8- Cam Clay and Modified Cam Clay Material Models

The Cam-Clay and modified Cam-Clay models are an elastic plastic strain hardening models that are based on Critical State theory and the basic assumption that there is a logarithmic relationship between the mean stress and the void ratio. The first critical state models for describing the behavior of soft soils such as clay, the Cam-Clay (CC) and Modified Cam-Clay (MCC) were formulated by researchers at Cambridge University. Both models describe three important aspects of soil behavior, strength, compression or dilatancy (the volume change that occurs with shearing), and Critical State at which soil elements can experience unlimited distortion without any changes in stress or volume.

A large proportion of the volume occupied by a soil mass consists of voids that may be filled by fluids (primarily air and water). As a result, deformations in soil are accompanied by significant, and often non-reversible, volume changes. A major advantage of cap plasticity models, a class to which the CC and MCC formulations belong, is their ability to model volume changes more realistically.

The primary assumptions of the CC and MCC models are described next. In critical state mechanics, the state of a soil sample is characterized by three parameters, mean stress, deviatoric stress), and specific volume. The specific volume is defined as $\upsilon = 1 + e$, where e is the void ratio.

8.1- Virgin Consolidation Line and Swelling Line

The models assume that when a soft soil sample is slowly compressed under isotropic stress conditions, and under perfectly drained conditions, the relationship between specific volume and mean stress consists of a straight virgin consolidation line (also known as the normal compression line) and a set of straight swelling lines (see Figure 8.1). Swelling lines are also called unloading-reloading lines.

The virgin consolidation line in Figure 8.1 is defined by the equation

$$\upsilon = N - \lambda \ln\left(-p\right) \tag{8.1}$$

while the equation for a swelling line has the form

$$\upsilon = \upsilon_s - \kappa \ln\left(-p\right) \tag{8.2}$$

The values λ, κ and N are characteristic properties of a particular soil. λ is the slope of the normal compression (virgin consolidation) line on $\upsilon - \ln p$ plane, while κ is the slope of swelling line. N is known as the specific volume of normal compression line at unit pressure, and is dependent on the units of measurement. As can be seen on Figure 8.1, υ_s differs for each swelling line, and depends on the loading history of a soil.

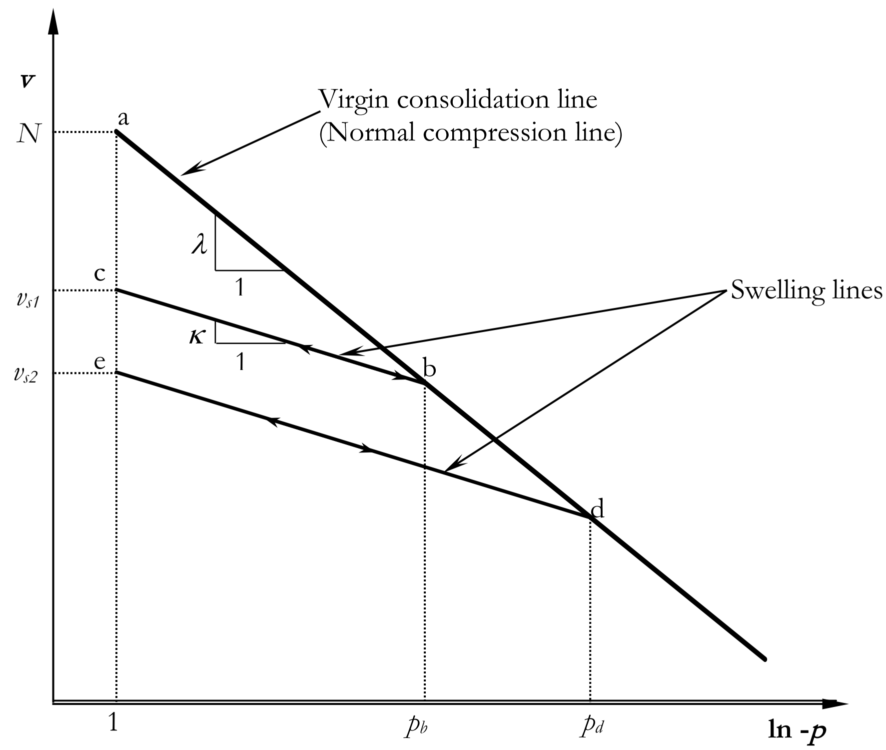

Figure 8.1. Typical behavior of clays in consolidation (oedometer) test

If the current state of a soil is on the virgin consolidation line the soil is described as being normally consolidated. If the stress state is below the line, it becomes overconsolidated. In general, soil does not exist outside the virgin consolidation line; when it does that state is unstable.

