rocscience

RS3

Consolidation

Verification Manual

Table of Contents

1. One Dimensional Compression of a Finite Layer

1.1. Problem Description

This problem analyzes a one-dimensional consolidation of an elastic soil body. A ramp load is applied at the top surface of soil over a time period, t_0, and remained constant, as illustrated in Figure 1-1. The rate of load is described as a dimensionless time factor, T_{v0}, which, in this problem, is 0.0001 (the rate is inverse of the magnitude). The soil is under drained conditions with seepage only allowed at the top surface. The material properties of soil are outlined in Table 1.1.

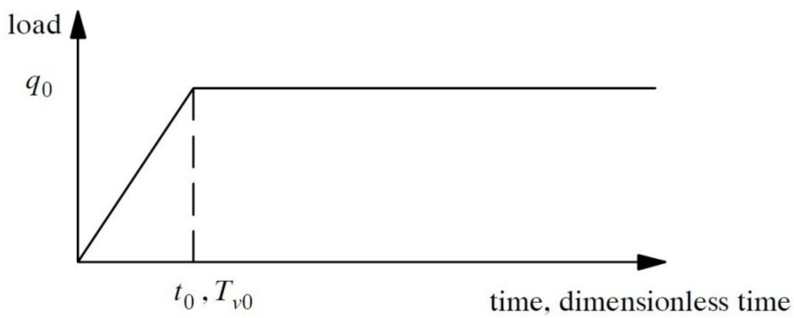


Figure 1-1: Load vs. Time

Table 1.1: Model parameters

Parameter	Value
Young's modulus (E)	200 kPa
Poisson's ratio (v)	0.3
Permeability (k)	0.01 m/s
Thickness (H)	10 m
Coefficient of consolidation (c_v)	0.2744 m^2/s

The dimensionless time of consolidation is determined by the following equation

$$T_v = \frac{c_v t}{H^2}$$

where c_v is the coefficient of consolidation described by the following equation

$$c_v = \frac{kE(1-v)}{\gamma_w(1+v)(1-2v)}$$

where k, E, and v being the drained permeability, the drained Young's modulus, and the drained Poisson's ration of the soil, respectively.

The degree of consolidation is defined by the following equation

$$U = \frac{S_c}{S_{c,max}}$$

where S_c is the settlement of the soil layer at each time step, measured from H = 0m.

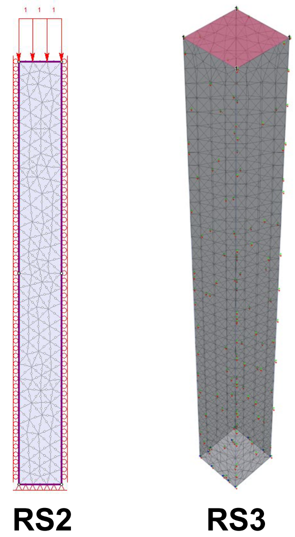


Figure 1-2 Model Geometry and Applied Boundary Conditions.

1.2. Analytical Solution

From the work by Terzaghi [1], several assumptions are made to determine the rate and degree of consolidation. They include that the coefficient of permeability and volume compressibility remains the same at every point in the layer; and for a consolidated compressed layer with uniform thickness the volume of drained water per unit time exceeds the amount that enters, which is equivalent to the change in volume.

Based on these assumptions, and using the void equation and Darcy's law, the differential equation for consolidation under linear drainage is as following

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

Furthermore, the percentage of settlement, $U\%$, is described as a function of T_v, with the following equation

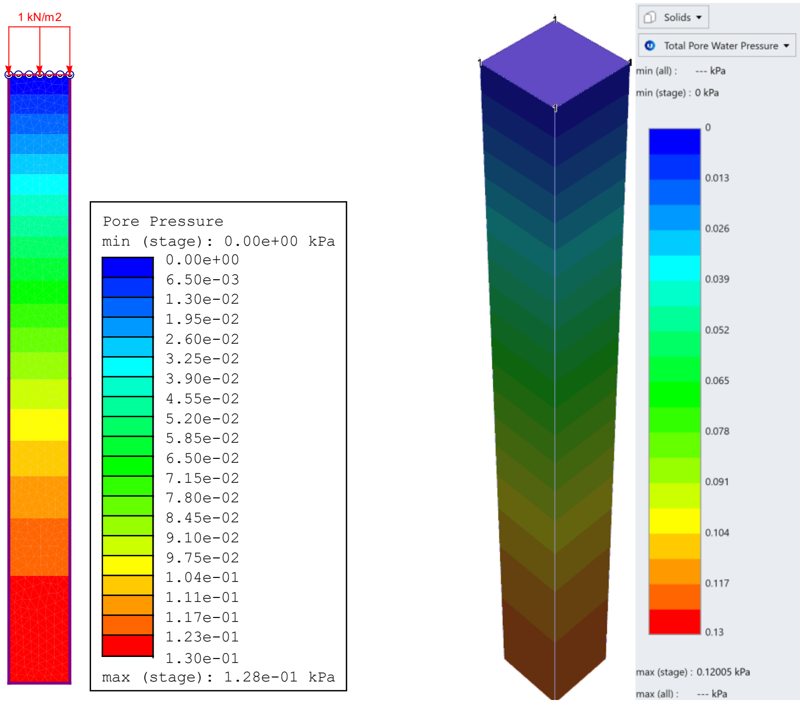
$$U\% = f(T_v)$$

where $U\%$ is the same for the consolidation of every layer under specific loading and drainage conditions, to determine the relation between the time factor and degree of consolidation.

Although the solution does not include the secondary time effect, where the solution approaches a horizontal asymptote, it can be used to determine upper and lower limit values for the rate of settlement. Additionally, the analytical solution used for this example is specific for the boundary conditions of a permeable surface and impermeable base.

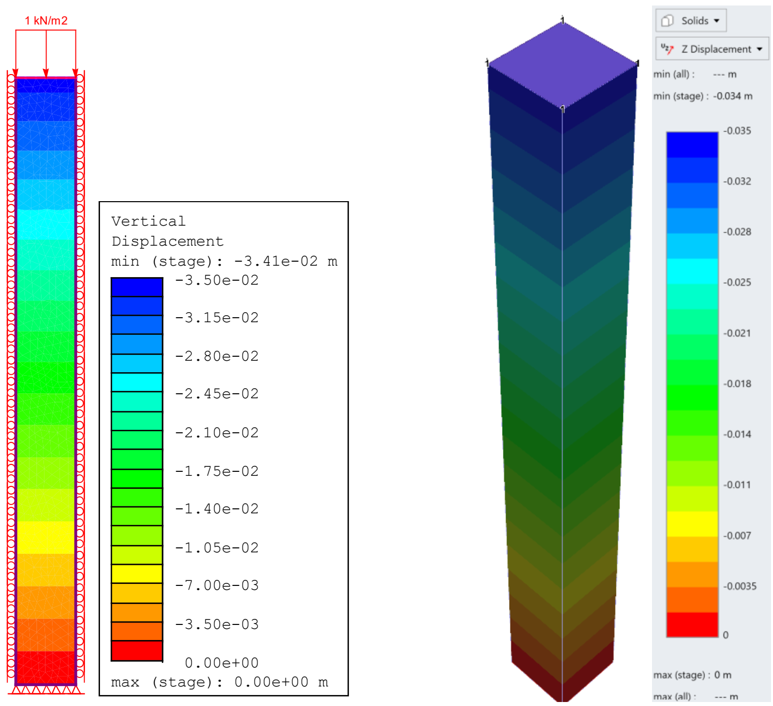
1.3. Results

Time dependent soil consolidation and settlement problems can be analyzed using the coupled stress/deformation analysis. The analysis is conducted over 10 stages, including Initial steady state stage, and Tv of 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, and 1. With the escape of water at the seepage face, the lowest pore water pressure is ensued at top with the gradual increase towards the bottom, which is best defined at last stage, Tv = 1 (Figure 1-3). For this reason, an opposite trend is manifested in deformation, where a vertical deformation occurs at highest magnitude from the top and gradually decreasing towards the bottom (Figure 1-4). The modeling results from 2D and 3D analyses show a close agreement.



(a) (b)

Figure 1-3: Pore pressure contour for T_v = 1 as modeled in (a) RS2 and (b) RS3



(a) (b)

Figure 1-4: Vertical displacement contour for T_v = 1 as modelled in (a) RS2 and (b) RS3

The vertical displacement data sampled at the top surface over the stages are plotted on the displacement-Tv graph, which shows an incrementing trend in deformation over time (Figure 1-5). The pore pressure data was also sampled along the length of the model to monitor its change with respect to time, but also to track along the length. The pore pressure-distance graph (Figure 1-6) shows well the consolidation effect on pore pressure. Both graphs show the Finite Element solution is in agreement with the analytical results.

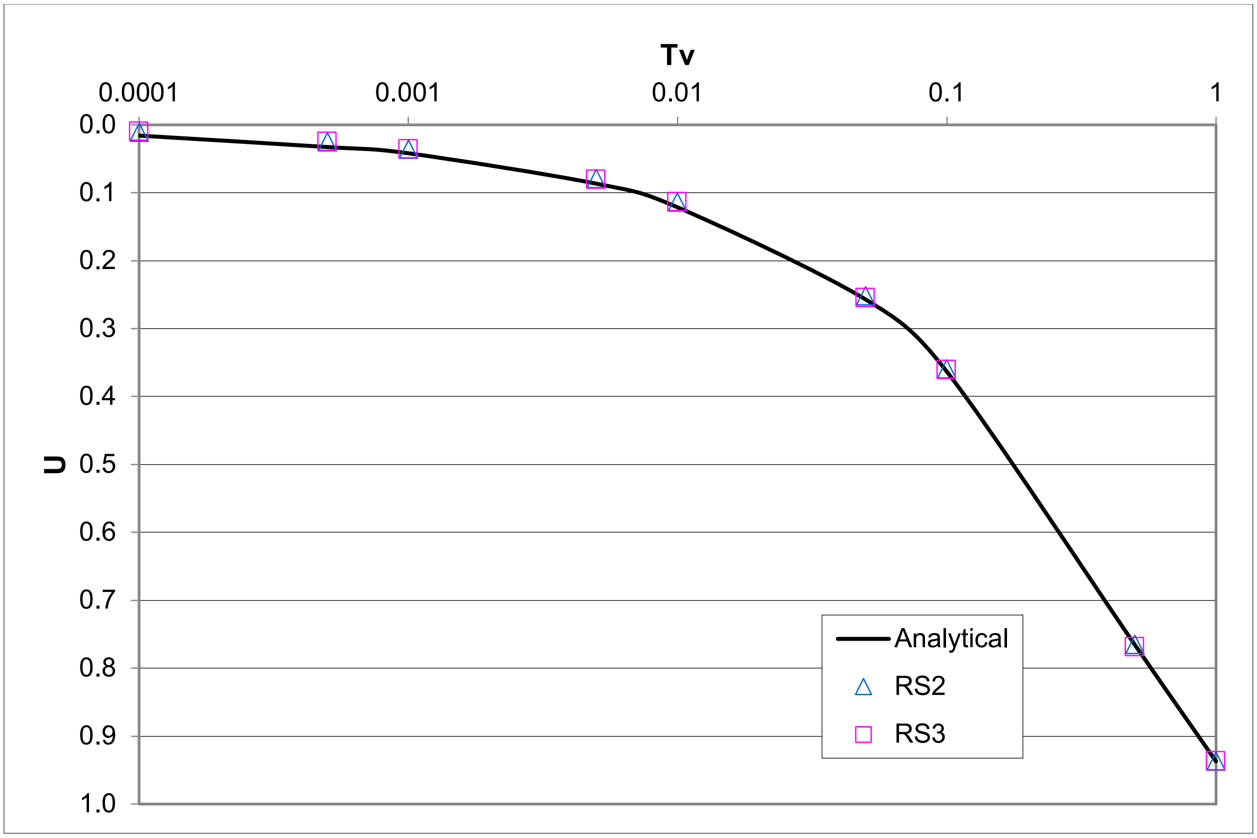


Figure 1-5: Degree of consolidation versus time factor

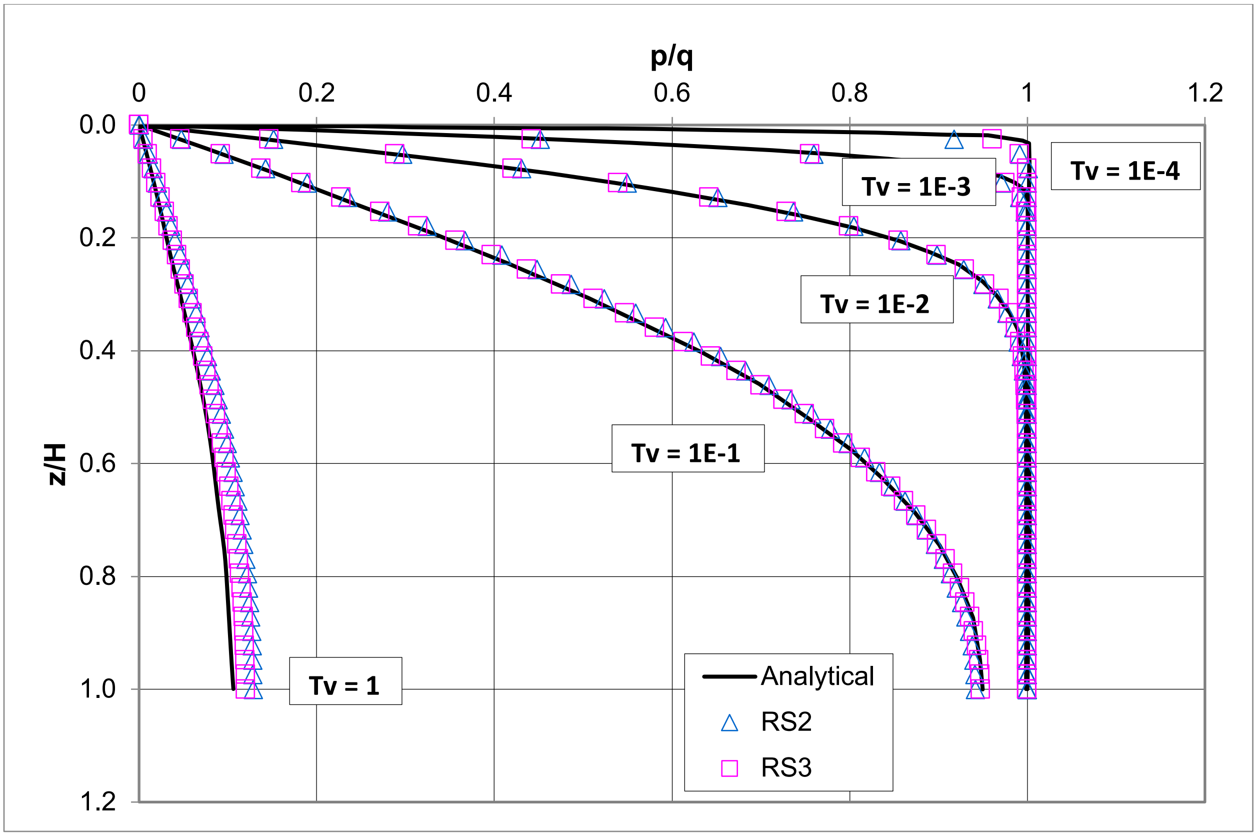


Figure 1-6: Pore pressure versus ratio of depth

1.4. References

1. Terzaghi, K. 'Die Berechnung der Durchlassigkeitsziffer des Tones aus demVerlauf der hydrodynamischen Spannungsersceinungen', Originally published in 1923 and reprinted in From Theory to Practice in Soil Mechanics, John Wiley and Sons, New York, 133-146, 1960.

