7- Duncan-Chang Hyperbolic Material Model

The mechanical behavior of soils is highly nonlinear and their exhibit stress dependency in their stiffness. The Duncan-Chang model (Duncan and Chang, 1970) is a model developed to address these characteristics. There are more advanced models that are based on the theory of elasto-plasticity that can capture these features in ways that are more mathematically elegant, but this model has been included in RS^2 and RS^3 since it is well-known and has a history in soils mechanics.

This model is based on stress-strain curve in drained triaxial compression tests of both clays and sands which can be approximated by a hyperbolae function.

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{a + b\varepsilon} \tag{7.1}$$

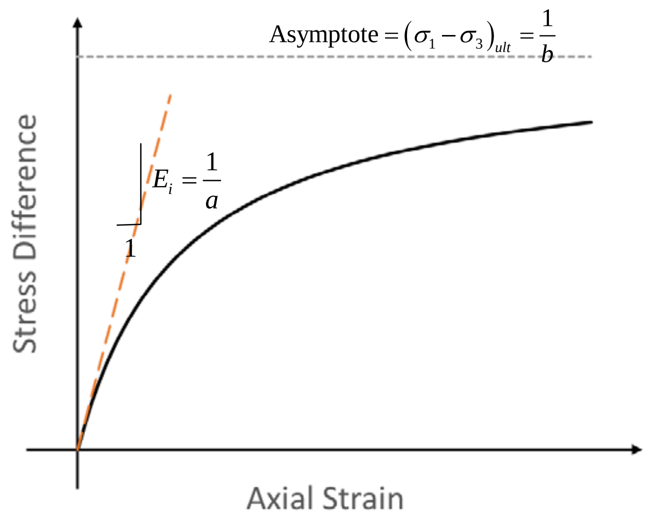

Figure 1- Hyperbolic stress-strain curve

This model is basically a nonlinear elastic model with loading and unloading/reloading elastic modulus that are stress dependent and formulated using power law functions. Its failure criterion is based on Mohr-Coulomb model.

The Mohr-Coulomb failure criterion in terms of principal stresses can be expressed as

$$F_s = \frac{1}{2}(\sigma_1 - \sigma_3) + \frac{1}{2}(\sigma_1 + \sigma_3)\sin\varphi - c\cos\varphi = 0 \tag{7.2}$$

where φ is in the angle of internal friction and c is the cohesion. Considering the Mohr-Coulomb material properties, the elastic modulus of Duncan-Chang model in loading condition, E_t, is

$$E_t = K_e P_{atm}\left(\frac{-\sigma_1}{P_{atm}}\right)^n\left(1-\frac{R_f(1-\sin\varphi)(\sigma_1-\sigma_3)}{2c\cos\varphi-2\sigma_1\sin\varphi}\right)^2 \tag{7.3}$$

In above K_e is the modulus number, this dimensionless parameter represents Young's modulus (range of values: 350-1120); n is the modulus exponent and governs the stress dependency of E_t (range of values: 0-1); R_f is the failure ratio (usually in the range of 0.6-0.95) and P_{atm} is the atmospheric pressure which is used to normalize stress inputs. The failure ratio is the ratio of deviatoric stress at failure to the asymptotic value of deviatoric stress that is obtained from the Mohr-Coulomb criterion.

There are two options for the Poisson's ratio, constant and stress dependent. Poisson's ratio is a direct input if the selected option is constant. Otherwise the bulk modulus is calculated from the equation below and from the combination of the elastic modulus and bulk modulus the Poisson's ratio will be calculated internally.

$$K_t = K_B P_{atm}\left(\frac{-\sigma_1}{P_{atm}}\right)^m \tag{7.4}$$

where K_B is the bulk modulus number.

The elastic modulus for unloading and reloading cases, E_u, is

$$E_u = K_u P_{atm}\left(\frac{-\sigma_1}{P_{atm}}\right)^n \tag{7.5}$$

where K_u is the unloading modulus number.

The dialog for defining this constitutive model is shown in Figure 7.1. The stiffness has a special option for defining this Duncan-Chang model, and when this option selected the strength tab will be automatically set to Mohr-Coulomb to define the rest of the parameters for the model.

Sample stress paths of drained triaxial compression tests on loose and dense sand samples Figure 4.2 and 4.3. The experimental results are from an article by Duncan and Chang (1970). Both tests start form a hydrostatic confinement of $p = p' = 294.3\ \text{kPa}$. The material properties of the sands are presented in Table 7.1.

