rocscience

RS3

Application of Hybrid Mesh for Open Pit Stability Analysis

Examples

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1. Application of Hybrid Mesh for Open Pit Stability Analysis

1.1. Problem Description

The accuracy of Finite Element modeling results heavily relies on the mesh type employed to discretize the domain. The level of accuracy can be improved by using denser mesh or by employing higher-order element. RS3 offers two different types of elements, namely 4-noded tetrahedron and 10-noded tetrahedron. The comparison between the two element types are presented in Table 1-1. It is generally recommended to use 10-noded elements, but especially for SSR analysis or the model with high complexity aiming to enhance result accuracy. Nonetheless, such recommendation presents a paradox in terms of computation efficiency as it introduces additional computation load to an already resource-intensive analysis.

In this context, meshing the entire model with 10-noded elements may not always be imperative. When the model encompasses a specific part of the domain with low geometric/mechanical complexity, where linear analysis suffices, or when there is only a limited area of interest, employing 10-noded elements across the entire domain might not be essential. To cope for these cases, RS3 introduces a hybrid mesh approach, providing users with the flexibility to selectively apply 10-noded elements to specific critical areas within the domain. This approach allows for the optimization of computational resources and the enhancement of result accuracy by strategically employing higher-order elements only in regions where their benefits are most needed.

Table 1-1 Comparison between 4-Noded and 10-Noded Tetrahedron Elements

4-Noded Tetrahedron	10-Noded Tetrahedron
• Forms a smaller number of Degrees of Freedoms (DOFs) • Linear Shape Function • Less accuracy in capturing non-linear material behaviour (deformation and stress distribution) • Requires less computation resource, hence a shorter computation time	• Forms a larger number of DOFs • Quadratic Shape Function • Higher accuracy in capturing non-linear material behaviour (deformation and stress distribution) • Requires more computation resource, hence a longer computation time

A comparison study was undertaken to evaluate the efficacy of three different meshing techniques, i.e. 4-noded, 10-noded, and mixed-noded (hybrid) tetrahedra elements. This study demonstrates the nuanced trade-offs associated with the adoption of diverse meshing techniques. Specifically, the focus is spotlighted on evaluation of the performance of the hybrid mesh in two critical aspects: computational efficiency and its result accuracy.

1.2. Model Setup

The numerical investigation is carried out with an open pit mine model with pit height of approximately 150 m and overall slope angle of 40° and 20° Southern and Northern regions, respectively. The geometry of the open pit model is presented in Figure 1-1. A boundary is established within the model to specify a region to conduct Shear Strength Reduction (SSR) analysis (SSR Area). This specific region is selected since this region is identified to be the most prone to failure based on slope stability analysis conducted with Slide3, which applies 3D Limit Equilibrium Method to calculate FS and produce the corresponding slipping surface (Figure 1-5a)

