rocscience

RS3

Joint

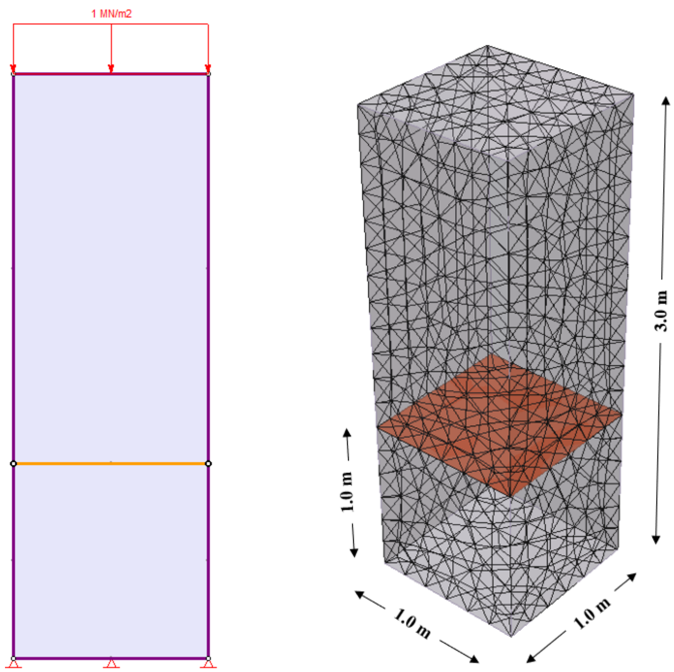
Verification Manual

Table of Contents

1. Single Jointed Rock Column Under Axial Pressure

1.1. Problem Description

This problem concerns a one-dimensional bar of elastic rock material subjected to a uniaxial load. The bar is loaded vertically with a uniform pressure P = 1 MPa and contains a joint at some distance z from the ground surface. In RS3, this situation was modeled using a narrow three-dimensional column with the dimensions given in. The 2D and 3D models are created in RS2 and RS3 and the results are compared with an analytical solution. In RS3: 4-node and 10-node tetrahedra solid elements are used, and in RS2: 4-node and 8-node quadrilateral elements are used to discritize the model.



a) b)

Figure 1-1 Problem Geometry (a) RS2 and in (b) RS3

The problem is using the same parameters that are listed in Table 1-1.

Table 1-1: Model Parameters

Parameter	***Value***
Young's modulus (E)	2000 MPa
Poisson's ratio	0.01
Height	3 m
Width and Depth	1 m
Joint properties	
Normal stiffness (k_{nn})	10 GPa/m
End condition	Open

1.2. Analytical Solution

A modified version of the analytical solution presented was used to verify the results obtained from RS2 and RS3 simulations [1]. The displacement along the bar is given by:

$$\mu(y) = \frac{Py}{E} + \sum H(\frac{P}{k_{nm}})$$

where y is the distance of the considering point to the bottom of the rock column, P is the applied pressure, E is the elastic modulus of the column, and k_{nn} is the normal stiffness of the joint. H is a form of the Heaviside function; it takes on the value of its argument when y exceeds the height of the joint and otherwise returns zero.

1.3. Results

Figure 1-2 shows the displacement field of the single-jointed rock column along its vertical axis obtained by RS2 and RS3. The analytical results are also shown for reference. It can be seen that the results obtained from RS2 and RS3 agree well with the analytical values.

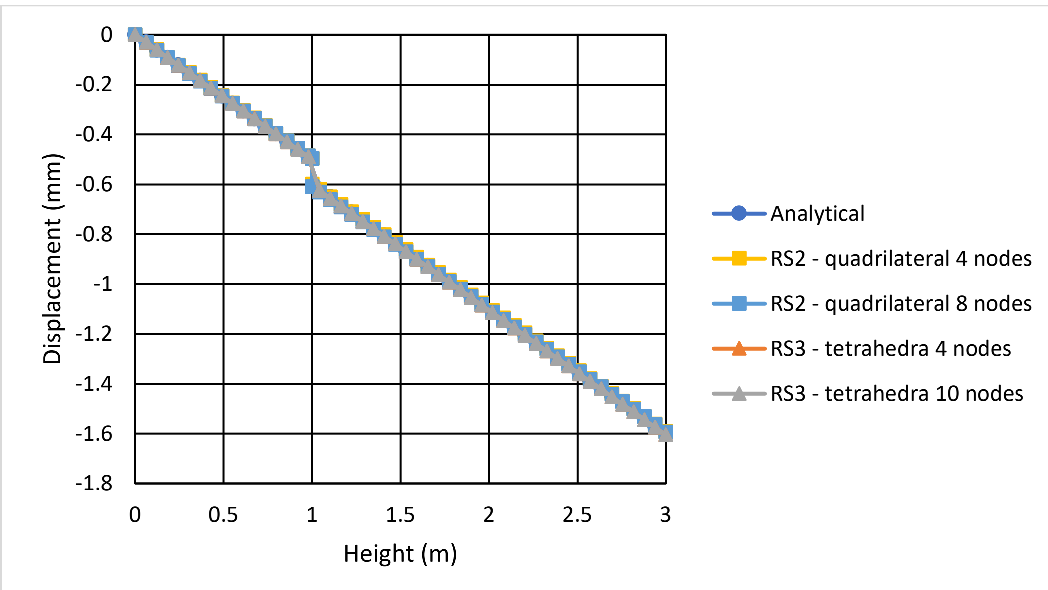


Figure 1-2 RS2 and RS3 Analytical and Numerical Displacement Fields Along Vertical Axis

1.4. References

1. Deb, Debasis & Das. Kamal Ch (2010), “Extended finite element method for the analysis of discontinuities in rock masses”. Geotech. Geol. Eng., Vol. 28, pp. 643-659

1.5. Data Files

The input data file **JointVerification-01.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

2. Single Jointed Rock Column Under the Distribution Load Along Vertical Axis

2.1. Problem Description

This problem concerns a one-dimensional bar of elastic rock material subjected to an axial distributed load along the vertical axis. The bar is loaded vertically with a uniform distributed load Pa = 10MPa/m and contains a joint at some distance z from the ground surface. In RS3, this situation was modeled using a narrow three-dimensional column with the parameters shown in Table 2-1. The uniform distributed load of 10 MPa/m was introduced into this model by changing the unit weight to 10 MPa/m. Figure 2-1 shows the completed model in RS2 and RS3. The problems are simulated with two types of elements available in RS3: 4-node and 10-node tetrahedra solid elements. The problems are simulated with two types of joint elements available in RS2: 4-node and 8-node quadrilateral solid elements.

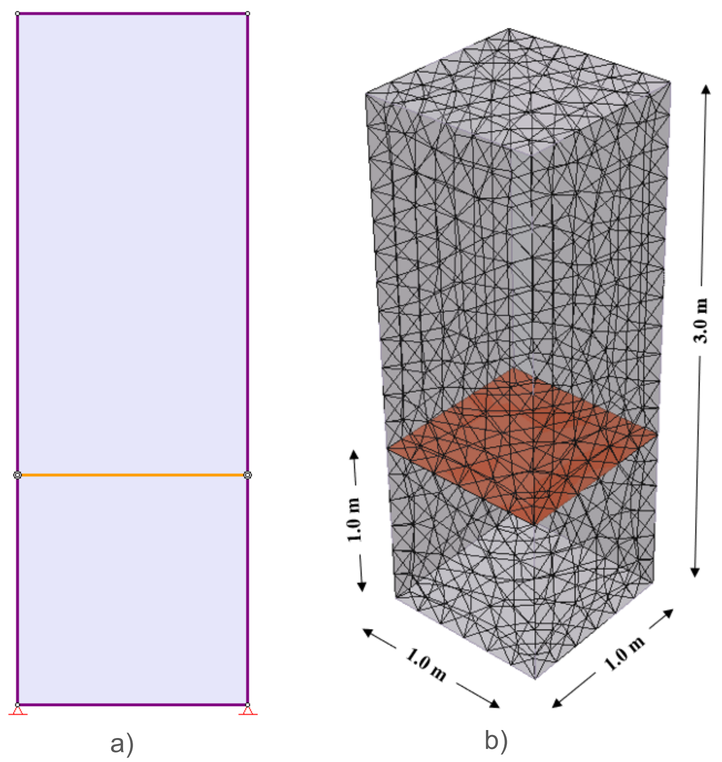


Figure 2-1 Problem Geometry (a) RS2 and in (b) RS3

The problem is using the same parameters that are listed in Table 2-1.