1.5. Data Files

The RS3 input file **consolidation #001.rs3v3** (graded mesh) can be downloaded from RS3 Online Help page.

2. Consolidation of Finite layer Compressed Between Rigid Plates

2.1. Problem Description

This problem investigates the consolidation of an elastic drained soil that is free to flow at two lateral ends when compressed by smooth, impermeable plate. Material parameters used for this problem are outlined in Table 2.1, and the parameters for the composite liner and the interface between the liner and soil are outlined in Table 2.2 and Table 2.3 respectively.

Table 2.1: Model parameters

Parameter	Value
Young's modulus (E)	200 kPa
Poisson's ratio (v)	0.3
Permeability (k)	0.0001 m/s
Half-width (a)	1.25 m
Thickness (H)	1 m
Coefficient of consolidation (c_v)	0.002744 m^2/s

Table 2.2: Elastic composite liner parameters

Parameter	Value
Young's modulus	200 kPa
Poisson's ratio	0.3
Thickness	1 m

Table 2.3: Liner-soil interface parameters

Parameter	Value
Normal Stiffness	1×10^5 kPa/m
Shear Stiffness	0 kPa/m

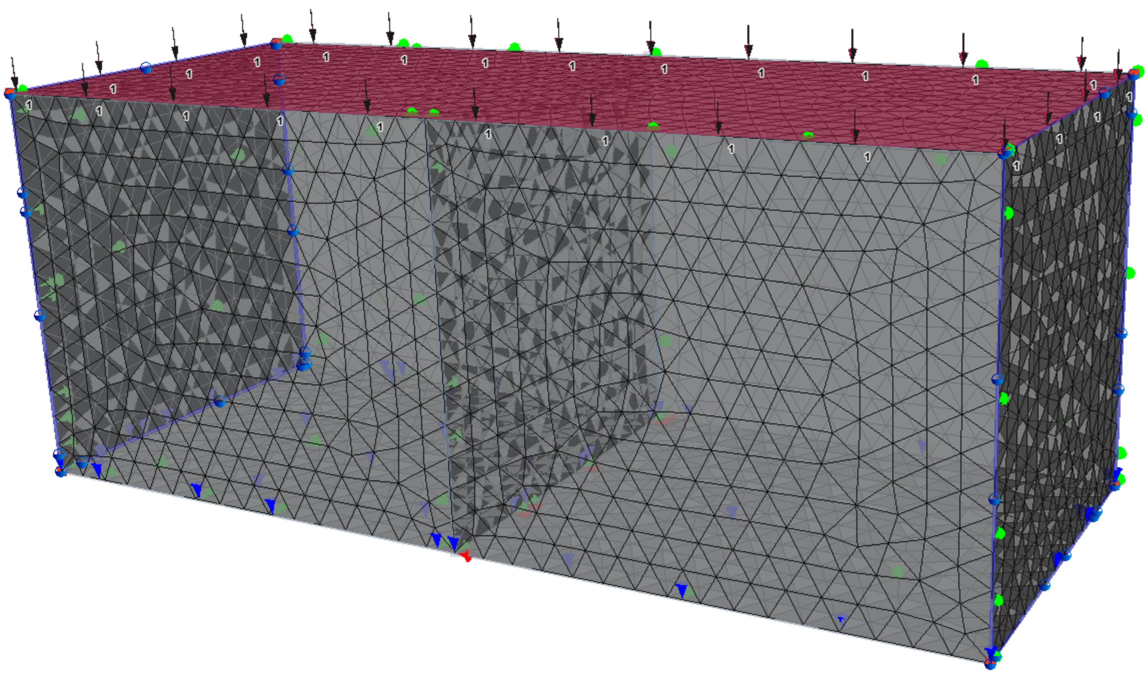
The dimensionless time factor for this example is described by the equation:

$$T_v = \frac{c_v t}{3a^2}$$

Where a is the half-width of the layer.

Similar to the previous example, 6 noded graded mapped mesh is used for RS2 model and 10 noded graded mesh for RS3 model (Figure 2-1). The soil is loaded with a constant pressure of 1 kPa applied on the top plate and vertically supported by restraints applied at the bottom.

(a)



(b)

Figure 2-1: Finite layer compressed between two rigid plates with graded mesh in (a) RS2 and (b) RS3

2.2. Analytical Solution

According to Mandel [1] the problem of an isotropic, elastic, and drained layer in the plane strain state under an axial load, $P_0H(t)$, between two smooth, rigid, impermeable plates involve the following components:

Displacement $u_x = u_x(x, t)$

$u_z = u_z(z, t)$

Strain $\varepsilon_x = \varepsilon_x(x, t)$

$\varepsilon_z = \varepsilon_z(t)$

Stress $\sigma_x = \sigma_x(x, t)$

$\sigma_z = \sigma_z(x, t)$

Pore fluid pressure $p = p(x, t)$

with the following boundary conditions

$$x = \pm a$$

$$\sigma_x = 0$$

$$p = 0$$

where x is the horizontal distance from centre of layer, σ_x is the horizontal stress applied, and p is the pore fluid pressure. The relationship between vertical stress and axial loading can be de described as following

$$2\int_0^a \sigma_z(x, t)dx = -P_0 H(t)$$

where σ_z is the vertical stress applied and $H(t)$ is the Heaviside unit step function.

Additionally, the equations for fluid-saturated, isotropic, poroelastic materials satisfying an irrotational condition has the following strain-displacement relations:

$$\sigma_{ij} = 2G\left(\varepsilon_{ij} + \frac{v}{1-2v}\varepsilon_{ll}\delta_{ij}\right) - \alpha p \delta_{ij}$$

$$\frac{\partial \sigma_{ij}}{\partial x_i} = 0$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i}\right) = 0$$

where σ_{ij}, ε_{ij}, p, and u_i are the total stress, average strain, pore fluid pressure, and average strain, respectively; G, v, and α are the shear modulus, drained Poisson's ratio, and Biot's coefficient of effective stress described by the equation

$$\alpha = \frac{3(v_u - v)}{B(1+v_u)(1-2v)}$$

where B and v_u are Skempton's pore pressure coefficient and undrained Poisson's ratio.

The diffusion equation of the pore fluid satisfying the irrotational condition is given by the following equation

$$\frac{\partial p}{\partial t} = c\Delta^2 p - \frac{\alpha}{S}\frac{d}{dt}g(t)$$

with the storage coefficient S and diffusivity coefficient c both given by

$$S = \frac{\alpha^2(1-2v)^2(1-v)}{B(v_u - v)(1-v)}, \qquad c = \frac{\kappa}{S}$$

where κ is the permeability coefficient, $g(t)$ is the auxiliary function of t, with the following relations between volumetric strain and fluid pressure

$$\varepsilon_{ll} = \frac{\eta}{G}p + g(t), \qquad \eta = \frac{\alpha(1-2v)}{2(1-v)}$$

Finally, the 2D Mandel solution (in solved in Laplace space) has the following final equations

$$\frac{\tilde{p}(\chi,\xi)}{(P_0/2a)(a^2/c)} = \frac{B}{3}(1+v_u)(1-v)\frac{1}{\xi D(\xi)}\left\{1 - \frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\},$$

$$\frac{\tilde{\sigma}_z(\chi,\xi)}{(P_0/2a)(a^2/c)} = -\frac{1}{\xi D(\xi)}\left\{(1-v) - (v_u - v)\frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\},$$

$$\frac{\tilde{\sigma}_x(\chi,\xi)}{(P_0/2a)(a^2/c)} = 0,$$

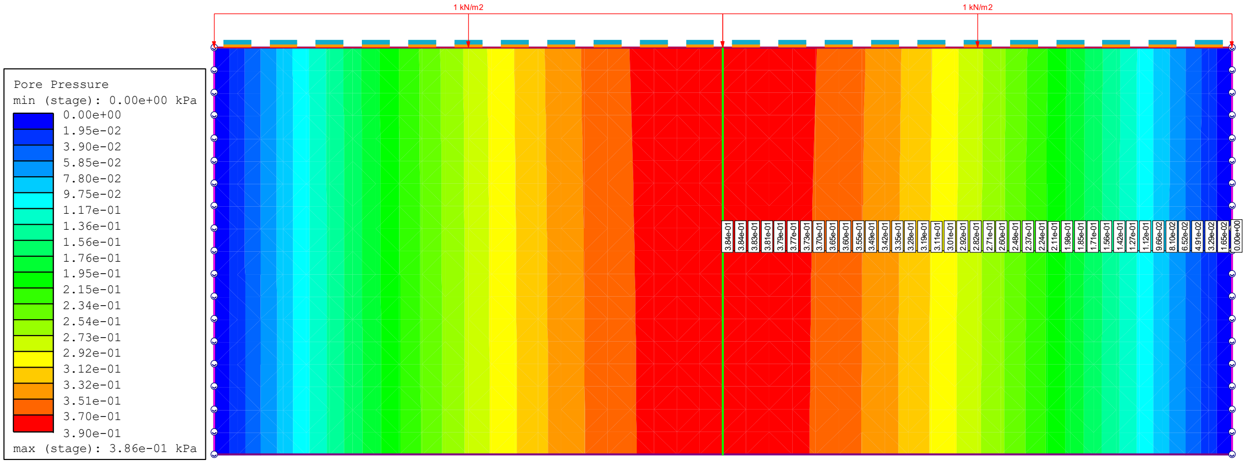
$$\frac{2G\tilde{\varepsilon}_z(\xi)}{(P_0/2a)(a^2/c)} = -(1-v)(1-v_u)\frac{1}{\xi D(\xi)},$$

$$\frac{2G\tilde{\varepsilon}_x(\chi,\xi)}{(P_0/2a)(a^2/c)} = \frac{1}{\xi D(\xi)}\left\{v_u(1-v) - (v_u - v)\frac{\cosh(\sqrt{\xi}\chi)}{\cosh(\sqrt{\xi})}\right\}.$$

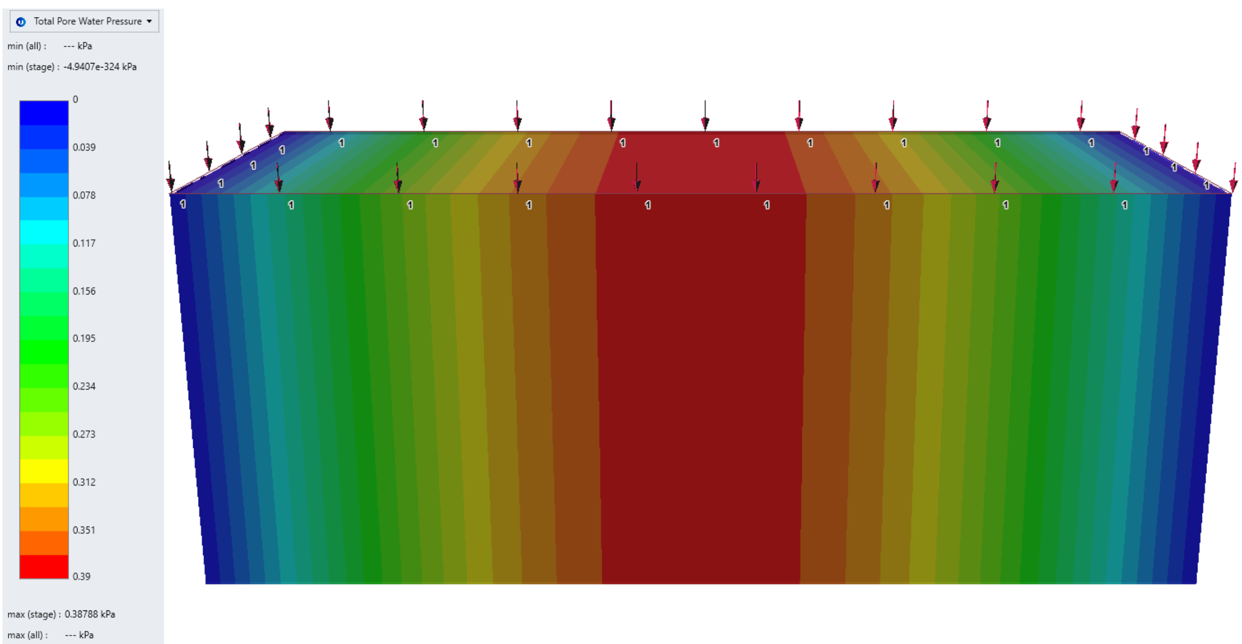
2.3. Results

Time dependent soil consolidation and settlement problems can be analyzed using the coupled stress/deformation analysis. The analysis was conducted over 11 stages, including Initial steady state stage at first second (1 s) stage; and Tv of 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, and 1. With the presence of seepage face on left and right surfaces, the lowest pore water pressure is ensued at those two surfaces with the gradual increase towards the centre (Figure 2-2). The modeling results from 2D and 3D analyses show a close agreement.

The pore pressure data was sampled at the centre of the model from the right end to the middle. The pore pressure-distance graph (Figure 2-3) shows reduction in pore pressure closer to the seepage face and overall reduction with respect to time, which demonstrates the consolidation effect on pore pressure. The graph shows that the Finite Element solutions are in agreement with the analytical results.



(a)



(b)

Figure 2-2: Pore pressure contour plot at T_v = 0.1 as modeled in (a) RS2 and (b) RS3

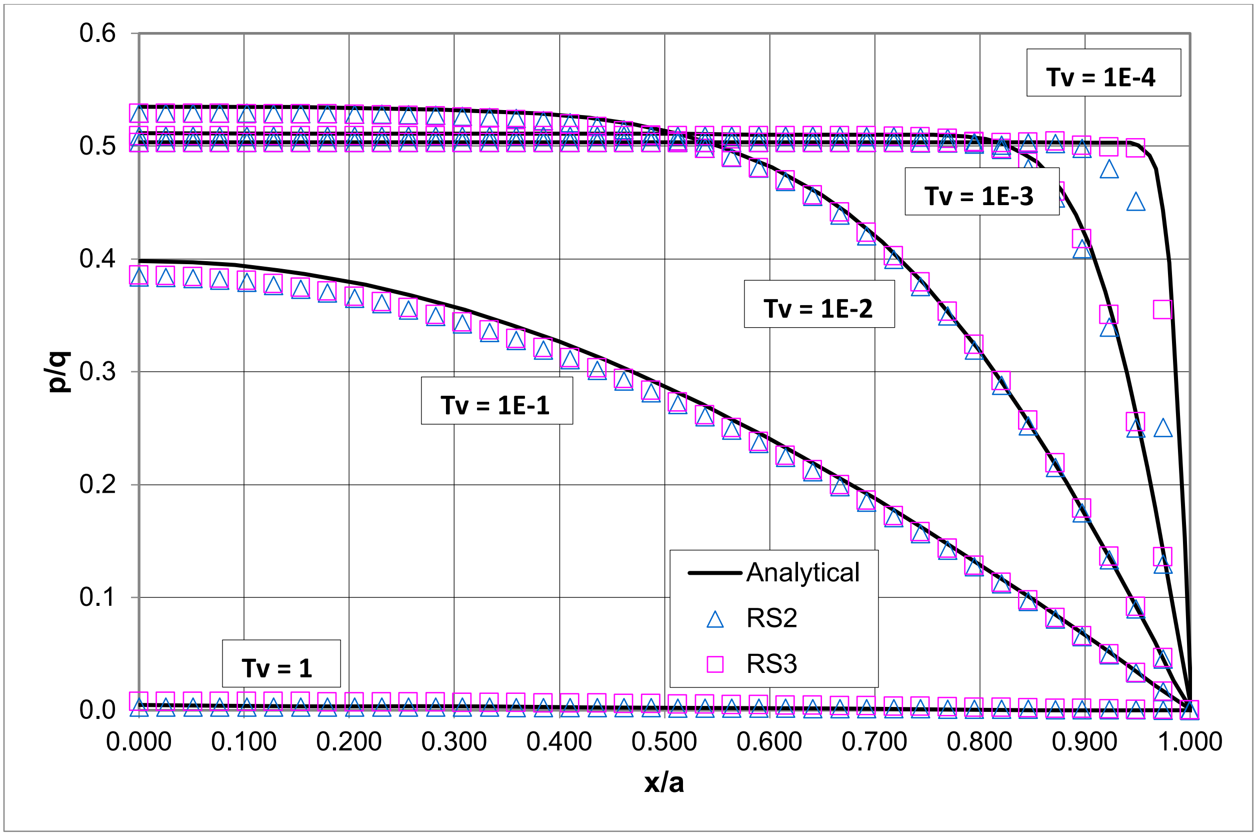


Figure 2-3: Pore pressure versus distance ratio from centre

2.4. References

1. Mandel, J. 'Consolidation des sols', Geotechnique, III, 287-299, 1953

2.5. Data Files

The RS3 input file **consolidation #002.rs3v3** (graded mesh) can be downloaded from RS3 Online Help page.

