

RS3

Groundwater

Verification Manual

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1. Shallow Unconfined Flow with Rainfall

1.1. Problem Description

The problem considered in this section involves the infiltration of water downward through soil. It is characterized by a boundary of flow domain also known as a free surface. Such a problem domain is said to be unconfined.

Water may infiltrate downward through the soil due to rainfall or artificial infiltration. Rainfall can be presented as a uniform discharge P (m/s), defined as the amount of water per unit area that enters the aquifer per unit time. Figure 1-1 shows the problem of flow between two long and straight parallel rivers, separated by a section of land. The free surface of the land is subjected to rainfall.



Figure 1-1: Unconfined flow under rainfall

This problem was modelled in both RS2 and RS3. The RS2 model is taken from verification problem #001 in the RS2 groundwater verification manual. The mesh and groundwater boundary conditions are shown in Figure 1-2. The RS2 model uses three-noded triangular finite elements while the RS3 model uses four-noded tetrahedron elements. RS2 model has a 5 m (height) x 10 m (width) rectangular external boundary and the external boundary of RS3 model was constructed extruding that profile by 1 m. The ground water boundary conditions and material parameters applied to both models are presented in Table 1-1.



Figure 1-2: Shallow unconfined flow under rainfall as constructed in (a) RS2 and (b) RS3

Table 1-1 summarizes other relevant model parameters.

Parameter	Value
Total head at left boundary (h1)	3.75 m
Total head at right boundary (h ₂)	3.0 m
Width (L)	10.0 m
Infiltration rate (P)	2.5e-6 m/s
Hydraulic conductivity (k)	1.0e-5 m/s

1.2. Analytical Solution

The equation for flow can be expressed as

(1.2.1)
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \nabla^2 \phi = -P$$

For one-dimensional flow, such as that encountered in the present example, solution of equation (1.2.1) after application of the appropriate boundary conditions yields the horizontal distance, x_a , at the maximum elevation of the free surface, which can be calculated as

(1.2.2)
$$x_a = \frac{L}{2} \left(1 - \frac{k}{p} \frac{h_1^2 - h_2^2}{L^2} \right)$$

The corresponding maximum height for the free surface, h_{max} , can be calculated as

(1.2.3)
$$h_{max} = \sqrt{h_1^2 - \frac{x_a}{L}(h_1^2 - h_2^2) + \frac{P}{k}(L - x)x}$$

1.3. Results

The developed RS2 and RS3 models were used to determine x_a and h_{max} and further compared with those values determined analytically. The pressure head contour plots superposed by phreatic line or surface from RS2 and RS3 models are presented in Figure 1-3 (a) and (b), respectively.

2.5e-008	2.5e-005	2.5e-005	2.5e-008	2.5e-008	2.5e-008	2.5e-008	2.5e-008	2.5e-005	2.5e-008	2.5e-006	2.5e-005	2.5e-008	2.5e-008	2.5e-005	2.5e-008	2.5e-008	2.5e-008	2.5e-005
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	Pressure H	lead																
	n _1	000																
3.75	-0.	700																
1	-0.	400																
	-0.	200																
	0.	500																
3.75	0.	800																
	1.	400																
	1.	700																
3.75	2.	000																
•	2.	600																
	2.	900																
	3.	200																
3.75	3.	800																
Ť	4.	100																
	4.	400																
	4.	700																
a.ya	5.																	
3.75																		
4	-											-						

(a)



(b)



The comparison of x_a and h_{max} yielded from RS2 and RS3 models and analytical solution are tabulated in Table 1-2.

Table 1-2: Maximum height of free surface co-ordinates

	RS2	RS3	Analytical
Xa	4.22	4.25	3.99
h _{max}	4.52	4.46	4.25

Both the RS2 and RS3 results are in close agreement with the analytical solution. If necessary, a finer mesh discretization could be used to improve the results.

1.4. References

1. Harr, M. E. (1990) Groundwater and Seepage, 2nd Edition, Dover

1.5. Data Files

The RS3 input file groundwater #001.rs3v3 can be downloaded from RS3 Online Help page.

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2. Flow Around a Cylinder

2.1. Problem Description

This example examines the problem of uniform fluid flow around a cylinder of unit radius as depicted in Figure 2-1. The problem domain was reproduced in RS2 and RS3 as shown in Figure 2-2.



Figure 2-1: Fluid flow surrounding impermeable cylinder







(b)

Figure 2-2: Model geometry in (a) RS2 and (b) RS3

Owing to the symmetry of the problem around the x-axis, only half of the domain is constructed for modelling. The domains are discretized with 442 six-noded triangular elements and 5340 10-noded tetrahedron elements for RS2 and RS3, respectively. The ground water boundary conditions and material parameters applied to both models are presented in Table 2-1.

Parameter	Value			
Head at left boundary (\u03c61)	1.0 m			
Head at right boundary (\u03c62)	0 m			
Domain length (L)	10.0 m			
Hydraulic conductivity (k)	1.0e-5 m/s			
Cylinder radius (a)	1.0 m			

2.2. Analytical Solution

The closed form solution for this problem is discussed in [1]. This analytical solution gives the total head values at any point (r, θ) at the circumference of the cylinder or outside in the problem domain as

(2.2.1)
$$\phi = U\left(r + \frac{a^2}{r}\right)\cos\theta + 0.5$$

where U is the uniform undisturbed velocity, defined by

(2.2.2) $U = \frac{\phi_1 - \phi_2}{L}$

2.3. Results

Figure 2-3 shows total head contour plots with phreatic surface and query points at the excavation wall and along the boundary. Comparison study was conducted with the data sampled at these points from RS2 and RS3 and another finite element results provided in [2].





(b)

Figure 2-3: Total head contour plot in (a) RS2 and (b) RS3

The comparison between total head values at different location within the problem domain determined from RS2 and RS3 models, analytical solution and presented in [2]. The modeling results were within 5% of those provided in [2].

Coord	dinate	Total Head					
X	У	RS2	RS3	Analytical	Ref. [2]		
4	1	0.500	0.500	0.500	0.500		
4.5	0.866	0.381	0.381	0.375	0.378		
5	0	0.263	0.262	0.250	0.277		
6	0	0.203	0.203	0.188	0.213		
8	0	0.000	0.000	-0.031	0.000		

Table 2-2: Total head at selected points in problem domain

2.4. References

- 1. Streeter, V.L. (1948), Fluid Dynamics, McGraw Hill, pp. 373-377.
- 2. Desai, C. S., Kundu, T. (2001), Introductory Finite Element Method, Boca Raton, Fla. CRC Press.

2.5. Data Files

The RS3 input file groundwater #002.rs3v3 can be downloaded from RS3 Online Help page.

3. Confined Flow Under Dam Foundation

3.1. Problem Description

The problem considered is a simple example of confined flow. It was selected to help assess the validity of both RS2 and RS3 models on confined flow problems.

Figure 3-1 shows a dam that rests upon a homogeneous isotropic soil [1]. In the example, the walls (entity 1) and base (entity 2) of the dam are assumed to be impervious. The water level is 5 m upstream of the dam and 0 m downstream.



Figure 3-1: Model geometry

The confined flow problem domain as shown in Figure 3-1 was reproduced with RS2 and RS3 considering the following conditions:

- The total head along the line segment, upstream of the dam, that lies between points A and B (Figure 3-1), is equal to 5 m
- The total head along the line segment, downstream of the dam, that lies between points C and D, is equal to 0 m

The RS2 model was discretized using 398 three-noded triangular finite elements. The RS3 model was discretized using 2015 four-noded tetrahedra finite elements. The geometries with ground water boundary conditions applied to the models are presented in Figure 3-2.



(b)

Figure 3-2: Model geometry in (a) RS2 and (b) RS3

3.2. Analytical Solution

The flow is considered to be two-dimensional with negligible flow in the lateral direction. The flow equation for isotropic soil can be expressed as

(3.2.1)
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

This equation can be solved either using a numerical procedure or a flow net. Flow net techniques are well documented in groundwater references.

The numerical modeling results for the problem is controlled by the boundary conditions applied. For the particular example in this document, following boundary conditions are applied such that:

- No flow occurs across the impermeable base, and
- The pressure heads at the ground surface upstream and downstream of the dam are solely due to water pressure.