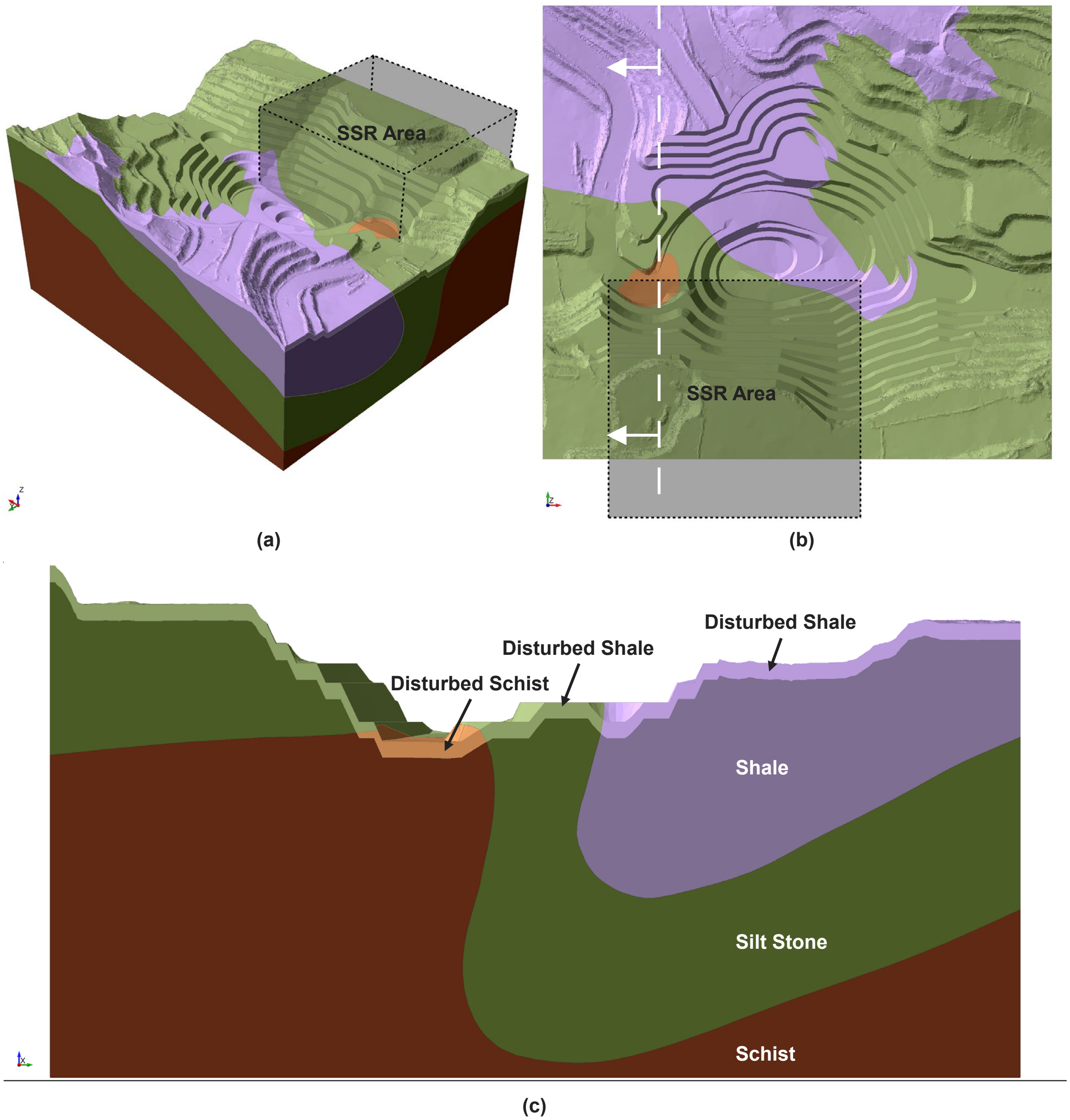


Figure 1-1 RS3 Model Geometry Showing (a) towards South-East Perspective and (b) Top-Down View with Dashed Line Representing Cross Section presented in (c)

The entire domain is meshed in uniform distribution. It is important to note that uniform mesh gradation option does not strictly apply a certain mesh size throughout the entire geometry, instead, if necessary, program flexibly assigns with finer mesh to preserve the geometry of the model. Moreover, a finer mesh is applied within the SSR area to divide the domain with more elements and achieve a higher DOF within the area of focus (Figure 1-2). The numerical simulation is repeated three times, each time (case) employing different meshing techniques, but uniformly applying identical element distribution (Table 1-2).

Table 1-2 Number of Elements and DOFs by Element Types

Element Types	Number of Elements	Number of DOFs
4-Noded	804,097	431,313
10-Noded	804,097	3,340,158
Mixed 4-Noded and 10-Noded Tetrahedra (Hybrid)	804,097	1,287,780