Table 2-1: Model Parameters

Parameter	***Value***
Young's modulus (E)	2000 MPa
Poisson's ratio	0.01
Height	3 m
Width and Depth	1 m
Joint properties	
Normal stiffness (k_{nn})	10 GPa/m
End condition	Open

2.2. Analytical Solution

Using integration along the column and accounting for the joint stiffness, the following equation was obtained and used to calculate the displacement along the rock column:

$$\mu(y) = \frac{PA(h^2 - y^2)}{2E} + \sum H(\frac{P}{k_{nm}})$$

where h is the column height, y is the distance of the considering point to the bottom of the rock column, P is the applied pressure, A is the area of the rock column, E is the elastic modulus of the column, and k_{nn} is the normal stiffness of the joint. H is a form of the Heaviside function; it takes on the value of its argument when y exceeds the height of the joint and otherwise returns zero.

2.3. Results

Figure 2-2 shows the displacement field of the single-jointed rock column along its vertical axis obtained by RS2 and RS3. The analytical results are also shown for reference. The jump in displacement at the joint matches well with the analytical solution. It can be seen that for each type of joint elements used the results obtained from RS2 and RS3 agree well with the analytical values.

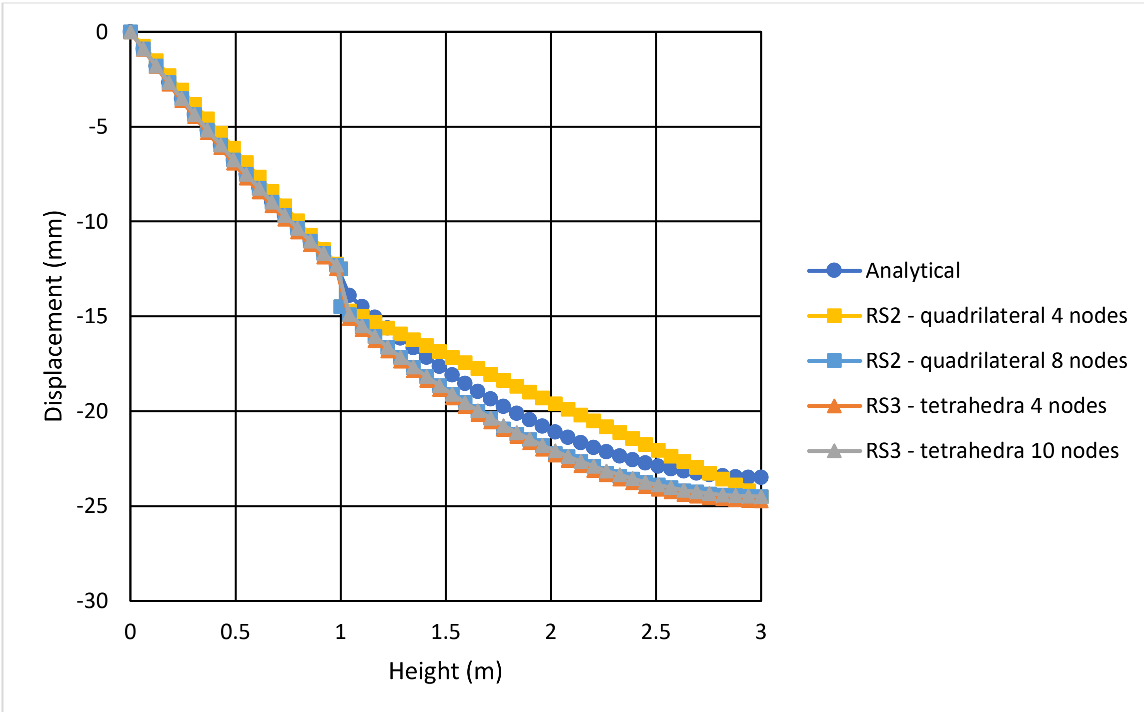


Figure 2-2 RS2 and RS3 Analytical and Numerical Displacement Fields Along Vertical Axis

2.4. References

1. Deb, Debasis & Das. Kamal Ch (2010), “Extended finite element method for the analysis of discontinuities in rock masses”. Geotech. Geol. Eng., Vol. 28, pp. 643-659

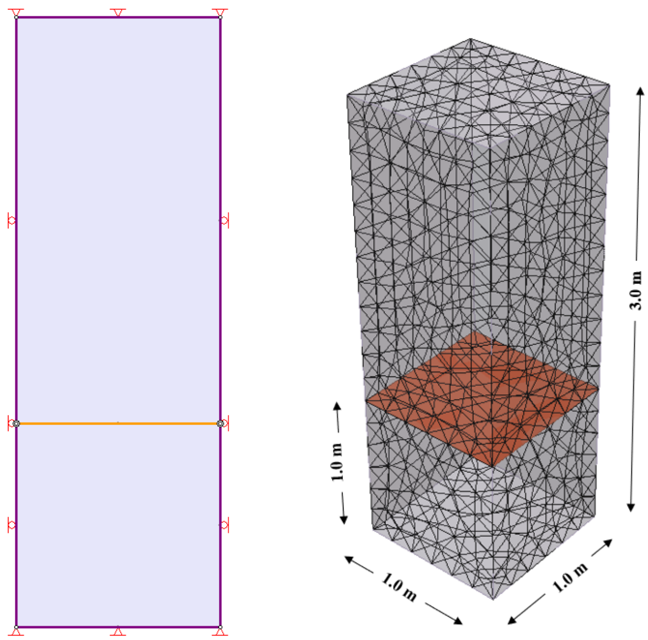
2.5. Data Files

The data folder **JointVerification-02** can be downloaded from the RS3 Online Help page for Verification Manuals.

3. Pressurized Joint

3.1. Problem Description

This problem concerns a one-dimensional bar of elastic rock material subjected to a pressure of P_a = 10 MPa at the joint. In RS3, this situation was modeled using a narrow three-dimensional column with the parameters shown in Table . Figure 3-1 shows the completed model in RS2 and RS3. The problems are simulated with two types of joint elements available in RS3: 4-node and 10-node tetrahedra solid elements. The problems are simulated with two types of joint elements available in RS2: 4-node and 8-node quadrilateral solid elements.



a) b)

Figure 3-1 Problem Geometry (a) RS2 and in (b) RS3

The problem is using the same parameters that are listed in Table 3.3-1

Table 3.3-1: Model Parameters

Parameter	***Value***
Young's modulus (E)	2000 MPa
Poisson's ratio	0.01
Height	3 m
Width and Depth	1 m
Joint properties	
Normal stiffness (k_{nn})	10 GPa/m
End condition	Open

3.2. Analytical Solution

Consider the joint and the solid elements acting as three springs, displacement can be obtained along the vertical axis of the rock column easily. k_1, k_2 and k_3 are the equivalent stiffness of the upper part, joint, and lower part respectively. The displacement will be the total of the pressure P in the two opposite directions as shown in Figure 3-2.

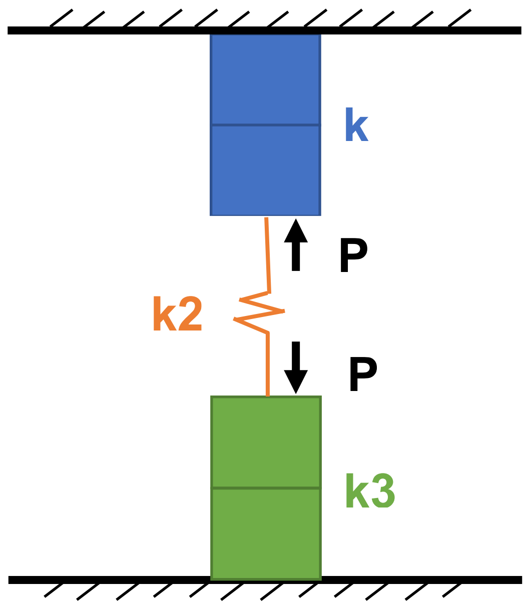


Figure 3-2 Analytical Model

3.3. Results

Figure 3-3 shows the displacement field of the single-jointed rock column along its vertical axis obtained by RS2 and RS3. The analytical results are also shown for reference. The displacement fields agree well with the analytical solution.