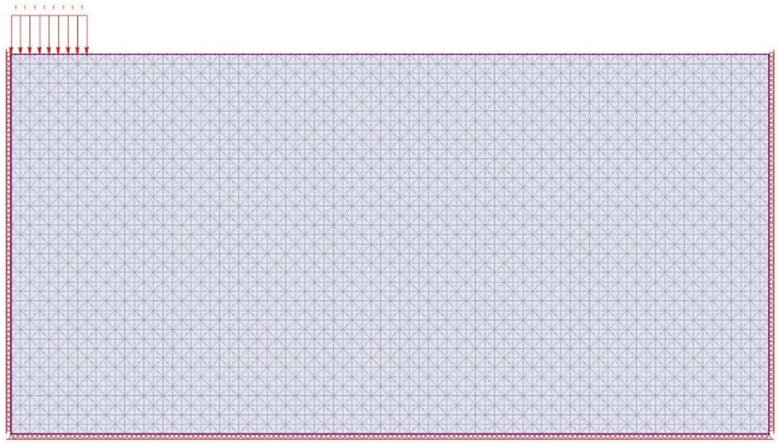
3. Flexible Strip Footing on Finite Layer

3.1. Problem Description

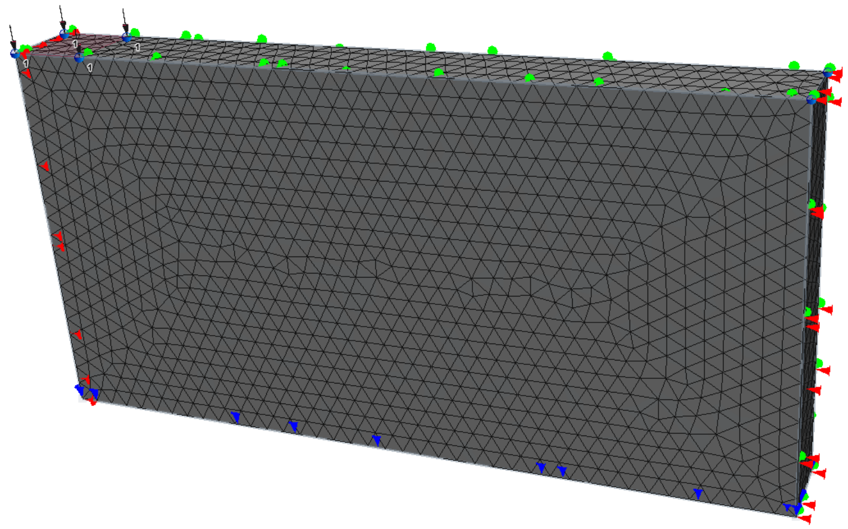
This problem analyzes the consolidation of a flexible strip footing on a porous elastic soil layer with the model parameters described in Table 3.1. The model geometry is shown in Figure 3-1.

Table 3.1: Model parameters

Parameter	Value
Young's modulus (E)	200 kPa
Poisson's ratio (v)	0
Permeability (k)	0.0001 m/s
Thickness (H)	1 m
Width	10 m
Coefficient of consolidation (c_v)	0.002744 m^2/s



(a)



(b)

Figure 3-1: Model geometry constructed using (a) RS2 and (b) RS3

The dimensionless time of consolidation is determined by the equation

$$T_v = \frac{c_v t}{H^2}$$

The degree of consolidation defined by the following equation

$$U = \frac{S_c}{S_{c,max}}$$

where S_c is the settlement of the soil layer. Additionally, the top surface of the models are permeable with a constant pressure applied over the 1m long footing strip, shown in Figure 3-1.

3.2. Analytical Solution

Determined by Booker [1], the equation for a uniformly loaded strip developed through Laplace and Fourier transformations is as following

$$w = \frac{1}{4}\pi^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[p_0 + \sum_{n=t}^{\infty} p_n e^{Sn(\alpha,\beta)t}\right] e^{(i\alpha x + i\beta y)}\, d\alpha\, d\beta$$

which, when evaluated, gives the following solutions

$$z = z_1 \begin{cases} \sigma_{zz} = q & -a \leq x \leq a \\ \sigma_{zz} = 0 & elsewhere \end{cases}$$

$$q^F = q\frac{sin\alpha a}{\alpha}\delta(\beta)$$

where q is the uniform pressure applied, z is the layer depth, a is the footing width, α and $\beta$are coordinate points, $\delta(\beta)$ is the Dirac delta function, and σ_{zz} is the vertical stress given by the following equation

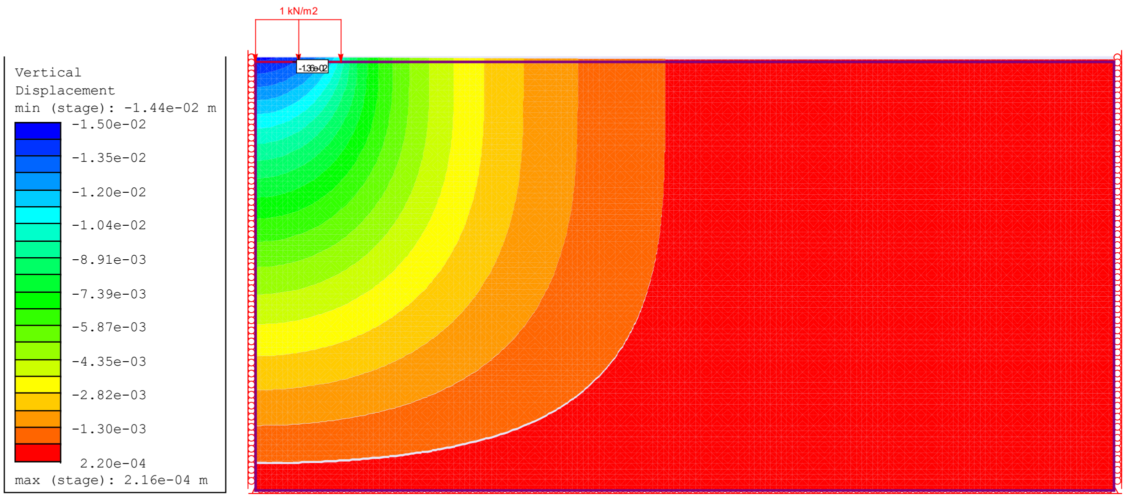
$$\sigma_{zz} = \sigma + \lambda e_v - 2G\frac{dw}{dz}$$

where λ and G are Lame's parameters for the soil skeleton, e_v is the void ratio, and w is the soil deformation.

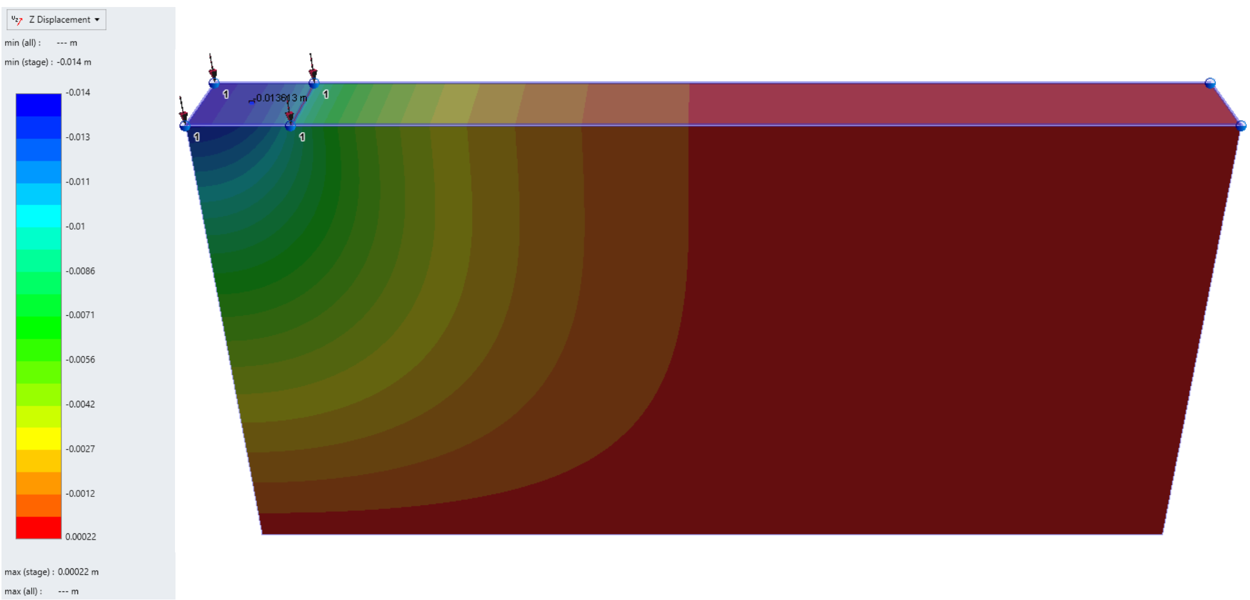
3.3. Results

Time dependent soil consolidation and settlement problems can be analyzed using the coupled stress/deformation analysis. The analysis was conducted over 12 stages, including Initial steady state stage; initial 1 second stage; and Tv of 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.05, 0.1, 0.5, 1, and 10. With the presence of strip loading at the top surface, the excess pore water pressure develops below the loading area, which dissipates over the stage.

The vertical displacement contour plots in Figure 3-2 shows displacement concentration below the footing, demonstrating soil settlement. The contour plots show a close agreement between the 2D and 3D modeling results. A query point is placed at the end of the strip load to collect the modeling result data at that location. The displacement data collected by the query point in all stages is plotted on the dimensionless time versus normalized data (representing the settlement) graph in Figure 3-3. The graph shows a close agreement between the results from RS2 and RS3, and more importantly with the analytical solution by Booker [1].



(a)



(b)

Figure 3-2: Vertical Displacement contour at the last stage in (a) RS2 and (b) RS3

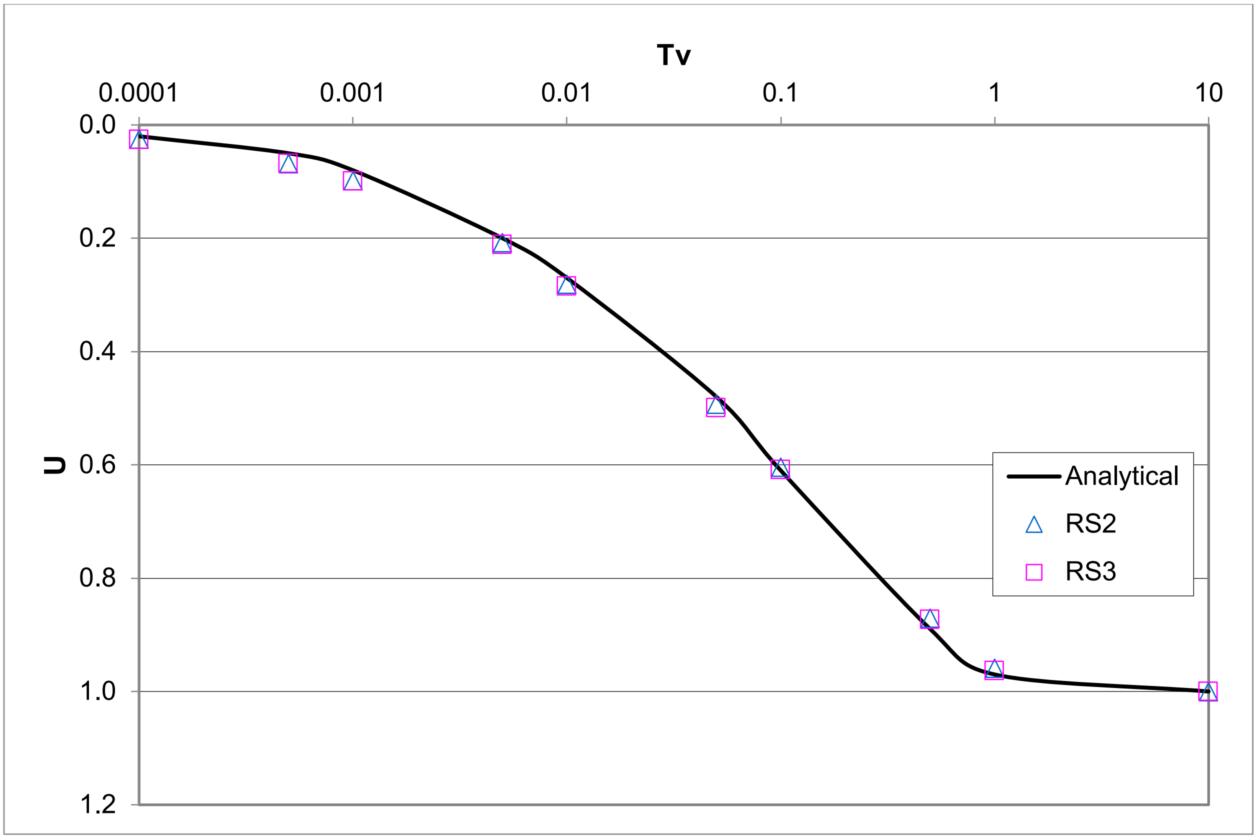


Figure 3-3: Degree of consolidation versus time factor for elastic strip footing

3.4. References

1. Booker, J.R. ‘The consolidation of a finite layer subject to surface loading’, International Journal for Solids and Structures, 10, 1053-1065, 1974.

3.5. Data Files

The RS3 input file **consolidation #003.rs3v3** can be downloaded from RS3 Online Help page.

4. Analysis of Consolidation of Thick Cylinder

4.1. Problem Description

This problem analyzes elastoplastic consolidation through the expansion of a thick cylinder under drained and undrained loading conditions. The cylinder has an inner radius a and an outer radius b; and is subjected to an internal pressure p, with zero external pressure. Material properties and modeling geometries used for this exercise are provided in Table 4.1 and Figure 4-1, respectively.