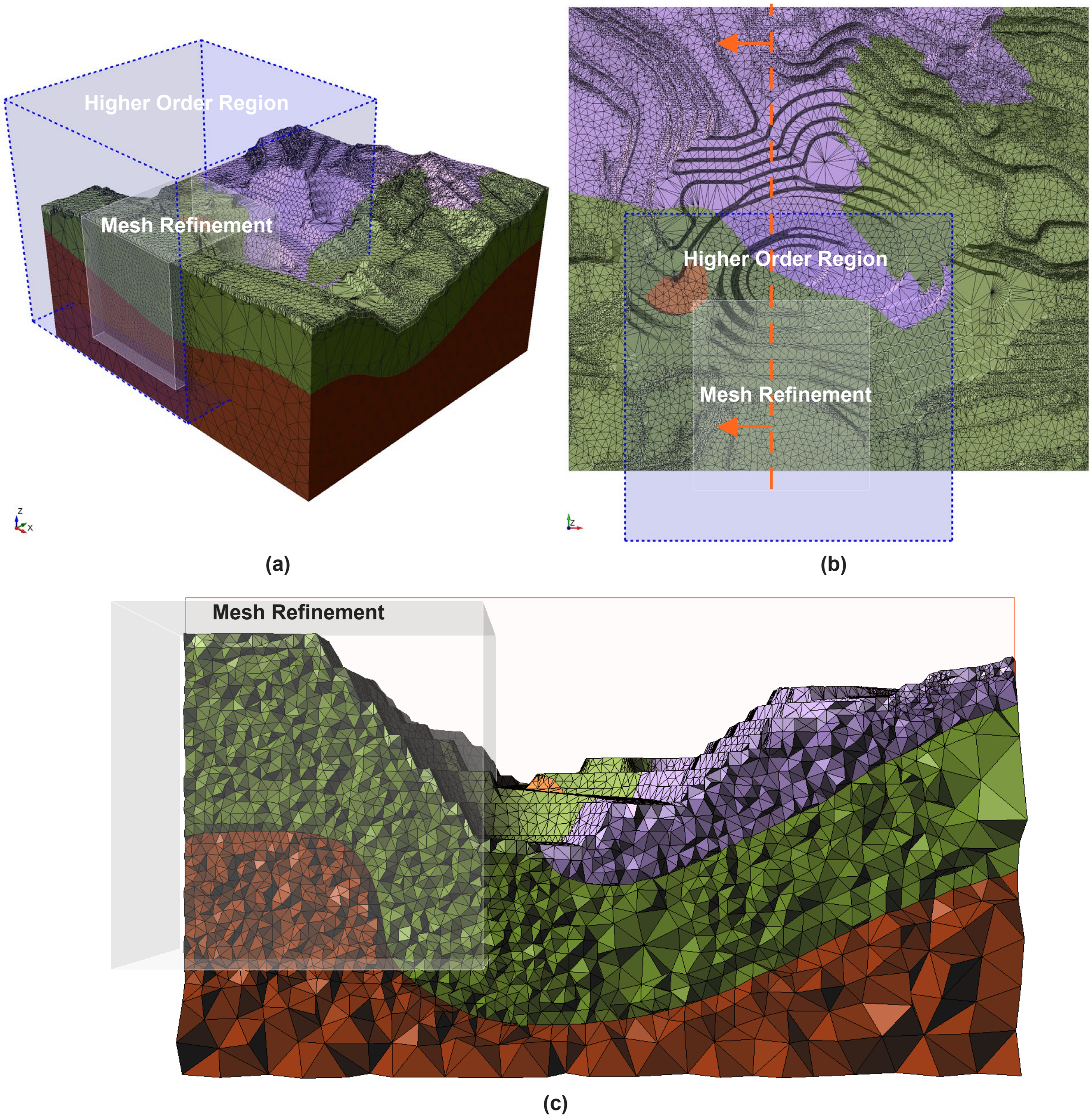


Figure 1-2 Mesh Distribution Showing (a) towards North-West Perspective and (b) Top-Down View with Dashed Line to Show Cross Section Presented in (c)

An implementation of hybrid mesh system is carried out in two-fold process:

1. Defining the higher order region(s), in another word, allocating the region(s) to be meshed with 10-noded elements (as defined in Figure 1-2a and b), followed by
2. Applying Mixed 4-Noded and 10-Noded Tetrahedra meshing in the mesh setting.

To ensure a rigorous validation process and avoid unwarranted assumptions about the correctness of the 10-noded element mesh, an initial slope stability analysis results obtained from Slide3 serves as a baseline to verify the accuracy of the modeling solution by RS3.

1.3. Material Properties

The open pit consists of three main folded lithological units, Shale, Silt stone, and Schist (Figure 2-1-c). Up to 10 m depth from the excavation, disturbed properties are assigned to those units. For the numerical exercise, Mohr Coulomb constitutive failure criterion is used to define the strength of rock masses. RSData is used to estimate the Mohr Coulomb parameters by computing the best fit line of the Generalized Hoek-Brown strength envelops defined for each unit (Table 1-3). Disturbed rock masses are assigned with equivalent properties, but with disturbance factor (D) of 0.7. The rock mass strength and elastic properties considered for the numerical analysis are provided in Table 1-3.

Table 1-3 Rock Mass Properties

		Elastic Properties		Generalized Hoek-Brown				Mohr Coulomb		
		E (MPa)	ν	σ_{ci} (MPa)	GSI	mi	D	c (MPa)	ϕ (°)	σ_t (MPa)
Siltstone	Undisturbed	3650	0.24	25.5	50	4.8	0	0.636	30.9	0.122
	Disturbed	1270					0.7	0.423	32.4	0.059
Schist	Undisturbed	4200	0.27	31.0	48	5.3	0	0.682	32.7	0.116
	Disturbed	1460					0.7	0.450	24.6	0.0543
Shale	Undisturbed	6140	0.25	40.8	61	6.2	0	1.19	39.4	0.345
	Disturbed	2450					0.7	0.818	33.4	0.196

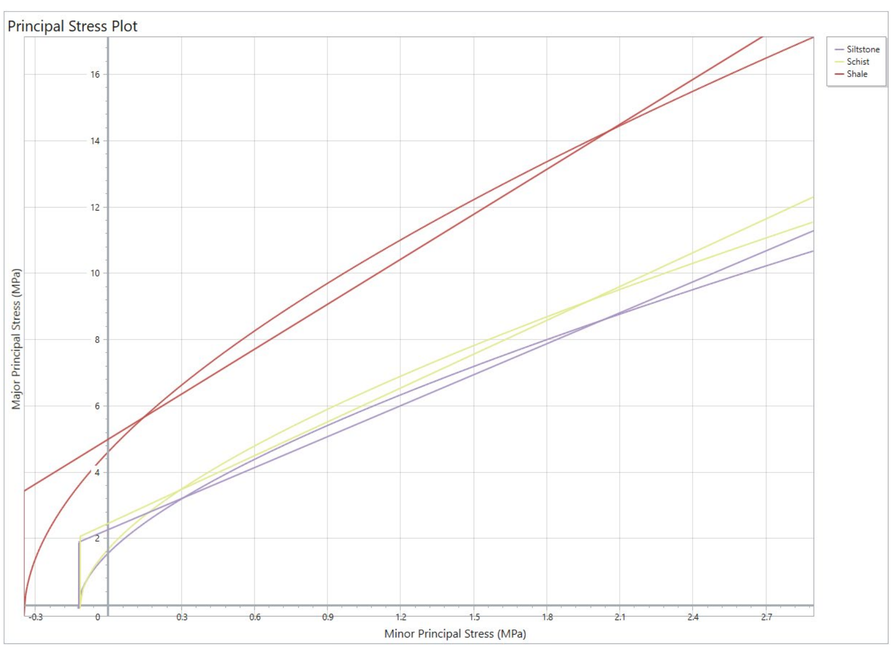


Figure 1-3 Rock Mass Failure Criteria Conversion from Generalized Hoek-Brown to Mohr-Coulomb

1.4. Modeling Results

Conducting SSR analysis iteratively computes stress analysis with updated strength properties to determine at which Strength Reduction Factor (SRF) the slope becomes unstable, which is referred as Critical SRF. Numerically, this is equivalent to the state of which energy balance in the system fails to reach equilibrium (convergence doesn't occur). SSR analysis is computed for all cases to find out the overall stability of the modeled sector, the critical region of failure, and to compare between cases quantitatively. Moreover, the rock mass deformation and failure mechanisms are investigated.