3.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour below the dam. As shown in pressure head plot (Figure 3-3) and total head plot (Figure 3-4), the numerical results from the two models show a high agreement.



(a)



(b)

Figure 3-3: Pressure head contour plot in (a) RS2 and (b) RS3





(b)



The total head values are calculated analytically by [1] at points along line 1-1, which is located 4 m below the dam base and along line 2-2, a vertical cross section passing through the rightmost base of the dam (see Figure 3-1). The total head along the two lines could be determined from RS2 and RS3 results. As shown in Figure 3-5 and Figure 3-6, The results from RS2 and RS3 agree closely with those provided in [1].



Figure 3-5: Total head variation along line 1-1



Figure 3-6: Total head variation along line 2-2

3.4. References

1. Rushton, K. R., Redshaw, S.C. (1979), Seepage and Groundwater Flow, John Wiley & Sons, U.K.

3.5. Data Files

The RS3 input file groundwater #003.rs3v3 can be downloaded from RS3 Online Help page.

4. Shallow Unconfined Flow Through Earth Dam

4.1. Problem Description

This example considers the problem of seepage through an earth dam (See Figure 4-1). The task of calculating the shape and length of the free surface (line of seepage) is quite complicated. Some analytical solutions based on presenting flow nets as confocal parabolas are available in [1] and [2].



Figure 4-1: Earth dam with trapezoidal toe drain

The RS2 model geometry and boundary conditions used in this example is shown in Figure 4-2 (a). The identical geometry was applied for the external boundary of RS3 model and extruded by 1 m out of plane.





Figure 4-2: Earth dam as modeled in (a) RS2 and (b) RS3

The total head on the upstream face of the dam was taken to be 4 m, and the toe drain was located at the downstream toe of the dam, i.e. total head at location (22,0) was taken to be 0. The boundary condition at the toe was assumed to be undefined, meaning that it initially either had flow, Q, or pressure head, P, equal to 0. A total number of 441 three-noded triangular finite elements and 44121 four-noded tetrahedra finite elements were used to model the problem in RS2 and RS3, respectively.

4.2. Analytical Solution

By defining the free surface as Kozney's basic parabola [1], we can evaluate y1, the vertical height of the underdrain, as

 $(4.2.1) y_1 = \sqrt{d^2 + L^2} - d$

Then the minimum horizontal length of the underdrain, x1, can be calculated as

 $(4.2.2) x_1 = \frac{y_1}{2}$

4.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the dam. As shown in pressure head plot (Figure 4-3) and total head plot (Figure 4-4), the numerical results from the two models show a high agreement. Figure 4-3 and Figure 4-4 show contours of pressure head and total pressure head, respectively.



(b)

Figure 4-3: Pressure head contour plot in (a) RS2 and (b) RS3



(a)



(b)

Figure 4-4: Total head contour plot in (a) RS2 and (b) RS3

The minimum length and height of the underdrain measured in RS2 and RS3 are shown in Table 4-1. The results from RS2 and RS3 agree closely with those derived analytically [1].

	RS2	RS3	Analytical
X 1	0.226	0.240	0.242
y 1	0.395	0.440	0.484

Table 4-1: Minimum drain dimensions (m)

4.4. References

- 1. Haar, M. E. (1990), Groundwater and Seepage, 2nd edition, Dover.
- 2. Raukivi, A.J., Callander, R.A. (1976), *Analysis of Groundwater Flow*, Edward Arnold.

4.5. Data Files

The RS3 input file groundwater #004.rs3v3 can be downloaded from RS3 Online Help page.

5. Unsaturated Flow Behind an Embankment

5.1. Problem Description

The geometry of the problem considered in this section is taken from the FLAC manual [1]. The example is modified slightly to handle two different materials with different coefficients of permeability. Figure 5-1 shows the geometries of the proposed models in RS2 and RS3.



Figure 5-1: Embankment model in (a) RS2 and (b) RS3

The saturated hydraulic conductivities of material 1 and material 2 are 1×10^{-10} m/s and 1×10^{-13} m/s, respectively. The problem is discretized into 1070 6-noded triangular finite elements and 30712 10-noded tetrahedra finite elements for RS2 and RS3, respectively. Total head boundary conditions of 10 m and 4 m are applied to the left and right boundaries of the model, respectively. Zero flow (impermeability) is assumed at the top and at the bottom of the embankment.

5.2. Finite Difference Solution from Reference

For this problem, RS2 and RS3 results are compared with those from FLAC presented in [1]. Figure 5-2 and Figure 5-3 show the contours of pressure head and flow lines produced by FLAC.



Figure 5-2: Pressure head contour plot produced by FLAC



Figure 5-3: Flow lines produced by FLAC

5.3. Results

Pressure head contour and flow line plots are produced with RS2 and RS3, as shown in Figure 5-4 and Figure 5-5, respectively. Both show a close agreement with FLAC.





(b)

Figure 5-4: Pressure head contour plot in (a) RS2 and (b) RS3





5.4. References

1. Coetzee, Hart, et al. (1995), Flac Basics: An introduction to FLAC and a guide to its practical application in geotechnical engineering. Minneapolis, MA.: Itasca Consulting Group, Inc.

5.5. Data Files

The RS3 input file groundwater #005.rs3v3 can be downloaded from RS3 Online Help page.

6. Steady-State Seepage Analysis Through Saturated-Unsaturated Soils

In this verification example, five earth dams with various properties are examined using RS2 and RS3. The models were verified based on the Pressure head data presented in Fredland & Rahardjo [1].

6.1. Problem Description

This problem concerns seepage through an unsaturated earth dam. The geometry of the problem considered in this section, which is shown in Figure 6-1, is taken from [1]. A series of comparison study were conducted to verify the Seepage analysis modeling result from RS2 and RS3 based on the Finite Element analysis result from the work by [1]. The pressure head along the line 1-1 was used as the major metrics for this verification exercise.



Figure 6-1: Isotropic earth dam with horizontal drain in (a) RS2 and (b) RS3

The problem is discretized into 336 3-noded triangular finite elements and around 9070 4-noded tetrahedra finite elements in RS2 and RS3, respectively. The mesh used for this example was created using mapped mesh option to nearly replicate that used in [1].

Five cases were investigated using the constructed model. These cases include Isotropic earth dam with a horizontal drain; Anisotropic earth dam with a horizontal drain; Isotropic earth dam under steady-state infiltration; and Isotropic earth dam with seepage face. The key ground water condition assigned for each case is briefed in Table 6-1.

Cases	Ground water condition	
Isotropic earth dam with a horizontal drain	12 m horizontal drain Isotropic soil with permeability function as shown in Figure 6-2 b	
Anisotropic earth dam with a horizontal drain	12 m horizontal drain Anisotropic soil where permeability coefficient in the horizontal direction is nine times larger than in the vertical direction, which has the permeability function assigned in vertical direction as shown in Figure 6-2 b	
Isotropic earth dam with a core and horizontal drain	12 m horizontal drain Isotropic soil (permeability function as Figure 6-2 b) with a core, which has lower coefficient of permeability as shown in Figure 6-2 c	
Isotropic earth dam under steady-state infiltration	12 m horizontal drain Isotropic soil with permeability function as shown in Figure 6-2 b Presence of infiltration, which is simulated by applying a flux boundary of 1x10 ⁻⁸ m/s along the boundary of the dam	
Isotropic earth dam with seepage face	Isotropic soil with permeability function as shown in Figure 6-2 b Presence of unknown boundary condition to represent seepage faces	

Table 6-1 Summary of groundwater condition applied for different cases







Figure 6-2 Permeability Function for Steady State Seepage through a Dam

6.2. Finite Element Solution from Reference

Total head distributions within the dam computed from finite element analysis conducted by Lam (1984) [2] for different cases are presented with two-dimensional contour plot. Figure 6-3 shows the results.





6.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the dam. Ground water flow behaviours determined from the two models are presented for all cases with respect to the flow direction, phreatic surface, and pressure head and total head contour plots. Moreover, the comparison for pressure head distribution along line 1-1 between the results from RS2, RS3, and analytical solution are provided.

6.3.1. Isotropic earth dam with a horizontal drain

Figure 6-5 and Figure 6-6 show the flow vectors, pressure head and total head fields calculated by RS2 and RS3 for the first case.