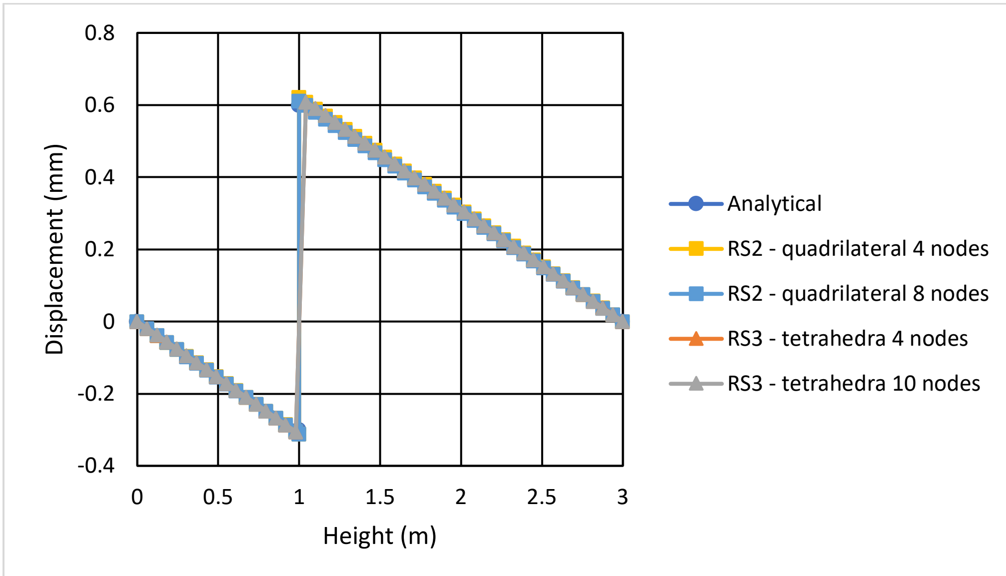


Figure 3-3 RS2 and RS3 Analytical and Numerical Displacement Fields Along Vertical Axis

3.4. References

1. Deb, Debasis & Das. Kamal Ch (2010), "Extended finite element method for the analysis of discontinuities in rock masses". Geotech. Geol. Eng., Vol. 28, pp. 643-659

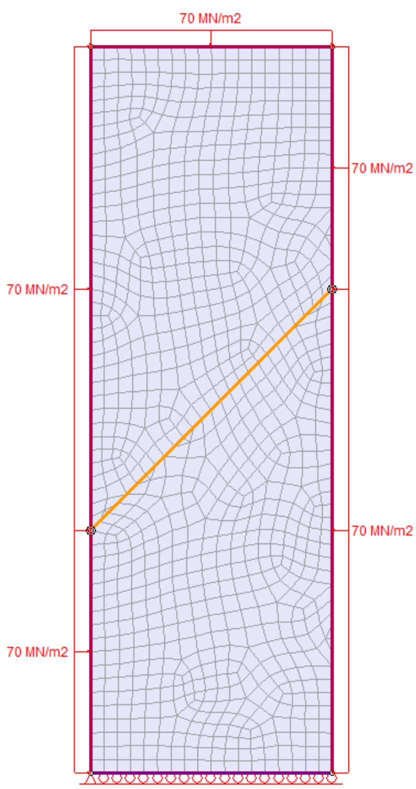
3.5. Data Files

The data folder **JointVerification-03** can be downloaded from the RS3 Online Help page for Verification Manuals.

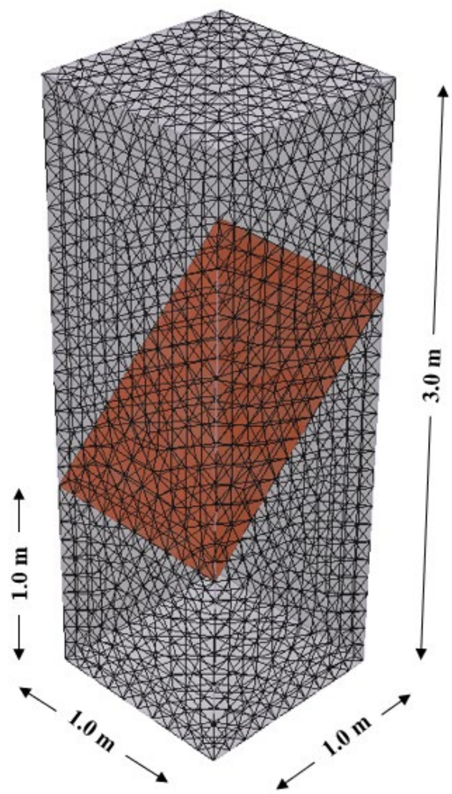
4. Triaxial Loading of a Jointed Rock Column (Mohr Coulomb Criterion)

4.1. Problem Description

This problem concerns an elastic rock column containing a single planar joint and subjected to triaxial loading. In RS3, this situation is modeled three-dimensionally as shown in Figure 4-1. The compressive strength of the column for various angles of the joint is of interest, assuming joint slip to be the mode of failure. Two cases were considered; both the secondary principal (horizontal) field stress and joint cohesion are varied. The shear strength of the joint is defined using the Mohr-Coulomb criterion. This problem was solved using an alternate computational method [1].



a)



b)

Figure 4-1 Problem Geometry (a) RS2 and in (b) RS3

Table 4-1 summarizes the material and joint properties used in the model.

Table 4-1: Model Parameters

Parameter	***Case 1***	***Case 2***
Young's modulus (E)	2000 MPa	
Poisson's ratio	0.3	
Height	3 m	
Width and Length	1 m	
Confining stresses	35 MPa	70 MPa
Joint properties		
Friction angle	30 Degrees	
Normal stiffness (k_{nn})	10 GPa/m	
Shear stiffness	1 GPa/m	
End condition	Open	

4.2. Analytical Solution

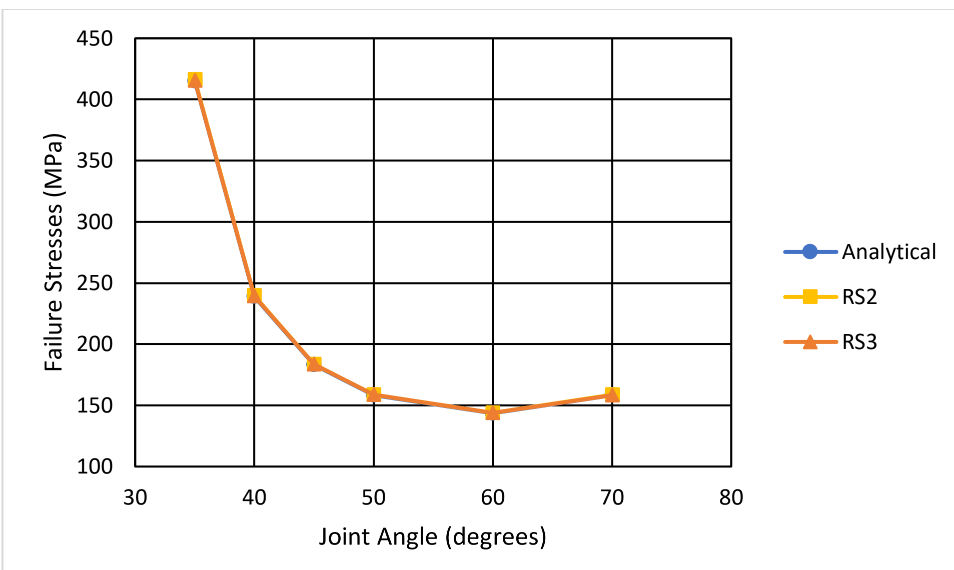
The primary (vertical) stress required for joint slip is given by the following equation [1]. Failure stress is a function of the friction angle and cohesion of the joint, as well as the joint angle.

$$\sigma_1 = \sigma_3 + \frac{2(\sigma_3 tan\emptyset + c)}{(1 - tan\emptyset cot\beta)sin2\beta}$$

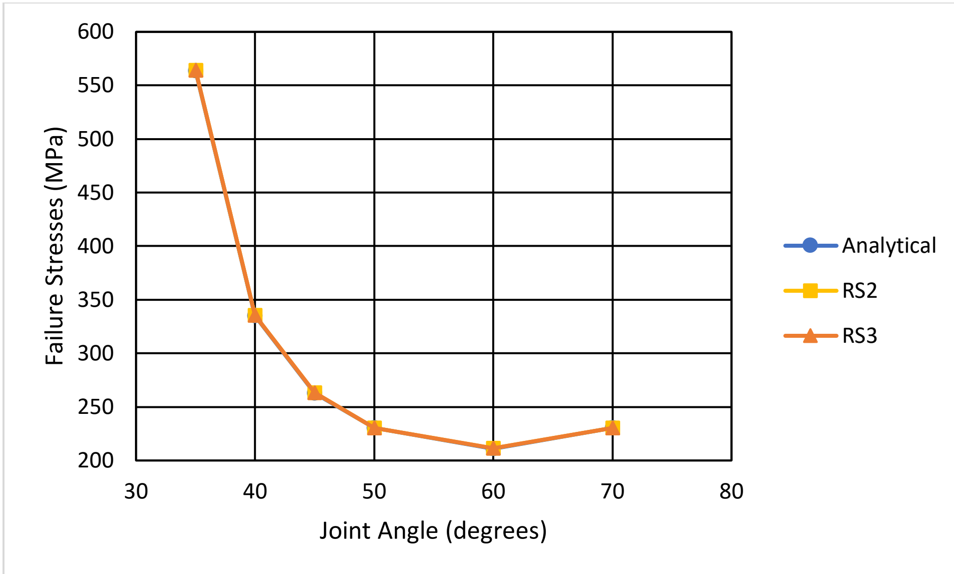
where c is the cohesion, ϕ is the friction angle of the joint, σ3 is the secondary principal stress, and β is the joint angle with reference to the horizontal axis.