The hardening behavior of the CC and MCC models is formulated based on the virgin consolidation line, whereas the swelling line is used in calculations of elastic properties.

8.2- The Critical State Line

Sustained shearing of a soil sample eventually leads to a state in which further shearing can occur without any changes in stress or volume. This means that at this condition, known as the critical state, the soil distorts at constant state of stress with no volume change. This state is called the Critical State and characterized by the Critical State Line (CSL). In $p'-q$ plane the CSL is a straight line passing through the origin with the slope equal to M , one of the characteristic of the material that is the main parameter in the definition of yield surface (see Figure 8.3).

The location of this line relative to the normal compression line is shown on Figure 8.2. As seen in the picture, the CSL is parallel to the virgin consolidation line in $\upsilon - \ln p$ space. The parameter Γ is the specific volume of the CSL at unit pressure. Like N , the value of Γ depends on measurement units.

There is a relationship between the parameter N of the normal compression line and Γ. For the Cam-Clay model the two parameters are related by the equation

$$\Gamma = N - (\lambda - \kappa) \tag{8.3}$$

while for the Modified Cam-Clay model the relationship is

$$\Gamma = N - (\lambda - \kappa)\ln 2 \tag{8.4}$$

Due to this relationship between N and Γ, only one of them needs to be specified when describing a Cam-Clay or Modified Cam-Cam material.

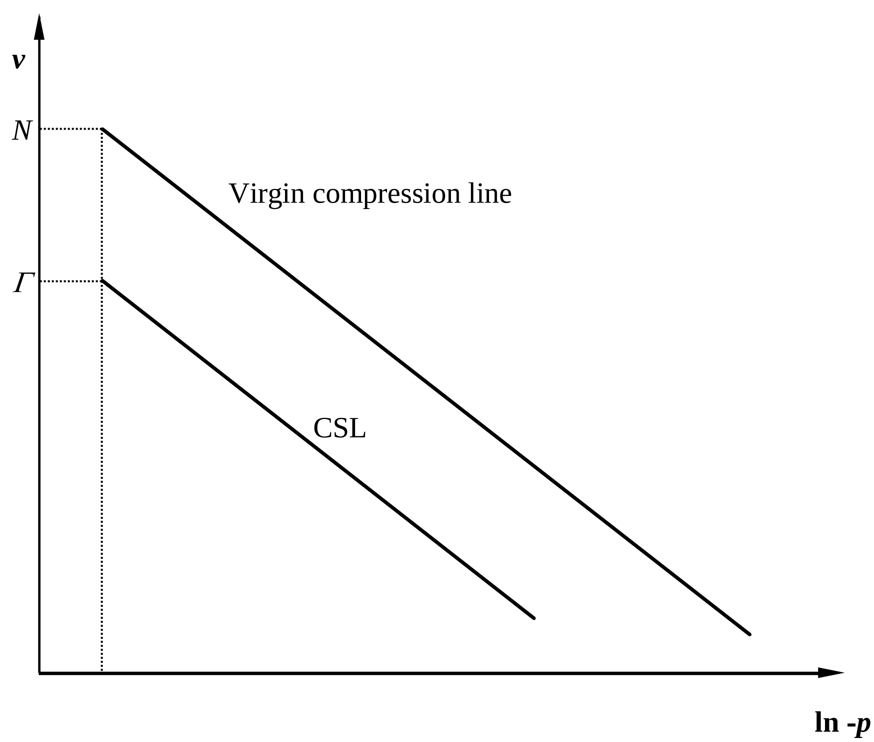

Figure 8.2. Typical Critical State Line and Virgin Compression Line of clays

8.3- Yield Functions

The yield functions of the Cam-Clay model is

$$F_c = q + Mp\ln\left(\frac{-p}{p_c}\right) = 0 \tag{8.5}$$

And the yield function for modified Cam-Clay

$$F_c = q^2 + M^2 p\left(p + p_c\right) = 0 \tag{8.6}$$

In $p-q$ space, the CC yield surface is a logarithmic curve while the MCC yield surface plots as an elliptical curve (Figure 8.3). The parameter p_c (known as the yield stress or pre-consolidation pressure) controls the size of the yield surface. The parameter M is the slope of the CSL in $p-q$ space. A key characteristic of the CSL is that it intersects the yield curve at the point at which the maximum value of q is attained.