(a)

(b)

Figure 6-5: Pressure head plot in (a) RS2 and (b) RS3



(b)



The pressure head distribution along line 1-1 (vertical line extending from the top to bottom at the center of the dam) calculated analytically was compared to that produced from RS2 and RS3 (see Figure 6-7). The modeling results show a good agreement with the analytical result.



Figure 6-7: Pressure head distributions along line 1-1 for isotropic earth dam

6.3.2. Anisotropic earth dam with a horizontal drain

Figure 6-8 presents the flow vectors and the location of the phreatic line from the RS2 and RS3groundwater model.



(b)

Figure 6-8: Flow vector plot and phreatic surface in (a) RS2 and (b) RS3







The pressure head distribution along line 1-1 calculated analytically was compared to that produced from RS2 and RS3 (see Figure 6-11). The modeling results show a good agreement with the analytical result.



Figure 6-11: Pressure head distributions along line 1-1

6.3.3. Isotropic earth dam with a core and horizontal drain

The results from the third case show that the hydraulic head change takes place largely in the zone around the core. The flow vectors show that the water flows upward into the unsaturated zone and around the core zone as shown in Figure 6-12. Pressure head and total head contours are presented in Figure 6-13 and Figure 6-14.




(b)





(a)



(b)

Figure 6-13: Pressure head plot in (a) RS2 and (b) RS3





Figure 6-14: Total head plot in (a) RS2 and (b) RS3

The pressure head distribution along line 1-1 calculated analytically was compared to that produced from RS2 and RS3 (see Figure 6-15). The modeling results show a good agreement with the analytical result.



Figure 6-15: Pressure head distributions along line 1-1

6.3.4. Isotropic earth dam under steady-state infiltration

Figure 6-16 plots the flow vectors and phreatic line calculated by RS2 and RS3 for the fourth case. Pressure head and total head contours are presented in Figure 6-17 and Figure 6-18, respectively.



(b)

Figure 6-16: Flow vector plot in (a) RS2 and (b) RS3



(a)



(b)



The pressure head distribution along line 1-1 calculated analytically was compared to that produced from RS2 and RS3 (see Figure 6-19). The modeling results show a good agreement with the analytical result.



Figure 6-19: Pressure head distributions along line 1-1

6.3.5. Isotropic earth dam with seepage face

The flow vectors and the phreatic surface for the fifth case are presented in Figure 6-20. Pressure head and total head contours are presented in Figure 6-21 and Figure 6-22, respectively.



(b)

Figure 6-20: Flow vector plot in (a) RS2 and (b) RS3



(b)





Figure 6-22: Total head plot in (a) RS2 and (b) RS3

The pressure head distributions were determined along the slope face with unknown hydraulic condition (seepage face) and line 1-1. Pressure head calculated analytically, and numerically from RS2 and RS3 are

max (stage) : 10 m



shown in Figure 6-23 and Figure 6-24. The graphs show good agreement between modeling results and the analytical solution.

Figure 6-23: Pressure head distributions along seepage face



Figure 6-24: Pressure head distributions along line 1-1

6.4. References

1. Fredlund, D.G. and H. Rahardjo (1993), Soil Mechanics for Unsaturated Soils, John Wiley.

2. L. Lam and D. G. Fredlund (1984), "Saturated-Unsaturated Transient Finite Element Seepage Model for Geotechnical Engineering," Adv. Water Resources, vol. 7, pp. 132-136.

6.5. Data Files

The RS2 and RS3 data files for the steady-state seepage analysis are:

File name	Description	Location	
groundwater #006_01.rs3v3	Isotropic earth dam with a horizontal drain	RS3 Online Help page	
groundwater #006_02.rs3v3	Anisotropic earth dam with a horizontal drain	RS3 Online Help page	
groundwater #006_03.rs3v3	Isotropic earth dam with a core and horizontal drain	RS3 Online Help page	
groundwater #006_04.rs3v3	Isotropic earth dam under steady-state infiltration	RS3 Online Help page	
groundwater #006_05.rs3v3	Isotropic earth dam with seepage face	RS3 Online Help page	

7. Seepage Within Layered Slope

7.1. Problem Description

This example considers the problem of seepage through a layered slope. Rulan and Freeze [1] studied this problem using a sandbox model. The material of the slope consisted of medium sand and fine sand with relatively lower permeability. The geometry of the problem is shown in Figure 7-1 and the two permeability functions used to model the soil are shown in Figure 7-2. These permeability functions are similar to those presented by Fredlund and Rahardjo [2]. Comparison study was conducted between the pressure head distribution presented in [2], based on finite element analysis, and that computed from RS2 and RS3 along line 1-1 and line 2-2.



Figure 7-1: Layered slope problem geometry



Figure 7-2: Permeability functions for materials used in model

The RS2 and RS3 model geometries used in this example are shown in Figure 7-3. The RS2 model uses three-noded triangles while the RS3 model uses four-noded tetrahedron elements. The RS2 model is taken from verification problem #007 in the RS2 groundwater verification manual.



(a)



(b)

Figure 7-3: Layered slope model in (a) RS2 and (b) RS3

A constant infiltration rate of $2.1 \cdot 10^{-4}$ m/s is applied to the top of the side of the slope. The water table is located at 0.3 m from the toe of the slope. The boundary condition at the slope face was assumed to be undefined, meaning that it initially either had flow, *Q*, or pressure head, *P*, equal to 0.

7.2. Finite Element Solution from Reference

Fredlund and Rahardjo present their own finite element analysis for this problem in [2]. The resultant pressure head data are shown in the figure below.

---- = Equipotential line (m)



Numbers are hydraulic heads in (m)

Figure 7-4: Hydraulic head data at t = 208 s for unsteady-state flow analysis in [1]

7.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the dam. Ground water flow behaviours determined from the two models are presented with respect to the flow direction, phreatic surface, and pressure head and total head contour plots. Figure 7-5 shows the location of the calculated water table location and the direction of the flow vectors. Resultant pressure head and total head contour plots are shown in Figure 7-6 and Figure 7-7.



(a)



Figure 7-5: Flow vectors and phreatic surface with constant infiltration in (a) RS2 and (b) RS3





(b)

Figure 7-6: Pressure head contour plot for isotropic earth dam with constant surface infiltration in (a) RS2 and (b) RS3



(b)

Figure 7-7: Total head contour plot for isotropic earth dam with constant surface infiltration in (a) RS2 and (b) RS3

The comparison for pressure head distribution along line 1-1 and line 2-2 between the results from RS2, RS3, and the finite element analysis by [2] are presented in Figure 7-8 and Figure 7-9. The modeling results show that seepage analysis conducted by RS2 and RS3 are in good agreement with the Finite Element solution presented in [2].



Figure 7-8: Total head variation along line 1-1



Figure 7-9: Total head variation along line 2-2

7.4. References

1. Fredlund, D.G. and H. Rahardjo (1993), Soil Mechanics for Unsaturated Soils, John Wiley.

7.5. Data Files

The RS3 input file groundwater #007.rs3v3 can be downloaded from the RS3 installation folder.

8. Flow Through Ditch-Drained Soils

8.1. Problem Description

In problems related to ditch-drained aquifers, numerical solutions are often used to predict the level of the water table and the distribution of soil-water pressure. The problem considered in this section involves the infiltration of water downward through two soil layers (Figure 8-1). The depth of the soil to the impermeable level is 0.5 m. The ditch is assumed to be water free.



Figure 8-1: Drainage through multi-layered soil

Table 8-1 summarizes the soil parameters of the layered system used in this example. This system simulates a coarse and fine soil. The lower layer has a thickness of 0.1 m. The rate of incident rainfall (infiltration) is taken to be equal to 4.4444e-5 m/s.

Table 8-1: Soil conductivity a	and Gardner's parameters
--------------------------------	--------------------------

Soil A	Relative Conductivity	1.11e-3 (m/s)	
	Gardner's parameters	a = 1000, n = 4.5	
Soil B	Relative Conductivity	1.11e-4 (m/s)	
	Gardner's parameters	a = 2777.7, n = 4.2	

RS2 and RS3 models for the problem are shown in Figure 8-2. The RS2 model is taken from verification problem #008 in the RS2 groundwater verification manual. The RS2 model uses a uniform mesh of three-noded triangles while the RS3 model uses a uniform mesh of four-noded tetrahedron elements. Both models have a height of 0.5 m and a length of 1 m. The *RS3* model has a depth of 0.1 m.



