RS3

3D finite element program for stress analysis and support design around excavations in soil and rock

Dynamic Module Verification Manual

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1 Natural Period of One-Dimensional Column

1.1 Problem Description

Verification problem #01 for RS2. This problem involves wave propagation in a soil column with a square cross section of width 1 m and height of 10 m subjected to gravity. we are using 10-noded tetrahedral elements to discretize 3D domain as presented in **Figure 1-1**. A summary of the parameters used to model the column are presented in **Table 1.1**.

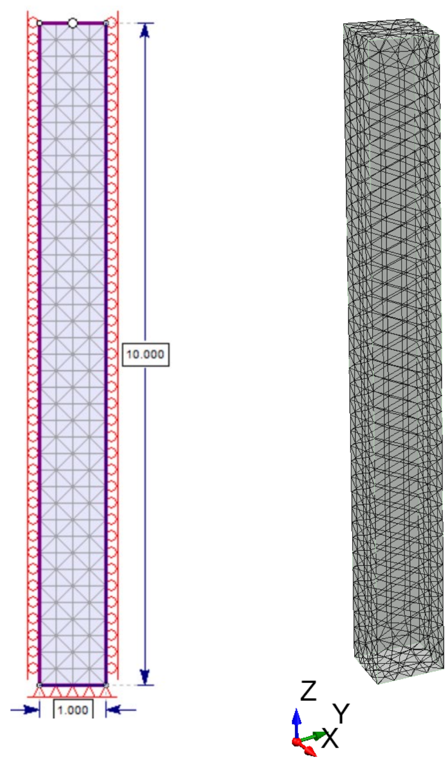

Figure 1-1: Soil column as constructed in **(left)** RS2 and in **(right)** RS3

Table 1.1: Input parameters for one-dimensional column model

Parameter	*Value*
Material type	Elastic
Young's modulus (*E*)	50000 kPa
Poisson's ratio (*v*)	0
Unit weight (*γ*)	20 kN/m^3
Height (*h*)	10.0 m
Width (*w*)	1.0 m

1.2 Analytical Solution

Natural period of an elastic column with a fixed base and an open top face is presented in Eq. (1.1).

$$T = 4L\sqrt{\frac{\rho}{E}} \tag{1.1}$$

where L is the height of the column, E is the elastic modulus of the soil column and ρ is the soil mass. Given the inputs from **Table 1.1**, the analytical natural period (T) of this column is 0.255 s.

1.3 Results

Figure 1-2 shows the vertical displacement at the top face of the soil column with respect to time as produced by RS2 and RS3. The natural period calculated in RS2 and RS3 is 0.256 s, which agrees well with the analytical solution presented earlier.

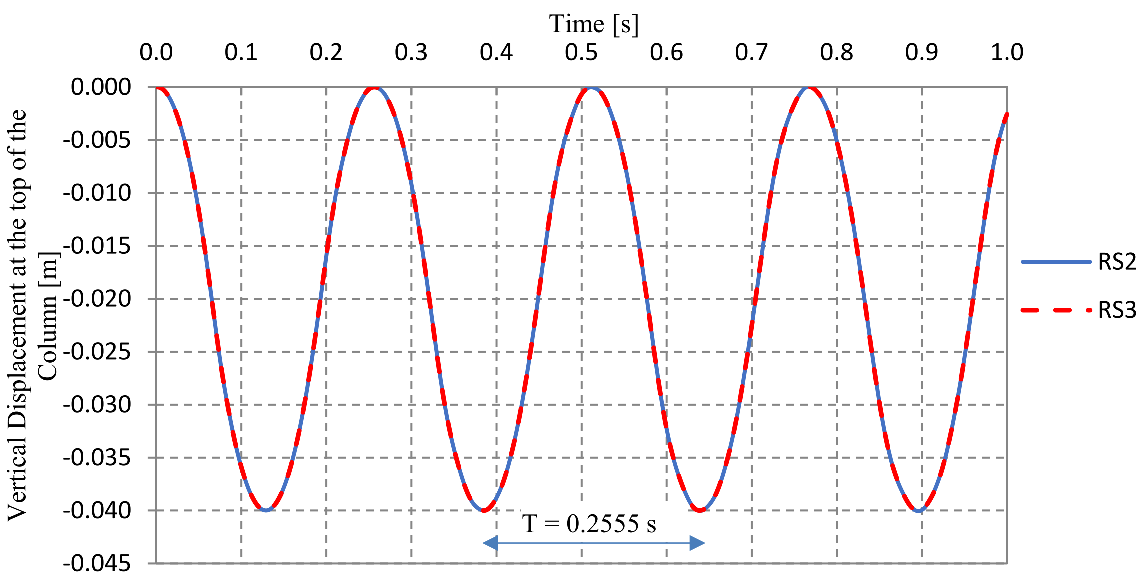

Figure 1-2: RS2 and RS3 Solution. Vertical Displacement-Time

1.4 References

1. Itasca Consulting Group (2005). *FLAC – Fast Langrangian Analysis of Continua, Version 5, User's Manual.* Itasca Consulting Group, Inc, Minneapolis, Minnesota.

1.5 Data Files

The input data file **dynamic #001.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

2 Simply Supported Beam Subjected to a Harmonic Point Load

2.1 Problem Description

Verification problem #05 for RS2. This problem concerns the dynamic behavior of a simply supported beam with a point load at midspan that changes amplitude harmonically. The beam has a length of 1 m and a rectangular cross section of 0.1 m by 0.05 m. The natural period of the beam is determined and compared with the analytical solution and RS2 results. The geometry of 2D and 3D models are presented in **Figure 2-1** and the material properties are provided in **Table 2.1**.

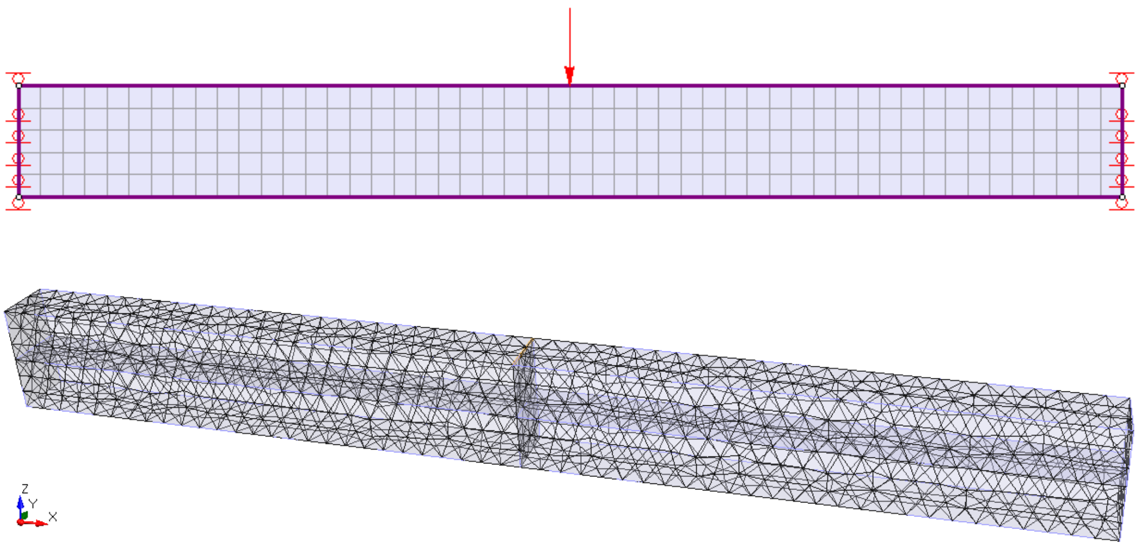

Figure 2-1: Simply Supported Beam Modeled in **(top)** RS2 and in **(bottom)** RS3

Table 2.1: Model parameters

Parameter	***Value***
Material type	Elastic
Young's modulus (*E*)	300000 kPa
Unit weight (γ)	9.81 kN/m^3
Poisson's Ratio (ν)	0
Length (L)	1.0 m
Height (*h*)	0.1 m
Width (*w*)	0.05 m

The beam is loaded with a harmonic load at the midspan of the beam with an amplitude of 1 kN and a forcing frequency, $\overline{\omega}$, of 40 Hz.

2.2 Analytical Solution

Any number of natural frequencies of the beam may be calculated using Eq. (2.1), where n is the number of the mode number.

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{m}} \tag{2.1}$$

Each of these frequencies corresponds to a unique modal response that contributes to the overall response of the beam that is being subjected to the harmonic point load. The analytical solution may be determined by calculating the modal responses of the beam and the sum of these responses will be the overall response of the beam since the beam is modelled linear-elastically. Eq. (2.2) is the analytical solution to the simply supported beam.

Each mode's dynamic response can be described by the response of an equivalent undamped single degree of freedom system's response to a harmonic load, with parameters determined from the mode's natural frequency. The expression within the square brackets of Eq. (2.2) contains the response function of the equivalent single degree of freedom system.

$$u(x,t) = \frac{2PL^3}{\pi^4 EI}\sum_{n=0}^{\infty}\frac{\phi\left(\frac{L}{2}\right)}{n^4}\left[R_n \sin(\overline{\omega}t - \theta) - \frac{\beta_n}{1-{\beta_n}^2}\sin\omega_n t\right]\sin\left(\frac{n\pi x}{L}\right) \tag{2.2}$$

Where,

$$\beta_n = \overline{\omega}/\omega_n$$

$$R_n = \frac{\beta_n}{\left|1-{\beta_n}^2\right|}$$

$$\theta = \begin{cases} 0, & \beta_n < 0 \\ \pi, & \beta_n > 0 \end{cases}$$

The function, ϕ, describes the shape of the beam for each mode. The mode shapes of the beam correspond to sinusoidal curves with half-periods that are fractions of the length of the total beam. The shape function is included in Eq. (2.2) as the final sinusoid expression that is dependent on x, distance along the beam, rather than time. At the midspan there exists only three options for the value of this function as described below:

$$\phi(L/2) = \begin{cases} 0 & n = 2,4,6,\ldots \\ 1 & n = 1,5,9,\ldots \\ -1 & n = 3,7,11,\ldots \end{cases}$$

The contribution of each mode needs to be determined and then summed together to determine the overall response of the system analytically.

The analytical solution provided above, accounts solely for deflection in the beam arising from bending and ignores shear deformations. The majority of displacement may be contributed by the beam bending in static analysis, however, in dynamic analysis accounting for shear deformation will lower the natural frequencies of the system and alter the response. Using a greater number of elements in the RS2/RS3 model will presumably capture the effect of shear deformation on the beam's dynamic response better.

The shortening of the frequency is described in Eq. (2.3).

$$\omega'_n = \omega_n \left[1 + \left(\frac{n\pi r}{L}\right)^2 \left(1 + \frac{E}{\kappa G}\right)\right]^{-0.5} \tag{2.3}$$

Where r denotes the modulus of gyration is equal to the square root of the ratio between a cross section's second moment of area and its area. This is shown in Eq. (2.4).

$$r = \sqrt{\frac{I}{A}} \tag{2.4}$$

G is the shear modulus of the beam and κ is the Timoshenko shear coefficient, a correction parameter introduced to account for non-uniform shear stress distribution along the cross section. For rectangular cross sections this coefficient is given a value of $5/6$. From Eq. (2.3) it becomes evident that the reduction in natural frequency becomes more prominent with higher modes due to the presence of the n value.