Presented in this section are numerical validation of 10-noded element model, result comparison between different mesh techniques, and sensitivity analysis of mesh density of 4-noded element model on result improvement. The numerical validation process involves the comprehensive result comparison between 10-noded element model and 3D LEM slope stability tool, Slide3. A comparison on different mesh techniques is done based on the computation time and result accuracy. Finally, a series of numerical exercise is carried out to assess the correlation between the result accuracy improvement and the 4-noded element density and further evaluate the worthiness with respect to the computation efficiency.

1.4.1. Numerical Validation

Based on the 3D method of columns, Slide3 generates potential slip surfaces with different geometries and locations and provide the surface with lowest Factor of Safety (FS). The risk of different scale of failure depends on the slope geometry, and the subsurface properties (Figure 1-4). For this pit design, the slip surface as shown in Figure 1-5a, with FS of 2.33 was determined to be the most probable form of failure.

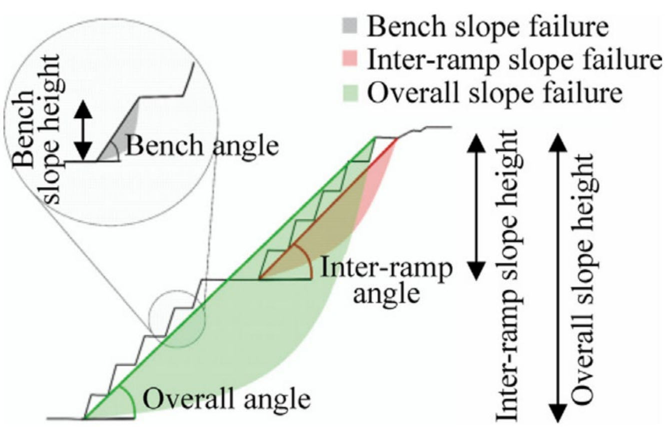
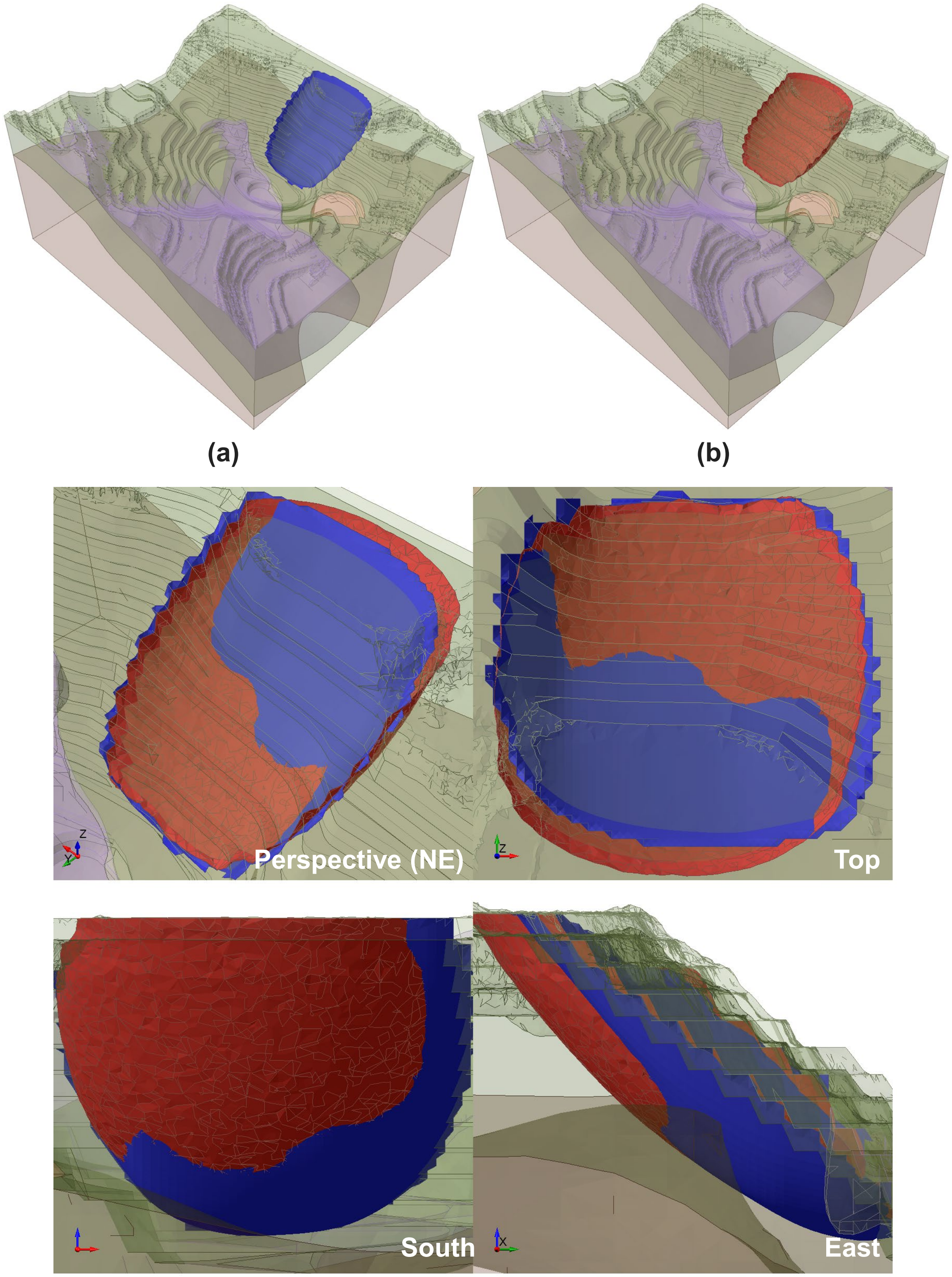