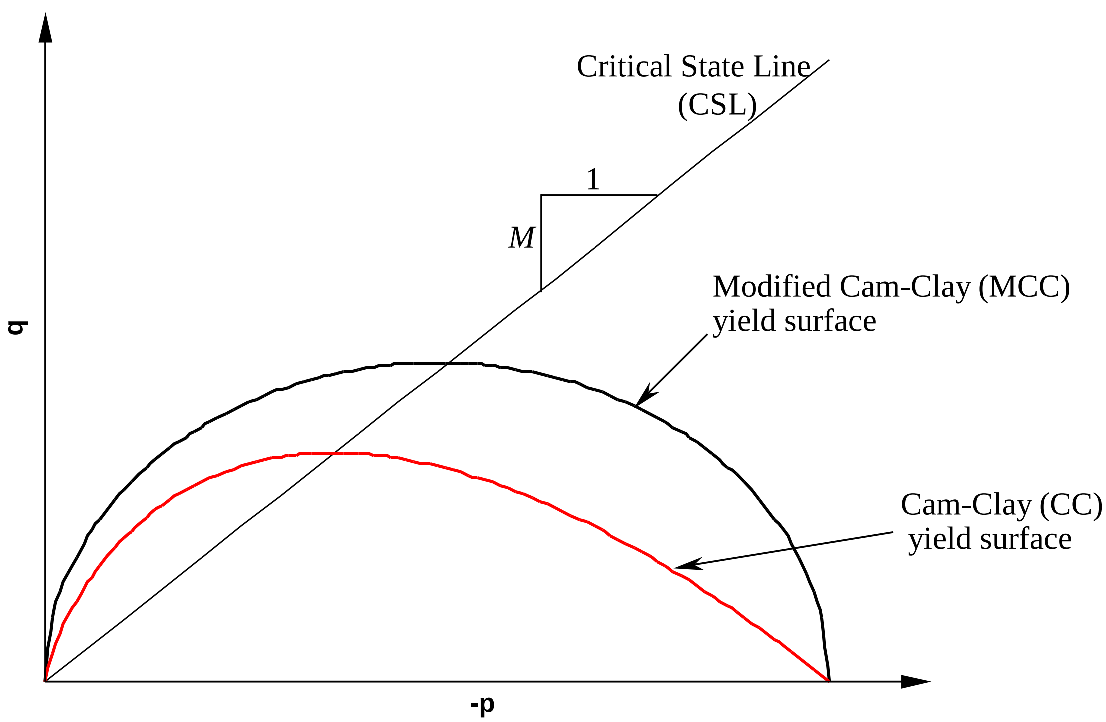

Figure 8.3. Yield surfaces of the Cam Clay and Modified Cam Clay models in p-q plane

In three-dimensional space $\upsilon - p - q$ the yield surface defined by the CC or MCC formulation is known as the *State Boundary Surface*. The State Boundary Surface for the Modified Cam-Clay model is shown in Figure 8.4.

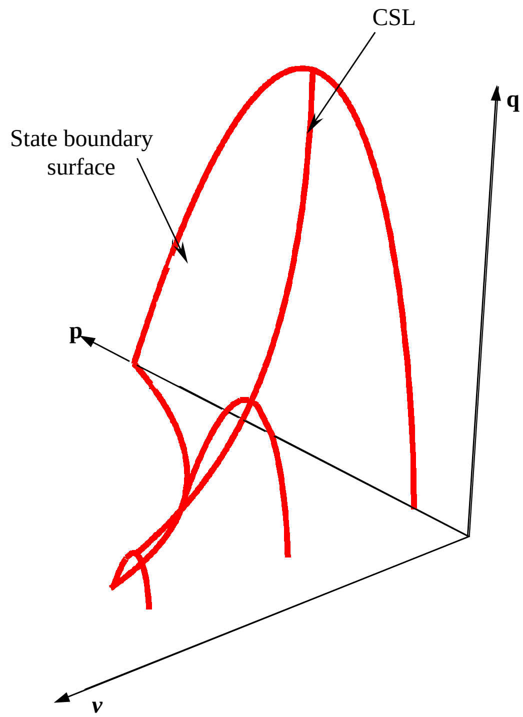

Figure 8.4. State Boundary Surface

8.4- Elastic Material Constants for Cam-Clay and Modified Cam-Clay

For Cam-Clay and Modified Cam-Clay soils, the bulk modulus is not constant. It depends on mean stress, specific volume, and the slope of the swelling line

$$K = -\frac{\upsilon p}{\kappa} \quad (8.7)$$

Cam-Clay and Modified Cam-Clay formulations require specification of either shear modulus G or Poisson's ratio ν.