4.3. Results

With joint angles of 35, 40, 45, 50, 60, and 70 degrees the analytical vertical stress σ_1 calculated through the analytical solution were compared to results acquired through RS2 as well as RS3. The results from RS2 and RS3 have a difference of less than 1% and are shown in Figure 4-2.



(a)



(b)

Figure 4-2 Failure Vertical Pressure. a) Case 1 b) Case 2

4.4. References

1. Deb, Debasis & Das. Kamal Ch (2010), "Extended finite element method for the analysis of discontinuities in rock masses". Geotech. Geol. Eng., Vol. 28, pp. 643-659

4.5. Data Files

The data folder **JointVerification-04** can be downloaded from the RS3 Online Help page for Verification Manuals.

5. Joint Constitutive Model: Hyperbolic Synthetic

5.1. Problem Description

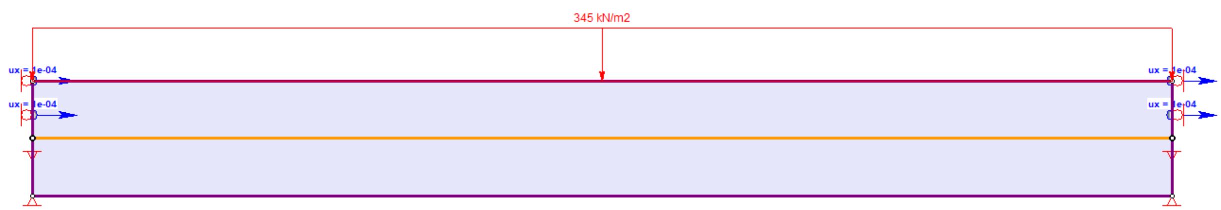
The geosynthetic Hyperbolic slip criterion can be used for modeling the shear strength of the interface between a geosynthetic (e.g geotextile or geogrid) and soil [1]. The model accounts for the softening of the geosynthetic by two methods: displacement softening and plastic work softening. Both methods were implemented in RS3. Generally, shear strength is defined by the following equation:

$$\tau = \frac{\sigma_\infty \sigma_n tan\emptyset_0}{\sigma_\infty + \sigma_n tan\emptyset_0}$$

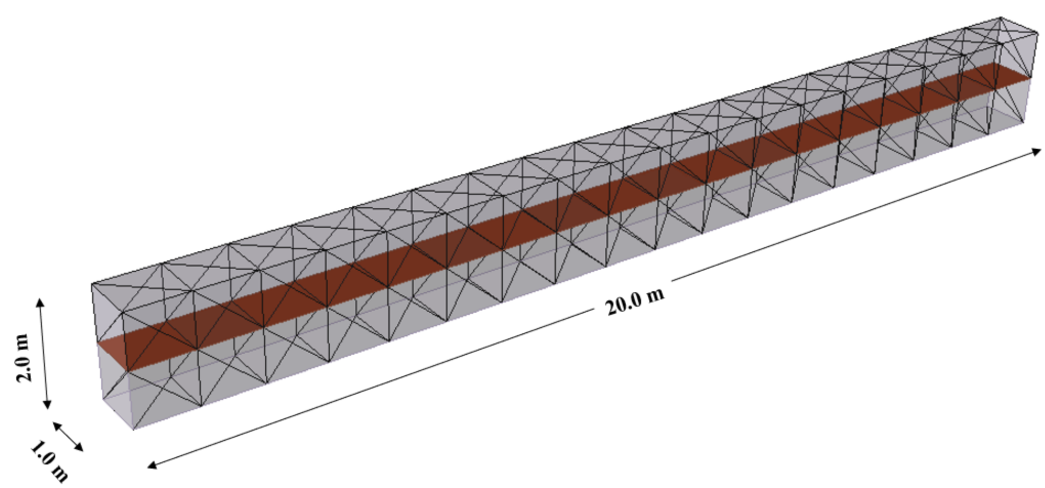
where σ_n is normal stress, σ_∞ is adhesion at $\sigma_n = \infty$, and $\emptyset_0$ is the interface friction angle at σ_n=0.

In addition to mentioned parameters, the following parameters are required for the model: residual friction angle $(\emptyset_r)$, residual adhesion (σ_r) , initial curve of the stress-strain displacement from experiment (k) and the plastic shear displacement that must take place to reach the residual strength (δ_r^p).

In order to verify the joint constitutive model, direct simple shear tests with large displacements were simulated [1]. Displacement was applied to one face of the joint and stress was measured with the corresponding displacement. Material properties used in the simulation are shown in Table 5-1. The direct shear tests were simulated in two cases: constant pressure (P = 345 kPa) and different pressures (P = 35 kPa and 345 kPa). Note that only in cases of vertical pressure changing dramatically, should the work softening method be chosen in order to capture soil-geosynthetic behavior.



(a)



(b)

Figure 5-1 Problem Geometry (a) RS2 and in (b) RS3

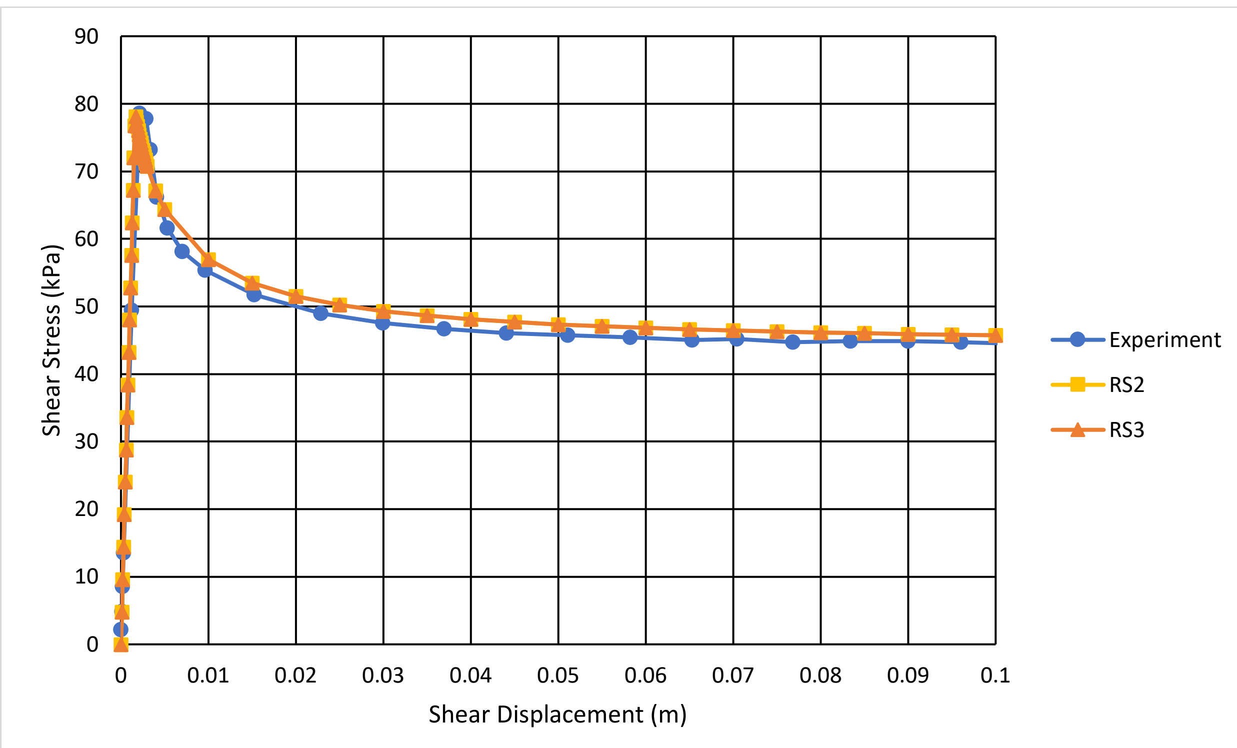
The problem is using the same parameters that are listed in Table 5-1.