Table 4.1: Model parameters

Parameter	Value
Young's modulus (E')	200 kPa
Poisson's ratio (v')	0
Friction angle (Φ')	30°
Cohesion (c')	1 kPa
Dilatancy angle (ψ')	0°
Permeability (k)	0.01 m/s
Internal radius (a)	2 m
Outer radius (b)	4 m
Coefficient of consolidation (c_v)	0.2039 m^2/s

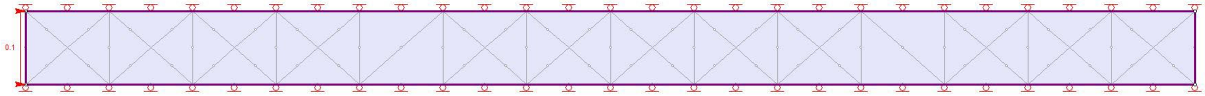
The dimensionless time factor for this example is described by the equation:

$$T_v = \frac{c_v t}{a^2}$$

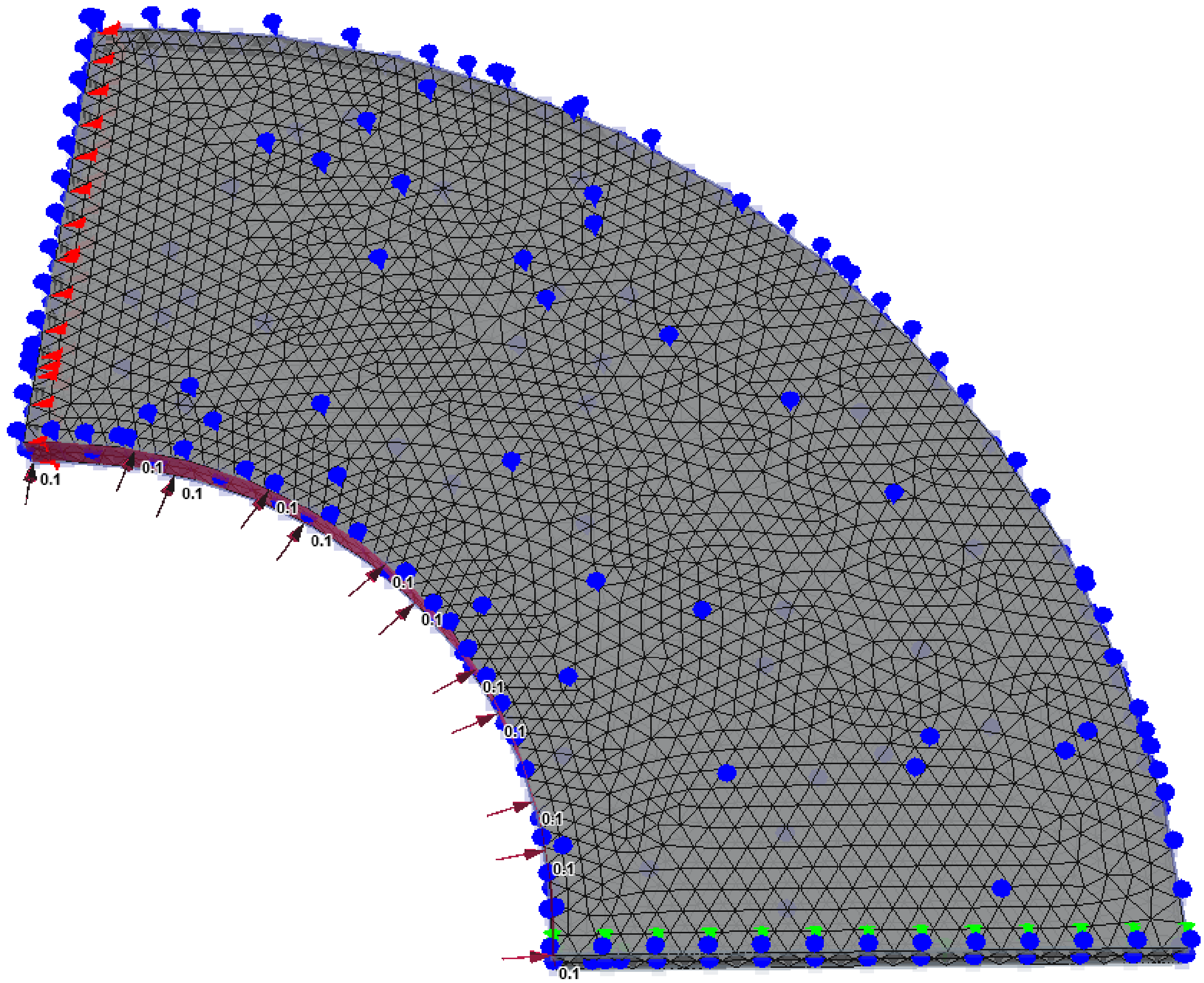
and a load rate parameter with the equation

$$\omega = \frac{\Delta q / c'}{\Delta T_v}$$

To simulate drained conditions, a slow loading rate of ω = 0.01 was used to apply the internal pressure over 12-time steps. Undrained loading conditions were developed in two models with different approach taken for each. The two undrained conditions were implemented with one applying rapid loading rate of ω = 10,000 however keeping the material behaviour drained; and the other assigning undrained material behaviour to the constitutive model.



(a)



(b)

Figure 4-1: Drain thick cylinder as modelled in (a) RS2 and (b) RS3

4.2. Analytical Solution

According to Yu [1], a cylinder with an elastic-perfectly plastic material behaviour induced by an internal pressure, p, should obey Hooke's law until yielding occurs.

Under the elastic stage, the radial and tangential stress, and displacement are given by the respective equations:

$$\sigma_r = -p_0 + (p - p_0)\left[\frac{1}{\left(\frac{b}{a}\right)^2 - 1} - \frac{1}{\left(\frac{r}{a}\right)^2 - \left(\frac{r}{b}\right)^2}\right]$$

$$\sigma_\theta = -p_0 + (p - p_0)\left[\frac{1}{\left(\frac{b}{a}\right)^2 - 1} + \frac{1}{\left(\frac{r}{a}\right)^2 - \left(\frac{r}{b}\right)^2}\right]$$

$$u = \frac{p - p_0}{2G\left(\frac{1}{a^2} - \frac{1}{b^2}\right)}\left[\frac{1 - 2v}{b^2} r + \frac{1}{r}\right]$$

where a and b represents inner and outer radii boundaries, respectively. The yield equation is provided as following:

$$\alpha\sigma_\theta - \sigma_r = Y$$

The elastic-plastic stage is analyzed by two separate regions which have the following respective equations:

Plastic Region: $a \le r \le \rho$

$$\sigma_r = \frac{Y}{\alpha - 1} + A r^{-\frac{(\alpha-1)}{\alpha}}$$

$$\sigma_\theta = \frac{Y}{\alpha - 1} + \frac{A}{\alpha} r^{-\frac{(\alpha-1)}{\alpha}}$$

Elastic Region: $b \ge r \ge \rho$

$$\sigma_r = -p_0 + B\left(\frac{1}{b^2} - \frac{1}{r^2}\right)$$

$$\sigma_\theta = -p_0 + B\left(\frac{1}{b^2} + \frac{1}{r^2}\right)$$

When the entire cylinder becomes plastic ($\rho = b$), the internal pressure can be calculated as following

$$p_f = \frac{Y + (\alpha - 1)p_0}{\alpha - 1}\left[\left(\frac{b}{a}\right)^{\frac{\alpha-1}{\alpha}} - 1\right] + p_0$$

In terms of displacement analysis, the displacement in the elastic zone has the following equation:

$$u = \frac{1 + v}{M}\left[\frac{1 - 2v}{\alpha - 1 + (1 + \alpha)\left(\frac{b}{p}\right)^2} r + \frac{1}{(\alpha - 1)\left(\frac{r}{b}\right)^2 + (1 + \alpha)\left(\frac{b}{p}\right)^2} r\right]$$

and displacement in the plastic zone follows the plastic flow rule with the equation:

$$\beta\dot{\epsilon_r} + \dot{\epsilon_\theta} = \frac{1 - v^2}{E}\left\{\left(\beta - \frac{v}{1 - v}\right)\sigma^r + \left(1 - \frac{\beta v}{1 - v}\right)\sigma_\theta + \left(\beta + 1 - \frac{v(1 + \beta)}{1 - v}\right)p_0\right\}$$

Additionally, according to Small [2] the undrained parameters for the thick cylinder model was determined from the drained parameters according to the following equations

$$E_u = \frac{3E'}{2(1 + v')}$$

$$c_u = \frac{2c'\sqrt{N_\phi}}{1 + N_\phi}$$

where

$$N_\phi = \frac{1 + \sin\phi}{1 - \sin\phi}$$

Finally, the collapse pressure of the cylinder with drained condition is given by the following equation

$$\frac{p}{c'} = 1.02$$

and the collapse pressure under undrained condition is given by the following equation

$$\frac{p}{c'} = 1.2$$

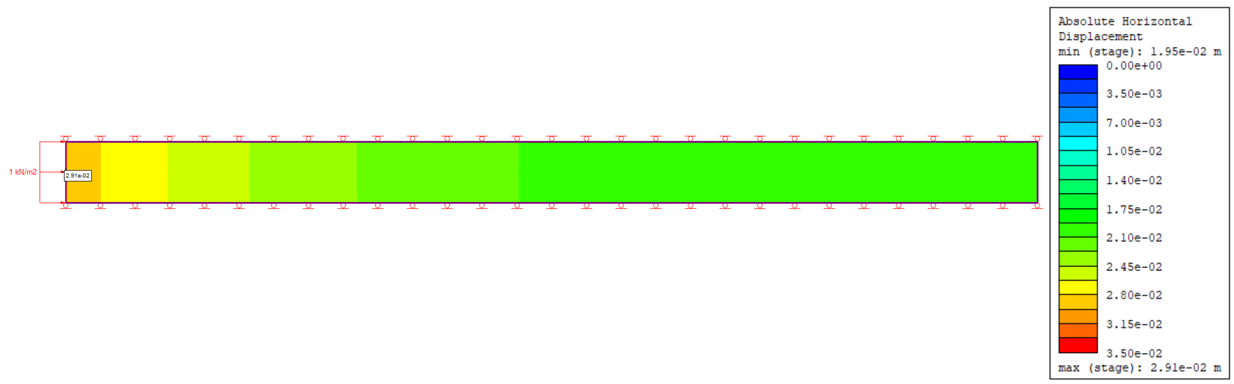
or

$$\frac{p}{c_u} = 1.4$$

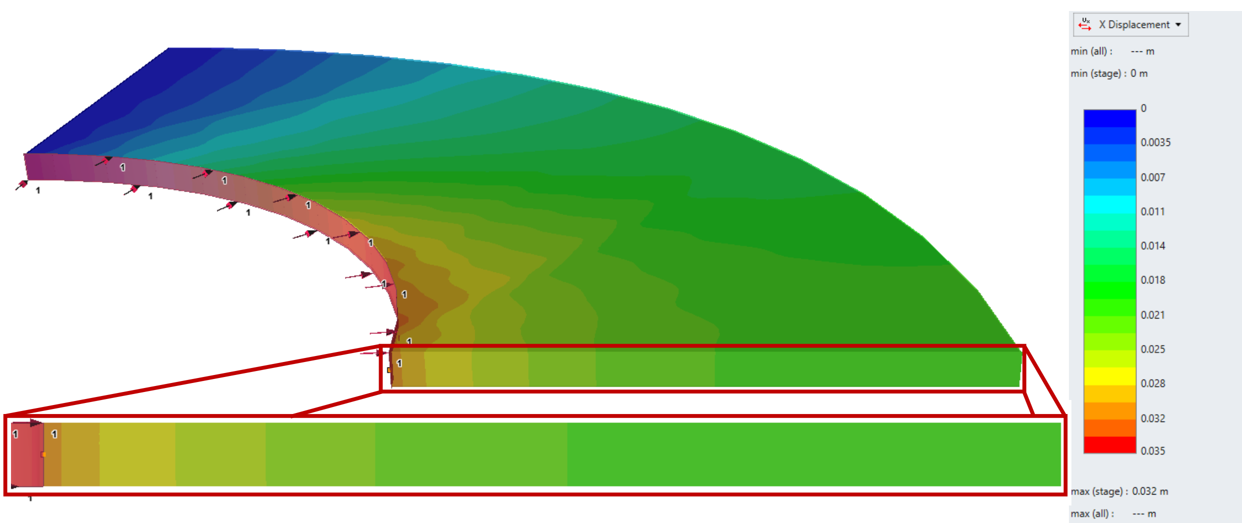
4.3. Results

4.3.1. Drained conditions

The expansion of cylinder due to the applied load is demonstrated by the horizontal displacement contour plots in Figure 4-2, which shows the modeling results at p/c' = 1. Since axisymmetric analysis is implemented in RS2, the plotted horizontal displacement contour is equivalent to radial displacement acting within the cylinder. Note that, RS3 contour plot does not show systematic distribution and only shows similarity to RS2 contour plot on cross section surface from front view (as magnified in red box in Figure 4-2b) because it is plotting the X displacement contour, not radial displacement, which is not a default contour option in RS3. It shows that the deformation is concentrated close to the inner radius boundary and dissipates towards outer radius, indicating that the expansion of internal radius.



(a)



(b)

Figure 4-2: Horizontal displacement contour plot before peak load in (a) RS2 and (b) RS3

A query point is placed at the inner boundary of cylinder, where load is applied to collect the modeling result data at that location. The displacement data collected by the query point in all stages is plotted on the pressure versus displacement ratio graph in Figure 4-3. The graph shows a close agreement between the numerical and analytical solutions by Small [2]. The displacement ratio is normalized displacement values by inner radius, a, and drained shear modulus, which is defined as following