8.4 The Overconsolidation Ratio and Initial State

The current state of a soil can be described by its stress state (p, q), specific volume υ, and yield stress, p_c (also known as preconsolidation pressure is a measure of the highest stress level the soil has ever experienced). The ratio of preconsolidation pressure to current pressure is known as the overconsolidation ratio (OCR).

The in-situ distribution of preconsolidation pressure for a Cam-Clay or Modified Cam-Clay material can be generated using the OCR. An OCR value of 1 represents a normal consolidation state; a state in which the maximum stress level previously experienced by a material is not larger than the current stress level. OCR > 1 describes an overconsolidated state indicating that the maximum stress level experienced by the material is larger than the present stress level.

To compute models involving Cam-Clay or Modified Cam-Clay materials, non-trivial initial effective stresses must be specified. RS^2 and RS^3 allow specification of gravity in-situ stresses or a constant stress field. The initial yield surfaces for all stress states must be specified by finding the corresponding p_c. This can be done by assigning the preconsolidation pressure directly or by specification of the OCR.

If a current stress state completely lies within a specified yield surface, the soil will initially respond elastically to loading. This implies that it is overconsolidated. If, however, the initial stress state is located on the yield surface, the soil will respond elasto-plastically upon loading, indicating that it is normally consolidated.

Since initial stress states that lie outside yield surfaces have no physical meaning for Cam-Clay and Modified Cam-Clay models, the programs will readjust the preconsolidation pressure to accommodate for the current level of stress

8.6- Hardening and Softening Behavior

The hardening of the material is attributed to plastic volumetric strain or the compaction of material, which in terms is equivalent to reduction in void ratio and specific volume.

Considering an increment of load from step n to $n+1$ the expansion of the yield surface is defined by the increase in preconsolidation pressure

$$(p_c)_{n+1} = (p_c)_n \exp\left(\frac{v_n \Delta\varepsilon_v^p}{\lambda - \kappa}\right) \tag{8.8}$$

If yielding occurs to the right of the point at which the CSL intersects a yield surface, hardening behavior, accompanied by compression, is exhibited. This side of the yield surface is known as the wet or subcritical side.

If yielding occurs to the left of the intersection of the CSL and yield surface (called the dry or supercritical side), the soil material exhibits softening behavior, which is accompanied by dilatancy (increase in volume). In softening regimen the yield stress curve decreases after the stress state touches the initial envelope.

8.7- Verifications and Examples

Analytical solution for Modified Cam Clay model in triaxial stress and strain states are presented by Peric (2006). For verification purposes the analytical results of some triaxial tests are compared to simulation results in this section. Figure 8.5 shows the results of a triaxial test that starts form a hydrostatic confinement of $p = p' = 200$ kPa on a normally consolidated clay using Modified Cam Clay model with constant Poisson's ratio. Similar results are presented for the case of constant shear modulus in Figure 8.6. The final yield surface is also included in the plot of the stress path in p-q plane. The material properties of the Modified Cam Clay model are presented in Table 8.1.

The observer behavior in both cases includes the gradual hardening of the material as the sample accepts higher levels of deviatoric stress and continuous compaction until reaching to the critical state.

Figure 8.7 shows the analytical and numerical results of triaxial test on an over consolidated clay with OCR=2.0. The test starts from an initial confinement of $p = p' = 100$ kPa, but until the stress path reaches to the initial yield surface with $p_c = 200$ kPa the behavior is elastic. After that the elastoplastic behavior will start and the sample will be sheared to the critical state condition.

The case of highly over consolidated clay with OCR=5.0 is presented in Figure 8.8. In this case the initial behavior is elastic until the stress path reaches the yield surface on the left side of its peak. The behavior is then shows a softening branch and then the stress path approaches to the critical state.

The 3D yield surfaces of the Cam Clay model and Modified Cam Clay model are presented in Figures 8.9 and 8.10 respectively.

Parameter	*Value*
N	1.788
M	1.2
λ	0.077
κ	0.0066
G (for the case of constant elasticity)	20000 kPa
μ (for the case of variable elasticity)	0.3
Initial State of the Normally Consolidated Clay	
Preconsolidation pressure, p_o	200 kPa
Initial mean volumetric stress, p'	200 kPa
Initial shear stress, q	0 kPa
Initial State of the Lightly Over Consolidated Clay	
Preconsolidation pressure, p_o	200 kPa
Initial mean volumetric stress, p'	100 kPa
Initial shear stress, q	0 kPa
Initial State of Highly Over Consolidated Clay	
Preconsolidation pressure, p_o	500 kPa
Initial mean volumetric stress, p'	100 kPa
Initial shear stress, q	0 kPa