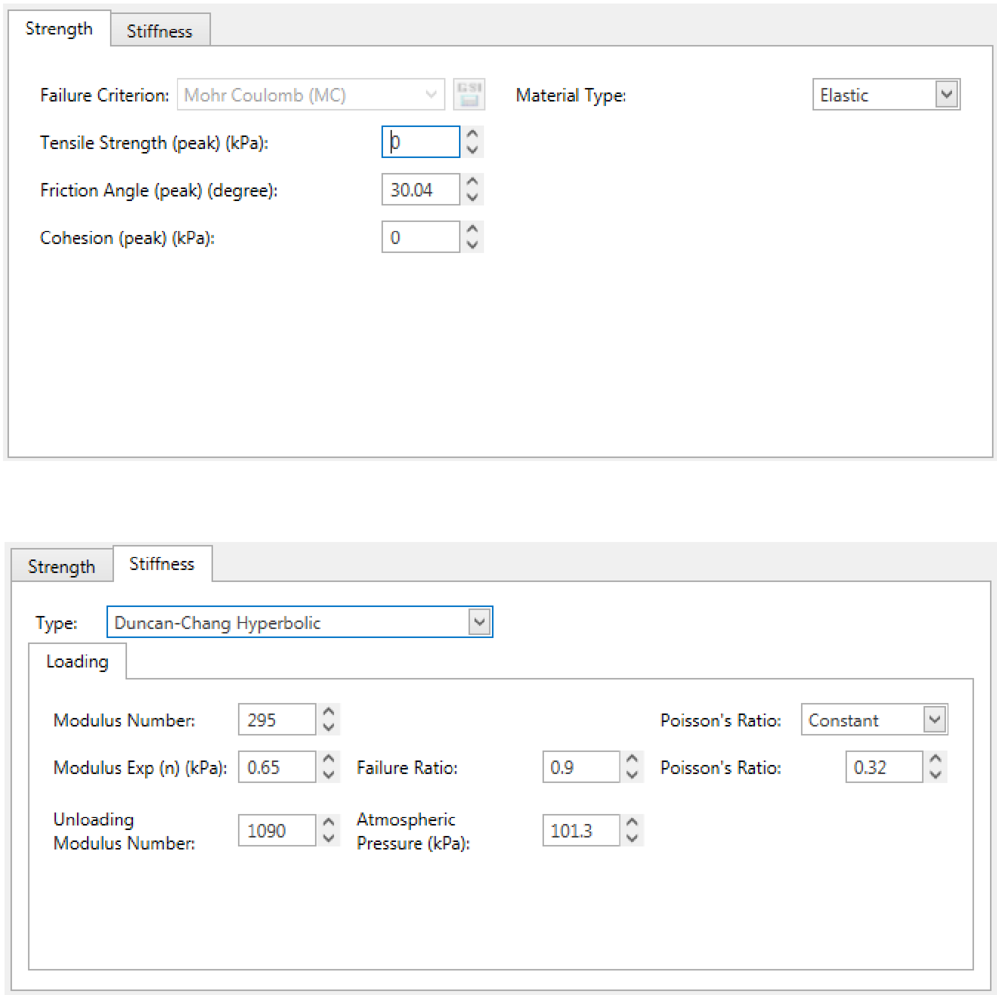

Figure 7.1. Dialog for defining Duncan-Chang model

Parameter	Dense Silica Sand	Loose Silica sand
Modulus number (K_e)	2000	295
Unloading modulus (K_u)	2120	1090
Modulus exponent (n)	0.54	0.65
Failure ratio (R_f)	0.91	0.90
Cohesion (c)	0	0
Friction angle (φ)	36.5º	30.4º
Poisson's ratio (ν)	0.32	0.32

Table 7.1. Duncan-Chang model parameters for loose and dense Silica sand (Duncan and Chang, 1970)

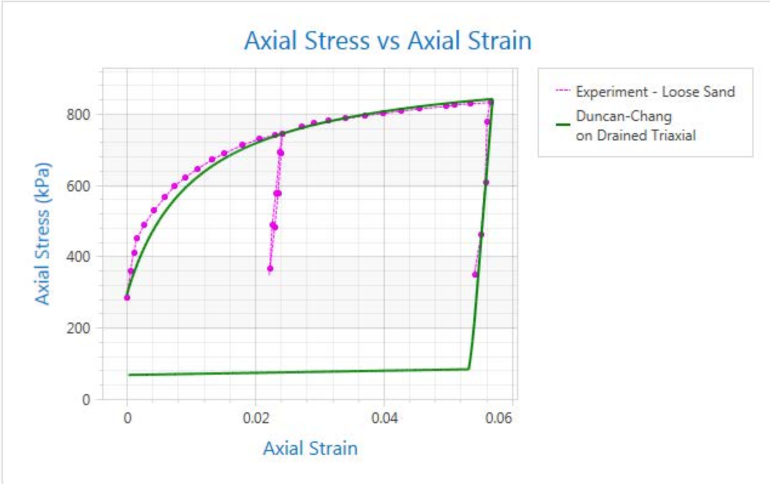

Figure 7.2. Drained Triaxial test on loose sand; comparison of experimental results with numerical results from Duncan-Chang model

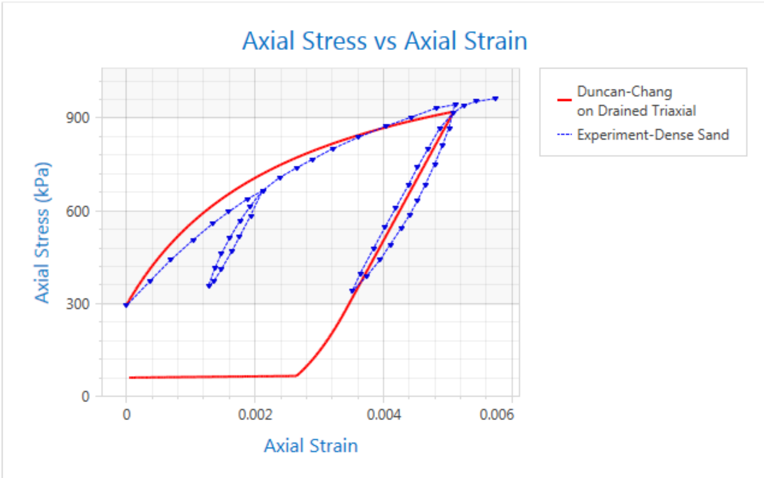

Figure 7.3. Drained Triaxial test on dense sand; comparison of experimental results with numerical results from Duncan-Chang model

References

Duncan, J. M., & Chang, C. Y. (1970). Nonlinear analysis of stress and strain in soils. Journal of Soil Mechanics & Foundations Div.