Figure 1-4 Schematic of slope failure at different scale

The outcome of the FE solution from RS3 (10-noded element case) achieves a close congruence with Slide3 result, not only in terms of stability parameters but, also in the generated slipping surface. The critical Strength Reduction Factor (SRF) computed with the 10-noded element case at 2.38, exhibits a discrepancy of only 0.05 when compared to the Factor of Safety (FS) computed with Slide3. The projected slip surface can be plotted in RS3 with isosurface interpolating a certain displacement value or the shear strain anomaly. In this case, the isosurface that interpolates total displacement of 0.22 m is plotted to represent slip surface. As illustrated in Figure 1-5c, the two surfaces from Slide3 and RS3 show a close agreement. Achieving the alignment for both the quantitative indicator (e.g. FS) and the failure pattern is of great importance as meeting one of the two criteria does not convey any meanings for validation.

Table 1-4 Critical SRF/FS Comparison between Slide3 and RS3

Method	Critical SRF/FS
Slide3 (3D LEM; Spencer)	**2.33**
RS3 (3D FEM 10-Noded; SSR)	**2.38**



(c)

Figure 1-5 (a) Potential slip surface computed from Slide3, (b) Inferred slip surface generated with isosurface of total displacement of 0.22 m at SRF = 2.39 from RS3, and (c) both surfaces

1.4.2. Meshing Technique Comparisons

To evaluate the performance of hybrid mesh, the results of models constructed with the three meshing techniques are discussed in this section. The comparison study was made with respect to two main measures, namely total computation time and result accuracy.

The numerical results of 4-noded element, 10-noded element, and hybrid mesh models are presented in Figure 1-6, Figure 1-7, and Figure 1-8 respectively. All figures represent the rock mass reactions captured at the state of SRF 0.01 point above critical SRF. This means that the plotted results represent the slope behaviour at material strength reduced to cause system convergence failure due to unacceptably large deformation. Also, contour plots are referenced at stage 1 (equivalent to SRF = 1) to zero the mechanical behaviour at its original strength.

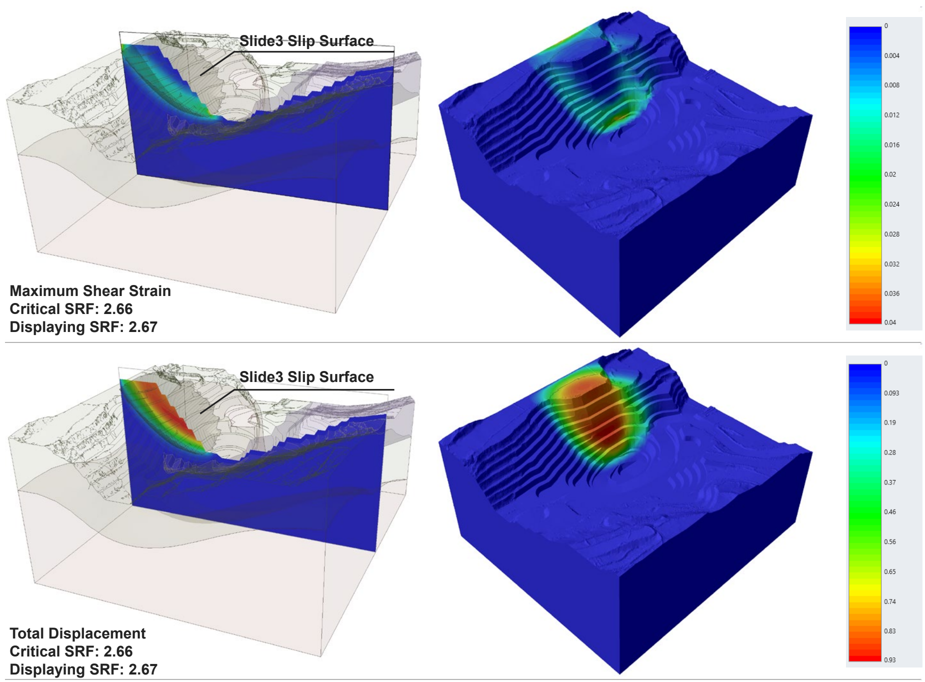


Figure 1-6 Maximum Shear Strain and Total Displacement Contour Diagrams of 4-noded Element Model

Both the 4-noded element model and the hybrid mesh model produce the slope instability at the overall slope scale. It is primarily driven by the shear failure in rock mass. Both 4-noded element model and hybrid mesh model show similar failure mechanism and failure geometry as the 10-noded element model.