Table 8.1. Cam Clay model parameters

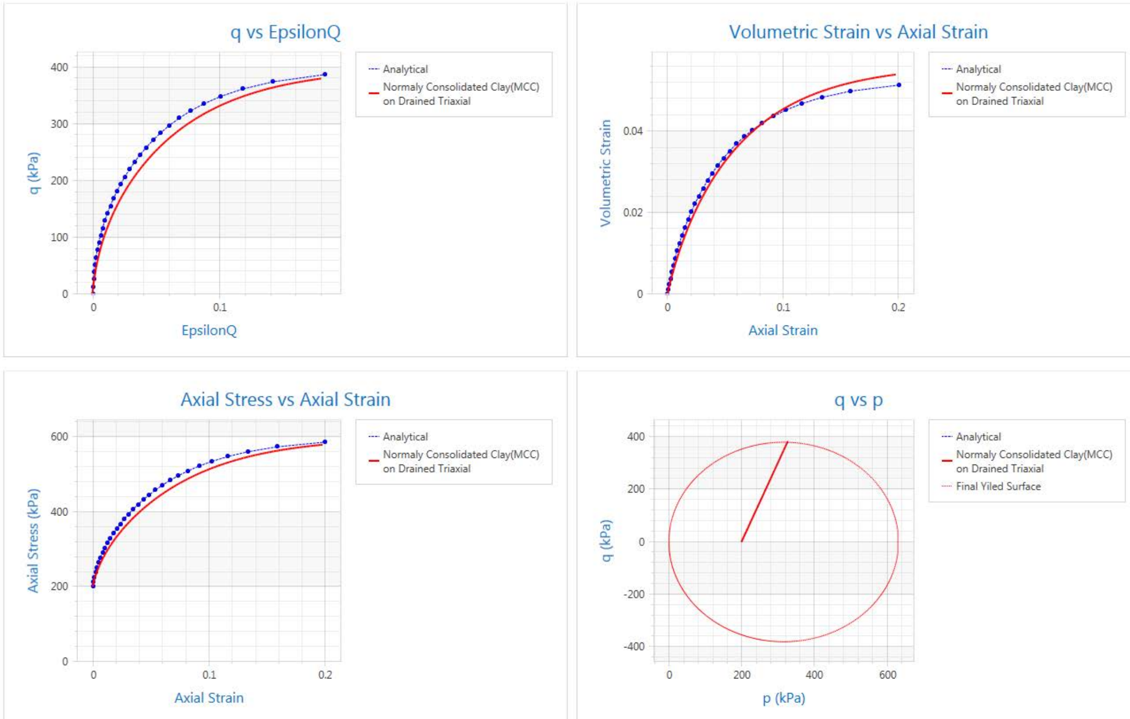

Figure 8.5. Stress paths of drained triaxial tests on normally consolidated clay with Modified Cam Clay model (constant Poisson’s ratio)