Table 5-1: Model Parameters

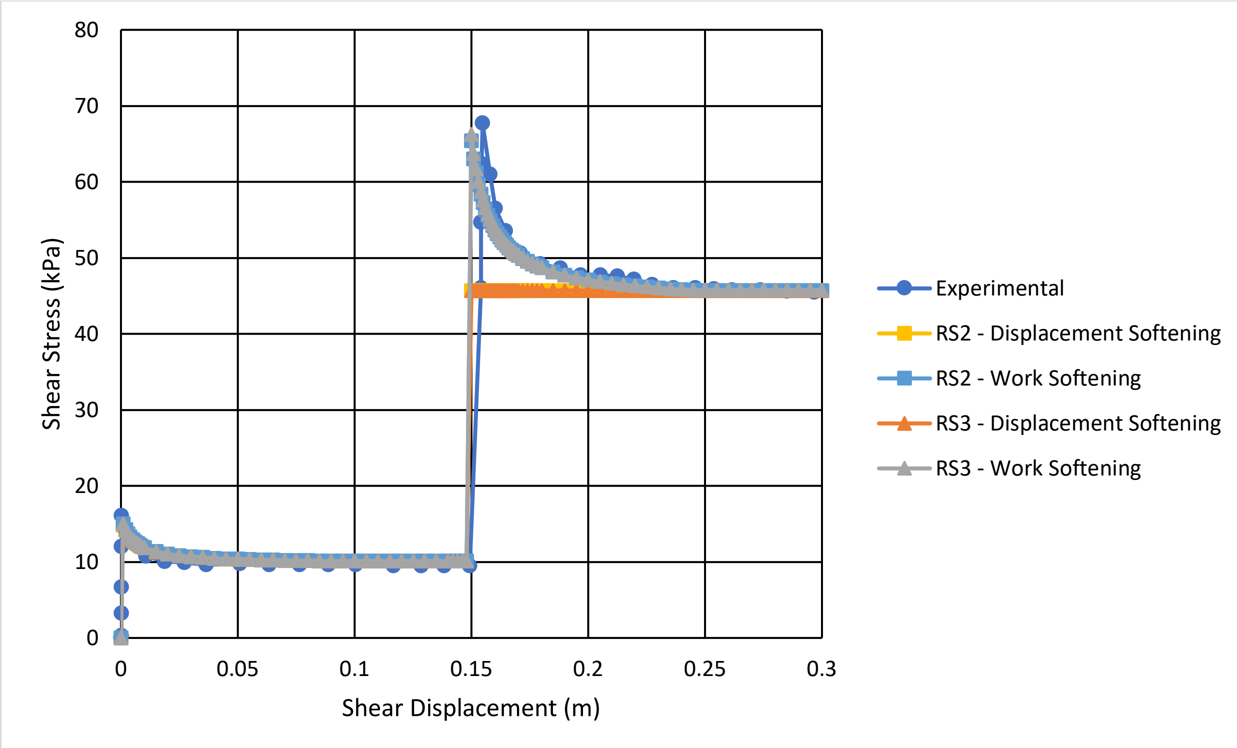
Parameter	***Value***
Peak adhesion (σ_{∞})	143 kPa
Residual adhesion (σ_r)	76 kPa
Peak friction angle ($\emptyset_0$)	26.8 Degrees
Residual friction angle ($\emptyset_r$)	18.4 Degrees
Initial stress strain curve slope (k)	20
Plastic displacement to reach residual strength (δ_r^p)	100 (mm)
Shear stiffness (K_s)	48 MPa/m

5.2. Results

Results obtained from RS2 and RS3 were compared with the experimental results in 2 cases: constant vertical stress and varied vertical stresses [1]. The results agree well with the experimental data. The displacement softening failed to capture the geosynthetic behavior when the applied pressure changed from 35 kPa to 345 kPa. Work softening can supplement for the displacement softening. The use of the work softening; however, it is only recommended when the vertical stress varies considerably because of the computational load associated with the work softening.



(a)



(b)

Figure 5-2 Stress-shear Displacement Curve. a) Constant pressure; b) Varied Pressures

5.3. References

1. Esterhuizen, J., Filz, G.M. and Duncan, J.M., Constitutive Behaviour of Geosynthetic Interfaces, Journal of GeoTechnical and Geoenvironmental Engineering, October 2001, pp 834-840

5.4. Data Files

The data folder **JointVerification-05** can be downloaded from the RS3 Online Help page for Verification Manuals.

6. Joint Constitutive Model: Mohr Coulomb with Residual Strength and Dilation

6.1. Formulation and Problem Description

6.1.1. Formulation

The joint constitutive model is the generalization of the Coulomb friction law. Both shear and tensile failure are considered, joint dilation and residual strength are also included.

In the elastic range, the behavior is governed by the joint normal and shear stiffnesses, k_n and k_s. Compression is negative.

The contact displacement increments are used to calculate the elastic force increments. The normal force increment and the shear force increment are updated using the following equations:

$$\Delta\sigma_n = k_n \Delta u_n$$

$$\Delta\tau = k_s \Delta u_s$$

The instantaneous loss of strength approximates the "displacement-weakening" behavior of a joint. The new force is corrected for

By tensile failure if $\sigma_n > T_{max}, \sigma_n = T_{residual}$

By shear failure if $\|\sigma_s\| > S_{max}, \|\sigma_s\| = S_{residual}$

where $S_{max} = c - \sigma_n tan\emptyset$ and $S_{residual} = c_{residual} - \sigma_n tan\emptyset_{residual}$

Dilation takes place only when the joint is at slip. The plastic shear displacement magnitude (Δu_s) is then calculated and the dilation displacement in the normal direction is then calculated by:

$$\Delta u_{n(dil)} = \Delta u_{s(plastic)} \tan(\gamma)$$

where γ is the dilation angle.

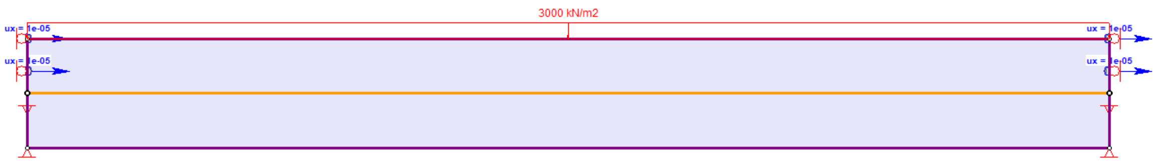
The normal force must be corrected to account for the effect of dilation:

$$\sigma_n = \sigma_n - k_n \Delta u_n$$

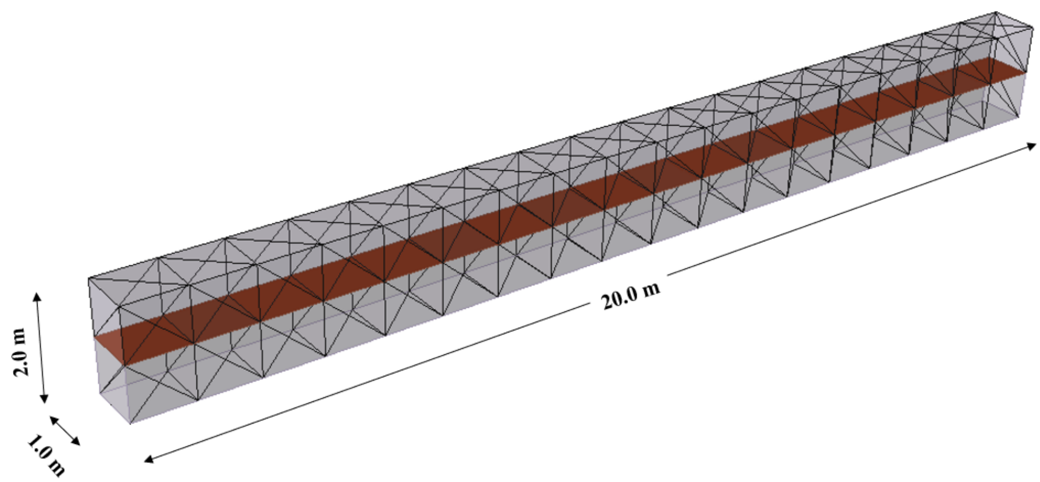
In RS3, directional dilation can be accounted for or can be ignored (i.e. joint will shrink if sheared in the opposite direction)

6.1.2. Problem Description

In order to verify the joint constitutive model, direct simple shear tests with large displacement were simulated in Figure 6-1. Displacement was applied to one face of the joint and stress was measured with the corresponding displacement.