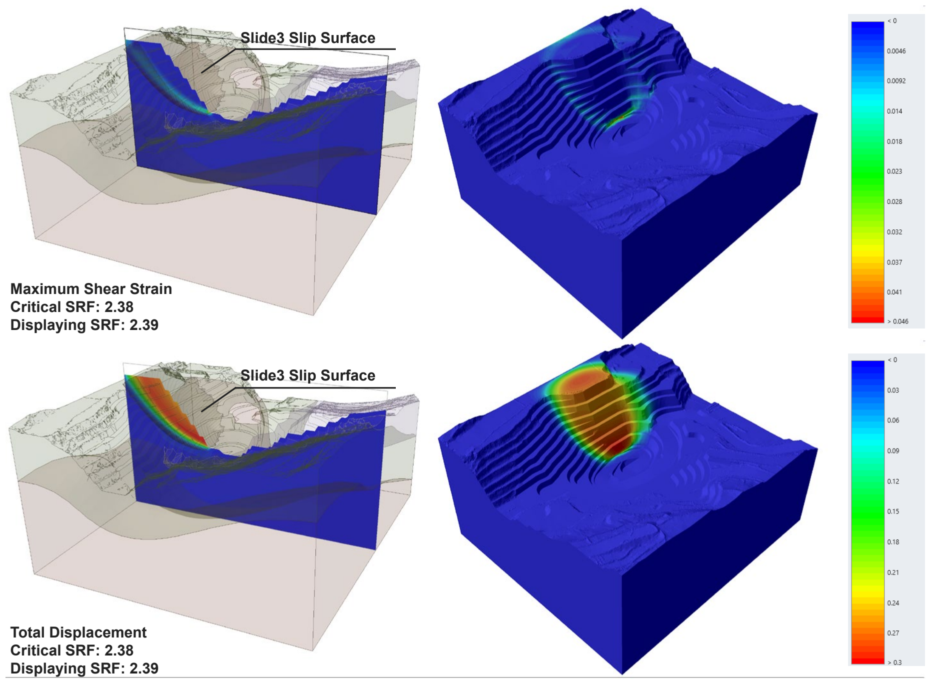


Figure 1-7 Maximum Shear Strain and Total Displacement Contour Diagrams of 10-noded Element Model

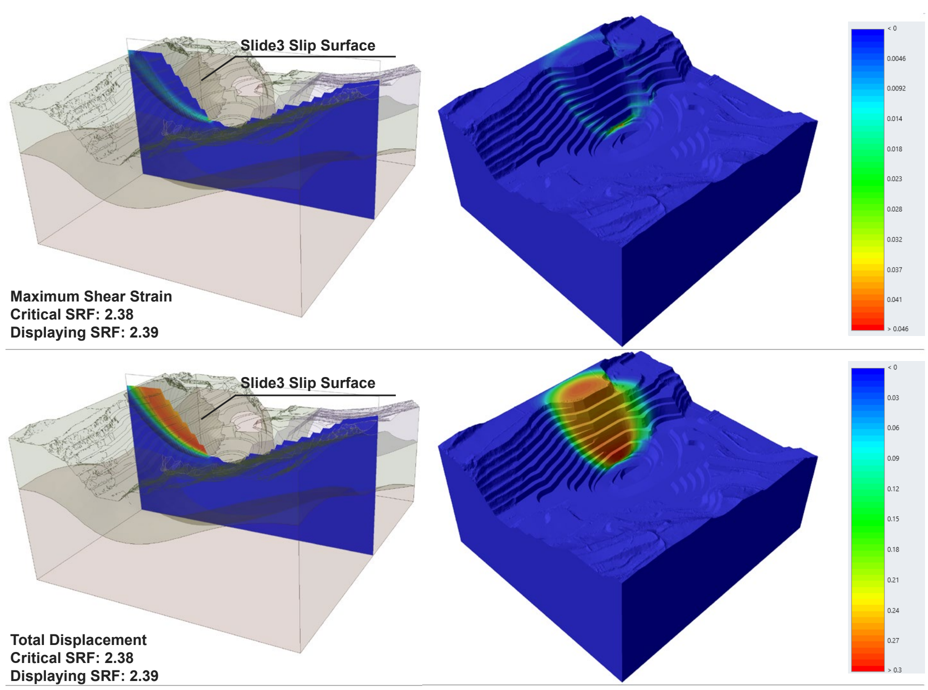


Figure 1-8 Maximum Shear Strain and Total Displacement Contour Diagrams of Hybrid Mesh Model

The computed results show that the critical SRF of the 4-noded element model is 11 % higher than that of the 10-noded element model. Considering the lower order function (and the lower DOF) the 4-noded element model employs compared to 10-noded element model, such overestimation makes sense.