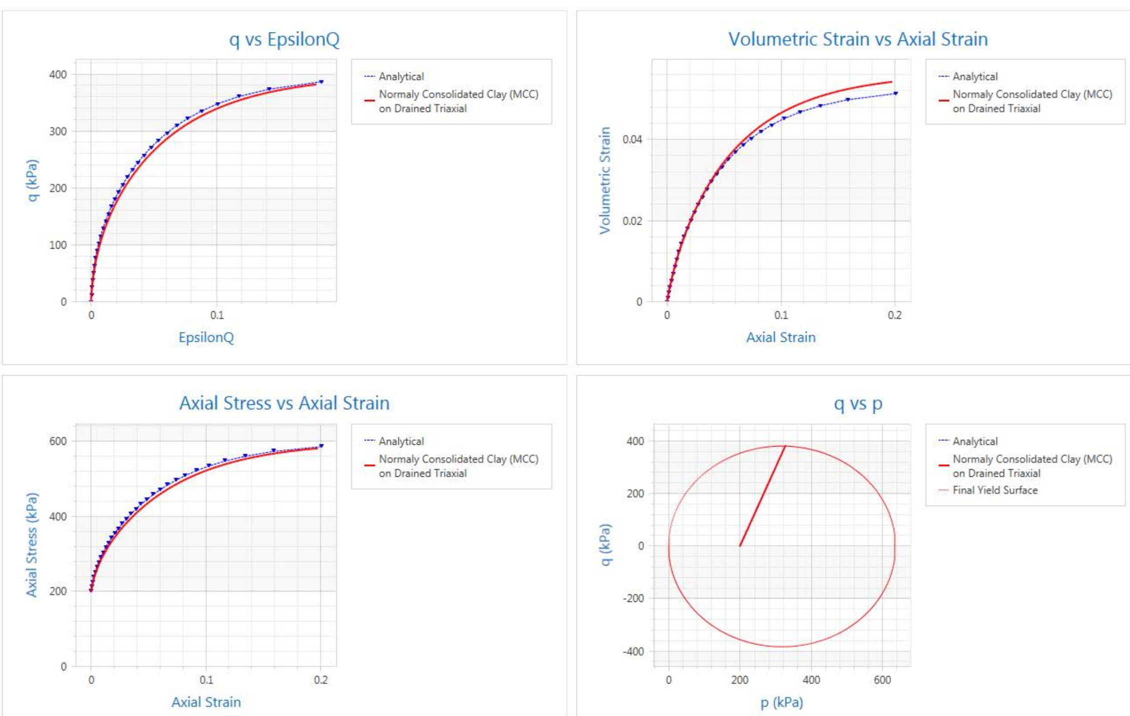

Figure 8.6. Stress paths of drained triaxial tests on normally consolidated clay with Modified Cam Clay model (constant shear modulus)

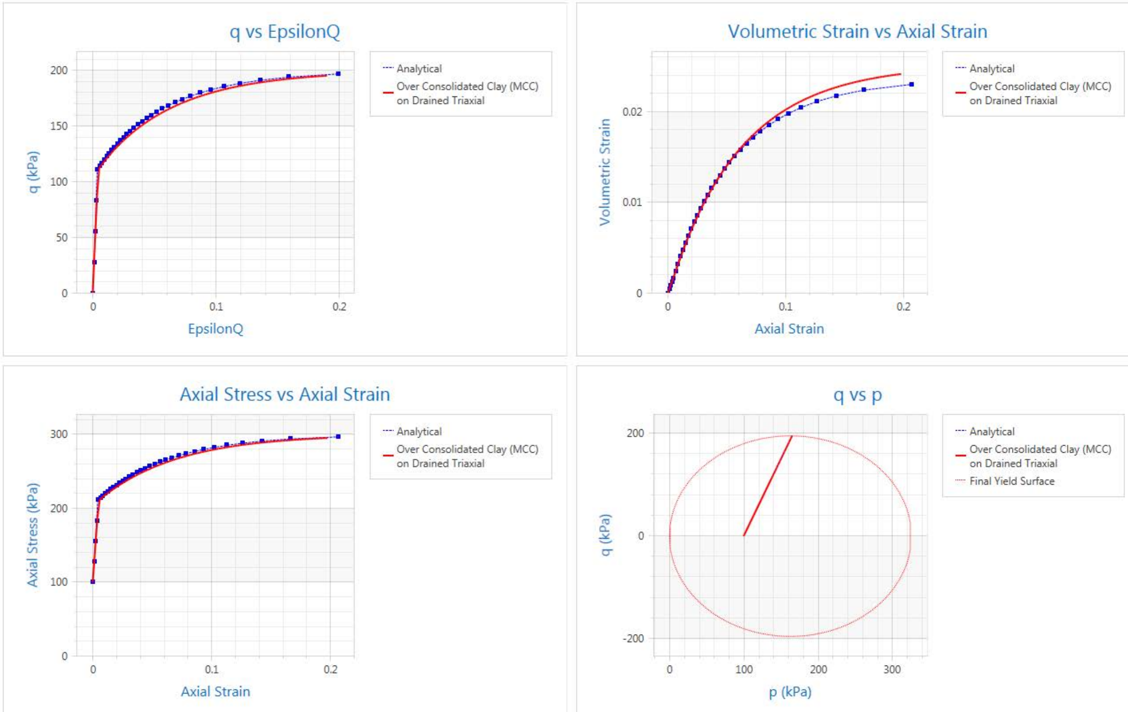

Figure 8.7. Stress paths of drained triaxial tests on over consolidated clay (OCR=2) with Modified Cam Clay model (Constant Poisson's ratio)