Figure 8-2: Multi-layered soil model in (a) RS2 and (b) RS3

8.2. Finite Element Solution from Reference

An alternative finite element solution for this problem is presented by Gureghian [1]. A sketch of the problem with pressure head contours is shown in Figure 8-3.



Figure 8-3: Pressure head contours for layered soil problem, as developed in [1]

8.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the ditch. Ground water flow behaviours determined from the two models are presented with respect to the phreatic surface, and pressure head and total head contour plots.

As presented the pressure head and total head in Figure 8-4 and Figure 8-5, the ground water behaviour above the unsaturated regime captured in RS2 and RS3 show a close agreement. Moreover, the modeling results show a close agreement with finite element solution from the work by [1].



(a)



(b)



Figure 8-4: Pressure head contour plot for multi-layered soil in (a) RS2 and (b) RS3

Figure 8-5: Total head contour plot for multi-layered soil in (a) RS2 and (b) RS3

8.4. References

1. Gureghian A. (1981), "A two dimensional finite element solution scheme for the saturatedunsaturated flow with application to flow through ditch drained soils:" J. Hydrology. (50), 333-353.

8.5. Data Files

The RS3 input file groundwater #008.rs3v3 can be downloaded from the RS3 Online Help page.

9. Seepage Through Dam

9.1. Problem Description

Seepage flow rate through earth dams is examined in this section. The geometry and material properties for two earth dams are taken from Bowels' Physical and geotechnical properties of soils [1].

9.1.1. Homogeneous dam

The Finite Element analysis by [1] (presented on p. 295) on seepage rate through a homogeneous dam is verified in this section. Figure 9-1 shows detailed geometry of the model. The permeability function as described in [1] is used to model the hydraulic conductivity for the saturated-unsaturated zone (Figure 9-2).



Figure 9-1: Homogenous earth dam as modeled in (a)RS2 and (b)RS3



Figure 9-2: Permeability function for the isotropic earth dam

9.1.2. Dam with impervious core

The second problem in this section considers a dam with an impervious core (Figure 9-3). The hydraulic permeability for the dam and the drain material are assumed to follow the functions shown in Figure 9-4.





(b)

Figure 9-3: Dam with impervious core geometry detail in (a)RS2 and (b)RS3



Figure 9-4: Permeability function for isotropic earth dam and drain

9.2. Finite Element Solution from Reference

Bowles [1] calculated the leakage flow rate through these dams using flow net techniques, which neglect the unsaturated flow. Chapuis et. al. [2] solved the same examples using SEEP/W, a finite element software package. The analysis results conducted by the two for each case are presented in Figure 9-5 and Figure 9-6.



Figure 9-5 Steady-state conditions in a homogeneous dam





9.3. Results

9.3.1. Homogeneous dam

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the dam. Ground water flow behaviours determined from the two models are presented with ground water table, flowline, pressure head and total head contour plots. Figure 9-7 presents the pressure head contour plots superposed by water table line or surface. Figure 9-8 shows the contours of total head with flow lines in the homogenous dam. The modeling results show a close agreement to the work by [1] and [2] as shown in Figure 9-5Figure 9-6



(b)

Figure 9-7: Pressure head contours for homogenous dam in (a) RS2 and (b) RS3



Figure 9-8: Total head contours and flow lines for homogenous dam in (a) RS2 and (b) RS3

9.3.2. Dam with impervious core

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the dam. Ground water flow behaviours determined from the two models are presented with ground water table, flowline, pressure head and total head contour plots. Figure 9-9 presents the pressure head contour plots superposed by water table line or surface. Figure 9-10Figure 9-8 shows the contours of total head with flow lines in the homogenous dam. The modeling results show a close agreement to the work by [1] and [2] as shown in Figure 9-6



Figure 9-9: Pressure head contours for isotropic dam with impermeable core in (a) RS2 and (b) RS3



(b)

Figure 9-10: Total head contours and flow lines for isotropic dam with impermeable core (a) RS2 and (b) RS3

9.4. References

- 1. Bowles J.E. (1984), Physical and geotechnical properties of soils. 2nd Ed. McGraw Hill, New York.
- 2. Chapuis, R., Chenaf D, Bussiere, B. Aubertin M. and Crespo R. (2001), "A user's approach to assess numerical codes for saturated and unsaturated seepage conditions", Can Geotech J. 38: 1113-1126

9.5. Data Files

The RS3 input file **groundwater #009_01.rs3v3** and **groundwater #009_02.rs3v3** can be downloaded from the RS3 Online Help page.

10. Steady-State Unconfined Flow Using Van Genuchten Permeability Function

10.1. Problem Description

Unconfined flow in a rectangular domain was analyzed in this section. The sensitivity of seepage face height to the downstream head is examined. The Van Genuchten [1] closed form equation for the unsaturated hydraulic conductivity function is used to describe the soil properties for the soil model. A variably saturated flow model [2], which assumes equipotential surfaces are vertical and flow is essentially horizontal, is also used for comparison.

10.2. Model Properties

A 10 m x 10 m square embankment has no-flow boundary conditions on the base and at the top. The water level at the left boundary is 10 m. Four different water levels (2, 4, 6 and 8 m) at the downstream boundary are considered in different stages. The soil has a saturated conductivity of $K_s = 1.1574 \cdot 10^{-5}$ m/s. The values of the Van Genuchten soil parameters are $\alpha = 0.64$ m⁻¹, n = 4.65.

The geometry and the mesh discretization of RS2 and RS3 models are shown in Figure 10-1. The RS2 model is taken from verification problem #010 in the RS2 groundwater verification manual. The RS2 and RS3 models use a mesh of 8-noded quadrilateral elements 10-noded tetrahedron elements for the discretization.







10.3. Analytical Solution

A study conducted by [2] shows the solutions to unconfined flow problems solved using fully saturated flow, variably saturated flow, and the Dupuit-Forchheimer models. Based on the variably saturated flow model, the sensitivity analysis on the downstream water level to the water table was examined. The phreatic lines determined for each downstream head developed by [2] are drawn in Figure 10-2.



Figure 10-2: Phreatic surfaces with variable downstream head [2]

10.4. Results

The variation of the phreatic surface with changing downstream water level predicted by RS2 and RS3 is presented in Figure 10-3. Moreover, Table 10-1 presents comparison of discharge values and seepage face from [2], RS2 and RS3. The modeling results show that the absolute length of the seepage face decreases significantly with an increase in the water level at the downstream boundary. In overall, the RS2 and RS3 modeling results show a close agreement with the results from work by [2].



Figure 10-3: Phreatic surfaces for various downstream water levels computed analytically, in RS2 and in RS3

	Model Dimensions	Downstream Water Level (m)	Discharge (m/sec)	Seepage face (m)
Clement et. al. [2]	10m x 10m	2	6.0764x10⁻⁵	4.8
RS2	10m x 10m	2	6.0659x10⁻⁵	5.0
RS3	10m x 10m	2	6.0708x10⁻⁵	5.0

Table 10-1: Discharge velocities and seepage face dimensions

10.5. References

- 1. Genuchten, V. M (1980), "A closed equation for predicting the hydraulic conductivity of unsaturated soils", Soils Sci Soc Am J. 44: 892-898
- 2. Clement, T.P, Wise R., Molz, F. and Wen M. (1996), "A comparison of modeling approaches for steady-state unconfined flow", J. of Hydrology 181: 189-209

10.6. Data Files

The RS3 input file groundwater #010.rs3v3 can be downloaded from the RS3 Online Help page.
11. Earth and Rock-Fill Dam Using Gardner Permeability Function

11.1. Problem Description

Seepage in a uniform earth and rock-fill dam is examined in this section. Nonlinear modeling is used to represent the seepage flow above and below the free surface. Gardner's nonlinear equation [1] between permeability function k_w and pressure head is used in this section and it can be presented as

$$(11.1.1) k_w = \frac{k_s}{1+ah^n}$$

Where *a* and *n* are the Gardner parameters

h = pressure head (suction)

 k_w = permeability

 k_s = saturated permeability

11.1.1. Uniform earth and rock-fill dam

Figure 11-1 shows detailed geometry of the first dam studied. The upstream elevation head is 40 m and the downstream elevation head is 0 m. The geometry of the dam is taken from [2]; the slope of the upstream face is 1:1.98 and the slope of the downstream face is 1:1.171 (Figure 11-1). Gardner's parameters are assigned values of a = 0.15 and n = 6.