$$G' = \frac{E'}{2(1 + v')}$$

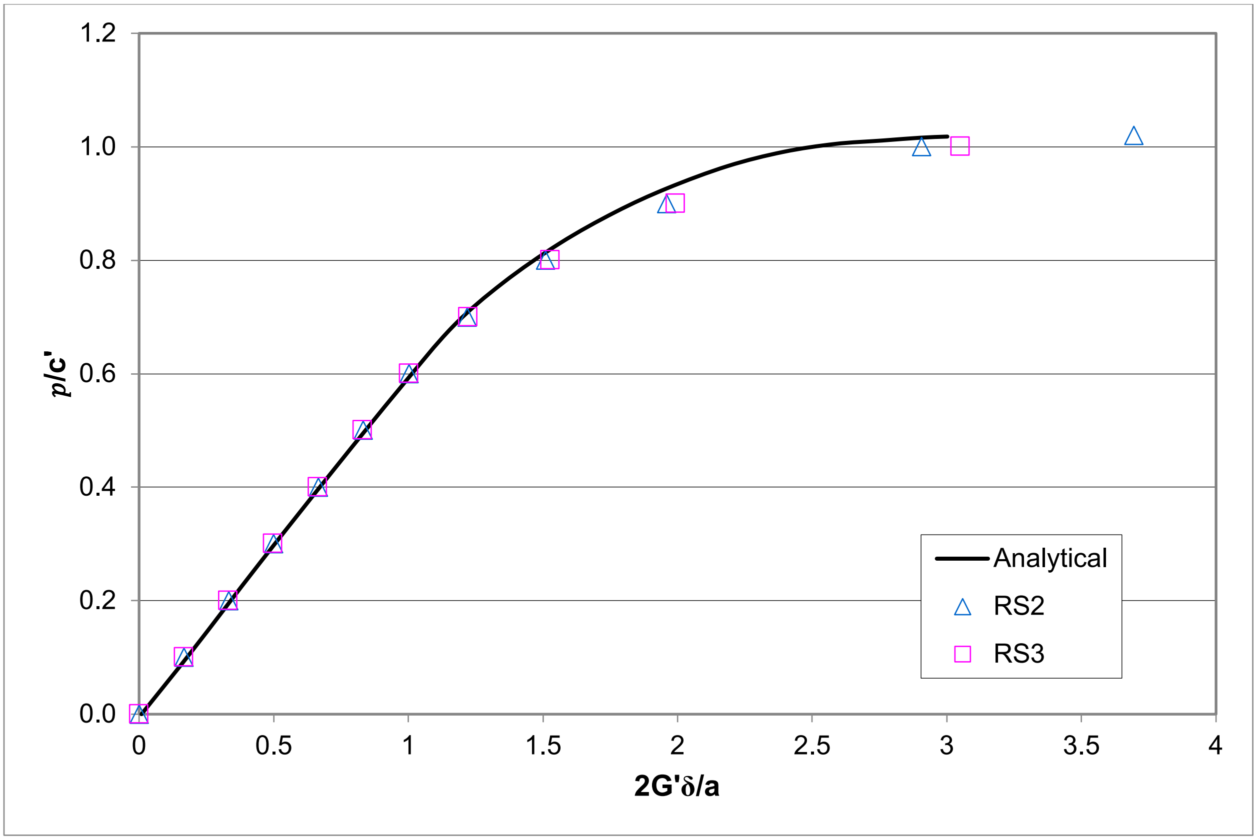
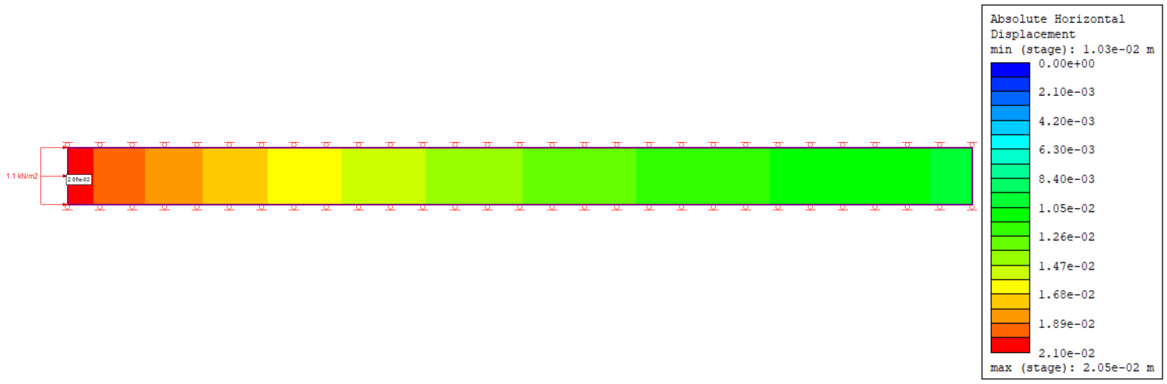


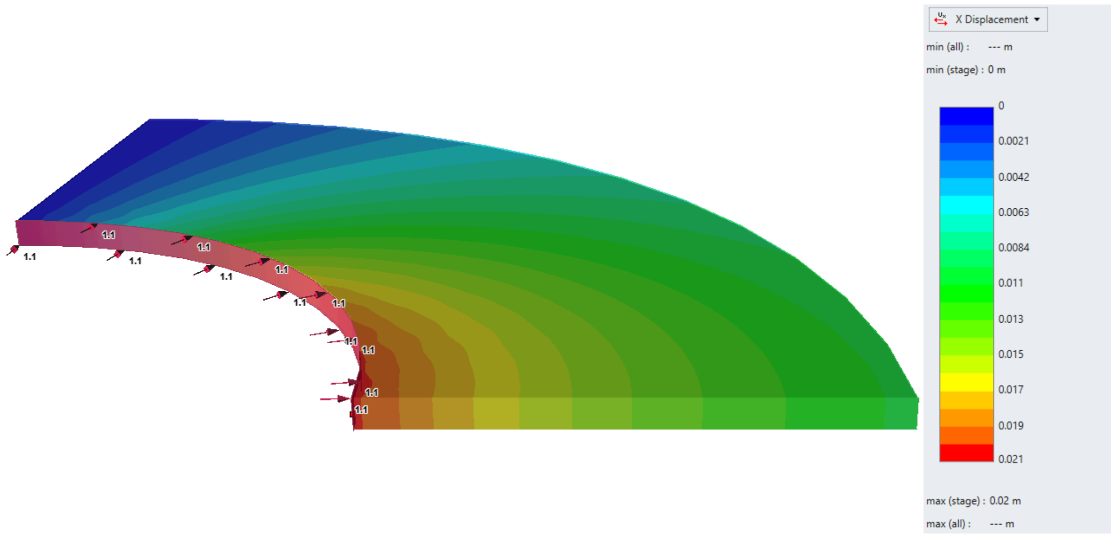
Figure 4-3: Pressure versus displacement for drained loading

4.3.2. Undrained Conditions

A similar deformation behaviour as captured in drained condition, however at lower scale, is shown from the modeling results under undrained condition (Figure 4-4). As expected, less deformation has occurred under undrained condition as volume change or dissipation of pore pressure is strictly restricted. As noted earlier, the only comparable displacement distribution captured in RS3 with RS2 is with respect to the surface parallel to front view (XZ axes) because the X displacement is plotted in RS3, which is equivalent to radial displacement only at that surface.



(a)



(b)

Figure 4-4: Horizontal displacement contour plot before peak load in (a) RS2 and (b) RS3

Based on the data collected from the query point located at the same location as the drained condition model, the deformation of RS2 and RS3 models under undrained condition show close agreement (Figure 4-5). The modeling results are also in accordance with the analytical solution by Hill [3].

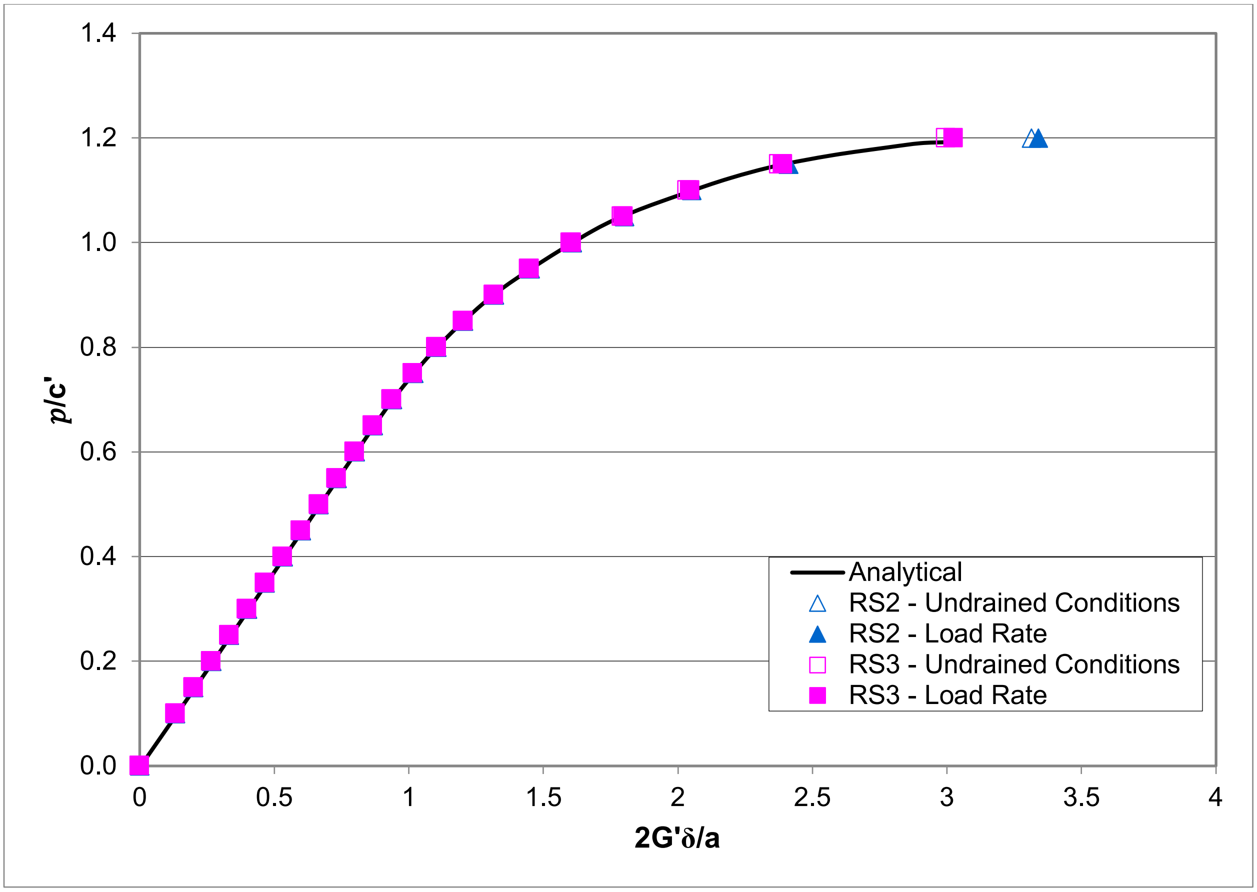


Figure 4-5: Pressure versus displacement for undrained loading

4.4. References

1. Yu, H.S, ‘Expansion of a thick cylinder of soil’, Computers and Geotechnics, 14, 21-41, 1992.
2. Small, J.C., Elasto-plastic consolidation of Soils, PhD thesis, University of Sydney, 1977.
3. Hill, R., The Mathematical Theory of Plasticity, Clarendon Press, Oxford, 1950.

4.5. Data Files

The RS3 input file **consolidation #004_01.rs3v3**, and **consolidation #004_02 (undrained conditions).rs3v3** and **consolidation #004_02 (load rate).rs3v3** can be downloaded from RS3 Online Help page.

5. Undrained Analysis of Strip Footing

5.1. Problem Description

This problem analyzes consolidation of a smooth flexible strip footing to which a constant pressure is applied under undrained conditions. Undrained conditions are modeled directly and indirectly by enforcing undrained material behaviour (from the stage setting) in order that the fluid cannot move into and out of the element (direct) or applying rapid load rate of ω = 150 (indirect). The model parameters of the soil model are outlined in Table 5.1.

Table 5.1: Model parameters

Parameter	Value
Young's modulus (E')	200 kPa
Poisson's ratio (v')	0.3
Friction angle (Φ')	20°
Cohesion (c')	1 kPa
Dilatancy angle (ψ')	0°
Permeability (k)	0.01 m/s
Internal radius (a)	8 m
Outer radius (b)	16 m
Coefficient of consolidation (c_v)	0.1960 m^2/s

The load rate can be defined with following equation

$$\omega = \frac{\Delta q/c'}{\Delta T_v}$$

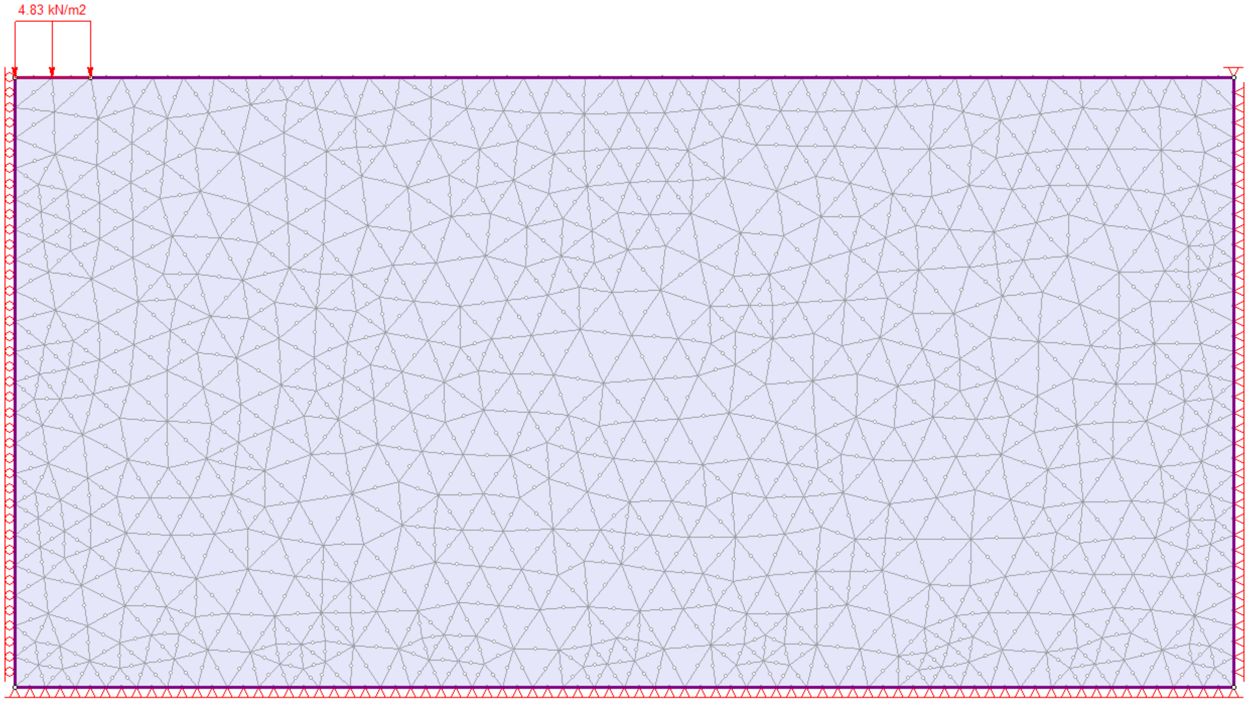
where q represents the applied load and the dimensionless time factor, T_v, is defined as following

$$T_v = \frac{c_v t}{B^2}$$

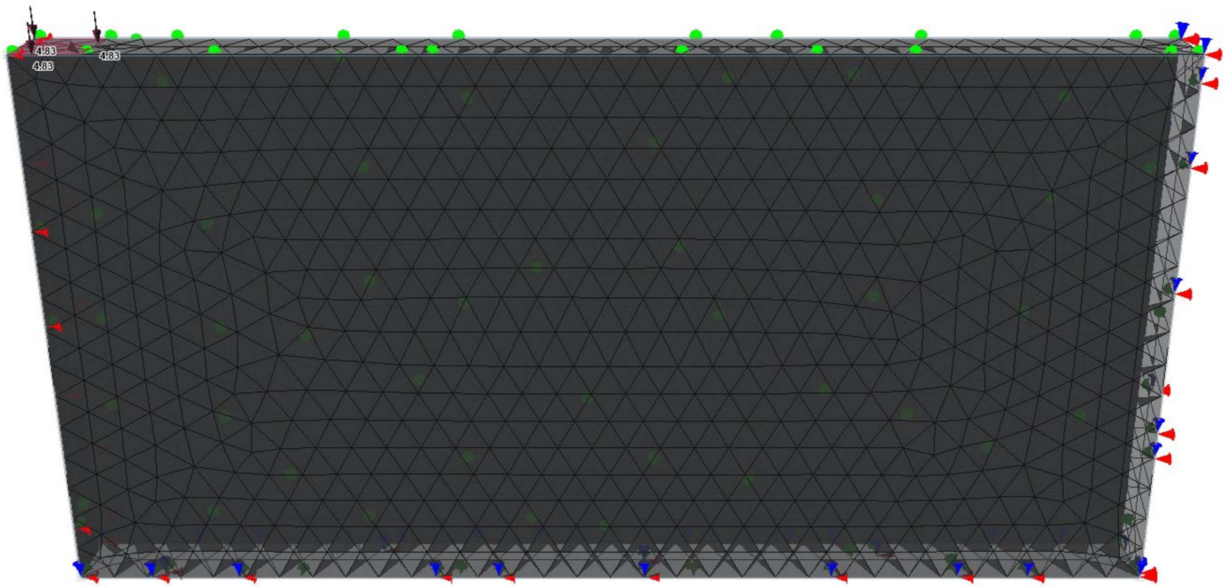
where B is the half-width of the footing strip and c_v is the two-dimensional consolidation coefficient that can be calculated with following equation

$$c_v = \frac{kE}{2\gamma_w(1+v')(1-2v')}$$

To simulate this problem, a constant pressure is applied over the footing strip, as shown in Figure 5-1.