The increase in displacement will also decrease the modal stiffness of the system. This diminished stiffness can be calculated by following Eq. (2.5) which uses the new reduced frequency.

$$K'_n = M_n(\omega'_n)^2 = \frac{mL}{2}(\omega'_n)^2 \tag{2.5}$$

The analytical solution may now be calculated using Eq. (2.2) but with reduced values for the stiffness and natural frequencies to calculate the modal response parameters.

2.3 Results

The accuracy of the model will depend on how many elements the beam contains. With fewer number of elements, the response of the system will resemble more the response of the simply supported beam without considering shear deformation.

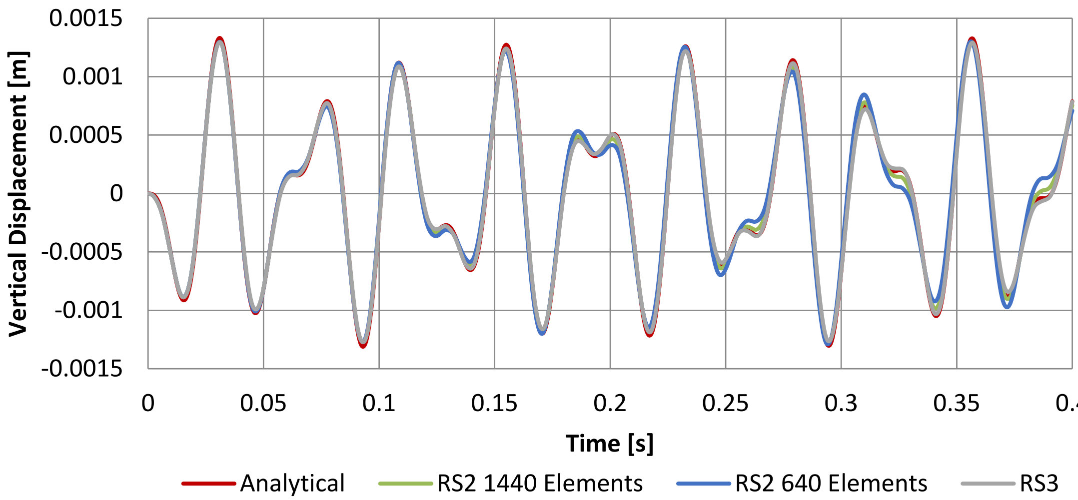

Figure 2-2: Vertical Displacement Response of the Midspan of the Beam

The results of RS2 analyses are presented in **Figure 2-2** that considered 640 and 1440 finite elements and compared with a dense mesh in RS3. The model with the greater number of elements exhibited a displacement response similar to that of the analytical solution that considered shear deformations. The coarser model was unable to capture that phenomenon as predicted.

The results demonstrate RS3's ability to capture complex material behavior during dynamic analysis provided that a sufficient number of elements have been used in the model.

2.4 References

1. Chopra, A. K. (1995). *Dynamics of Structures*. New Jersey: Prentice Hall.

2.5 Data Files

The input data file **dynamic #002.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

3 Cantilever Beam Under Harmonic Load

3.1 Problem Description

Verification problem #06 for RS2. This problem demonstrates behavior of a cantilever beam under harmonic load. The cantilever beam has a length of 1 m and a square cross section of 0.1 m by 0.1 m. The geometry of the models in both RS2 and RS3 are presented in **Figure 3-1**. Material properties are presented in **Table 3.1**.

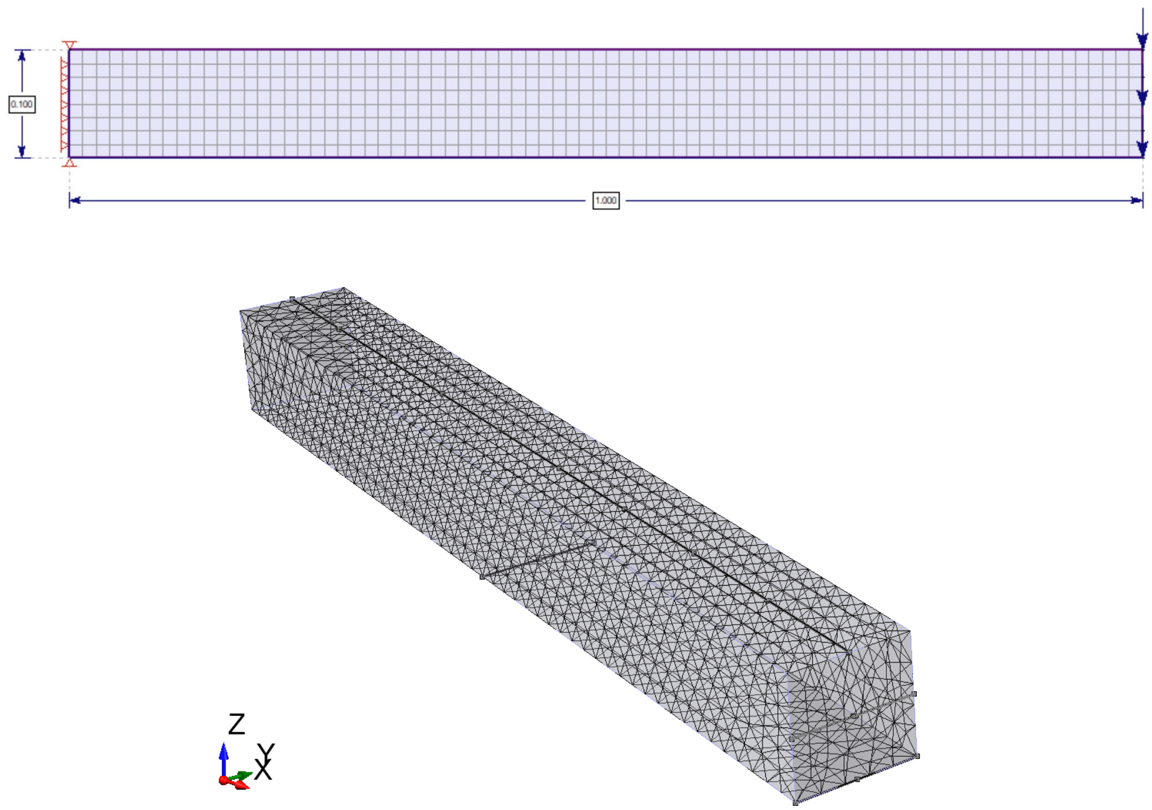

Figure 3-1: Cantilever beam modeled in **(top)** RS2 and in **(bottom)** RS3

The harmonic load is a function with respect to time as described in Eq. (3.1) and presented in **Figure 3-2**.

$$P(t) = \begin{cases} P(t) = 3.1941\sin(\pi t/2) & \text{if} \quad 0 < t \le 2 \\ P(t) = 0 & \text{if} \quad t > 2 \end{cases} \tag{3.1}$$

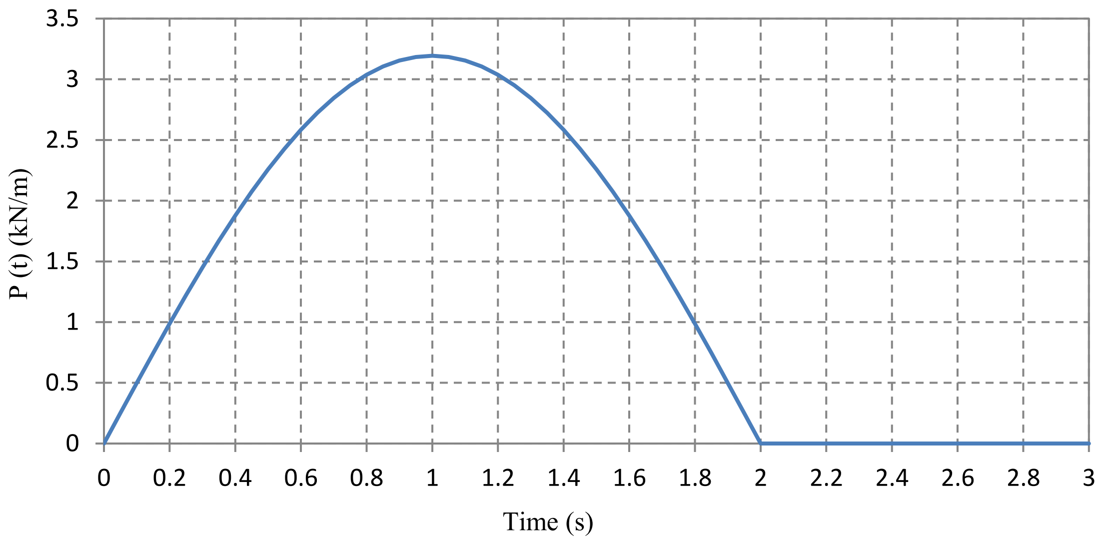

Figure 3-2: Harmonic load acting on the beam

Table 3.1: Model parameters

Parameter	***Value***
Material type	Elastic
Young's modulus (*E*)	38329.2 kPa
Poisson's ratio (*v*)	0.3
Unit weight (*γ*)	9.81 kN/m^3
Length (L)	1.0 m
Height (*h*) & Width (*w*)	0.1 m

3.2 Results

The beam displacement obtained using RS3 is compared with the solutions calculated by RS2 and commercial FEA software MIDAS GTS. In the first attempt, linear elements are used in both RS2 and RS3 models as it is shown in **Figure 3-3**. The results from RS2 and RS3 are slightly different from the MIDAS GTS results. By using a higher order element in RS3 (10-noded tetrahedral elements) the results matched with the provided results by MIDAST GTS.

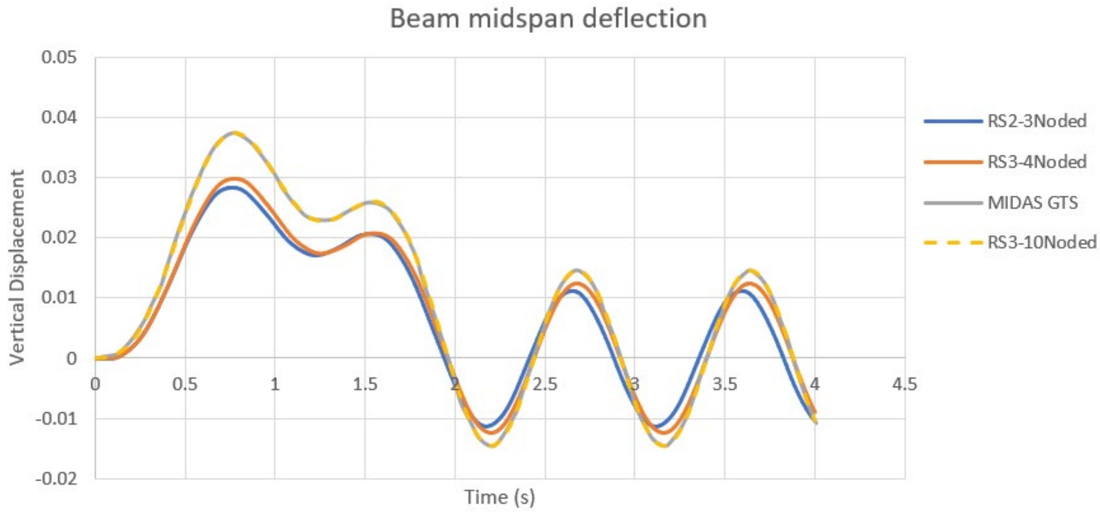

Figure 3-3: Midspan Displacement Response

3.3 References

1. J. M. Duncan and C. Y. Chang (1970), *Nonlinear analysis of stress and strain in soils*, J. of Soil Mech. and Foundation Division, ASCE, 96 (SM5), pp. 1629-1653.