Figure 11-1: Uniform earth and rock-fill dam model geometry in (a) RS2 and (b) RS3

11.1.2. Heterogeneous earth and rock-fill dam

Figure 11-2 shows a dam with a permeable foundation and toe drain [2]. The permeability coefficient of the foundation of sand layer is 125 times that of the earth dam and blanket. The toe drain has a permeability coefficient 10000 times larger than that of the dam. Table 11-1 shows Gardner's parameters for the different model layers.



(b)

Figure 11-2: Heterogeneous dam with permeable foundation and toe drain in (a) RS2 and (b) RS3

Layer	K _s (m/s)	Α	n
Dam	1x10 ⁻⁷	0.15	2
Foundation	1.25x10 ⁻⁵	0.15	6
Toe drain	1x10 ⁻³	0.15	6

Table 11-1: Material parameters for heterogeneous dam

11.2. Finite Element Solution from Reference

For this problem, RS2 and RS3 results are compared to those obtained using ABAQUS commercial software, which are presented by Zhang et al. in [2].

11.3. Results

11.3.1.Uniform earth and rock-fill dam

Figure 11-3 shows the pressure head contour plot produced by RS2 and RS3, which indicates that the elevation of the release point on the downstream face is 19.404 m and 19.464 m, respectively. This is comparable to the results presented by [2], which predict an elevation of 19.64 m for identical dam geometry.



(b)

Figure 11-3: Pressure head contour plot in (a) RS2 and (b) RS3

11.3.2. Heterogeneous earth and rock-fill dam

Presented in this section is the comparison between seepage analysis results for the Heterogeneous earth and rock-fill dam problem obtained from ABAQUS [2], RS2, and RS3. The distribution of the total head contours and phreatic line from [2] are shown in Figure 11-4. As presented in Figure 11-5, RS2 and RS3 modeling results show a good agreement with the result obtained from ABAQUS.



Figure 11-4: Total head contours for heterogeneous dam [2]. Units in m·10²



(a)



Figure 11-5: Total head contour plot in (a) RS2 and (b) RS3

11.4. References

- G Gardner, W. (1956), "Mathematics of isothermal water conduction in unsaturated soils." Highway Research Board Special Report 40 International Symposium on Physico-Chemical Phenomenon in Soils, Washington D.C. pp. 78-87.
- 2. Zhang, J, Xu Q. and Chen Z. (2001), "Seepage analysis based on the unified unsaturated soil theory", Mechanics Research Communications, 28 (1) 107-112.

11.5. Data Files

The RS3 input file **groundwater #011_01.rs3v3** and **groundwater #011_02.rs3v3** can be downloaded from the RS3 Online Help page.

12. Seepage from Trapezoidal Ditch into Deep Horizontal Drainage Layer

12.1. Problem Description

Seepage from a trapezoidal ditch into a deep horizontal drainage layer is analyzed in this section. The geometry of the problem is depicted in Figure 12-1.



Figure 12-1: Seepage from a trapezoidal ditch

Figure 12-2 presents the mesh and groundwater boundary conditions for the RS2 and RS3 models. The RS2 model is taken from verification problem #012 in the RS2 groundwater verification manual. Owing to symmetry, only half of the problem was modeled with RS2 and the external boundary of RS3 model was constructed extruding that profile by 1 m. The RS2 model uses a uniform mesh of three-noded triangles while RS3 uses a mesh of four-noded tetrahedron elements. The key model parameters are shown in Table 12-1.



Figure 12-2: Model of trapezoidal ditch and deep drainage layer in (a) RS2 and (b) RS3

Table	12-1.	RS2	and	RS3	model	parameters
rapic	12-1.	I LOZ	ana	1,00	mouci	parameters

Parameter	Value
Ditch half-width (B/2)	25 m
Ditch depth (H)	10 m
Bank slope (α)	45°
Soil conductivity (k)	10 ⁻⁵ m/s

12.2. Analytical Solution

The total head distribution can be determined based on a flow net drawn by hand using Vedernikov's boundary conditions (width of seepage zone, depth to horizontal equipotential lines) (Figure 12-3). Figure 12-4 shows the flow net used to obtain the analytical solution.



Figure 12-3: Theoretical flow net (from Harr, 1990 [1])



Figure 12-4: Hand-drawn flow net according to Vedernikov's boundary conditions

12.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour from the trapezoidal ditch. Ground water flow behaviours determined from the two models are presented with flowline and total head contour plots (Figure 12-5). RS2 and RS3 modeling results are in good agreement with the flow net shown in Figure 12-4.



(b)

Figure 12-5: Flow vector plot generated by (a) RS2 and (b) RS3

A material query was added at the center of the ditch to obtain the total head distribution along the vertical cross-section immediately underlying the ditch. Figure 12-6 plots total head as a function of depth and compares RS2 and RS3 results to those drawn from the flow net in Figure 12-4.



Figure 12-6: Comparison of RS2 and RS3 results to analytical solutions for total head distribution below centre of ditch

12.4. References

1. Haar, M. E. (1990), Groundwater and Seepage, 2nd Edition, Dover.

12.5. Data Files

The RS3 input file groundwater #012.rs3v3 can be downloaded from the RS3 Online Help page.

13. Seepage from Triangular Ditch into Deep Horizontal Drainage Layer

13.1. Problem Description

Seepage from a trapezoidal ditch into a deep horizontal drainage layer is analyzed in this section. The geometry of the problem is depicted in Figure 13-1.



Figure 13-1: Triangular ditch with deep underlying drainage layer

The RS2 and RS3 model for the problem described in the previous section is shown in Figure 13-2. Owing to symmetry, only half of the problems were modeled due to symmetry.



Figure 13-2: Triangular ditch as constructed in (a) RS2 and (b) RS3

13.2. Analytical Solution

The total head distribution can be determined based on a flow net drawn by hand using Vedernikov's boundary conditions (width of seepage zone, depth to horizontal equipotential lines) (Figure 13-3). Figure 13-4 shows the flow net used to obtain the analytical solution.



Figure 13-3: Theoretical flow net beneath triangular ditch (from Harr, 1990 [1])





13.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour from the trapezoidal ditch. Ground water flow behaviours determined from the two models are presented with flowline and total head contour plots (Figure 13-5). RS2 and RS3 modeling results are in good agreement with the flow net shown in Figure 13-4.





(b)

Figure 13-5: Flow vector plot generated by (a) RS2 and (b) RS3

Upon inspection of the flow vectors, the seepage zone appears to be approximately 21 m wide, which equates to a total seepage zone of 42 m when symmetry is accounted for. This is in close accordance with Vedernikov's solution.

A material query was added at the center of the ditch to obtain the total head values along the vertical axis of the model. Figure 13-6 compares the head distribution obtained using RS2 and RS3 with the analytical solution.



Figure 13-6: Comparison of RS2 and RS3 results to analytical solutions for total head beneath ditch

13.4. References

1. Haar, M. E. (1990), Groundwater and Seepage, 2nd Edition, Dover.

13.5. Data Files

The RS3 input file groundwater #013.rs3v3 can be downloaded from the RS3 Online Help page.

14. Unsaturated Soil Column

14.1. Problem Description

Steady-state capillary head distribution above the water table in a narrow soil column is analyzed in this example. The geometry of the problem and considered hydraulic properties of soil are provided in Figure 14-1 and Table 14-1, respectively.



Figure 14-1: Narrow soil column above the water table

Table 14-1: Model parameters

Parameter	Value
Column height (L)	1 m
Saturated soil conductivity (Ks)	10 ⁻⁷ m/s
Infiltration/Exfiltration rate (v)	±8.64 ·10 ⁻⁴ m/day
Sorptive number (α)	1 m ⁻¹

Soil column with 2mm width and 1 m depth is represented with RS2 and RS3, as shown in Figure 14-2. The model has infiltration and exfiltration condition applied at the top surface.