(a)



(b)

Figure 5-1: Flexible strip footing on elastoplastic layer as modelled in (a) RS2 and (b) RS3

5.2. Analytical Solution

According to Small [1], the Biot consolidation formulation combined with the simple elastoplastic Mohr-Coulomb model requires a zero-dilation angle, to prevent a change in volume and a gain in strength due to plastic shearing.

Biot's equations for soil consolidation include the following:

Equilibrium stresses

$$\frac{\partial \sigma_{ij}}{\partial x_i} - F_i = 0$$

Effective stresses in relation to Hooke's law

$$\sigma'_{ij} = \sigma_{ij} - \rho\delta_{ij} = -H_{ijkl}\epsilon_{kl}$$

Flow of water determined by Darcy's law

$$v_i = -\frac{k_{ij}}{\gamma_w}\frac{\partial p}{\partial x_j}$$

Incompressible pore water resulting in the rate of volume decreasing equaling the rate at which water is expelled

$$\frac{\partial v_i}{\partial x_i} = -\frac{\partial \theta}{\partial t}$$

where

x_i are coordinates

σ_{ij} are the components of the total stress tensor

F_i are the components of body force

p is the pore pressure

σ'_{ij} are components of effective stress

v_i are the components of superficial velocity vector

ϵ_{kl} are components of the strain tensor with the equation

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

θ is volume strain

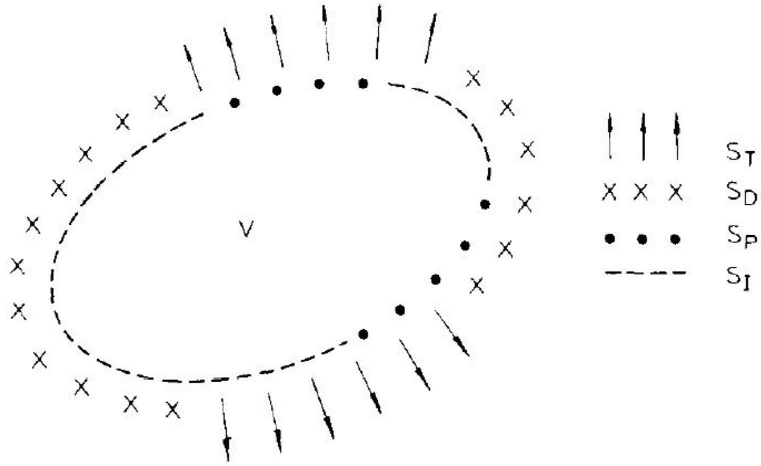
H_{ijkl} are elastic coefficients in generalized Hooke's Law

k_{ij} are the coefficients of permeability in generalized Darcy's Law

γ_w unit weight of water

t time

The equations are then integrated over the region V



with the following boundary conditions

$$\sigma_{ij} n_j = -T_i \qquad \text{applied to } S_t$$

$$u_i = 0 \qquad \text{applied to } S_D$$

$$p = 0 \qquad \text{applied to } S_p$$

$$n_i v_i = 0 \qquad \text{applied to } S_I$$

and the initial condition $\theta = 0$ when $t = 0^+$, where there are no instantaneous volume changes.

Additionally, according to Prandtl [2] the collapse pressure, the point to which the solution by Small [1] asymptotes head towards, can be calculated as following

$$\frac{q}{c_u} = 5.14$$

or

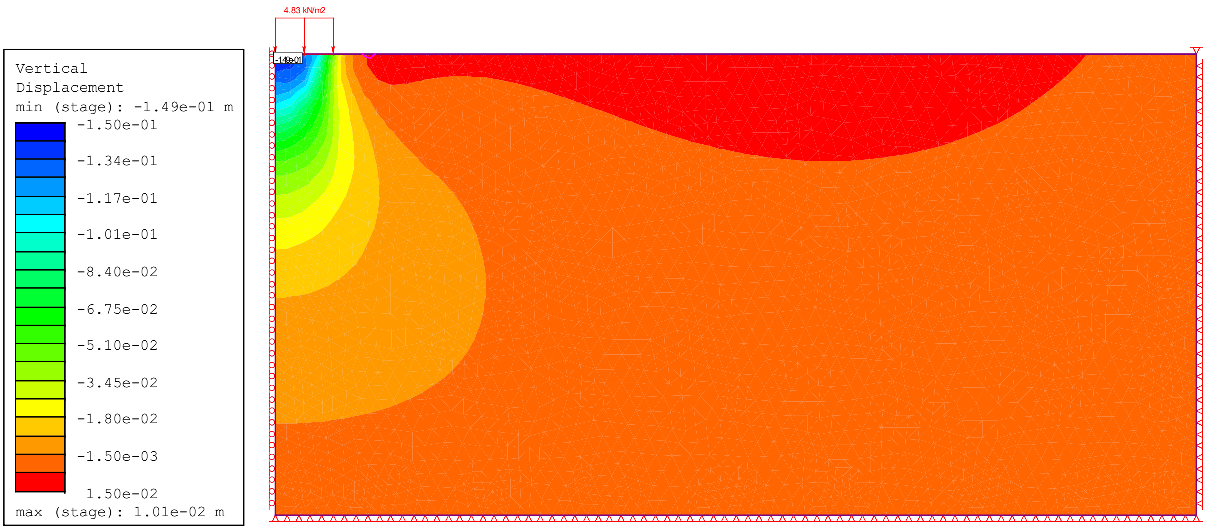
$$\frac{q}{c'} = 4.83$$

for simulated direct and indirect undrained conditions, respectively.

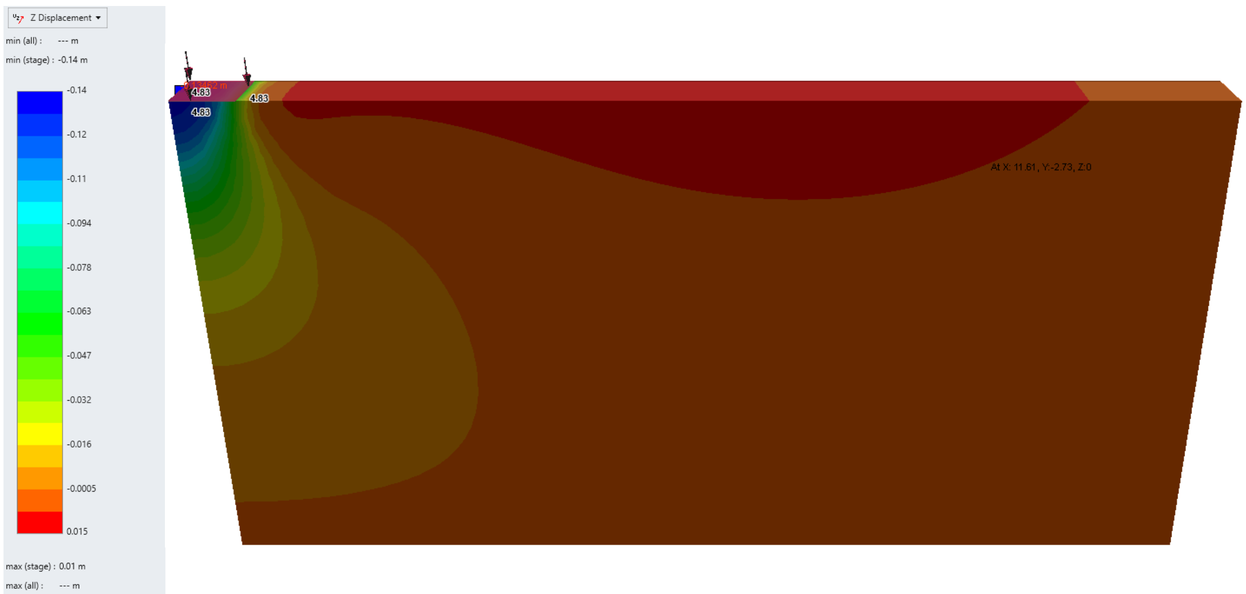
5.3. Results

The settlement of soil due to the applied load is demonstrated by the vertical displacement contour plots in Figure 5-2, which shows the modeling results at final load stage. The contour plot shows a settlement throughout the depth of soil underneath the strip footing. The downward displacement is concentrated closer to the strip footing and dissipates rather gradually along the depth, but abruptly in lateral direction, which produces upheaving behaviour from short distance away from the load. Moreover, deformation captured in 2D analysis shows an agreement with that captured in 3D.

A query point is placed at the end of the strip load to collect the modeling result data at that location. The displacement data collected by the query point in all stages is plotted on the pressure versus displacement ratio graph in Figure 5-3. The graph shows a close agreement between the results from RS2 and RS3, and more importantly with the analytical solution by Small [1].



(a)



(b)

Figure 5-2: Vertical displacement contour at final stage as modeled in (a) RS2 and (b) RS3 with rapid load rate

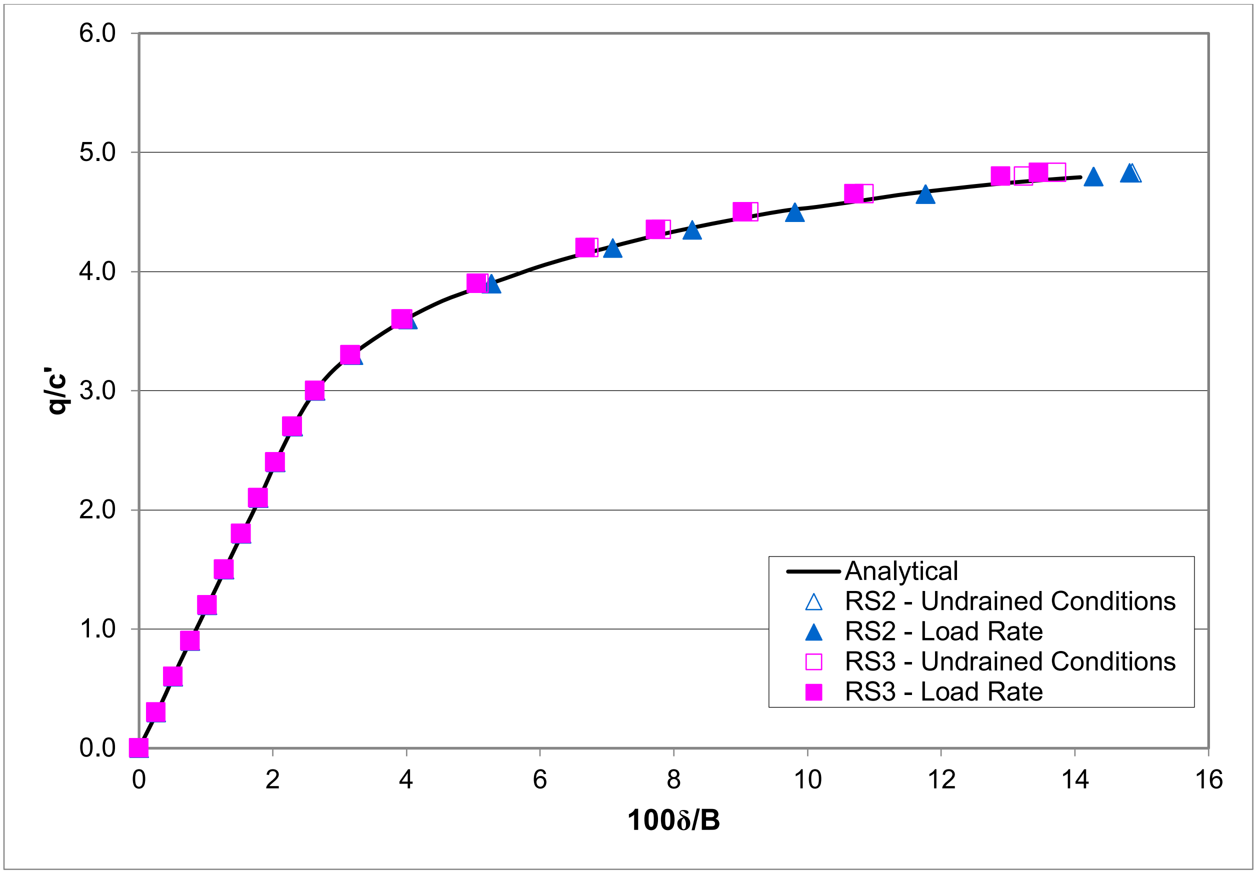


Figure 5-3: Pressure versus displacement for undrained loading

5.4. References

1. Small, J.C., Elasto-plastic consolidation of Soils, PhD thesis, University of Sydney, 1977.

2. Prandtl, L., 'Spannungsverteilung in plastischen Koerpern', in Proceedings of the 1st International Congress on Applied Mechanics, Delft, 43-54, 1924.

5.5. Data Files

The RS3 input file **consolidation #005 (undrained conditions).rs3v3** and **consolidation #005 (load rate).rs3v3** can be downloaded from RS3 Online Help page.