(a)



(b)

Figure 6-1 Problem Geometry (a) RS2 and in (b) RS3

6.1.3. Residual Strength

To verify the residual strength, the direction shear test was simulated with four stages. At the first stage, a normal pressure of 3 MPa was applied to the surface. The direct shear test was performed until the shear displacement reached the value of 1mm. Then the normal pressure was increased to 9 MPa. The shear test was then continued until the shear displacement reach 2 mm. Material properties are shown in the Table 6-1.

Table 6-1: Model Parameters

Parameter	*Value*
Poisson's ratio	0.01
Cohesion	10 kPa
Friction angle	30 degrees
Normal stiffness (k_n)	10 GPa/m
Shear stiffness (k_s)	10 GPa/m
Dilation angle	0 degree
Residual cohesion	1kPa
Residual friction angle	15 degrees

6.1.4. Dilation

To verify the dilation angle, the shear test was simulated with four stages. At the first stage, a normal pressure of 3 MPa was applied to the surface. The direct shear test was performed until the shear displacement reached the value of 1mm. Then the normal pressure was increased to 9 MPa. The shear test was then continued until the shear displacement reach 2 mm. Four simulations were performed with different values of dilation angles (0, 10, 20 and 30 degrees). Material properties are shown in the Table 6-2.

Table 6-2: Model Parameters

Parameter	*Value*
Poisson's ratio	0.01
Cohesion	10 kPa
Friction angle	30 degrees
Normal stiffness (k_n)	10 GPa/m
Shear stiffness (k_s)	10 GPa/m
Dilation angle	0, 10, 20 and 30 degrees
Residual cohesion	10 kPa
Residual friction angle	30 degrees

6.1.5. Directional dilation

In order to compare the difference when accounting for directional dilation, a direct shear test was performed with a directional option on and off. A similar direct shear test as the previous section was performed. The only difference is that in the last stage, the sample was sheared in the opposite direction until it reached the value of 1mm in that direction. Material properties are shown in Table 6-3.

Table 6-3: Model Parameters

Parameter	***Value***
Poisson's ratio	0.01
Cohesion	10 kPa
Friction angle	30 degrees
Normal stiffness (k_n)	10 GPa/m
Shear stiffness (k_s)	10 GPa/m
Dilation angle	20 degrees
Residual cohesion	10 kPa
Residual friction angle	30 degrees

6.2. Result and Discussion

6.2.1. Residual strength

As shown in Figure 6-2, as the shear displacement increases, the shear stress increase until it reaches a peak value of 1.7 MPa and then dropped to a residual value of 0.805 MPa. Using Mohr Coulomb criterion obtained the same values from the simulation. Increasing normal pressure to 9MPa results in an increase of shear stress failure value. The failure stress of the joint increase to approximately 2.41 MPa which is greater than the residual value at normal stress of 3 MPa but still smaller than the peak value at 9 MPa normal pressure.

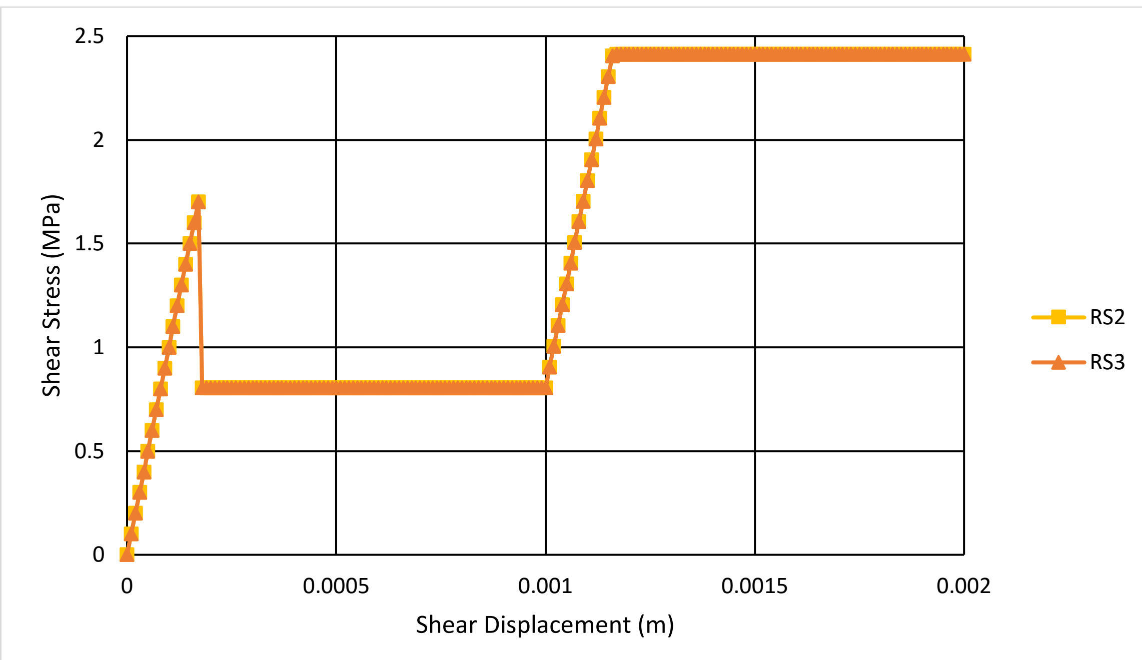


Figure 6-2 Mohr Coulomb Model: Residual Strength

6.2.2. Dilation

As shown in Figure 6-3 conducted through RS3, the angle between line of shear and normal displacement and the horizontal line is the dilation angle. At the first stage, the joint shrunk in the normal direction due to applied compressive pressure. As the joint slipped in stage 2, dilation occurred, and the normal displacement was proportional to the shear displacement. A higher dilation angle resulted in a more inclined line. Increasing normal pressure in stage 3 caused the joint to shrink. The shear displacement in stage 4 caused the dilation to occur again as it passed the critical shear displacement.

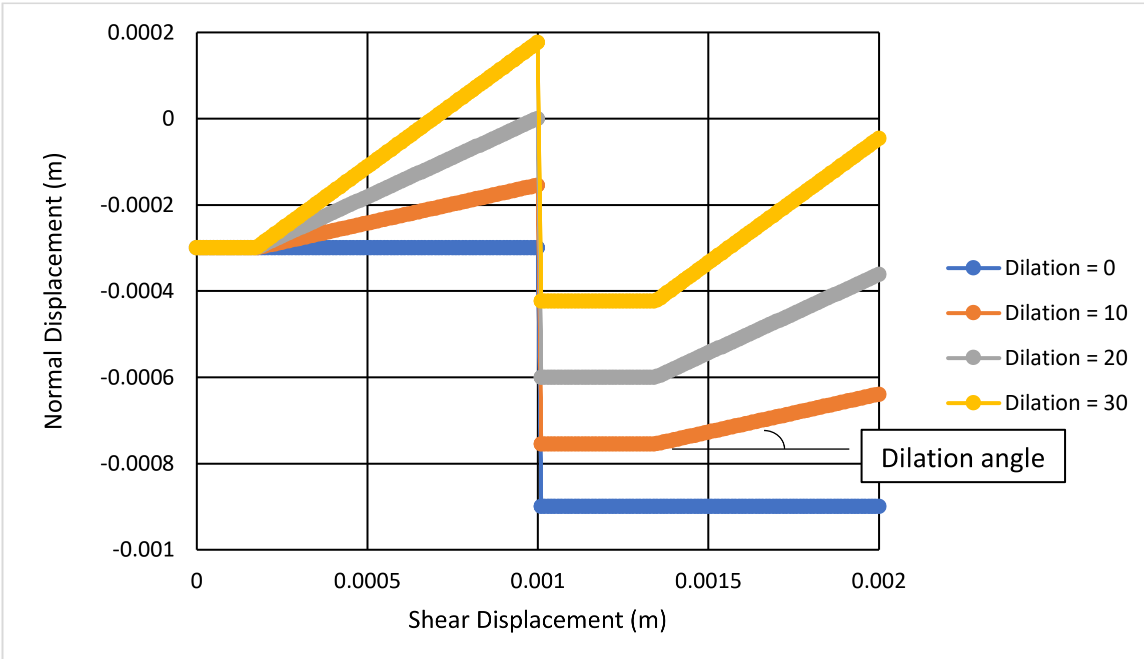


Figure 6-3 Mohr Coulomb Model: Dilation Angle (RS3)

6.2.3. Directional Dilation

Joint responses, calculated in RS3, corresponding to directional and non-directional dilation are shown in Figure 6-4. At the beginning, both options exhibited the same behavior until the shear displacement in the opposite direction was carried out. When the directional option was turned on, the joint shrunk if plastic shear displacement occurred in the opposite direction. As long as the plastic shear displacement in the opposite direction balanced to the plastic shear displacement in the previous direction, the joint dilated again. However, if the option was turn off, the joint kept dilating without considering the direction of the shear displacement.