Figure 14-2: Infiltration and exfiltration in a narrow column as modeled in (a) RS2 and (b) RS3

14.2. Analytical Solution

Gardner (1958) [1] proposed an analytical solution to this problem. Following equation was proposed to calculate capillary head:

(14.2.1)
$$\psi(z) = -\frac{1}{\alpha} \ln \left[\frac{1}{K_s} (v + (K_s - v)) e^{-\alpha(L-z)} \right]$$

where z is the vertical coordinate (m) and other parameters are as defined in Table 14-1.

14.3. Results

A material query was added throughout the depth of the column to plot the pressure head values. The output is depicted in Figure 14-3 for the constant infiltration case and Figure 14-4 for the constant exfiltration case. The RS2 and RS3 results are in good agreement with the analytical solution presented by [1].



Figure 14-3: Plot of pressure head against depth for constant infiltration case



Figure 14-4: Plot of pressure head against depth for constant exfiltration case

14.4. References

1. Gardner, W.R. (1959), Some Steady-State Solutions of the Unsaturated Moisture Flow Equation with Application to Evaporation from a Water Table, Soil Science 35 (1958) 4, 228-232.

14.5. Data Files

The RS3 input file **groundwater #014_01.rs3v3** and **groundwater #014_02.rs3v3** can be downloaded from the RS3 Online Help page.

15. Radial Flow to a Well in a Confined Aquifer

15.1. Problem Description

The problem concerns the radial flow towards a pumping well through a confined homogeneous, isotropic aquifer. The problem geometry illustrated in Figure 15-1 shows the right-hand side of the vertical well. Vertical dash dotted line at left hand side of the figure is the axis of symmetry, representing the centre line of the well.



Figure 15-1: Vertical well in confined aquifer

The RS2 and RS3 model used to simulate this problem is shown in Figure 15-2. To ensure highly accurate results, the model mesh was created with 6-noded triangular elements and 10-noded tetrahedra elements for RS2 and RS3, respectively. The discretization density and element density were increased near the well where high pore pressure gradients were expected.







Figure 15-2: Model in (a) RS2 and (b) RS3

Considering the axisymmetric geometry of this exercise, axisymmetric type analysis was selected for the solver option of RS2. The model was constructed with respect to the following geometrical/hydraulic parameters:

- Well radius $r_w = 0.15$ m
- Boundary radius *r*_e = 40 m
- Aquifer depth *b* = 5 m
- Water table height H = 16 m
- Volumetric pumping rate Q = 0.125 m³ /s
- Soil conductivity k = 0.002 m/s

The pumping boundary condition was simulated by applying a negative normal infiltration of q along the length of the well in order that the flow direction is casted towards the well. The magnitude of q was calculated by dividing the volumetric pump rate (Q) by the surface area of the well:

(15.1.1)
$$q = \frac{Q}{2\pi r_w l} = \frac{0.125}{2\pi (0.15)(5)} = 0.0265 \, m/s$$

where *l* represents the length of the well. In this case it fully penetrates the reservoir so l = b.

15.2. Analytical Solution

According to Davis (1966) [1] the head h at any radius r is given by the analytical solution [1]

(15.2.1)
$$h = H - \frac{Q}{2\pi k b} \ln\left(\frac{r_e}{r}\right)$$

where *H* is the head at the far boundary, r_e is the radius of the far boundary, *b* is the thickness of the aquifer, *k*, is the permeability in the aquifer and *Q* is the volumetric pumping rate.

15.3. Results

Seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the confined aquifer. The steady state solution for total head produced by RS2 and RS3 is shown in Figure 15-3.



Figure 15-3: Total head contour plot in (a) RS2 and (b) RS3

In order to verify the RS2 and RS3 modeling results, the total head retrieved along the length of the model was compared to the analytical solution. The total head values computed by RS2 and RS3 with those derived from the analytical solution in [1] is plotted on the graph (See Figure 15-4). The graph shows that both RS2 and RS3 results are in good accordance with the analytical solution.



Figure 15-4: Total head distribution with increasing radial distance from well

15.4. References

1. Davis, S.N. and DeWiest, R.J.M., (1966), Hydrogeology, John Wiley & Sons, Inc., New York.

15.5. Data Files

The RS3 input file groundwater #015.rs3v3 can be downloaded from the RS3 Online Help page.

16. Radial Flow to a Well in an UnConfined Aquifer

16.1. Problem Description

The problem concerns the radial flow from an aquifer towards a pumping well in a homogeneous, isotropic soil. The problem geometry illustrated in Figure 16-1 shows shows the right-hand side of the vertical well, where the vertical dash line at the left hand side of the figure represents the axis of symmetry. It is considered that the aquifer has an impermeable base but is unconfined at the top.



Figure 16-1: Radial flow to a well in an unconfined aquifer

The RS2 and RS3 model used to simulate this problem is shown in Figure 16-2. To ensure accurate results, the model mesh was created with 6-noded triangular elements and 10-noded tetrahedra elements for RS2 and RS3, respectively. The discretization density and element density were increased near the well where high pore pressure gradients were expected.



Figure 16-2: Pumping from a well in an unconfined aquifer as modeled in (a) RS2 and (b) RS3

Both models use the following input parameters:

- Well radius $r_w = 0.15$ m
- Boundary radius *r*_e = 40 m
- Water table height *H* = 16 m
- Volumetric pumping rate $Q = 0.125 \text{ m}^3 \text{/s}$
- Soil conductivity k = 0.002 m/s

The pumping boundary condition was simulated by applying a negative normal infiltration of q along the length of the well. The magnitude of q was calculated by dividing the volumetric pump rate (Q) by the surface area of the well:

(16.1.1)
$$q = \frac{Q}{2\pi r_w l} = \frac{0.125}{2\pi (0.15)(16)} = 0.00829 \, m/s$$

where *I*, representing the length of the well, is assigned with 16 m considering that water fully penetrates the aquifer.

16.2. Analytical Solution

The height of the water table *h* at any radius *r* can be obtained from the analytical solution [1]

(15.2.1)
$$h^2 = H^2 - \frac{Q}{\pi k} \ln\left(\frac{r_e}{r}\right)$$

where *H* is the head at the far boundary, r_e is the radius of the far boundary, *k*, is the permeability in the aquifer and *Q* is the volumetric pumping rate.

16.3. Results

Steady state seepage analysis was conducted using RS2 and RS3 to simulate the ground water behaviour within the confined aquifer. The pressure head contour plots produced by RS2 and RS3 are shown in Figure 16-3.







Figure 16-3: Pressure head contour plot with water table surface in (a) RS2 and (b) RS3

The steady state seepage analysis results from RS2 and RS3 are verified based on the total head data sampled along the length of the model. As demonstrated in Figure 16-4, the sampled data shows a close agreement to the analytical solution by [1].



Figure 16-4: Total head distribution with increasing radial distance from well

16.4. References

1. Davis, S.N. and DeWiest, R.J.M., (1966), Hydrogeology, John Wiley & Sons, Inc., New York.

16.5. Data Files

The RS3 input file groundwater #016.rs3v3 can be downloaded from the RS3 Online Help page.

17. 1-D Consolidation with Uniform Initial Excess Pore Pressure

17.1. Problem Description

In this problem, soil consolidation process is simulated in one way drainage system and two way drainage system using a 1 m height soil column (Figure 17-1). Two cases are developed to represent each system. The first case allows seepage at both top and bottom surfaces, simulating the two way drainage system, while the second case allows the seepage only at the top surface (one way drainage system).



Figure 17-1: Model Geometry

Terzaghi's consolidation equation can be written as

(17.1.1)
$$\frac{\partial^2 u_e}{\partial Z^2} = \frac{\partial u_e}{\partial T}$$

using the dimensionless variables

(17.1.2)
$$Z = \frac{z}{H}$$

and

(17.1.3)
$$T = \frac{C_{\nu}t}{H^2}$$

where

Ζ	=	depth from the top of the column
Η	=	maximum drainage path
C_v	=	coefficient of consolidation
t	=	time
u _e	=	excess pore pressure

An initial condition is imposed at t = 0 as following,

$$u_e = u_0$$
 for $0 \le Z \le 1$

where

 u_0 = initial excess pore pressure

Along surfaces and edges where flow is allowed to occur, ground water boundary condition is set to $u_e = 0$. At the initial steady state stage, an initial pressure head of 100 m is applied uniformly throughout the column to generate excess pore pressure at the seepage face equivalent to that. The following properties are assumed for the soil:

- $m_w = 0.01 \,/\text{kPa}$
- $C_v = 1.02e-4 \text{ m}^2/\text{s}$
- $k = C_v \gamma_w m_w = 1e-5 \text{ m/s}$

The maximum drainage paths are taken as L/2 = 0.5 m for Case 1 and L = 1 m for Case 2. The problem is modeled in RS2 and RS3 with 1580 three-noded triangular elements and 7744 four noded tetrahedra finite elements, respectively. The RS2 and RS3 models for Case 1 and Case 2 are shown in Figure 17-2 and Figure 17-3, respectively.