6. Strip Footing with Associated and Non-associated Flow Rules

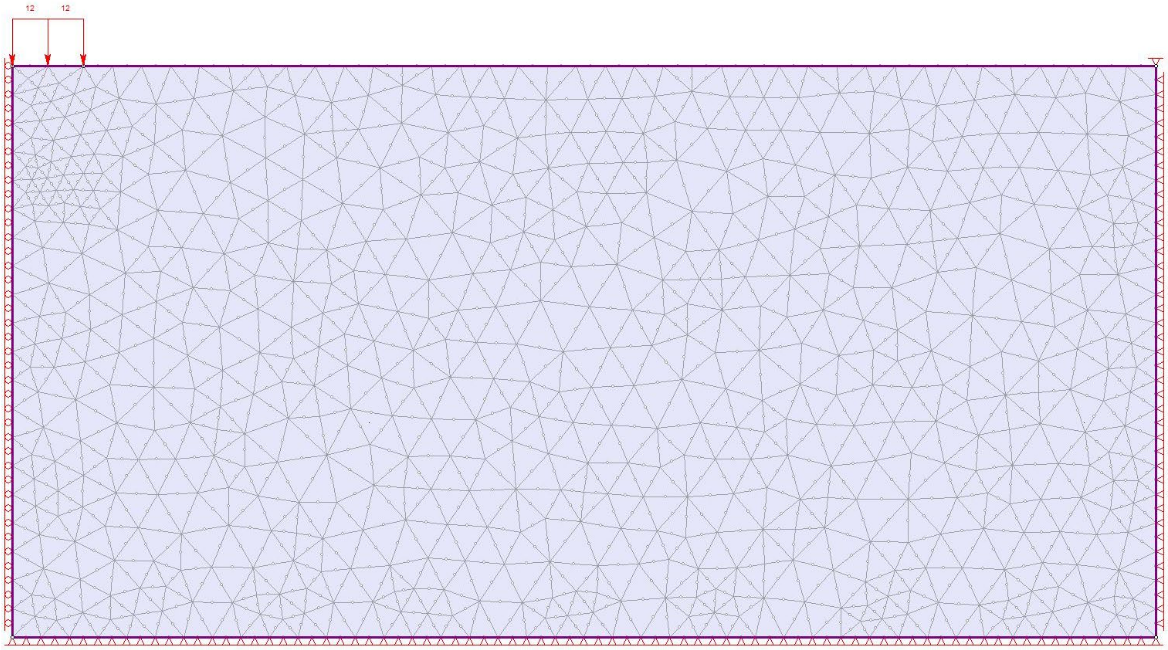
6.1. Problem Description

6.1.1. Associated Flow Rule

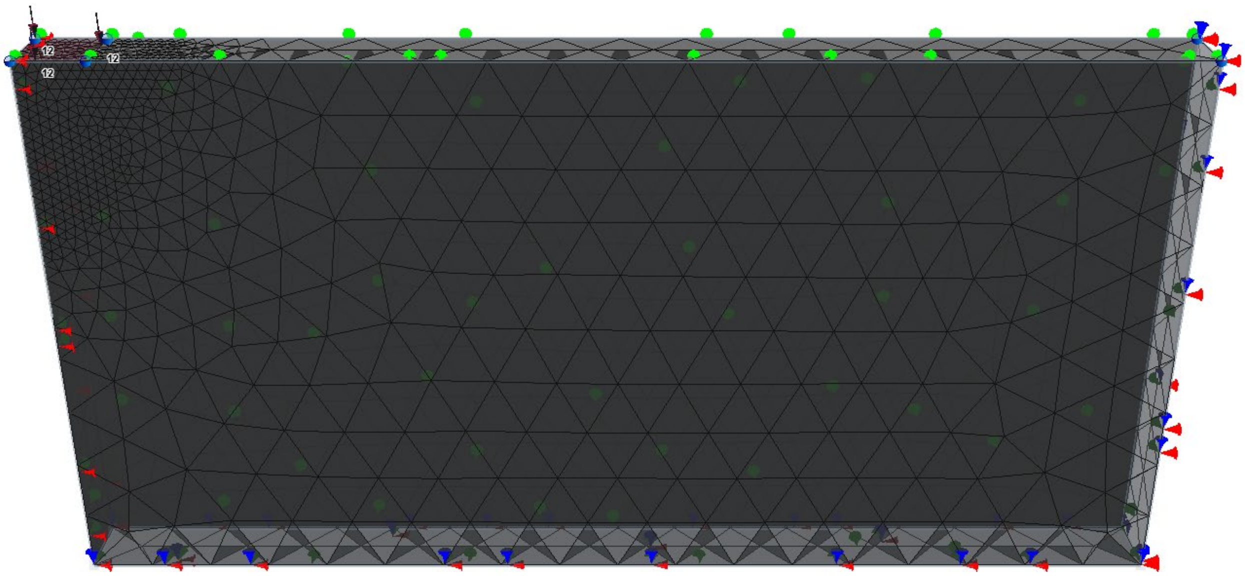
This problem analyzes the behaviour of a smooth flexible strip footing on an elastoplastic soil layer under drained conditions. The material properties assigned to the model is outlined in Table 6.1. The overall modeling geometry and equations for the time factor and coefficient of consolidation are the same as those in Section 5.1.

Table 6.1: Model parameters

Parameter	Value
Young's modulus (E')	200 kPa
Poisson's ratio (v')	0.3
Friction angle (Φ')	20°
Cohesion (c')	1 kPa
Dilatancy angle (ψ')	20°
Permeability (k)	0.01 m/s
Coefficient of consolidation (c_v)	0.1960 m^2/s



(a)



(b)

Figure 6-1: Strip footing on elastoplastic layer under drained conditions modelled in (a) RS2 and (b) RS3

A ramp load is imposed until T_{v0} = 0.01 at which it is held constant, displayed in Figure 6-2. By applying different load rate to each model, three cases with maximum load values of q_0/c' = 5, q_0/c' = 10, and q_0/c' = 15 are considered.

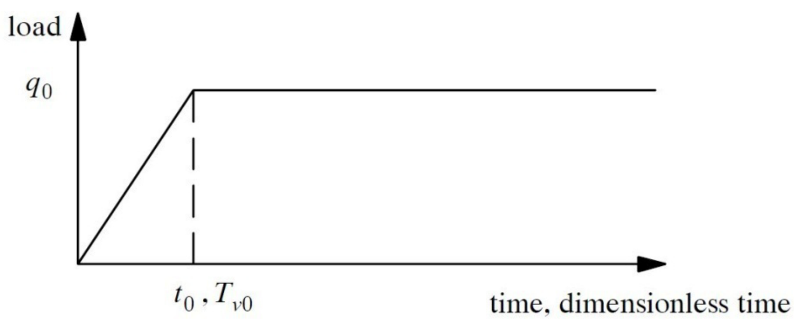


Figure 6-2: Load vs. Time

6.1.2. Non-associated Flow Rule

Another set of analysis is conducted to solve consolidation problem with dilation angle of 0° to simulate non-associated flow rule. In order to simulate drained condition, a slow loading rate of ω = 0.015 is applied. According to Prandtl, the collapse pressure can be calculated as following

$$\frac{q}{c'} = 14.83$$

6.2. Finite Element Solution from Reference

According to Manoharan and Dasgupta [1] regarding the finite element analysis of elastoplastic consolidation, is based on Biot's consolidation theory and has the following formulas:

Displacement and pore pressure vector:

$$\{\mathrm{u}\} = [N_u]\{\mathrm{u}_n\}$$

$$p = [N_p]\{\mathrm{p}_n\}$$

where $\{\mathrm{u}_n\}$ is the nodal displacement vector, $\{\mathrm{p}_n\}$ is the nodal pore pressure vector, and $[N_u]$ and $[N_p]$ are the shape functions.

Strain in terms of nodal displacement:

$$\{\epsilon\} = [B_u]\{\mathrm{u}_n\}$$

where $[B_u]$ is the strain-displacement matrix with the following equation

$$[B_u]^T = [N_u]^T \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \end{bmatrix}$$

Pore pressure derivatives:

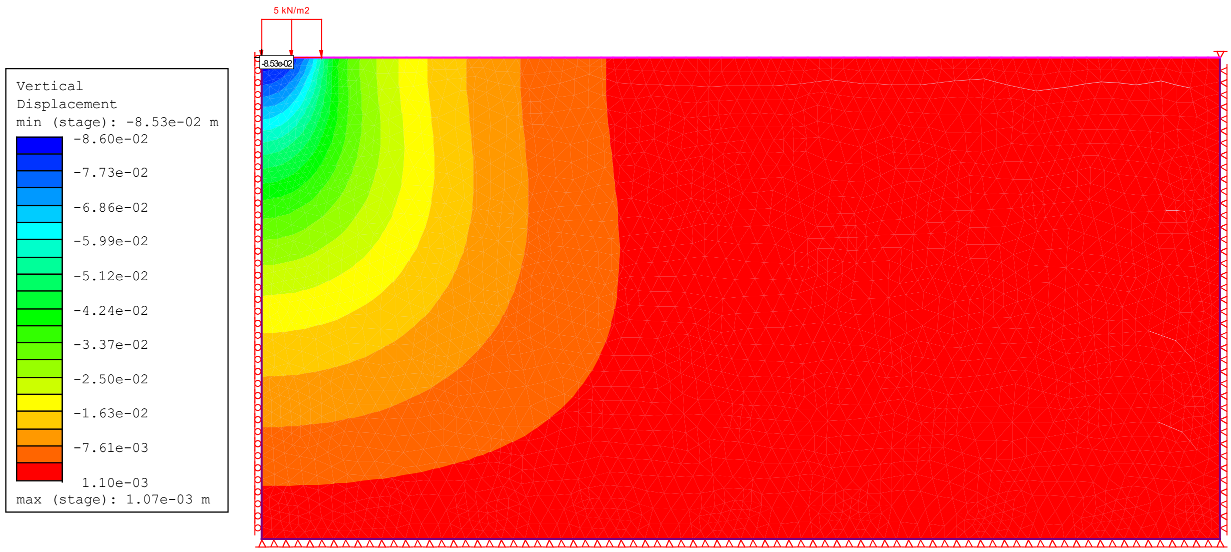
$$[B_p] = \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial z} \end{Bmatrix} [N_p]$$

6.3. Results

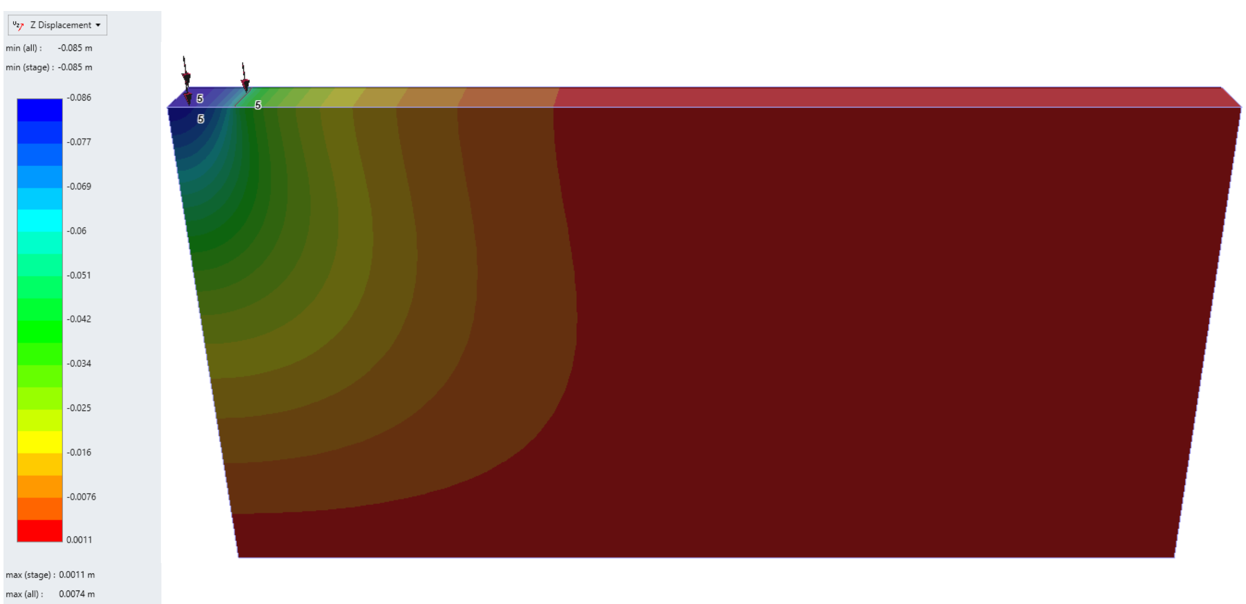
The verification for the modeling results is performed by comparing the settlement determined based on the reference solution and the modeling solution captured at the end of the strip load. A query point is placed at that location to collect the modeling result data.

6.3.1. Associated Flow Rule

The settlement of soil due to the applied load is demonstrated by the vertical displacement contour plots in Figure 6-3, Figure 6-4, and Figure 6-5, which show the modeling results for q/c' = 5, 10, and 15, respectively, at T_v = 70. As expected, the increase in the maximum load induces larger settlement, however a reduction in settling zone and increase in the upheaving area. Moreover, the deformation captured in 2D analysis shows an agreement with that captured in 3D.

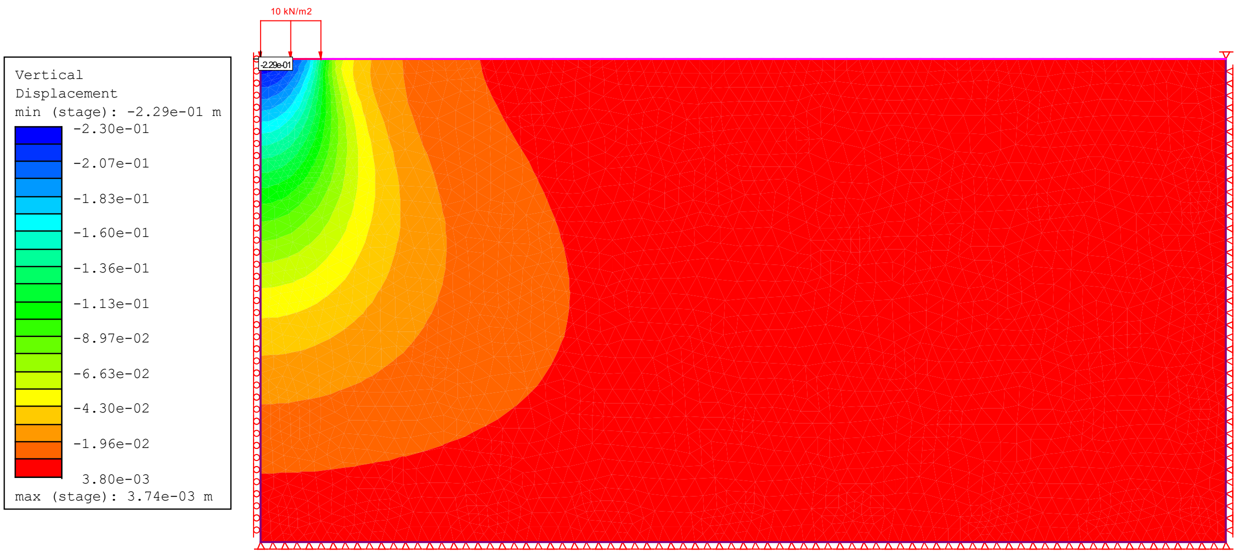


(a)

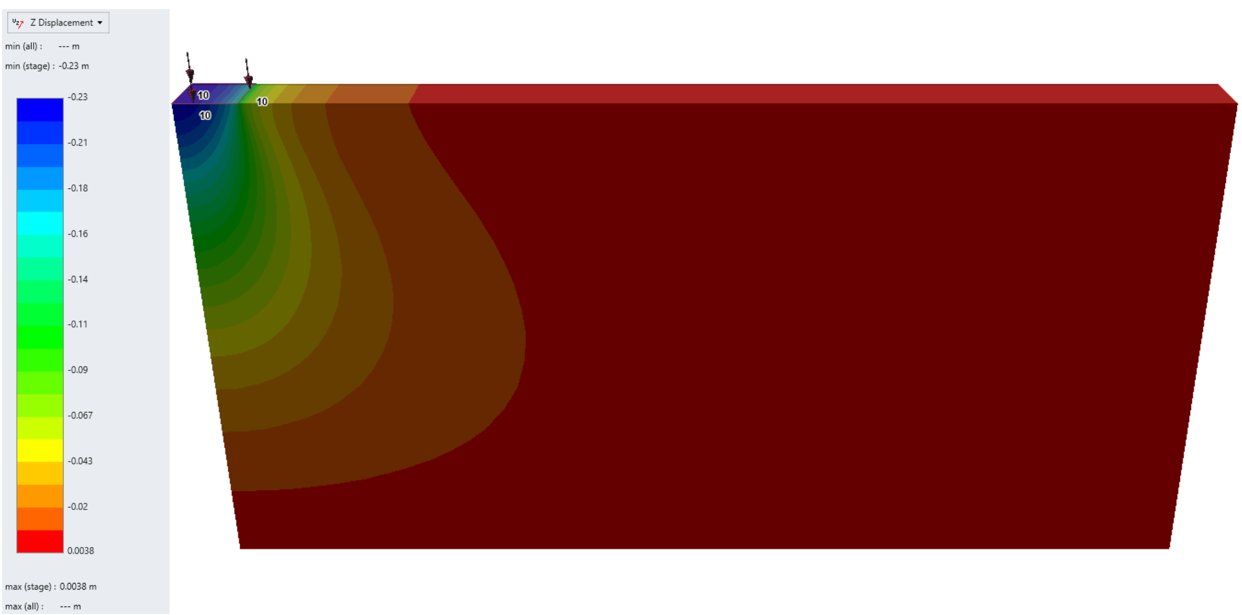


(b)