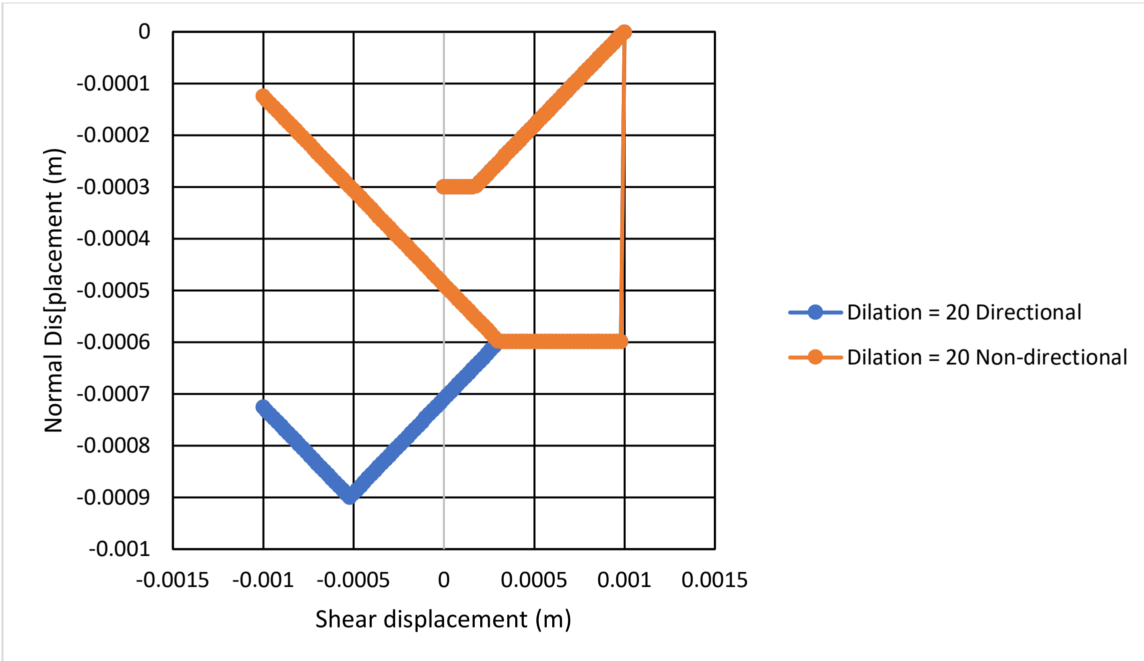


Figure 6-4 Mohr Coulomb Model: Directional Dilation (RS3)

6.3. References

1. Deb, Debasis & Das. Kamal Ch (2010), "Extended finite element method for the analysis of discontinuities in rock masses". Geotech. Geol. Eng., Vol. 28, pp. 643-659

2. Esterhuizen, J., Filz, G.M. and Duncan, J.M., Constitutive Behaviour of Geosynthetic Interfaces, Journal of GeoTechnical and Geoenvironmental Engineering, October 2001, pp 834-840.

6.4. Data Files

The data folder **JointVerification-06** can be downloaded from the RS3 Online Help page for Verification Manuals.

7. Alejano and Alonso Block Toppling

7.1. Problem Description

The work by Alejano and Alonso (2005) [1] presents the slope stability analysis of block toppling failure conducted using Goodman and Bray's limit equilibrium method (Figure 7-1). Furthermore, they discuss the result of that problem solved with 2D DEM (Figure 7-2). This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the block toppling failure mechanism at the safety factor specified by Alejano and Alonso (2005). Figure 7-3 shows the constructed RS2 and RS3 models.

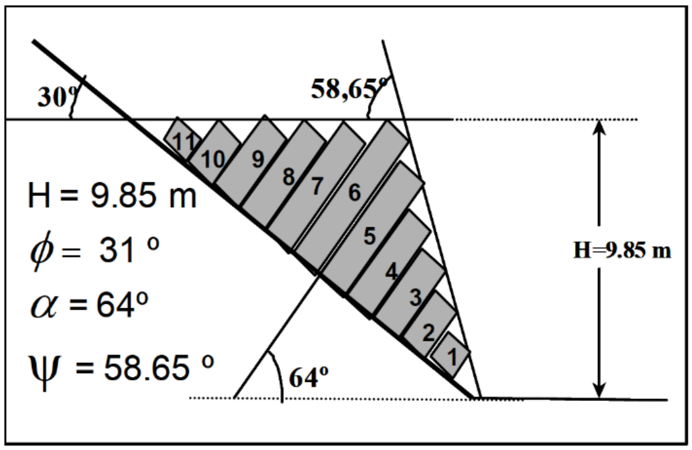


Figure 7-1 Goodman & Bray Geometry [1]

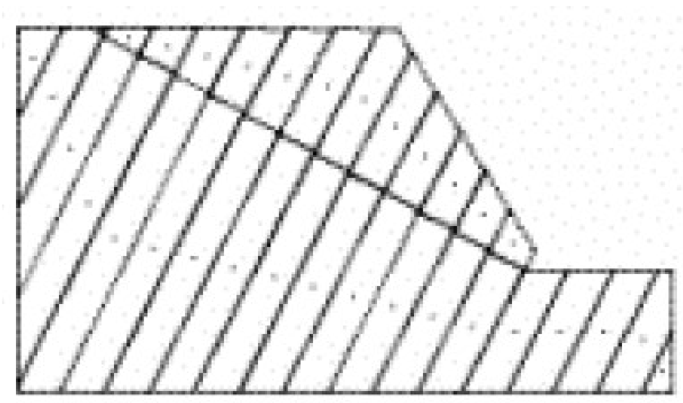
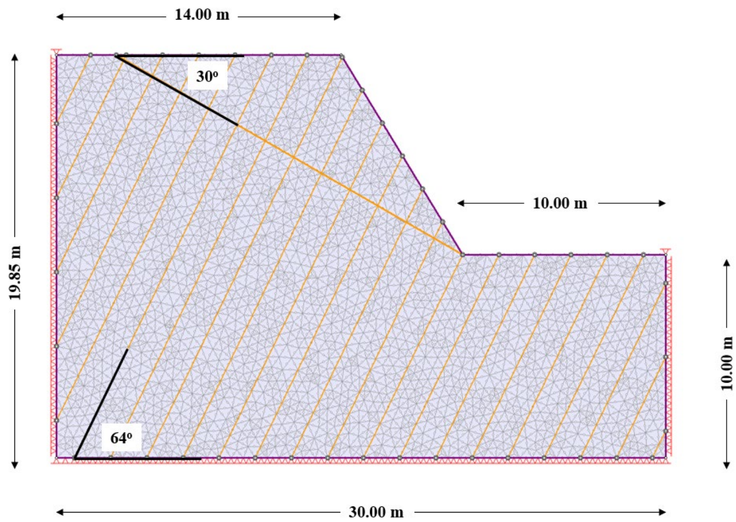
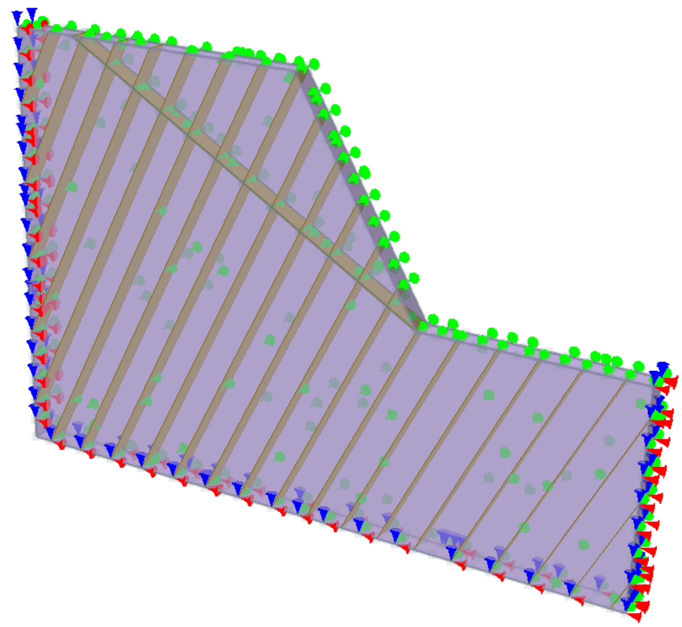