3.4 Data Files

The input data file **dynamic #003.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

4 Cantilever Beam Subjected to a Constant Point Load

4.1 Problem Description

Verification problem #07 for RS2. This problem demonstrates behavior of a cantilever beam under a constant load. The properties of the cantilever, shown in **Table 4.1**, are identical to the previous section's problem statement except that the Poisson's ratio here is zero, and that the beam is subjected to a constant load of 3.19 kN. However, the geometry is slightly different as the cross section of the beam is rectangular with a height of 0.1 m and a width of 0.02 m. The beam is fixed in the Y direction to prevent any movement in the out-of-plane direction. Loading the cantilever with a constant load the fundamental period may be ascertained from the deflection response and compared to the theoretical first mode period. The mode shapes of the cantilever beam are not simple and described using hyperbolic cosine functions which does not allow for a concise analytical response function to be generated. The model geometries are presented in **Figure 4.1**.

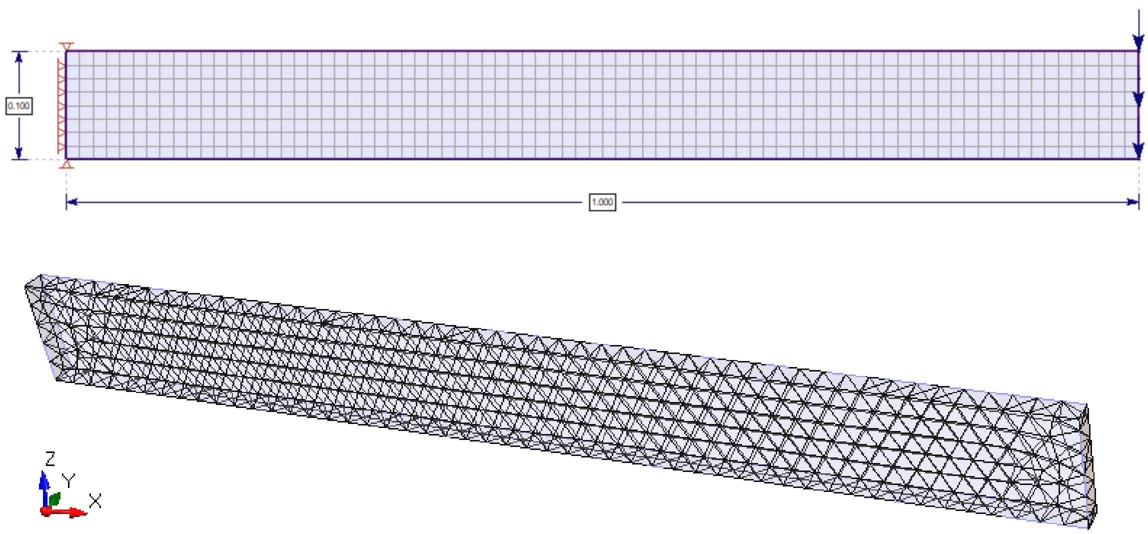

Figure 4-1: Cantilever beam modeled in **(top)** RS2 and in **(bottom)** RS3

Table 4.1: Model parameters

Parameter	*Value*
Material type	Elastic
Young's modulus (*E*)	38329.2 kPa
Poisson's ratio (*v*)	0
Unit weight (*γ*)	9.81 kN/m^3
Length (L)	1.0 m
Height (*h*)	0.1 m
Width (*w*)	0.02 m

4.2 Analytical Solution

The stiffness of a cantilever beam will be needed to determine the static stiffness of the system and it is presented below in Eq. (4.1).

$$K = \frac{3EI}{L^3} \tag{4.1}$$

The stiffness for this problem was determined to be 9.582 kN/m/m. Dividing the amplitude of the load by this stiffness produces a value of 0.033 m for the static stiffness.

The first natural period of a cantilever is defined by Eq. (4.2).

$$\omega_i = \frac{3.516}{L^2}\sqrt{\frac{EI}{m}} \tag{4.2}$$

Evaluating that expression, a fundamental period of 1.000 s was determined for this cantilever system. The analytical solution idealizes the system as a single degree of freedom system and the higher mode response of the cantilever is ignored. This is an imprecise idealization, but it allows one to evaluate the general shape of the cantilever response. The response of a single degree of freedom system to a constant load is described below in Eq. (4.3).

$$u(t) = u_{static}[1 - \cos(\omega t)] \tag{4.3}$$

The maximum amplitude of the displacement will be twice that of the static displacement.

4.3 Results

The beam displacement obtained using RS2 and RS3 is compared with the idealized response of a cantilever only containing a response from the first mode.

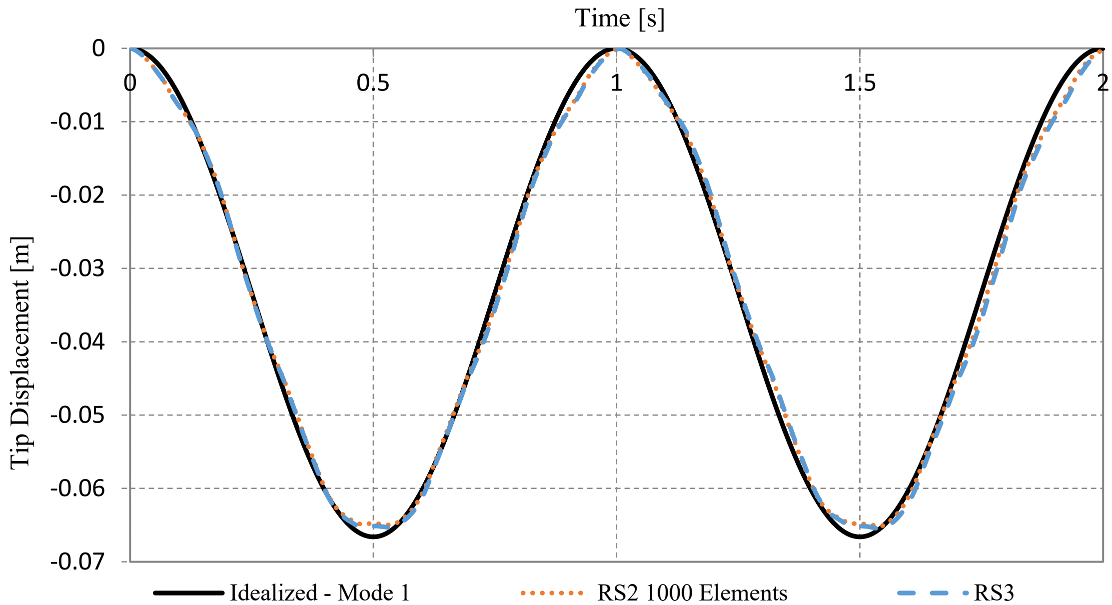

Figure 4-2: Cantilever Free End Vertical Displacement Response

The RS2 and RS3 responses display a minor discrepancy with the idealized response, especially a reduction in amplitude. The difference is likely due to the influence of higher mode responses. The deviation from a smooth sin curve is likely due to destructive interference of the higher modes. The response nevertheless exhibits a predominant period of 1 s as predicted analytically and revealing the influence of the fundamental period on the total response.

4.4 References

1. Chopra, A. K. (1995). *Dynamics of Structures*. New Jersey: Prentice Hall.

4.5 Data Files

The input data file **dynamic #004.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

5 Single Element with Spring and Damping

5.1 Problem Description

Verification problem #08 for RS2. This section attempts to validate the spring and dashpot elements as well as the mass-proportional damping that is presently implemented in the dynamic module of RS2 and RS3. In order to create a verifiable model, the problem had to be reduced to one that could be determined analytically, and for this reason the problem described within is that of a single degree of freedom system.

The model consists of a single quadrilateral element with near rigid stiffness that provides mass to the dynamic system. The geometry of the model is a 0.5 m by 0.5 m square element with a 0.1 m width. The model is restrained in the X and Y directions and is only supported vertically by springs that provide the system's effective stiffness. The element's rigidity is supposed to refrain it from deforming and allow the springs to solely combat the imposed loads. The configuration of the system is presented below in Figure 5-1 and the material properties are shown in Table 5.1.

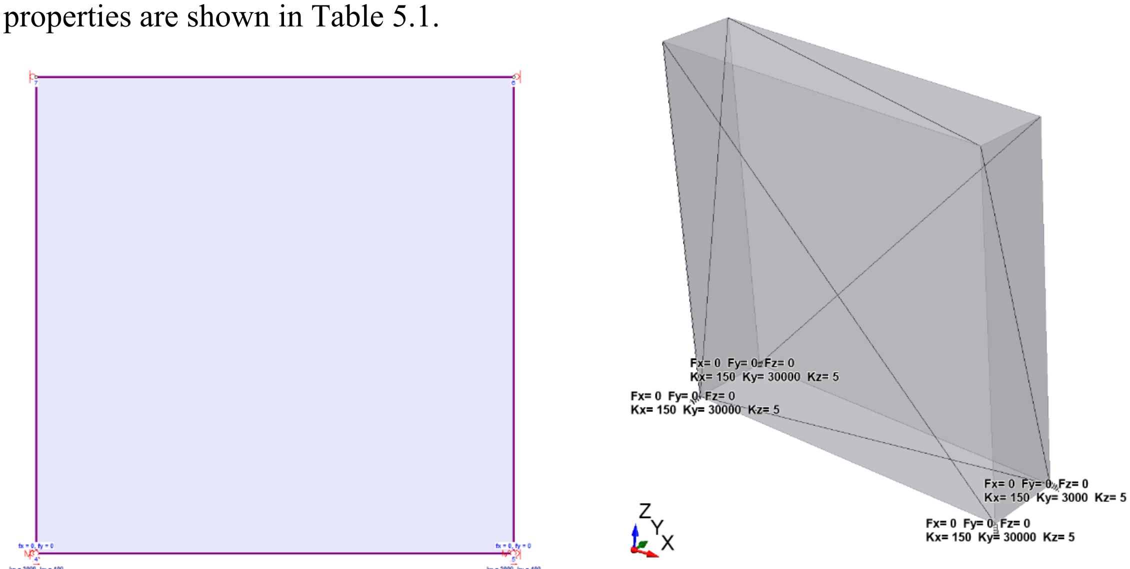

Figure 5-1: **(left)** RS2 model **(right)** RS3 model

Table 5.1: Model Parameters

Parameter	***Value***
Material type	Elastic
Spring Stiffness (K)	100 kN/m/m
Unit weight (γ)	9.81 kN/m^3
Poisson's Ratio	0
Length (L) & Height (*h*)	0.5 m
Width (*w*)	0.1 m

The bottom nodes of the element where the springs are attached will both be subjected to an identical harmonic force with an amplitude of 10 kN and a frequency of 10 Hz or 62.8 rad/s. Therefore, the effective stiffness of the system is 200 kN/m/m and the amplitude of the total load is 20 kN.

The system will be modeled:

- without damping
- with damping provided by mass-proportioned Rayleigh damping.

5.2 Damping Parameters

The coefficient of damper that is given to the dashpot damper is merely a fraction of the critical damping of the system and it is determined using Eq. (5.1).

$$C = \xi \times 2\sqrt{MK} \tag{5.1}$$

The mass of the system is 0.25 tons and the stiffness has been provided. assuming these values 10% for the damping ratio, the damping coefficient was determined to be 1.414 kNs/m/m.

To ensure that the mass-proportional Rayleigh damping provided equivalent damping, the damping parameter was determined by simply dividing the damping coefficient by the value of the mass. This results in a Rayleigh damping parameter of 5.657.

