed increment of deviatoric strain and $\underline{H}_{kl}$ is the transformed deviatoric strain history. $\underline{\dot{e}}_{kl}$ is later used to calculate the γ_{hist}.

Each principal direction is checked for a possible change in the loading direction. Reversed loading cases are detected when the signs of the corresponding diagonal elements in $\underline{\dot{e}}_{kl}$ and $\underline{H}_{kl}$ are different. To calculate the updated strain history according to the loading direction a diagonal transformation matrix, T_{kl} , is defined as

$$T_{ii} = \frac{1}{\sqrt{1+H_{ii}}}\left(1 + u\left(\lambda^{(i)} H_{ii}\right)\left(\sqrt{1 + H_{ii}} - 1\right)\right) \tag{16.11}$$

where $u(x)$ is the Heaviside step function

$$u(x) = \begin{cases} 0 & , \ x < 0 \\ 1 & , \ x \geq 0 \end{cases} \tag{16.12}$$

The updated deviatoric strain history is calculated as

$$\underline{H}^{*}_{kl} = T_{km}\left(\underline{H}_{mn} + \delta_{mn}\right)T_{lm} - \delta_{kl} \tag{16.13}$$

The updated deviatoric strain history, $\underline{H}^{*}_{kl}$, then is used in equation 16.5 to calculate the γ_{hist}.

The updated deviatoric strain then is transformed to the original frame of reference for the calculations of next increment.

16.3- Other Considerations in the Small Strain Range

Within the range of small strain the hardening rule for both deviatoric and volumetric mechanisms are accelerated. The model should keep track of the minimum value of the elastic modulus, E_{min} that is calculated based on the decay function in 16.1.

The hardening rule for the deviatoric hardening mechanism then is modified by replacing the plastic deviatoric strain that is multiplied by a factor as below

$$\varepsilon_q^{p-shear} \leftarrow \varepsilon_q^{p-shear} \left(\frac{E_{min}}{E_{ur}}\right)^{1+E_{ur}/E_i} \tag{16.14}$$

Similarly the hardening rule for the cap is modified by

$$\dot{p}_c \leftarrow \dot{p}_c \left(\frac{E_{min}}{E_{ur}}\right)^{1+E_{ur}/E_i} \tag{16.15}$$

16.4- Examples

Figure 16.4 and 16.5 show the numerical results of drained triaxial tests on Berlin sand-I. A comparison is made between the results obtained by Hardening Soil with Small Strain Stiffness model in PLAXIS and simulation results of the same model in Rocscience products. The model parameters are presented in Table 16.1.

Parameter	Values for Berlin Sand I
p_{ref} (kPa)	100
E_{50}^{ref} (MPa)	45
E_{ur}^{ref} (Ma)	180
E_{oed}^{ref} (MPa)	45
m	0.55
ν (Poisson's ratio)	0.2
G_0^{ref} (MPa)	168.75
$\gamma_{0.7}$	0.0002
K_0^{nc}	0.43
ϕ (degrees)	35
ψ (degrees)	5
c (kPa)	1.0
Failure ratio	0.9
Tensile strength (kPa)	0

Table 16.1. Hardening Soil with Small Strain Stiffness model parameters for Berlin sand-I (PLAXIS 2014)