Figure 17-2: Model for case 1 in (a) RS2 and (b) RS3



Figure 17-3: Model for case 2 in (a) RS2 and (b) RS3

17.2. Analytical Solution

The solution to the consolidation equation is given in [1] as:

(17.1.4)
$$u_e = \sum_{m=0}^{m=\infty} \frac{2u_0}{M} (\sin M Z) e^{-M^2 T}$$

where

$$M = \frac{\pi}{2}(2m+1)$$

17.3. Results

17.3.1.Case 1

Had conducted the seepage analysis using RS2 and RS3, it could be determined the pore pressure distribution within the soil column over time (transient state). Figure 17-4 shows excess pore pressure along the soil column at different times. The triangular and square data points represent the results from RS2 and RS3, respectively, while the solid lines represent values calculated using Equation 17.1.4. The close agreements are shown between the numerical and analytical solution.



Figure 17-4: Comparison of Pore Pressure Dissipation for Case 1

17.3.2.Case 2

The single drainage system, represented by Case 2 shows less effective water dissipation compared to the double drainage system, represented by Case 1 (Figure 17-5). Moreover, the modeling results are in close agreement with the Terzaghi consolidation equation values.



Figure 17-5: Comparison of Pore Pressure Dissipation for Case 2

17.4. References

1. T.W. Lambe and R.V. Whitman (1979) Soil Mechanics, SI Version, New York: John Wiley & Sons.

17.5. Data Files

RS3 input data file **groundwater #017_01.rs3v3** and **groundwater #017_02.rs3v3** can be downloaded from the RS3 Online Help page.

18. Pore Pressure Dissipation of Stratified Soil

18.1. Problem Description

The problem deals with 1D consolidation of stratified soils. Three cases are considered, which are shown in Figure 18-1. The properties for Soil A and Soil B are shown in Table 18-1. Both the pore fluid specific weight (γ_w) and the height of the soil profiles are assumed to be one unit. An initial pressure head of P = 1000 m is applied uniformly throughout the column.





	Soil A	Soil B
k	1	10
mv	1	10
Cv	1	1

18.2. Results

Had conducted the seepage analysis using RS2 and RS3, it could be determined the pore pressure distribution within the soil column over time (transient state). Figure 18-2 to Figure 18-4 show comparisons between excess pore pressures in the RS2 model, RS3 model, and values from the analytical solution presented in [1]. The triangular data points represent the RS2 interpretations, and the squares represent the RS3 interpretations, while the solid lines represent analytical values from [1]. As shown, the RS2 and RS3 results are in close agreement with the analytical solutions.



Figure 18-2: Comparison of Excess Pore Pressure for Case 1


Figure 18-3: Comparison of Excess Pore Pressure for Case 2



Figure 18-4: Comparison of Excess Pore Pressure for Case 3

18.3. References

1. Pyrah, I.C. (1996), "One-dimensional consolidation of layered soils", Géotechnique, Vol. 46, No. 3, pp. 555-560.

18.4. Data Files

RS3 input data file **groundwater #018_01.rs3v3**, **groundwater #018_02.rs3v3**, and **groundwater #018_03.rs3v3** can be downloaded from the RS3 Online Help page.

19. Transient Seepage Through an Earth Fill Dam with Toe Drain

19.1. Problem Description

In this problem, an earth fill dam with a reservoir on one side is modeled to investigate the transient seepage through the dam. The reservoir level is quickly raised, and transient seepage is investigated. The geometry and material properties for the dam are taken from the work by [1], which presents the use of FLEX PDE to compute seepage analysis.

The base of the earth fill dam is 52 m wide and there is a 12 m wide toe drain installed at the downstream side. The initial steady-state reservoir level is 4 m. For transient analysis, the reservoir level is quickly raised to a height of 10 m. It is assumed that the dam has isotropic hydraulic properties conditions and an m_v value of 0.003 /kPa. Figure 19-1 shows the coefficient of permeabilities used for dam material. The toe drain material has a coefficient of permeability equal to 0.0005 m/s.



Figure 19-1: Coefficient of Permeability Function for Dam Material

19.2. Model Geometry

The RS2 and RS3 models for initial steady state and transient analysis are shown in Figure 19-2 and Figure 19-3, respectively. The boundary conditions simulate the rise in the reservoir water level and the installed toe drain.





(b)

Figure 19-3: Model for Transient State in (a) RS2 and (b) RS3

19.3. Results

The water behaviour within the earth dam was investigated using the presented RS2 and RS3 models. The total head contour plots generated from RS2 and RS3 at times 15 hr and 16383 hr are presented in Figure 19-4 and Figure 19-5, respectively. The pink solid lines (RS2) or planes (RS3) represent water table, where pressure head is at 0. The black lines represent total head contour computed with FlexPDE from the work by [1], which shows a good agreement with the total head plot generated by RS2 and RS3.



(a)



(b)





(b)

Figure 19-5: Comparison of Total Head Contours for Time 16383 hr in (a) RS2 and (b) RS3

19.4. References

1. Pentland, et. al (2001), "Use of a General Partial Differential Equation Solver for Solution of Mass and Heat Transfer Problems in Geotechnical Engineering", 4th Brazilian Symposium on Unsaturated Soil, pp. 29-45.

19.5. Data Files

The RS3 input file groundwater #019.rs3v3 can be downloaded from the RS3 Online Help page.

20. Transient Seepage Through an Earth Fill Dam

20.1. Problem Description

In this problem, an earth fill dam with a reservoir on one side is modeled to investigate the transient seepage through the dam. This verification is extension of the Verification Example 19. Therefore, the geometry and material properties of the dam considered for this problem is similar to Verification Example 19. However, this problem does not include the toe drain. It is assumed that the dam has isotropic hydraulic properties conditions and a m_v value of 0.001 /kPa. Figure 20-1 shows the coefficient of permeabilities used for the dam material.





20.2. Model Geometry

The reservoir level is raised from 4 m to 10 m at the start of analysis time. The RS2 and RS3 models for initial steady state and transient analysis are shown in Figure 20-2 and Figure 20-3, respectively. The RS2 model is taken from verification problem #020 in the RS2 groundwater verification manual. The change in boundary conditions simulate the rise in the reservoir water level.





Figure 20-2: (a) RS2 and (b) RS3 Model - Initial Steady State





Figure 20-3: (a) RS2 and (b) RS3 Model - Transient

20.3. Results

Total head and pressure head contour plots superposed by water table are generated from RS2 and RS3. Figure 20-6 and Figure 20-7 show total head contours for the stages at 0.6 h and 19656 h, respectively. Figure 20-8 and Figure 20-9 show pressure head contours for the same times. The RS3 modeling results show a close agreement with RS2.



Figure 20-4: Total Head Contours at 0.6 h in (a) RS2 and (b) RS3



Figure 20-5: Total Head Contours at 19656 h in (a) RS2 and (b) RS3



Figure 20-6: Pressure Head Contours at 0.6 h in (a) RS2 and (b) RS3



Figure 20-7: Pressure Head Contours at 19656 h in (a) RS2 and (b) RS3

Total head values are sampled along the toe slope to verify the modeling result with respect to the work by [1] (Figure 20-5). The sampled data shows a good agreement with the data reported by [1].





20.4. References

1. Fredlund, D.G. and Rahardjo, H. (1993), Soil Mechanics for Unsaturated Soils, New York: John Wiley & Sons.

20.5. Data Files

The RS3 input file groundwater #020.rs3v3 can be downloaded from the RS3 Online Help page.