5.3 Analytical Solution

Since the system is effectively a single degree of freedom system the analytical solution for a damped system is readily available. The natural frequency was determined to be 28.28 rad/s by using Equation (5.2).

$$\omega_n = \sqrt{\frac{K}{M}} \tag{5.2}$$

For an undamped single degree of freedom system with no initial displacement or velocity the displacement response function is defined by Equation (5.3).

$$u(t) = \frac{Po}{K} R \sin(\overline{\omega} t - \theta) - \frac{\beta}{1-\beta^2} \sin \omega_n t \tag{5.3}$$

Where β is the ratio between the forcing and natural frequencies, R_d is the amplitude reduction factor of the particular solution and θ is the phase angle. These variables are defined as:

$$\beta = {}^{\overline{\omega}}/_{\omega_n}$$

$$R = \frac{\beta}{|1 - \beta^2|}$$

$$\theta = \begin{cases} 0, & \beta < 0 \\ \pi, & \beta > 0 \end{cases}$$

The first sinusoidal function represents the particular solution which oscillates at the forcing frequency whereas the second sinusoidal function is the complimentary solution that oscillates at the natural frequency. Typically, in a damped system the complimentary solution is the transient response of the system that dissipates over time, however in an undamped system it is ever present. For this system β was found to be 2.221, the reduction factor R had a value of 0.254 and the phase angle had the value of π.

A damped system possesses a similar response function except that the particular solution decays exponentially and there is some period elongation due to the damping. The steady-state solution retains the same form, but the reduction factor and phase angle definitions are modified to account for damping in the system. The damped response function of this system with no initial velocity or displacement is presented in Eq. (5.4).

$$u(t) = \frac{P_0}{K} R_d \sin(\bar{\omega} t - \theta_d) + e^{-\xi \omega t}[A \cos(\omega_d t) + B \sin(\omega_d t)] \quad (5.4)$$

Where the variables A and B are determined based on the initial conditions of the system. For a system that is initially stationary, the variables are defined below.

$$A = -\frac{P_0}{K} R_d \sin(-\theta_d)$$

$$B = \frac{\xi \omega A - \frac{P_0}{K} R_d \bar{\omega} \cos(-\theta_d)}{\omega_d}$$

The new damped parameters are defined below.

$$R_d = \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\xi\beta)^2}}$$

$$\theta_d = \tan^{-1}\left(\frac{2\xi\beta}{1 - \beta^2}\right)$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

For the damped problem the reduction factor R_d has a value of 0.254, the phase angle is -0.1124 rad and the damped natural frequency is 28.14 rad/s. The initial condition variables A and B were determined to be -0.002851 and -0.05667 respectively.

5.4 Results

The response of both the damped and undamped models show great agreement with both RS2 analysis and the analytical solution. As expected, the vertical displacement in an undamped model is higher than the model with damper. This behaviour is observed in both RS2 and RS3. This comparison demonstrates that the springs and dampers have been implemented correctly in RS3.

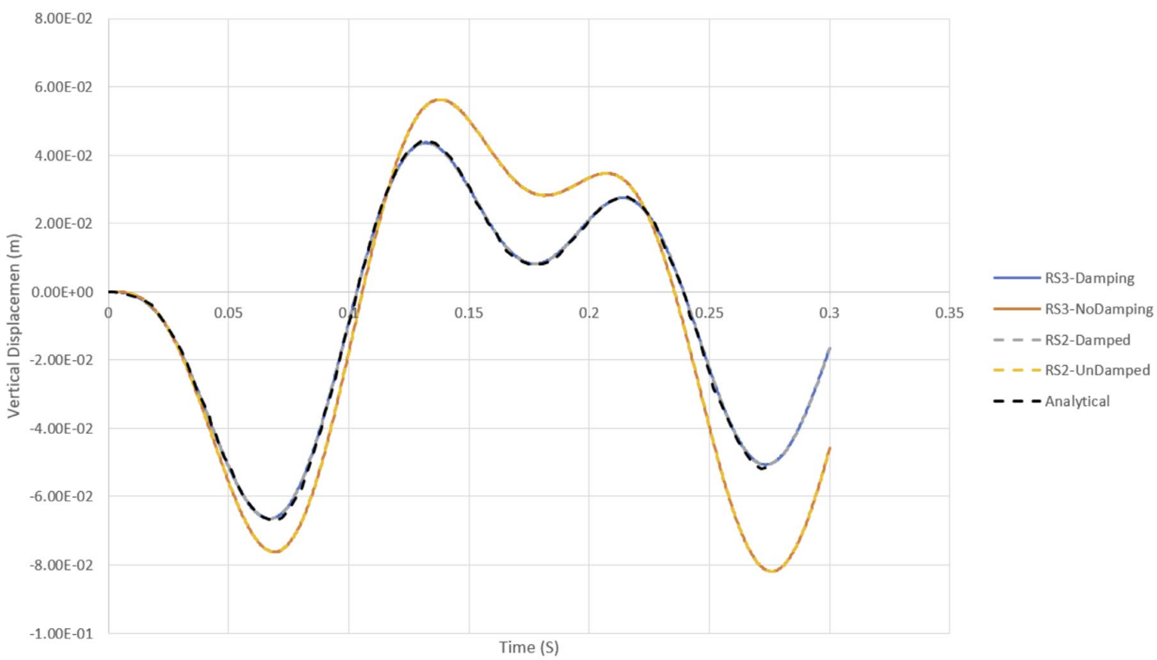

Figure 5-2: Damped and Un-Damped Displacement Response using Dashpot Dampers

5.5 Data Files

The input data file **dynamic #005.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

6 Dimensional S-Wave Propagation

6.1 Problem Description

Verification problem #10 for RS2. This problem addresses S-wave propagation in a one-dimensional soil column. The soil column has a height of 20 m and a rectangular cross section of 2 m by 5 m. The model is allowed to move only in the horizontal direction. A prescribed horizontal displacement of 0.01 m is applied to the bottom of the column. In order to test the absorb boundary condition, it is applied to the top of the soil column to replace the fixed boundary. The geometry of the problem is shown in Figure 6-1. The material properties used in the model are summarized in Table 6.1.

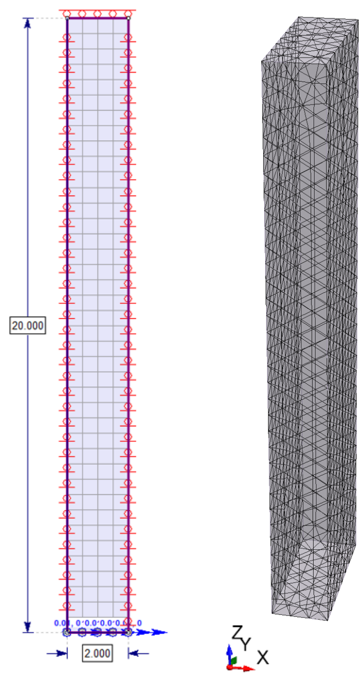

Figure 6-1: Model of a soil column in **(left)** RS2 and in **(right)** RS3

Table 6.1: Model parameters

Parameter	*Value*
Material type	Elastic
Young's modulus (*E*)	20000 MPa
Poisson's ratio (*v*)	0.25
Unit Weight (γ)	20 kN/m^3
Height (*h*)	20 m
Length (L)	2 m
Width (*w*)	5 m

6.2 Analytical Solution

Velocity of S-wave in the column is presented in Eq. (6.1).

$$\beta = \sqrt{\frac{G}{\rho}} \tag{6.1}$$

Where, G is the shear modulus as described in Eq. (6.2).

$$G = \frac{E}{2(1+\upsilon)} \tag{6.2}$$

In this problem, the analytical S-wave velocity is 62.64 m/s. The time necessary for the middle point of the bottom face to start moving is:

$$t = \frac{L/2}{\beta} = 0.16s \tag{6.3}$$

The shear wave is induced by moving the bottom face of the mode by 0.01 m and maintaining that imposed displacement for the entirety of the simulation. For the fixed top boundary case it is expected that the shear wave will reflect from the fixed boundary and repeatedly influence the midspan horizontal deflection. The viscous boundary should absorb the incoming shear wave and eliminate any reflection waves.

6.3 Results

Displacements of a point at the middle of the column are analyzed in two cases: no damping and with absorb boundary condition applied to the top of the column and compared with RS2. The results are shown in **Figure 6-2** and **Figure 6-3**. It can be seen that the middle point starts to move just before at 0.16s which agrees well with the analytical solution. The displacement at the point is dropped to zero at 0.48 s when the S-wave comes back due to lack of viscous boundaries. If an absorb boundary condition is used, a constant displacement of 0.01 m is observed after 0.16 s. Results of a similar problem calculated from Plaxis [1] are given in **Figure 6-4** and **Figure 6-5** for reference.

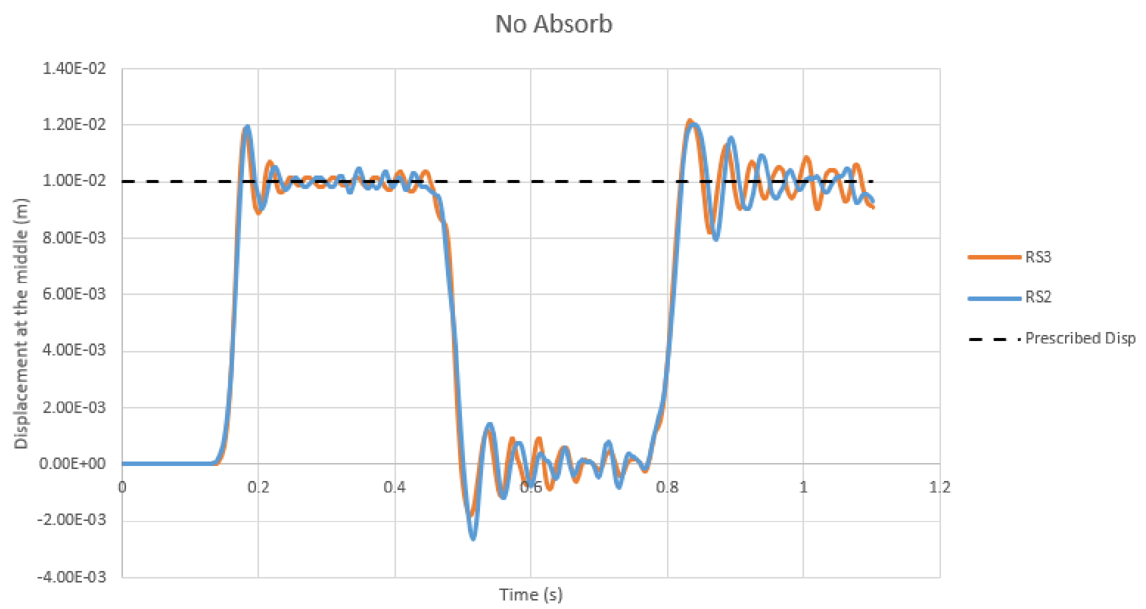

Figure 6-2: Displacement at the middle of the soil column-undamped fixed boundary