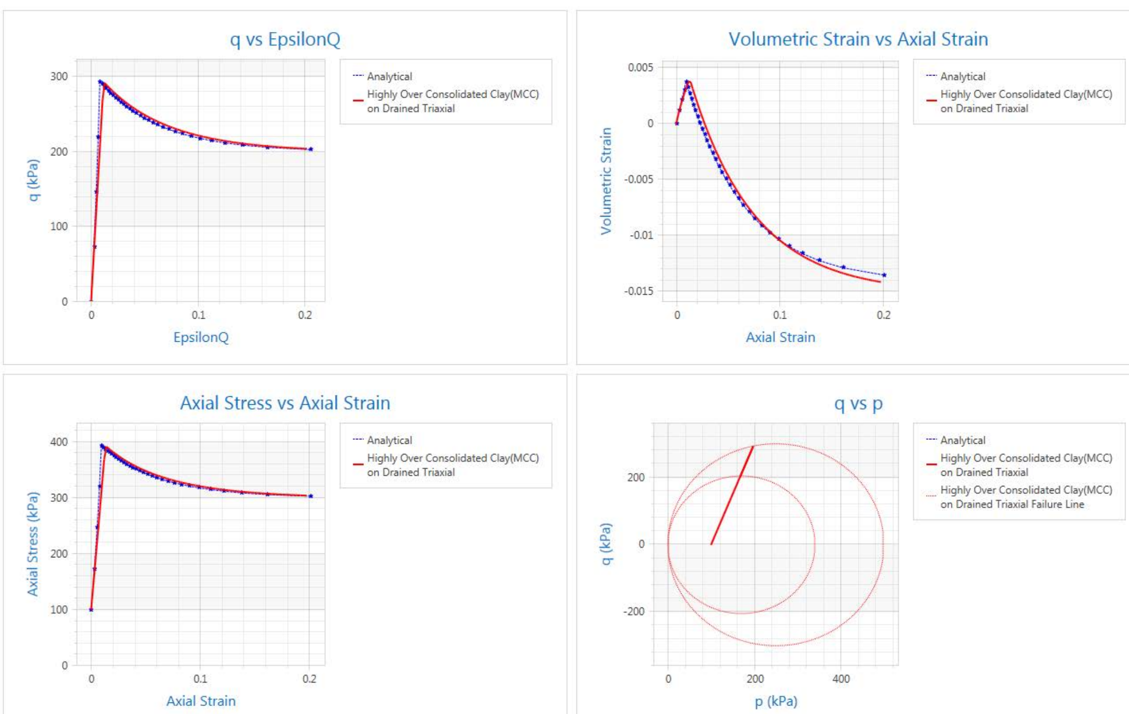

Figure 8.8. Stress paths of drained triaxial tests on highly over consolidated clay (OCR=5) with Modified Cam Clay model (Constant Poisson's ratio)