Furthermore, the shear strain plotted with 10-noded element model manifests a stronger definition of failure plane. In general, a lower-order finite element tends to exhibit stiffer behavior compared to a higher-order finite element and the incongruence in mechanical response grows with the increase in non-linearity by the increase in SRF (Figure 1-9). For this reason, the 4-noded element model shows a minimal deformation compared to 10-noded at SRF of 2.39 (Figure 1-10). However, it is important to note that the computation efficiency of 4-noded element model is extraordinarily advantageous (Table 1-5). This advantage becomes manifest especially for the case like this example with heavy meshing, because 4-noded element model can still be solved with direct solver as the total required memory to solve the system of equation is way less in compare with 10 noded elements (See note below Table 1-5). As a result, compared to the 10-noded element model, 4-noded element model completes computation for this example 42.8 times faster.

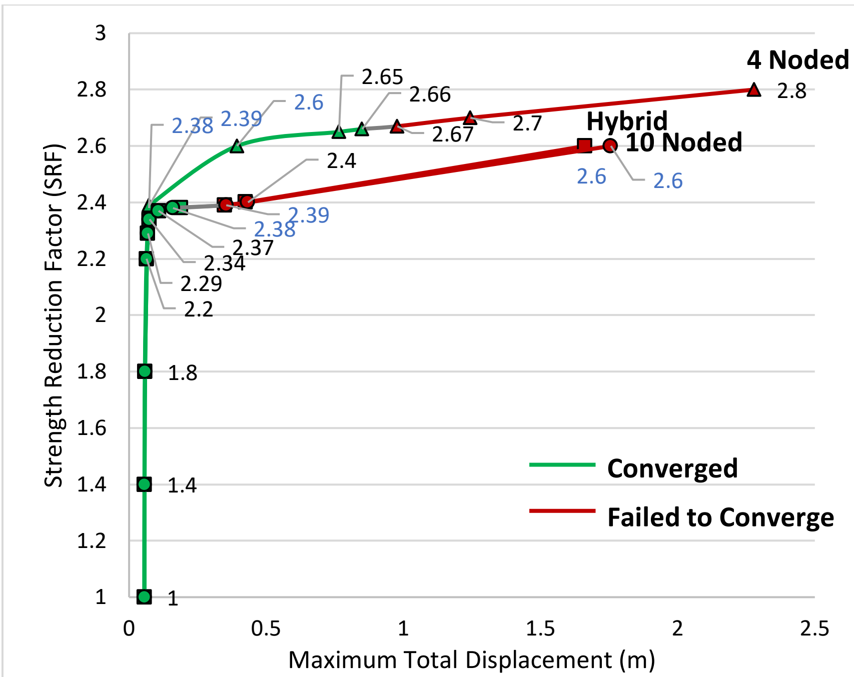


Figure 1-9 Shear Strength Reduction Graph

Table 1-5 and Figure 1-10 presents that the hybrid mesh technique is successfully demonstrated as a rigorous alternative approach. It merely costs 9 % of computation time the 10-noded element model requires. At the same time, the hybrid mesh model achieves a near-identical result as 10-noded element model. It shows no difference in the critical SRF; and the displacement and shear strain of hybrid mesh model are matching with the 10-noded element model.

Table 1-5 Result Summary of All Cases

Case	# of Elements	# of DoF	# of SSR Iterations	Total Time[1]	Normalized Time	Critical SRF
4-noded	804,097	431,313	10	**37m: 50s**	**1**	**2.66**
10-noded	804,097	3,340,158	11	**1620m: 8s**	**42.8**	**2.38**
Hybrid	804,097	1,287,780	11	**139m: 16s**	**3.68**	**2.38**

Note: [1] The total time difference is usually linear, meaning that the total time increases linearly with respect to the number of DoF. However, like this case, total computation time can increase exponentially when the matrix size becomes too large for the available memory that RS3 compute file automatically switches from direct solver to iterative solver.