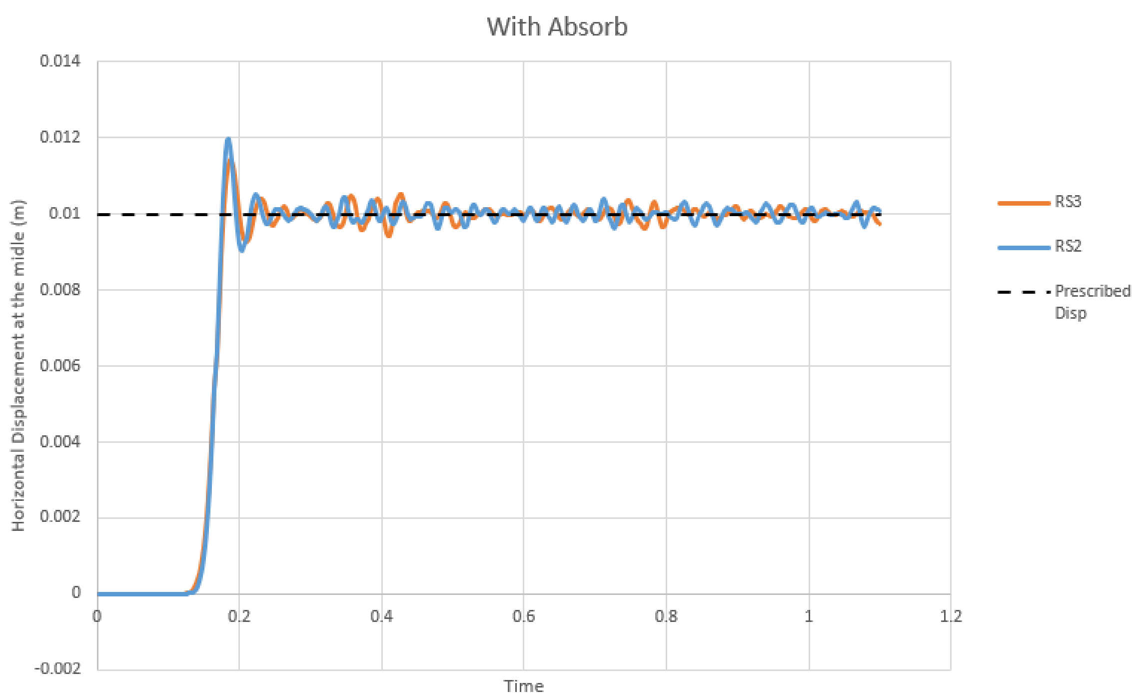

Figure 6-3: Displacement at the middle of the soil column-absorb boundary condition

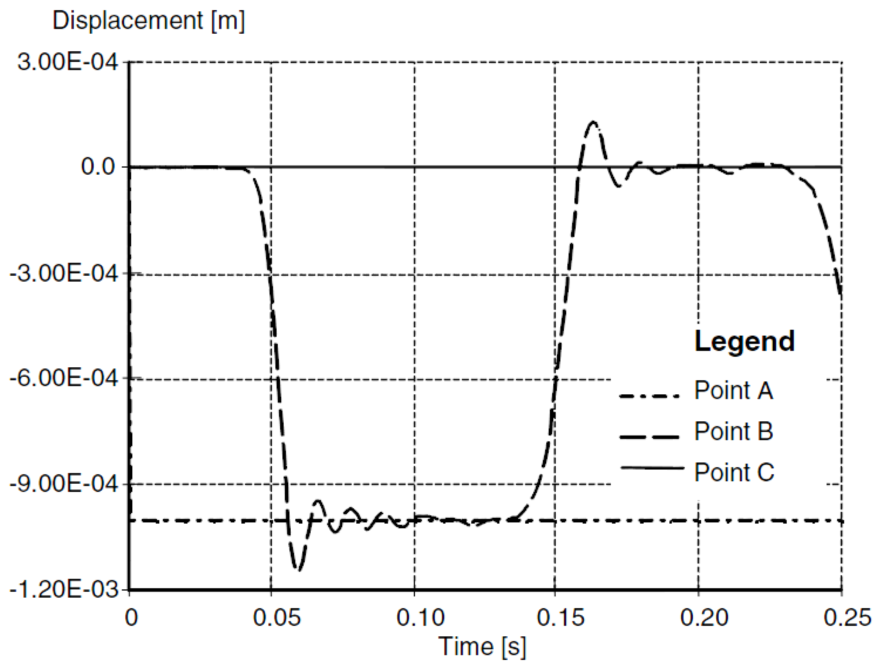

Figure 6-4: Displacements-undamped (from Plaxis)

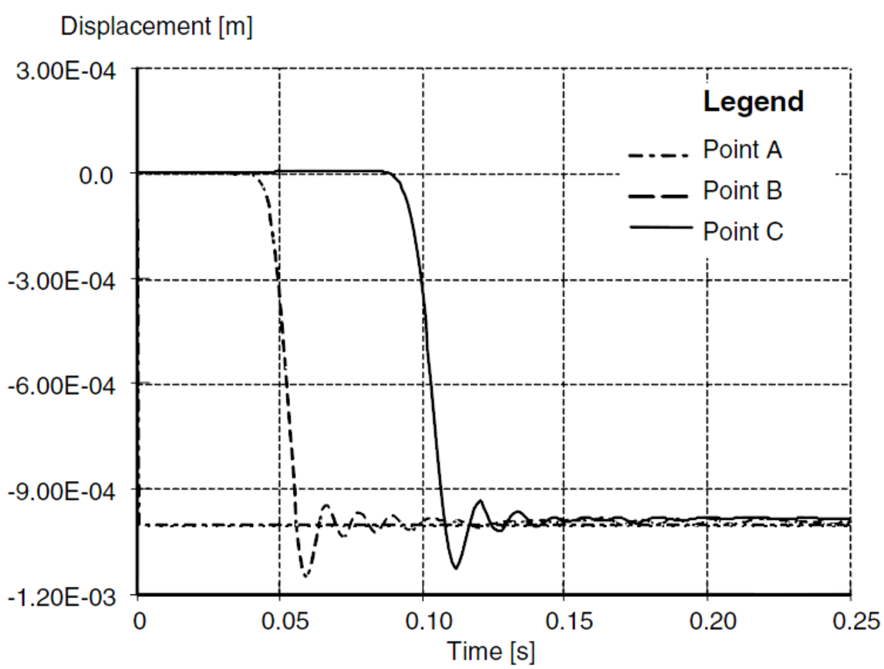

Figure 6-5: Displacements-viscous boundary (from Plaxis)

6.4 References

1. Brinkgreve, R. B. (2002) *Plaxis 2D Version 8.4: Reference, Scientific and Dynamic Manuals*, Lisse, Balkema.

6.5 Data Files

The input data file **dynamic #006.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

7 Lamb's Problem: S-Wave and P-Wave Propagation

7.1 Problem Description

Verification problem #11 for RS2. This problem addresses Lamb's problem [1] which is wave propagation in a semi-infinite elastic medium subjected to an impulsive force applied at the surface. Due to symmetry, only a quarter of the field problem is modeled in RS3 with the radius of 100 m in the horizontal direction and 30 m in the vertical direction. Viscous boundaries are introduced around the model. The geometry of the problem is shown in **Figure 7-1**. The point load acting on the top left of the model is approximated by a distributed load with the duration of 0.025 s and the load started after 0.05 s. The magnitude of the load is 50 kN. This point load is distributed in a radius of 2.5 m and applied to the model. The material properties used in the model are summarized in **Table 7.1**. No artificial damping was used in the simulation.

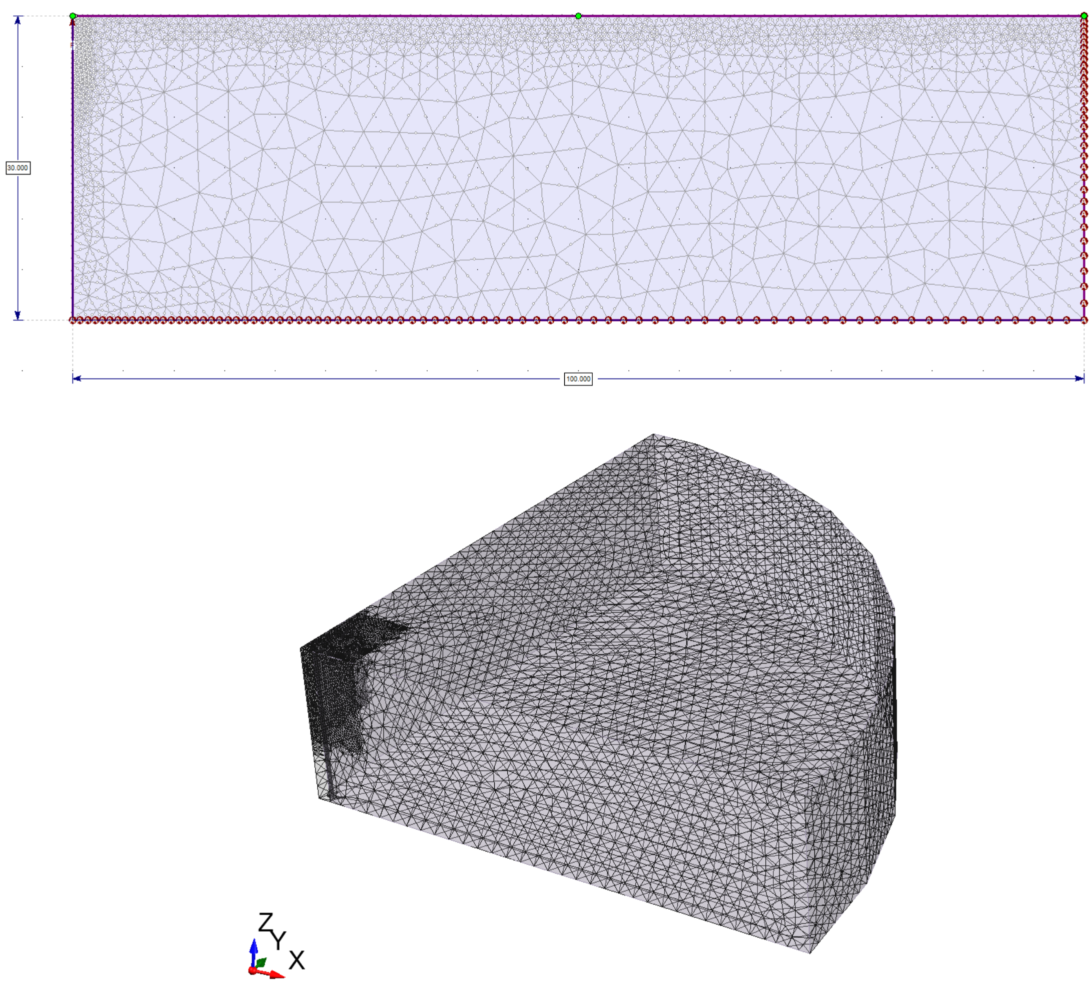

Figure 7-1: **(top)** RS2 **(bottom)** RS3 model of the problem

Table 7.1: Model parameters

Parameter	*Value*
Material type	Elastic
Young's modulus (*E*)	50000 MPa
Poisson's ratio (*v*)	0.25
Unit weight (*γ*)	20 kN/m^3
Radius (*r*)	100 m
Depth (*d*)	30 m

7.2 Results

Time-vertical displacement at relationship a point on the surface, 50 m away from the source as calculated by RS3 is shown in **Figure 7-2**. Results of a similar problem calculated from RS2 and Plaxis are also presented for reference. Please note that artifical damping values were employed in Plaxis to obtain the results. In all models, it is shown that the shock wave is approaching the desired point after 0.62 s of applying the initial load.