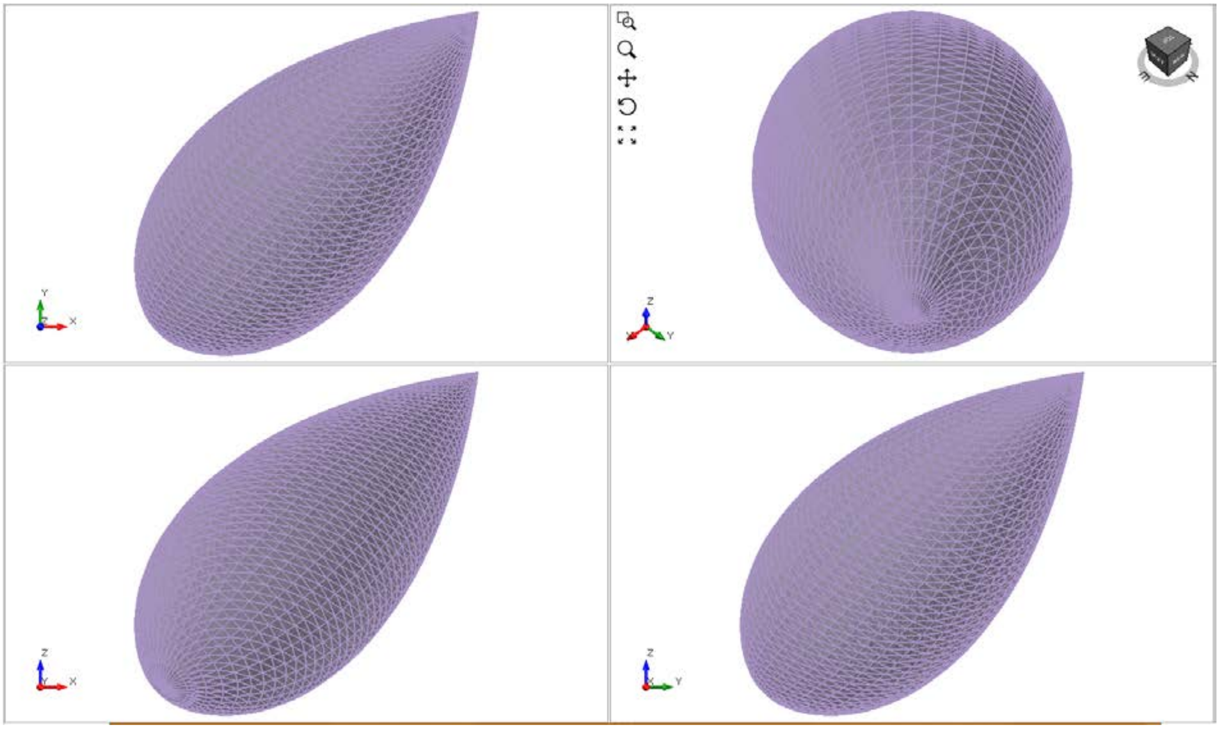

Figure 8.9. Yield surface of Cam Clay model in 3D stress space

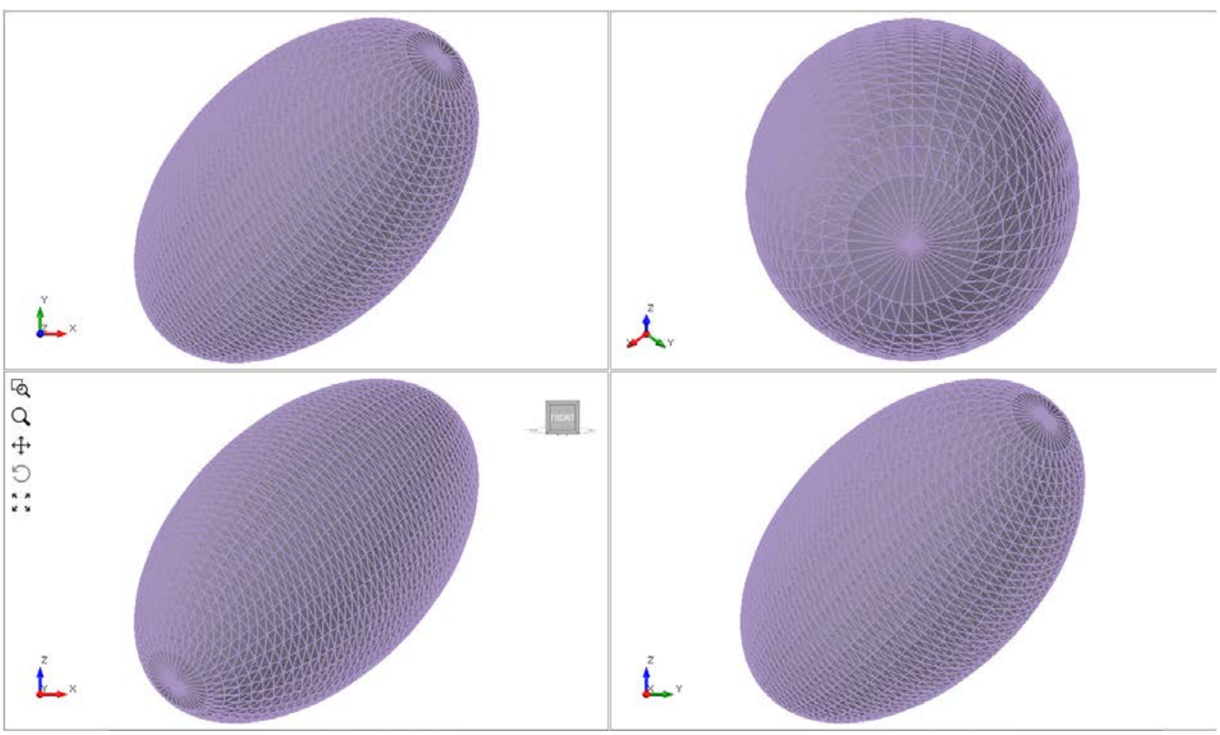

Figure 8.10. Yield surface of Modified Cam Clay model in 3D stress space

8.8- Remark on the application of Cam-Clay and Modified Cam-Clay models in FE analyses:

The Cam-Clay and Modified Cam-Clay models may allow for unrealistically large ratios of shear stress over mean stress when the stress state is above the critical state line. Furthermore, these models predict a softening behavior for the state of stress on the dry side of the yield surface. Without special considerations, the softening behavior leads to mesh dependency of a finite element analysis. The use of these two models in simulations of practical applications including general boundary valued problems is not recommended.

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Peric´ D. (2006), Analytical solutions for a three-invariant Cam clay model subjected to drained loading histories, Int. J. Numer. Anal. Meth. Geomech., 30, 363–387.