21. Seepage Below a Lagoon

21.1. Problem Description

This example deals with transient seepage below a lagoon. One half of the model geometry is considered since it is symmetrical. The section of the lagoon considered is 2 m wide. A 1 m deep soil liner is directly under the lagoon and the soil is assumed to extend 9 m below the soil liner before an impermeable boundary is encountered. An initial steady-state water table at a depth of 5 m from the ground surface is assumed. At analysis time zero, the water level in the lagoon is instantaneously raised to a height of 1 m. The model geometry for transient analysis at time zero is shown in Figure 21-1.



Figure 21-1: Model Geometry

An m_v value of 0.002 /kPa was assumed for both the soil and the liner. The permeability functions for the sands are shown in Figure 21-2.



Figure 21-2: Coefficient of Permeability Functions

21.2. Models

The RS2 and RS3 models for the initial steady state and transient analysis are shown in Figure 21-3 and Figure 21-4, respectively. The boundary conditions model the rise in water level in the lagoon. No flow is assumed across the lagoon centerline.



Figure 21-3: Model Initial Steady State in (a) RS2 and (b) RS3



Figure 21-4: Model Transient State in (a) RS2 and (b) RS3

21.3. Results

In order to investigate the ground water behaviour over time, the ground water seepage analysis was computed using transient method, whereby the model was staged at 73 minutes, 416 minutes, 792 minutes, and 11340 minutes. Figure 21-5 to Figure 21-8 show pressure head contours superposed by water tables, represented by pink planes for different transient analysis times.



(a)



(b)

Figure 21-5: Pressure Head Contours at 73 minutes in (a) RS2 and (b) RS3





Figure 21-6: Pressure Head Contours at 416 minutes in (a) RS2 and (b) RS3





Figure 21-7: Pressure Head Contours at 792 minutes in (a) RS2 and (b) RS3





Figure 21-8: Pressure Head Contours at 11340 minutes in (a) RS2 and (b) RS3

In order to verify the RS2 and RS3 modeling results, comparison study was conducted for the pressure head distribution along the top boundary of the model against those determined through analytical solution. The pressure head values sampled from the numerical models (See Figure 21-9) are plotted on the graph in Figure 21-10 that is superposed by the pressure head distribution curve from the work by [1]. The graph shows that both RS2 and RS3 results are in good accordance with the analytical solution.





Figure 21-9: Query Line in (a) RS2 and in (b) RS3. The RS3 Image Shows Pressure Head at 11340 Minutes

(b)



Figure 21-10: Comparison of Pressure Head Values along Top Boundary

21.4. References

1. Fredlund, D.G. and Rahardjo, H. (1993), Soil Mechanics for Unsaturated Soils, New York: John Wiley & Sons.

21.5. Data Files

The RS3 input file groundwater #021.rs3v3 can be downloaded from the RS3 Online Help page.

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22. Seepage in a Layered Slope

22.1. Problem Description

This problem deals with transient seepage in a layered slope. The slope consists of medium sand with a horizontal fine sand layer. At initial steady-state conditions, the water table is located at a height of 0.1 m from the toe of the slope. A constant infiltration of 2.1x10-4 m/s is applied at the top of the slope at time zero. An m_v value of 0.002 /kPa is assumed for both materials, and the permeability functions for the sands are shown in Figure 22-1.



Figure 22-1: Coefficient of Permeability Functions

22.2. Model Properties

Figure 22-2 shows the RS2 and RS3 models used to perform transient analysis with constant infiltration. Groundwater boundary conditions are also indicated. The RS2 model uses a uniform mesh of threenoded triangles with approximately 400 elements. The RS3 mesh consists of approximately 650 uniformly distributed four-noded tetrahedra elements. The slope has a top height of 1 m, toe height of 0.2 m and slope of 26.6°.



Figure 22-2: Model as constructed in (a) RS2 and (b) RS3

22.3. Results

In order to investigate the ground water behaviour over time, the ground water seepage analysis was computed using transient method, whereby the model was staged at 4.6 seconds, 31 seconds, and 208 seconds. Figure 22-3 to Figure 22-5 show the total head contour results from RS2 and RS3 superposed by water tables, represented by pink planes for different transient analysis times.









(b)

Figure 22-4: Total Head Contours for 31 seconds in (a) RS2 and (b) RS3





Figure 22-5: Total Head Contours for 208 seconds in (a) RS2 and (b) RS3

In order to verify the RS2 and RS3 modeling results, comparison study was conducted for the pressure head distribution along the query line shown in Figure 22-6 against those determined through analytical solution. The sampled pressure head values are plotted on the graph in Figure 22-7 that is superposed by the pressure head distribution curve from the work by [1]. The graph shows that both RS2 and RS3 results are in good accordance with the analytical solution.



Figure 22-6: Query Line



Figure 22-7: Comparison of Total Head Values

22.4. References

1. Fredlund, D.G. and Rahardjo, H. (1993), Soil Mechanics for Unsaturated Soils, New York: John Wiley & Sons.

22.5. Data Files

The RS3 input file groundwater #022.rs3v3 can be downloaded from the RS3 Online Help page.

23. Transient Seepage through a Fully Confined Aquifer

23.1. Problem Description

This problem deals with transient seepage through a fully confined aquifer. Two head conditions are examined. In both cases, the aquifer has an initial pore-water distribution that is changed through the introduction of five feet of hydraulic head to the left side of the aquifer. Seepage is then examined in the x-direction with time. The aquifer is 100 feet long and five feet thick. An illustration of the problem is presented in Figure 23-1.



Figure 23-1: Model geometry

23.1.1. Case 1 - No initial pore-water distribution

Figure 23-2 shows the RS2 and RS3 models used to perform a transient analysis with 0 feet of initial pore-water pressure.



Figure 23-2: Case 1 Model - 0 feet of Initial PWP in a) RS2 and in b) RS3

23.1.2.Case 2 - Initial pore-water distribution of 5 feet

Figure 23-3 shows the RS2 and RS3 models used to perform a transient analysis with 5 feet of initial head (assigned by setting the steady state boundary condition of the problem to 5 feet of head). Note that

the boundary condition on the left face is set to 10 feet (5 feet of initial PWP plus 5 feet of introduced hydraulic head).



Figure 23-3: Case 2 Model - 5 feet of Initial PWP in a) RS2 and in b) RS3

23.2. Analytical Solution

The equation for transient seepage through a fully confined aquifer can be expressed through the J.G. Ferris Formula [1] as,

(23.2.1)	$h(x,t) = h(x,0) + \Delta H \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{4t(T/S)}}\right)$
(23.2.2)	$T/S = \frac{k}{\gamma_{w} \cdot m_{v}}$

Where h(x,t) is the hydraulic head at position x at time t; ΔH is the head difference between the initial pore-water distribution and the introduced hydraulic head; and *erfc* is the complimentary error function.

23.3. Results

In order to investigate the ground water behaviour over time, the ground water seepage analysis was computed using transient method, whereby the model was staged at 1 hour, 12 hours, 24 hours, 48 hours, 72 hours, 120 hours, 240, hours, and 600 hours. Figure 23-4 and Figure 23-5 show the total head contour results from RS2 and RS3 at 600 hours superposed by water tables, represented by pink planes for different transient analysis times.



(b)

Figure 23-4: Total Head Contours, 600 hours, no initial PWP in a) RS2 and in b) RS3



(b)

Figure 23-5: Total Head Contours, 600 hours, 5 feet of initial PWP in a) RS2 and in b) RS3

In order to verify the RS2 and RS3 modeling results, comparison study was conducted for the total head distribution along the top surface of the material retrieved from the numerical and analytical solutions. The total head values sampled from RS2 and RS3 models are plotted on the graph with the curve drawn from the work by [1] in Figure 23-6 and Figure 23-7 for Case 1 and Case 2, respectively. The graph shows that both RS2 and RS3 results are in good accordance with the analytical solution.



Figure 23-6: Comparison of RS2 and RS3 results with the Analytical Solution - Case 1



Figure 23-7: Comparison of RS2 and RS3 results with the Analytical Solution - Case 2

23.4. References

1. Tao, Y. and Xi, D. (2006), "Rule of Transient Phreatic Flow Subjected to Vertical and Horizontal Seepage:" Applied Mathematics and Mechanics. (27), 59-65.

23.5. Data Files

RS3 input data files **groundwater #023_01.rs3v3** and **groundwater #023_02.rs3v3**can be downloaded from the RS3 Online Help page.