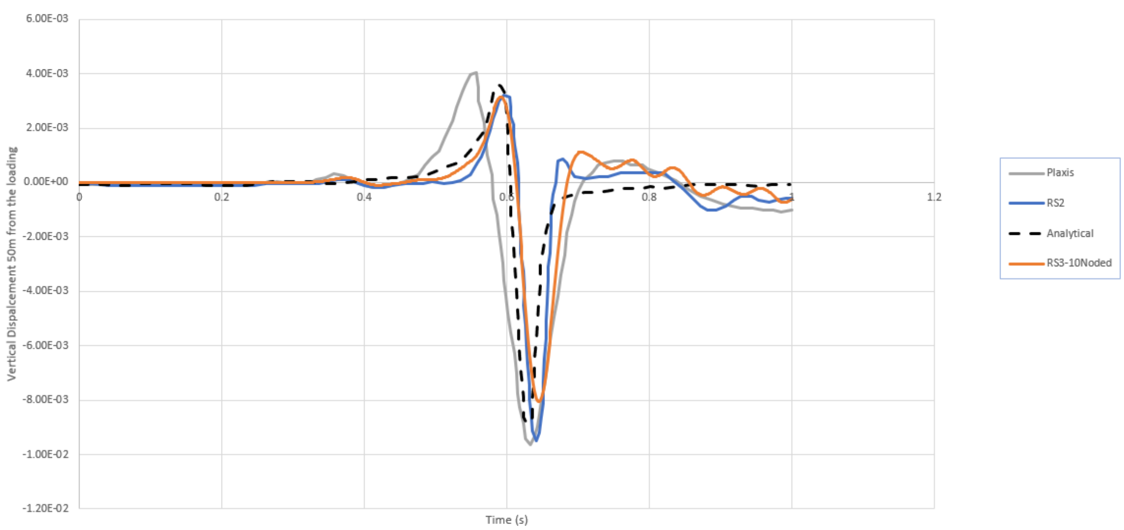

Figure 7-2: Vertical Displacement at x = 50m

7.3 Data Files

The input data file **dynamic #007.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

7.4 References

1. Brinkgreve, R. B. (2012) *Plaxis 2D Dynamic Module – Version 2011: Verification, Scientific and Dynamic Manuals*, Lisse, Balkema.

8 Harmonic Shear Wave

8.1 Introduction

This verification is from FLAC example 1.7.4 Slip Induced by Harmonic Shear Wave [1].

This case focuses on the energy dissipation given a homogeneous media under a shear wave, separated by a discontinuity in the middle. Absorb boundary is assigned to the top and bottom boundary of the model acting as non-reflective boundary, vertical restraints are assigned to the two lateral boundaries. The material model is elastic.

8.2 Problem Description

Verification problem #13 for RS2. A joint boundary is used to simulate the discontinuity in the middle of the media, three joint boundary cohesions are assigned to the model to simulate the non-slip surface (2.5 MPa) and slip surface (0.02 MPa, 0.1 MPa). The friction angle of the joint boundaries is equal to zero. The geometry of the model is presented in **Figure 8-1**. The model has a height of 200 m and a rectangular cross section of 80 m by 10 m.

A shear wave in terms of frequency w and time t is given as sin (wt) and applied in the horizontal direction at the bottom boundary of the models. Please note that the magnitude of the shear wave needs to be doubled in this case, taking consideration of two non-reflective boundary.

The RS3 response of the three models for displacement and velocity on top and bottom of the model are compared with RS2.

8.3 Geometry and Properties

The material properties used for this model is given in **Table 8.1**.

Table 8.1: Material Properties

Parameter	*Value*
Material type	Elastic
Young's modulus (E)	25000 MPa
Poisson's ratio (ν)	0.25
Unit Weight (γ)	26.5 kN/m^3
Height	200 m
Length	80 m
Width	10 m

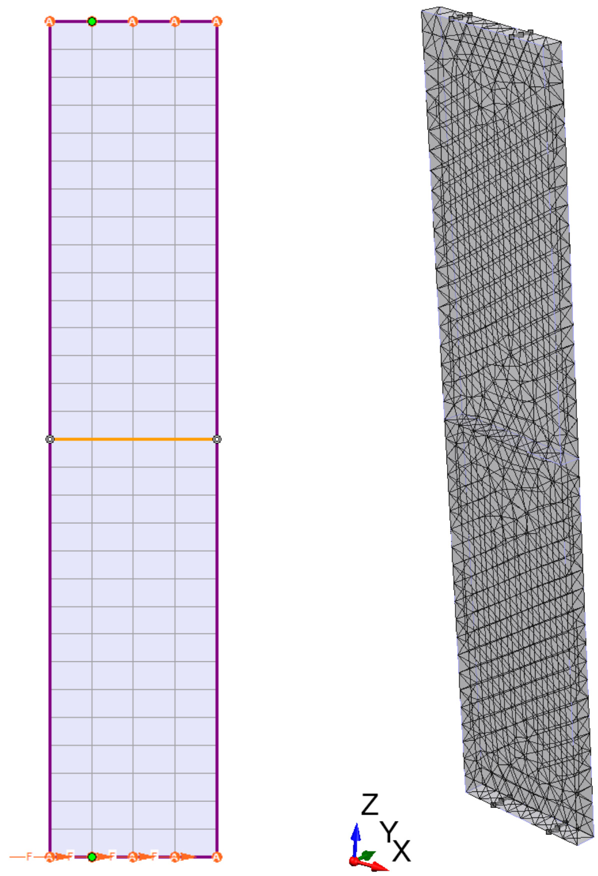

Figure 8-1: **(left)** RS2 **(right)** RS3 Model Geometry

8.4 Results

The following figures compare the horizontal displacement and velocity at the top and bottom of the column are compared between RS2 and RS3 for three cases (No Slip, Cohesion = 0.1 MPa, Cohesion = 0.02 MPa).

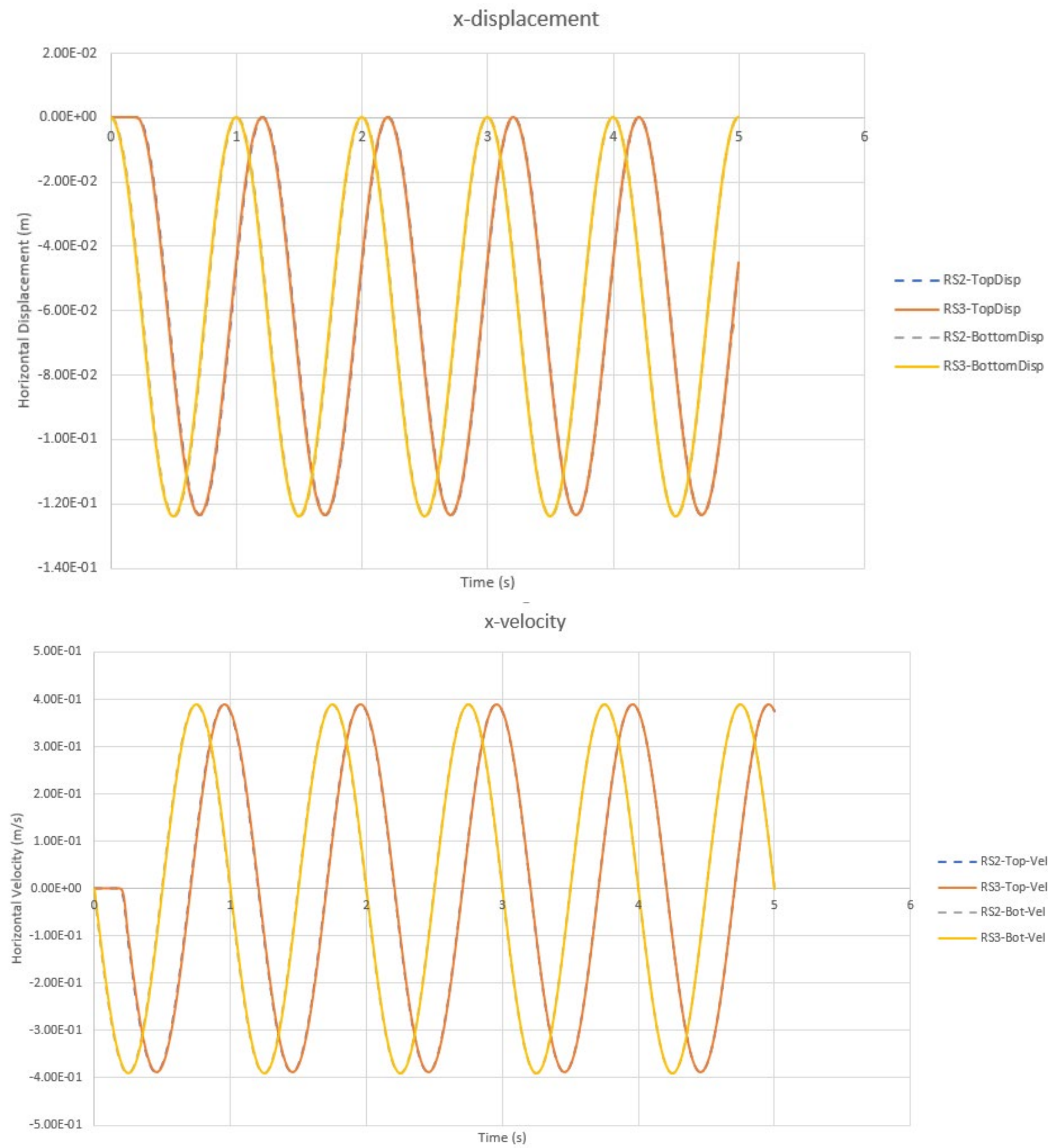

Figure 8-2: Comparison of horizontal displacement and velocity on top and bottom of the soil column for both RS2 and RS3 for the case of no slip

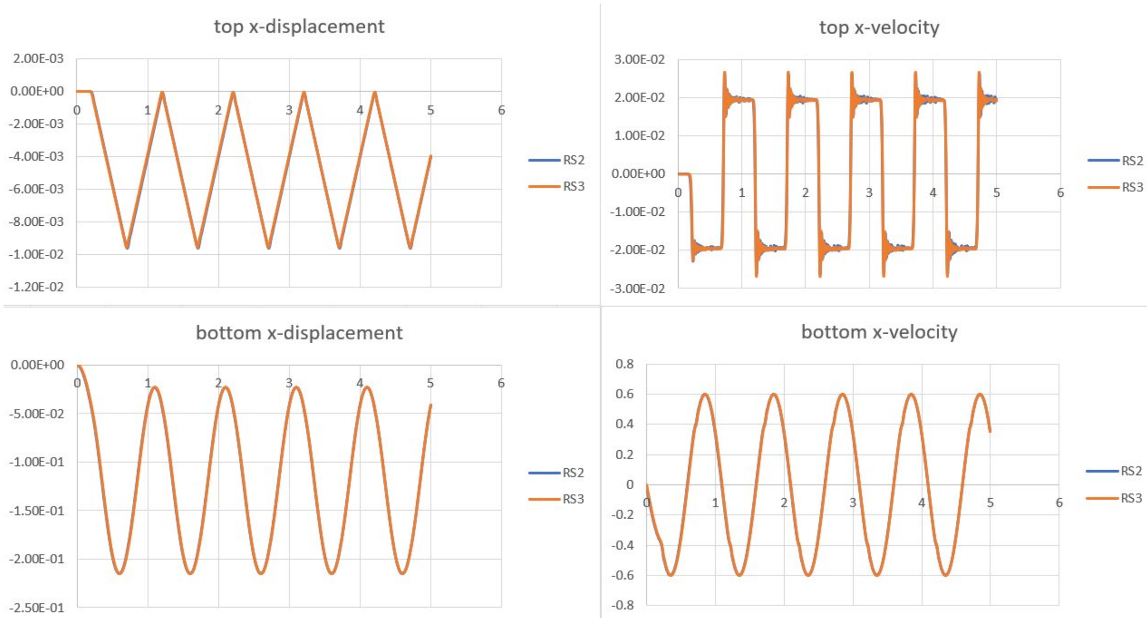

Figure 8-3: Comparison of horizontal displacement and velocity on top and bottom of the soil column for both RS2 and RS3 for the case of cohesion=0.1 MPa