Figure 6-3: Vertical displacement contour for q/c'=5 as modelled in (a) RS2 and (b) RS3

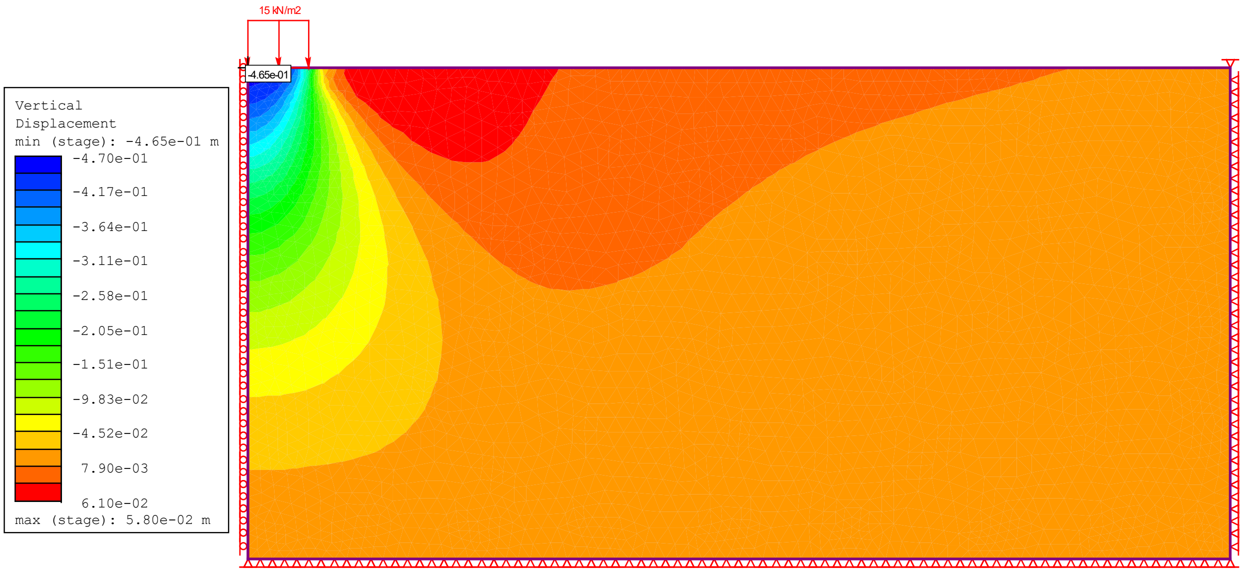


(a)



(b)

Figure 6-4: Vertical displacement contour for q/c’=10 as modelled in (a) RS2 and (b) RS3



(a)

(b)

Figure 6-5: Vertical displacement contour for q/c'=15 as modelled in (a) RS2 and (b) RS3

The vertical displacement data retrieved from query point for each load case is plotted on the settlement-time domain (Figure 6-6). The graph shows that RS2 and RS3 solutions are in accordance with the analytical solutions for each load case.

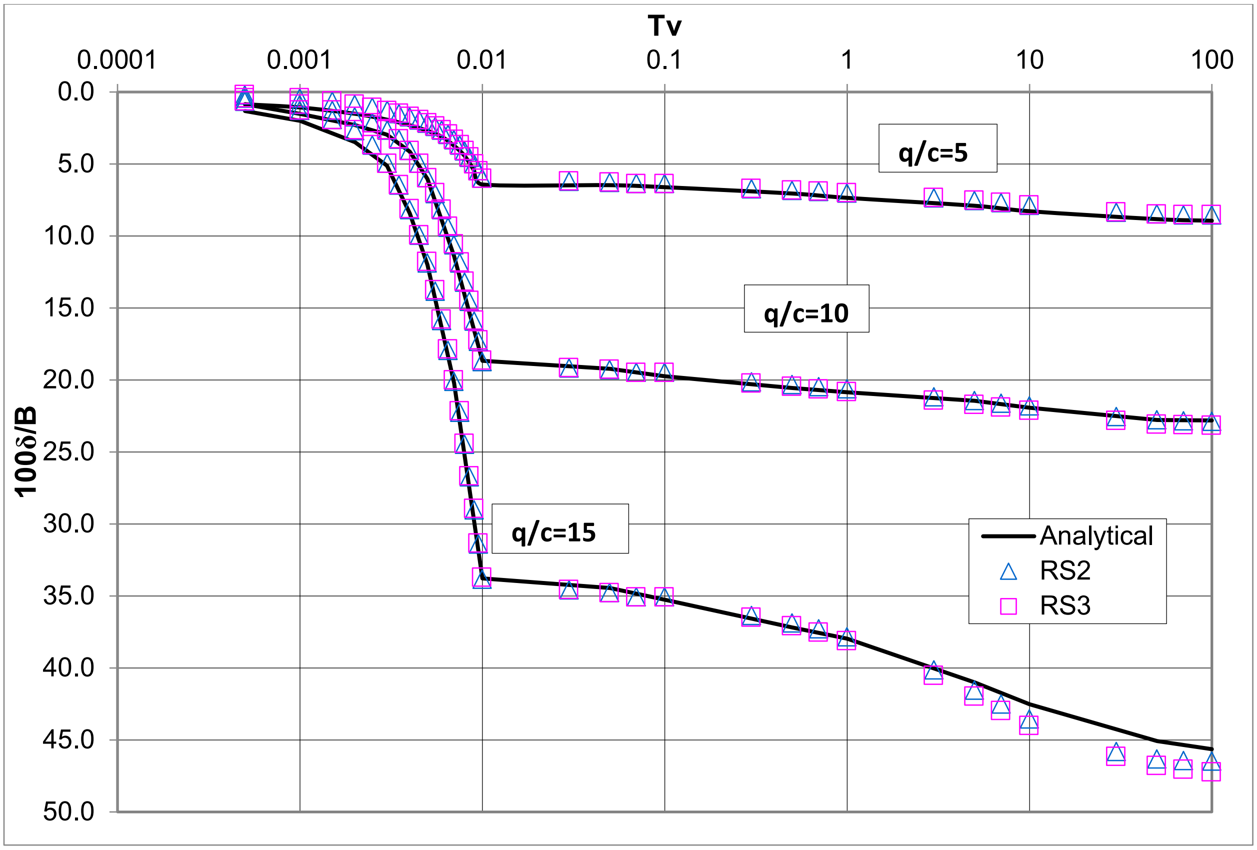
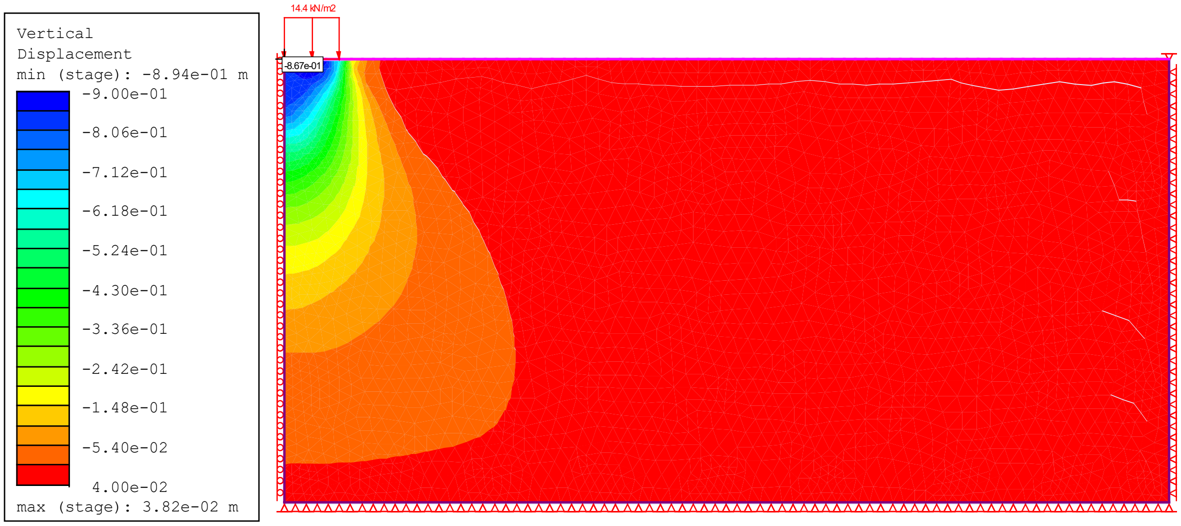


Figure 6-6: Settlement versus time factor for elastoplastic strip footing

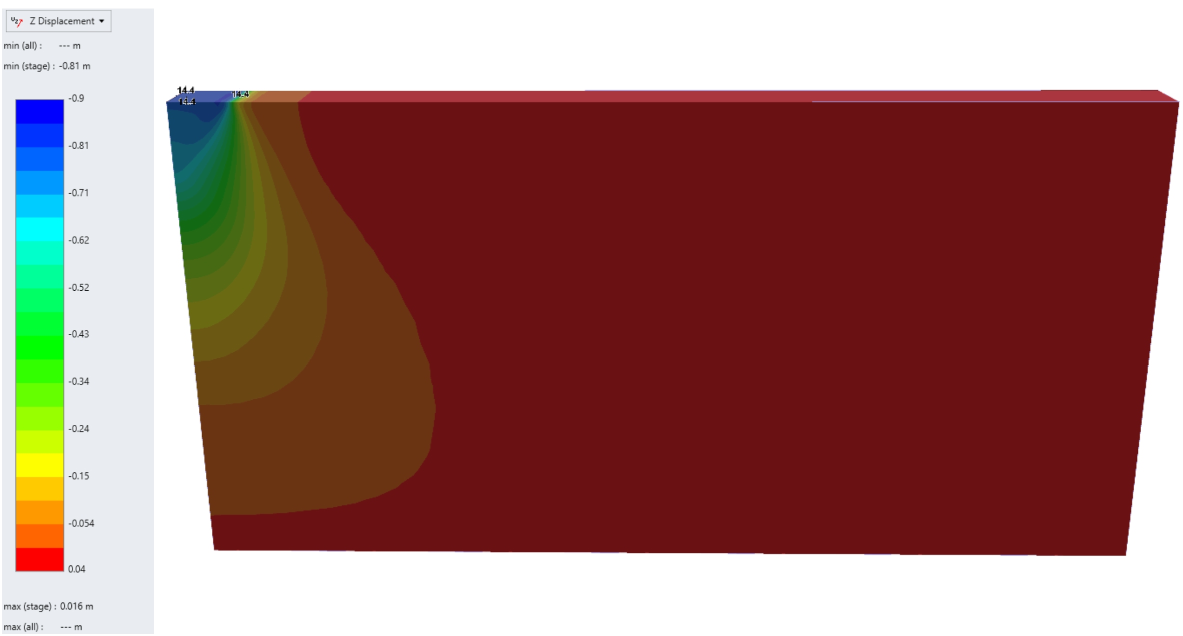
6.3.2. Non-associated Flow Rule

The settlement of soil due to the applied load is demonstrated by the vertical displacement contour plots in Figure 6-7, which shows the modeling results at final load stage. The contour plot shows a settlement throughout the depth of soil underneath the strip footing. The downward displacement is concentrated closer to the strip footing and dissipates rather gradually along the depth, but abruptly in lateral direction, which produces upheaving behaviour from short distance away from the load.

The displacement data collected by the query point in all stages is plotted on the pressure versus displacement ratio graph in Figure 6-8. The graph shows a close agreement between the results from RS2 and RS3, and more importantly with the analytical solution by Manoharan and Dasgupta [1].



(a)



(b)

Figure 6-7: Vertical displacement contour as modeled in (a) RS2 and (b) RS3

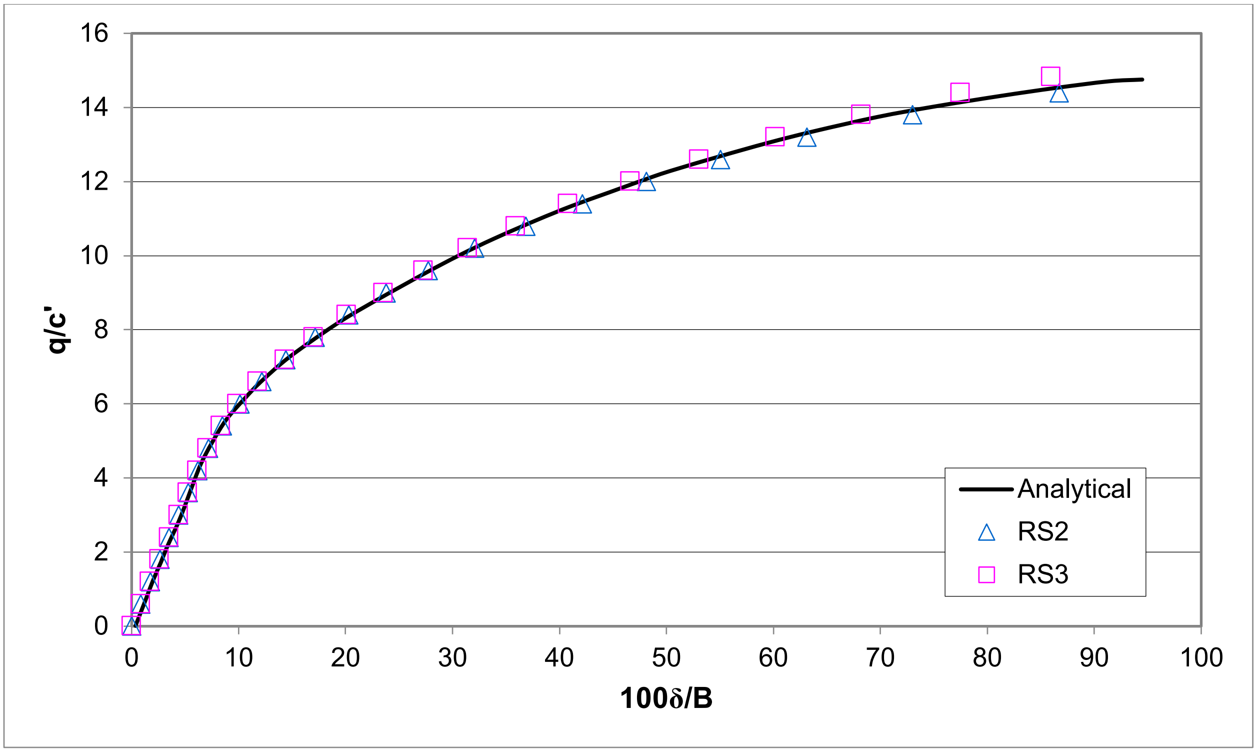


Figure 6-8: Pressure versus displacement for flexible strip footing with varying loading rates

6.4. References

1. Manoharan, N. and Dasgupta, S.P., ‘Consolidation analysis of elastoplastic soil’, Computers and Structures, 54, 1005-1021, 1995.

6.5. Data Files

The RS3 input files:

consolidation #006_01 (q=5).rs3v3

consolidation #006_01 (q=10).rs3v3

consolidation #006_01 (q=15).rs3v3

consolidation #006_02.rs3v3

can be downloaded from RS3 Online Help page.