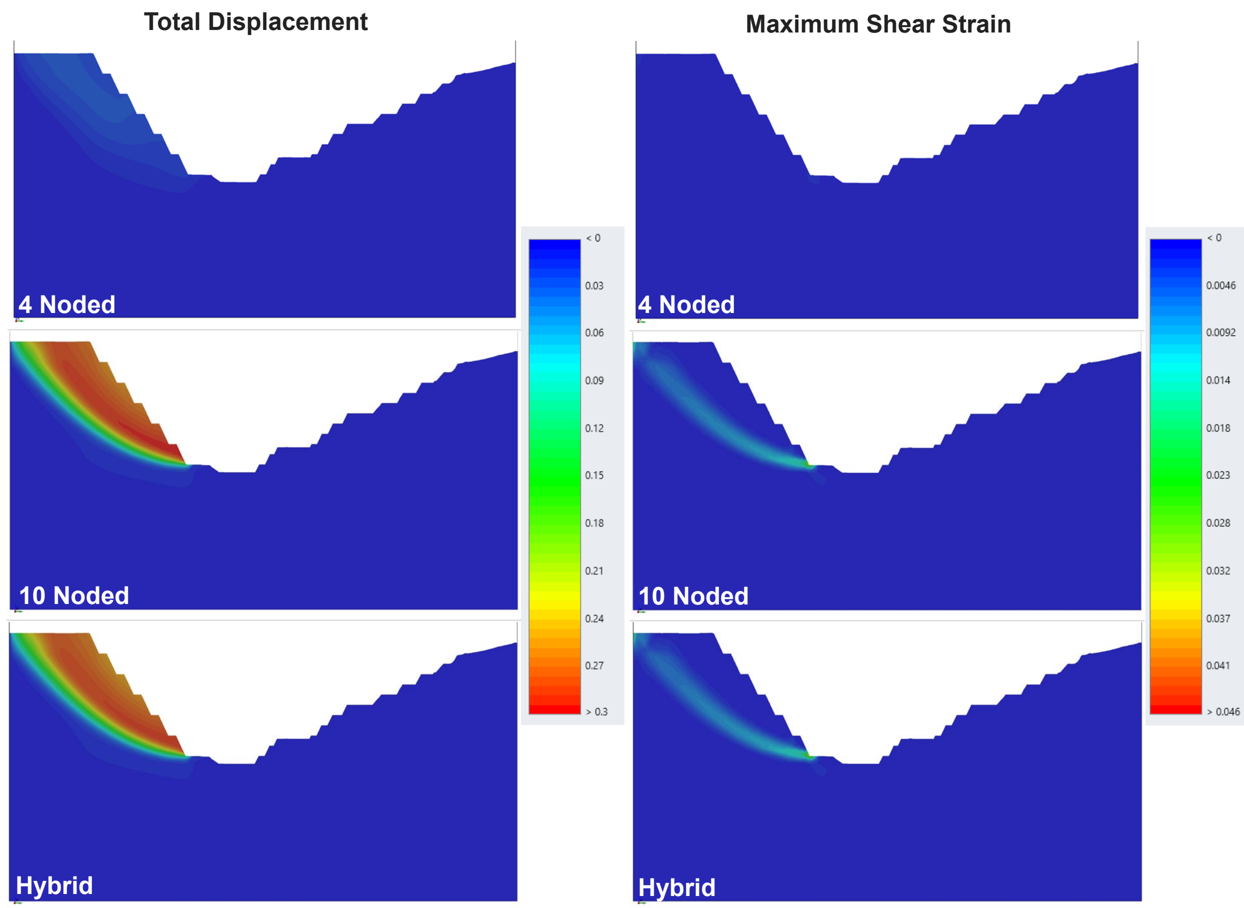


Figure 1-10 Result Comparison between Different Meshing Techniques for Modeling at SRF = 2.39

The numerical exercise is carried out a step further to assess the feasibility of achieving a comparable level of accuracy by improving the mesh density of 4-noded tetrahedral elements, akin to the precision exhibited by the hybrid mesh case in comparison to the 10-noded case. Three extra cases of 4-noded element models are developed assigning finer mesh within the mesh refinement region (Table 1-6). The modeling results successfully shows the narrowing gap in critical SRF to the 10-noded element model with the increase in the number of elements. However, despite both the degrees of freedom and the total computation time becomes similar to (R2 case) or well surpasses (R3 Case) those of hybrid mesh model, both models fall short of attaining the same level of accuracy as the hybrid configuration.

Table 1-6 Result Summary of Refined 4-Noded Cases

Case	# of Elements	# of DoF	# of SSR Iterations	Total Time	Critical SRF	% Deviation
4-noded_R1	1,046,761	553,902	11	**52: 41s**	**2.57**	**8**
4-noded_R2	1,924,071	996,189	13	**147m: 3s**	**2.49**	**5**
4-noded_R3	10,250,174	5,178,237	11	**2877m: 4s**	**2.42**	**2**

1.5. Discussion

In this example, a detailed numerical investigation is conducted to evaluate the performance of the two conventional meshing techniques, that universally assigns 4-noded or 10-noded element to the entire domain and the hybrid meshing technique. The hybrid meshing technique allows strategic assignment of higher-order elements in the focused regions.

Upon conducting the cross-validation exercise, the rock mass behaviour computed by the 10-noded element model and the 3D Limit Equilibrium slope stability solution (Slide3) show an agreement. This agreement instills confidence in establishing the results derived from the 10-noded element model as a benchmark for other models generated using different meshing techniques.

Irrespective to the adopted meshing techniques, RS3 achieves to provide the mechanical behaviour, RS3 consistently provides mechanical behavior aligned within a maximum difference of 11 % in critical SRF. However, the employment of hybrid mesh technique results in 0 % difference in SRF compared to the 10-noded element model and near-identical mechanical response with approximately of the 1/3 of DoF, which translates to 1/50 of the computation time.

The study findings suggest that employing hybrid mesh technique offers the benefit of achieving high result accuracy for the reasonable compromise in computation efficiency. Moreover, this balanced approach outperforms the limitations observed in the denser 4-noded case, emphasizing the practical advantages of adopting hybrid mesh strategies for effective geotechnical modeling.