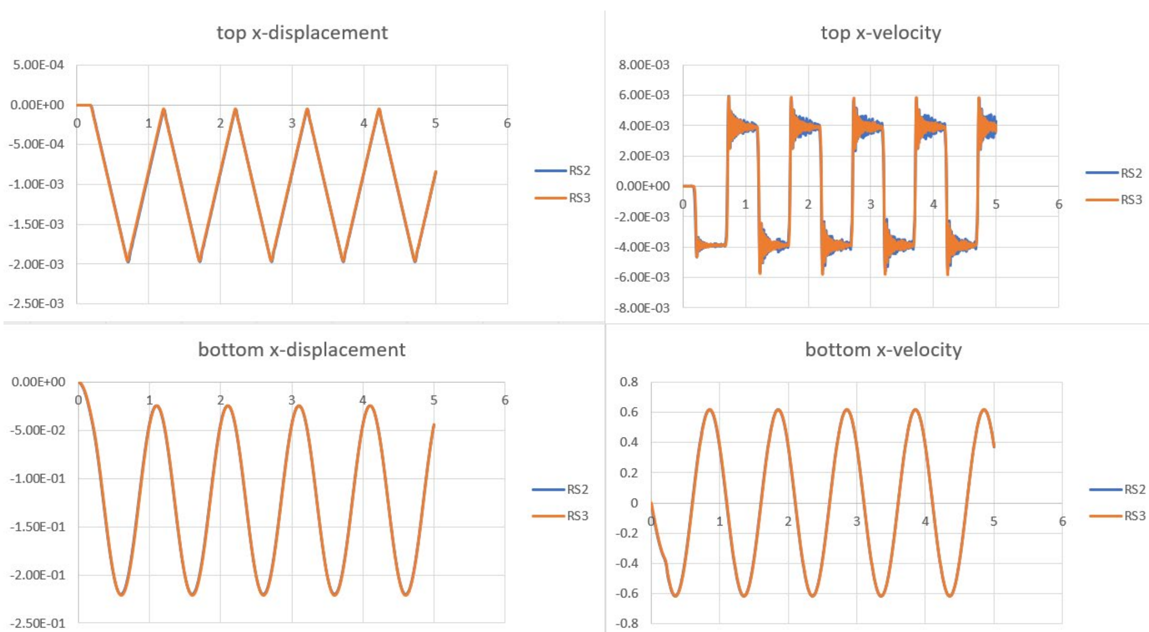

Figure 8-4: Comparison of horizontal displacement and velocity on top and bottom of the soil column for both RS2 and RS3 for the case of cohesion=0.02 MPa

8.5 References

1. Itasca Consulting Group Inc. (2011). *FLAC Dynamic Analysis Version 7.0 (pp. 1.252 – 1.261).*

9 Internal Blast

9.1 Introduction

This verification is from FLAC example 1.7.5 Hollow Sphere Subject to an Internal Blast [1].

This case demonstrates the propagation of a wave caused by a spherical internal pressure in a sphere and a rectangle. An absorb boundary is assigned to the outer boundary of the model. To model a half space in RS3, a spherical space is defined and to compare the results with RS2, an axisymmetric analysis is performed. The material model in this problem is elastic.

9.2 Problem Description

Verification problem #14 for RS2. The dynamic response of the half-spaced models is simulated in RS3. The sphere has a radius of 100 m and an inner boundary radius of 10 m. A pressure equals to 1 kPa is applied at the spherical inner boundaries of model. **Figure 9-1** indicates the model geometry in RS2 and RS3. The interest of this case is the propagation of the responsive wave translated by plotting the radial displacement at different time queries located at different distances from the internal pressure. The results are compared to the analytical solution by Blake [2], governed by an equation of compressional wave velocity C_p, time t, a potential function Ø and Laplacian operator ∇:

$$\frac{\partial^2 \o}{\partial t^2} = C_p^2 \nabla^2 \o \tag{9.1}$$

In this case, the potential function used to find the radial displacement can be expressed as following:

$$u_r = -\frac{p_0 a^3 k}{\rho C_p^2 r^2}[-1 + \sqrt{2-2v}\exp(-a_0\tau)\cos\left(w_0\tau - tan^{-1}\frac{1}{\sqrt{4k-1}}\right) + \frac{p_0 a^3 k}{\rho C_p^2 r}[\frac{a_0}{C_p}\sqrt{2-2v}\exp(-a_0\tau)\cos\left(w_0\tau - tan^{-1}\frac{1}{\sqrt{4k-1}}\right) + \frac{w_0}{C_p}\sqrt{2-2v}\exp(-a_0\tau)\cos\left(w_0\tau - tan^{-1}\frac{1}{\sqrt{4k-1}}\right) \tag{9.2}$$

Where,

p_0= pressure applied on the model;
a = Radius of the sphere;
v = Poisson's Ratio;
$K = \frac{1-v}{2(1-2v)}$;
r = Radial coordinate;
$a_0 = \frac{C_p}{2ak}$ = radiation damping constant;

$\tau = t - \frac{r-a}{C_p}$; and

$w_0 = \frac{c}{2aK}\sqrt{4K-1}$ = natural frequency.

9.3 Geometry and Properties

Table 9.1: Material Properties

Parameter	*Value*
Material type	Elastic
Young's modulus (*E*)	24975 MPa
Poisson's ratio (*v*)	0.25
Unit Weight (γ)	16.75 kN/m^3
Outer Sphere Radius (r_o)	100 m
Inner Sphere Radius (r_i)	10 m

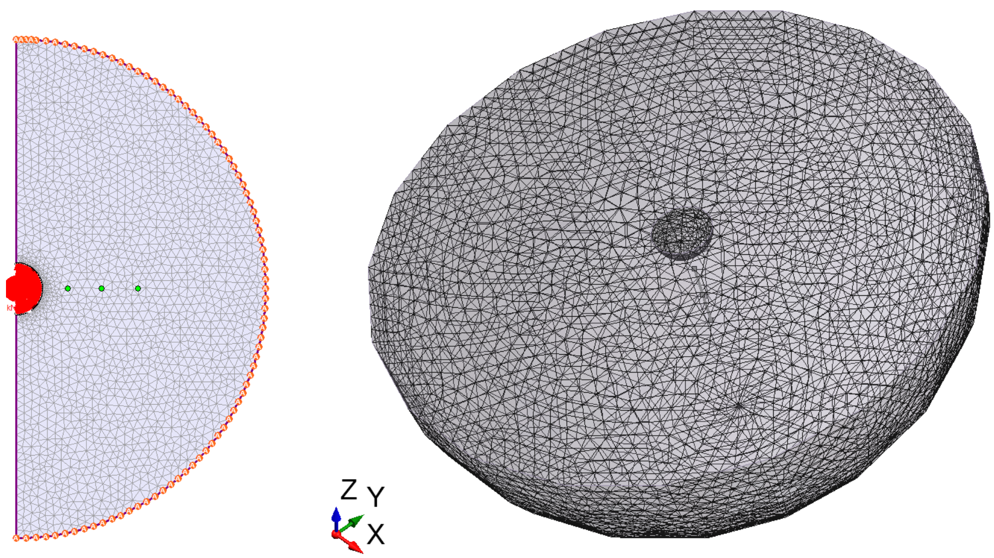

Figure 9-1: **(left)** axi-symmetric RS2 model **(right)** RS3 Model Geometry

9.4 Results

The response in **Figure 9-2** illustrates that farther locations transmit less wave. The values for vertical displacement were assessed at three points in the model (P1, P2 and P3). The locations of these points are at radii 20.5 m, 34.2 m, and 48.7 m, respectively. **Figure 9-3** demonstrates that the resulted vertical displacement from RS3 is almost match the analytical solution derived by Blake [2] and RS2 analysis.

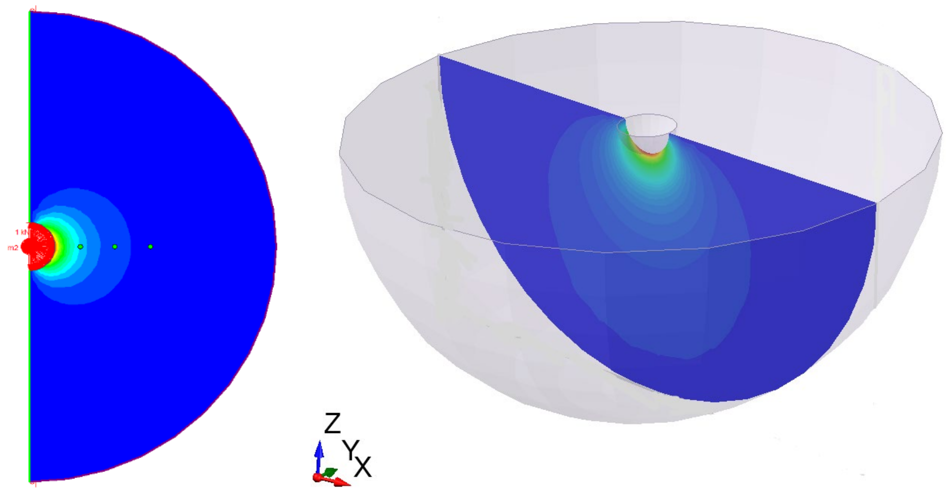

Figure 9-2: **(left)** Horizontal displacement in RS2 and **(right)** Vertical displacement in RS3

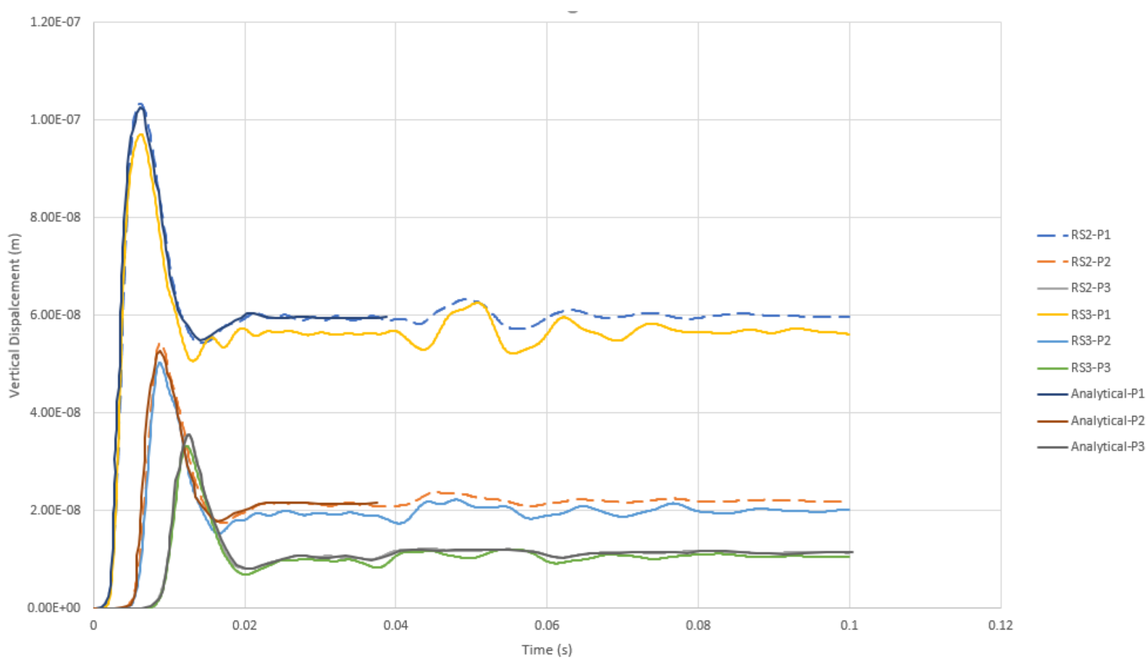

Figure 9-3: Vertical-displacement vs time at different time queries in comparison with the RS2 and analytical Solution by Blake (1952)

9.5 References

1. Itasca Consulting Group Inc. (2011). *FLAC Dynamic Analysis Version 7.0 (pp. 1.262 – 1.271).*

2. Blake, F.G. (1952). "Spherical Wave Propagation in Solid Media", J.Acous. Soc. Am., 24(2), 211-215.

10 Machine Foundation

10.1 Introduction

This verification is from FLAC example 1.7.6 Vertical Vibration of a Machines Foundation [1].