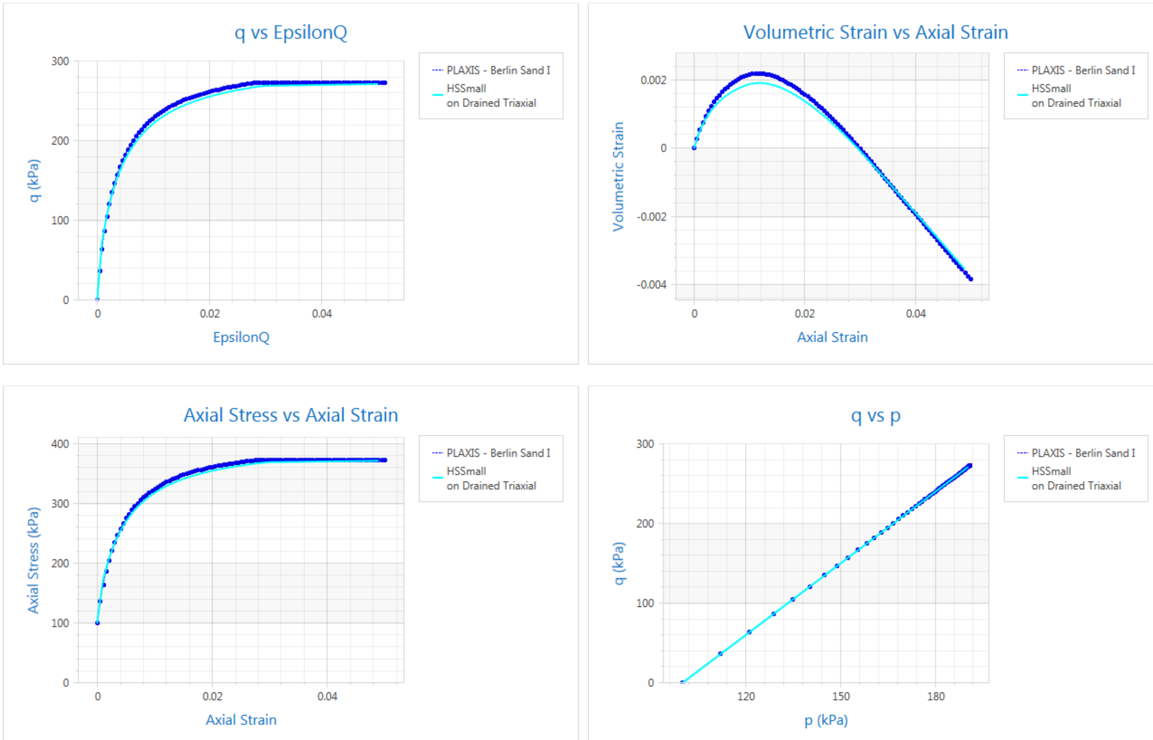

Figure 16.4. Stress paths of drained triaxial tests on Berlin sand-I

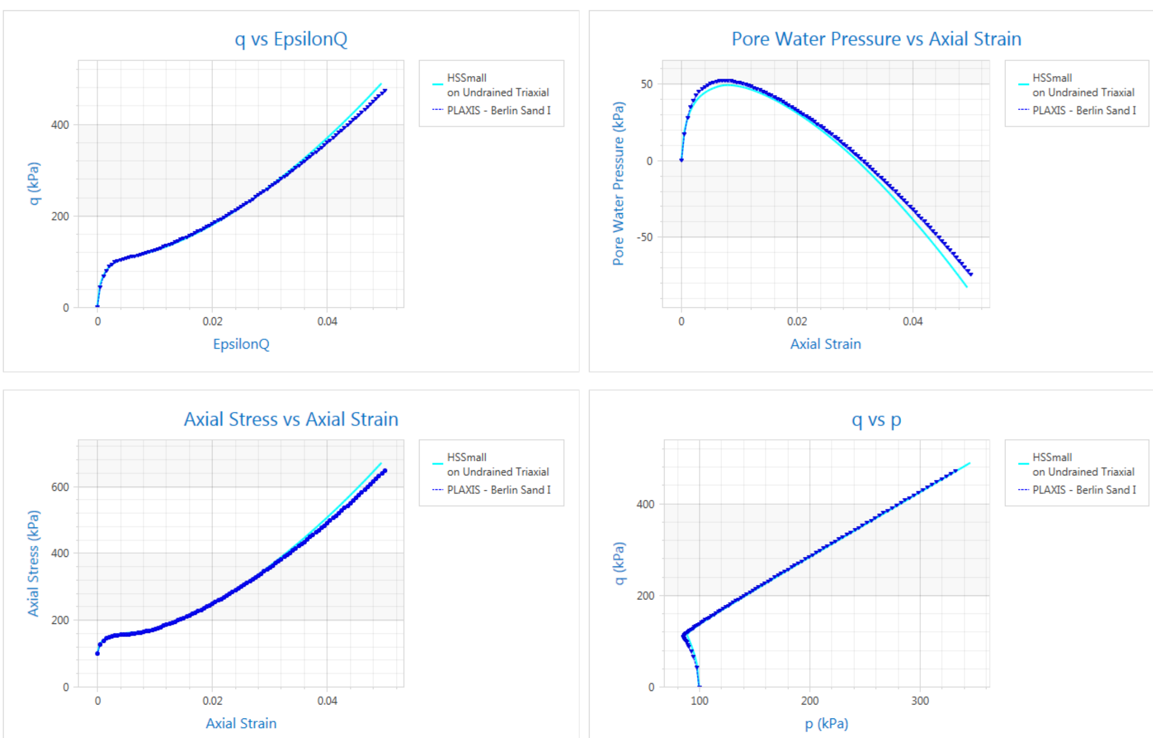

Figure 16.5. Stress paths of undrained triaxial tests on Berlin sand-I

References

Schanz, T., P. A. Vermeer, and P. G. Bonnier. "The hardening soil model: formulation and verification." Beyond 2000 in computational geotechnics (1999): 281-296.

Plaxis, "User's manual of PLAXIS." (2014).