This case concerns the vertical response of the soil directly underneath a rigid strip footing, under the cyclic loading applying on the footing. Half of the model is simulated by taking advantage of the model's symmetry and therefore it is fixed in the x-direction on the left boundary. Absorb boundary is assigned to the right and bottom boundary of the model. Five different frequency ratio a_0 (0.5, 1, 1.5, 2, 2.5) is used in five models. The stiff footing is modelled as beam element with a very large young's modulus to result in uniform vertical response and limit the horizontal and rotational movement of the soil directly underneath the footing. The material model in this problem is elastic.

10.2 Problem Description

Verification problem #15 for RS2. The dynamic response of the five models is simulated in RS3. The cyclic loading applied on the footing is in terms of a_0 and therefore different in five models. Material properties for this model are presented in **Table 10.1**. The cyclic loading P is expressed as P = $P_0 \sin(wt)$, where P_0 is the force amplitude, w is the operational frequency and t is the run time for each model as shown in **Table 10.2**. Noting that in this table α and β are damping parameters.

Figure 10-1 indicates the geometry of model in RS2 and RS3. The model has a length of 160 m with a rectangular cross section of 80 m by 4 m. The focus of this study was to compare the vertical displacement of a point under the load with RS2 results for the five different frequency ratios.

10.3 Geometry and Properties

Table 10.1: Material Properties

Parameter	***Value***
Material type	Elastic
Young's modulus (E)	11200 MPa
Poisson's ratio (v)	0.4
Unit Weight (γ)	128.8 pcf
Length (L)	160 m
Height (h)	80 m
Depth (d)	4 m

Table 10.2: Dynamic Properties

a₀	*W*	β	α	Time (s)
0.5	50.00	0.001000	2.50	1.2566
1	100.00	0.0005000	5.00	0.6283
1.5	150.00	0.000333	7.50	0.4189
2	200.00	0.000250	10.00	0.3142
2.5	250.00	0.000200	12.50	0.2513

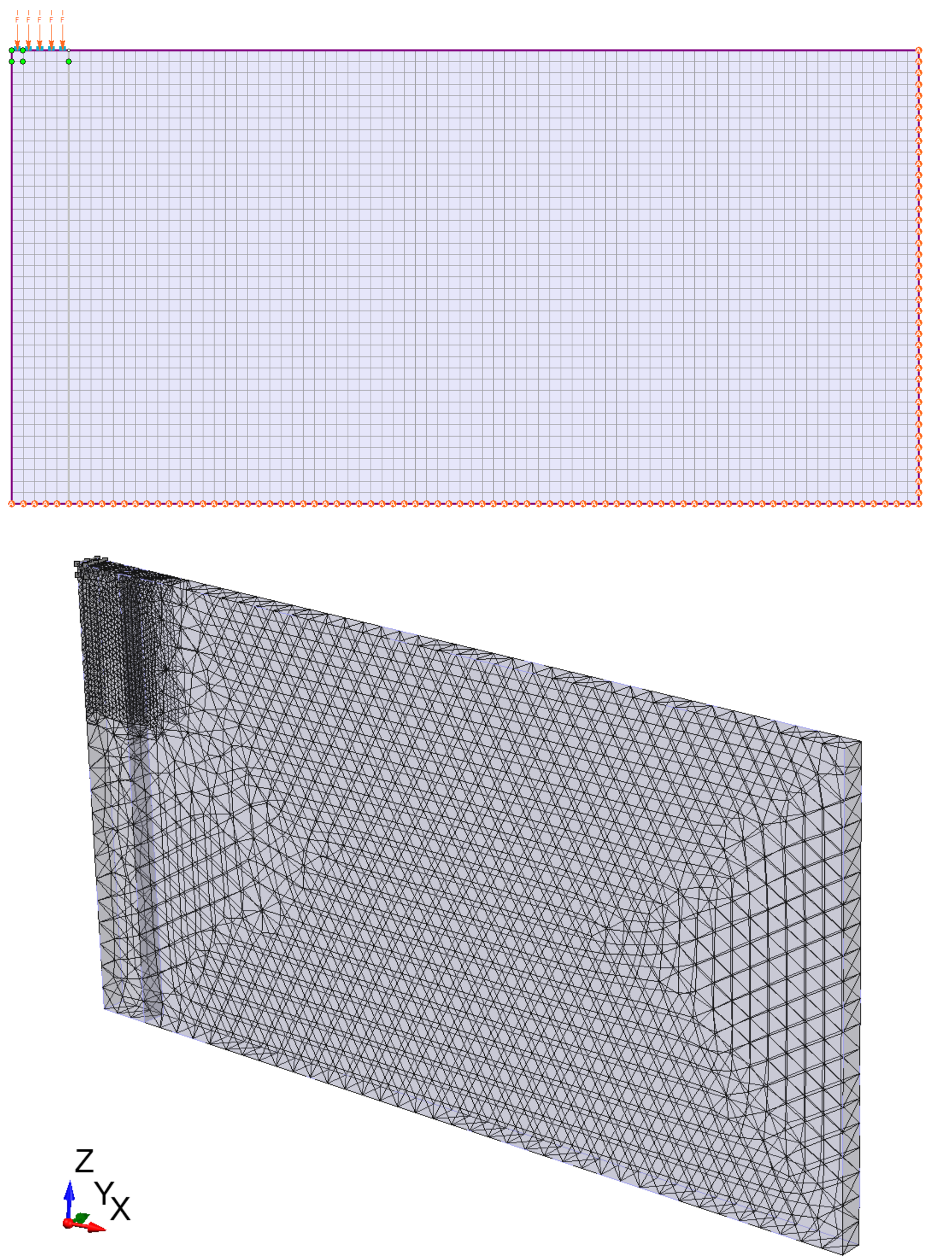

Figure 10-1: **(top)** RS2 and **(bottom)** RS3 Models Geometry

10.4 Results

The results for the vertical displacements in RS3 are presented as contours in **Figure 10-2. Figure 10-3** depicts variation of vertical displacement with the unified time (calculation time x a_0) for all five cases in both RS2 and RS3 where the results are in good agreement. As expected, when damping parameter is reduced, the higher vertical displacement is observed in both RS2 and RS3.

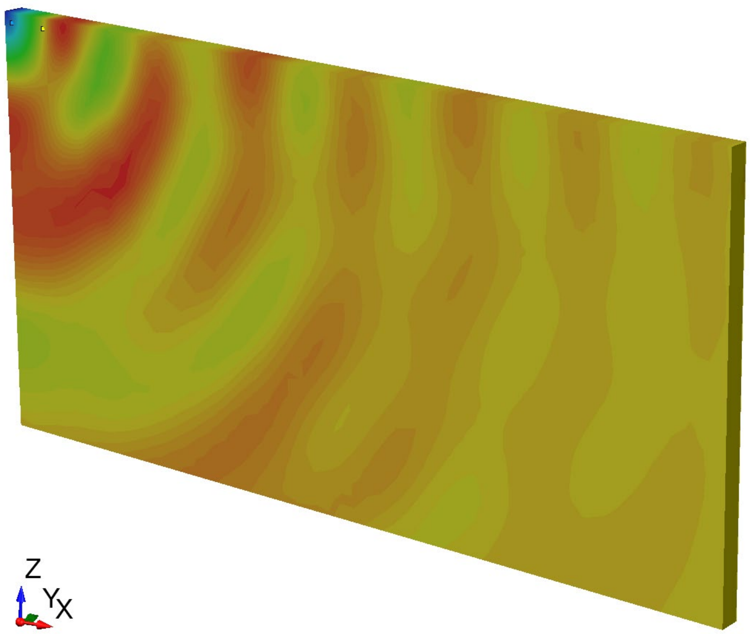

Figure 10-2: Vertical Displacement Results in RS3 for a_0=2.5

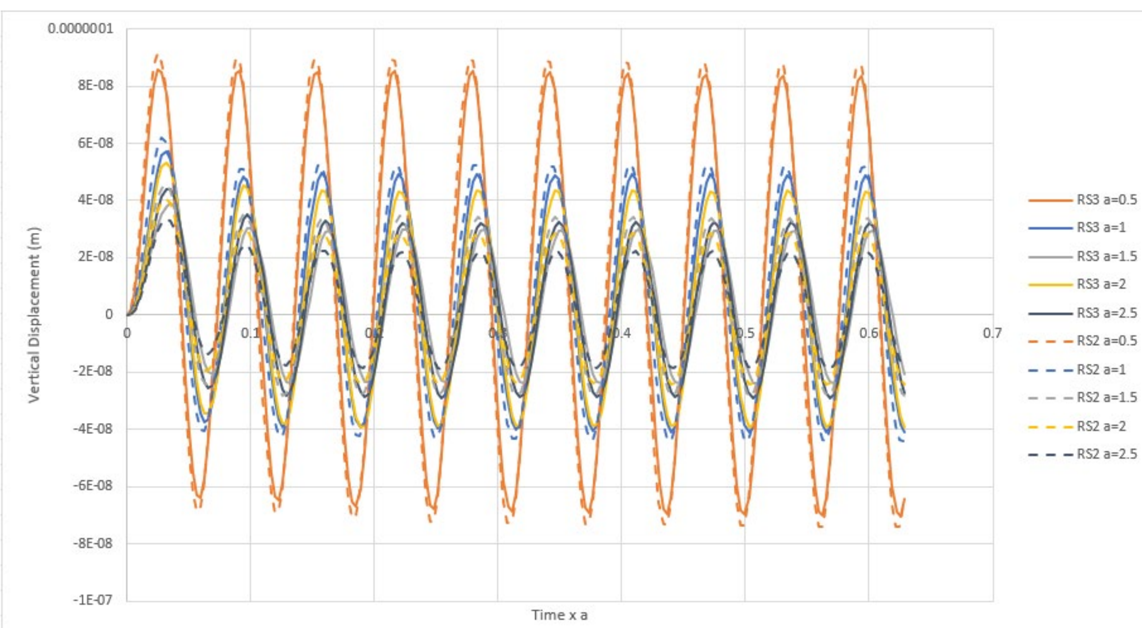

Figure 10-3: Comparison of vertical displacement between RS2 and RS3 for five study cases

10.5 References

1. Itasca Consulting Group Inc. (2011). *FLAC Dynamic Analysis Version 7.0 (pp. 1.272 – 1.278).*