Figure 7-2 DEM model Geometry [1]



(a)



(b)

Figure 7-3 Problem Geometry (a) RS2 and in (b) RS3

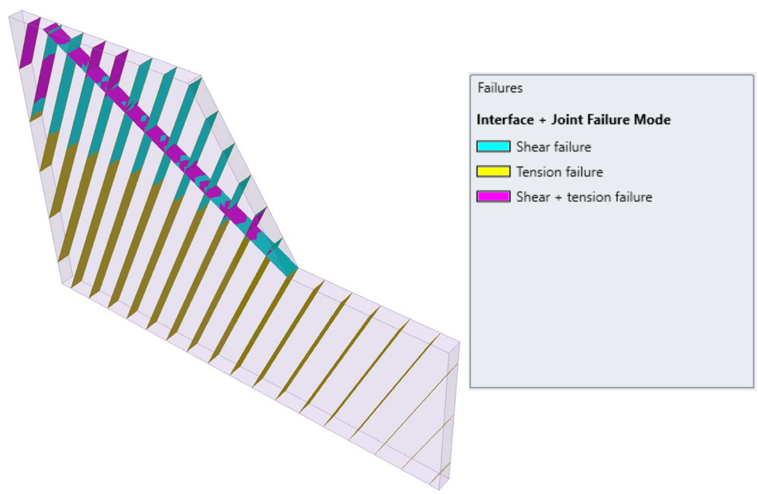
This problem focuses on the failure solely induced by joints. Hence, the failure of solid material is strictly restricted by implementing perfectly elastic material model. Considered geometrical and mechanical parameters are provided in Table 7-1.

Table 7-1: Model Parameters

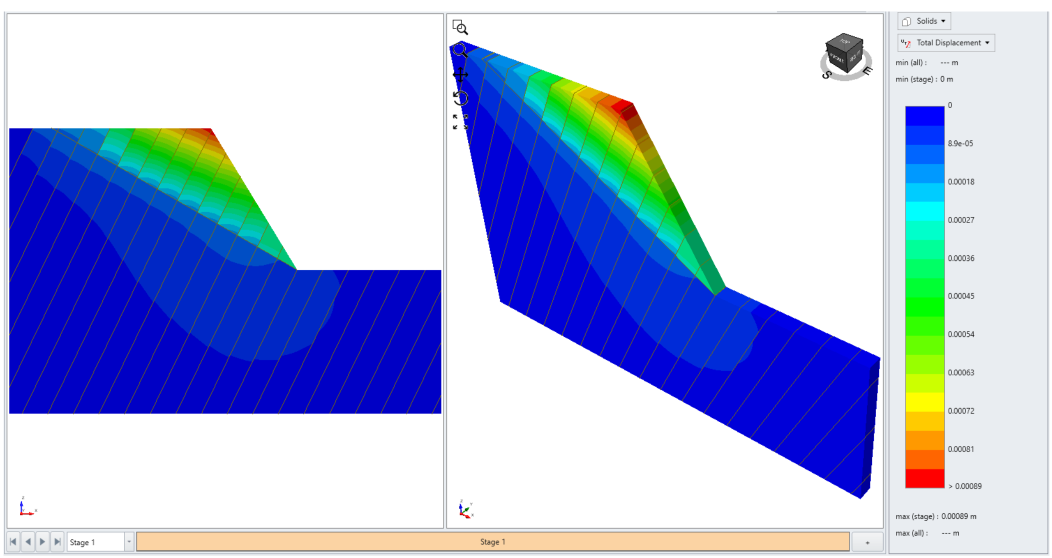
Parameter	***Value***
Slope height	9.85 m
Slope angle	58.65 degrees
Joint angle	64 degrees
Step surface	30 degrees
Peak friction angle (ϕ')	31 degrees
Unit weight (γ)	25 kN/m^3

7.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, the RS3 modeling results show that the slope reaches at its critical state at the Strength Reduction Factor (SRF) of 0.78 (critical SRF). Furthermore, the modeling results shows that the overall instability is caused by the failure of joint dipping at 30 degrees and large deformation is manifested on the blocks sitting above that joint (Figure 7-4).



(a)



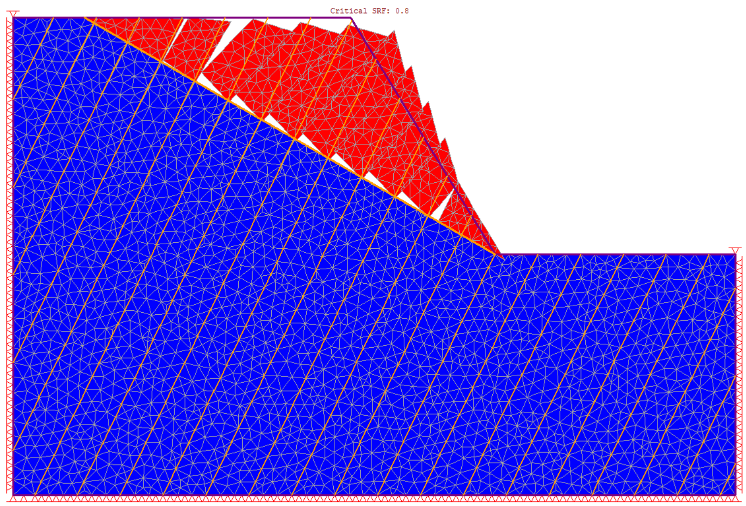
(b)

Figure 7-4 RS3 Modeling Result at SRF = 0.79, Showing (a) Joint Failure and (b) Total Displacement

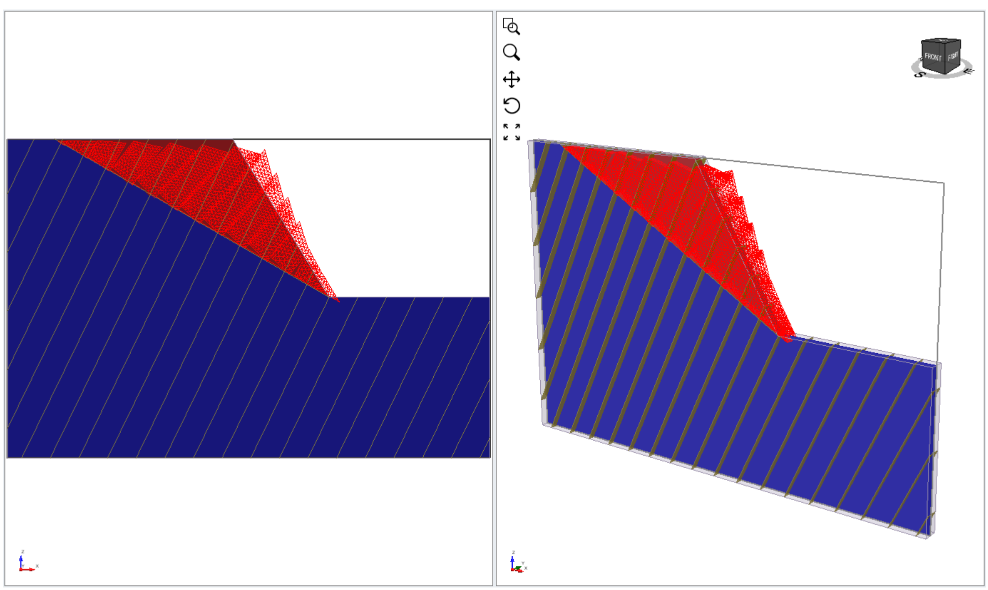
As presented in Table 7-2, factor of safety computed by RS3 (Critical SRF) is in agreement with that obtained from RS2 and other analysis methods. It is important to note that the forward block toppling failure mechanism is successfully captured (Figure 7-5) as the numerical investigation by [1] (Figure 7-6).

Table 7-2: Problem Factor of Safety

	Factor of Safety
Goodman	0.76
RS2	0.80
RS3	0.78
Alejano and Alonso (2005) [1]	0.87



(a)



(b)

Figure 7-5 Deformation captured from RS2 (a) and RS3 (b)

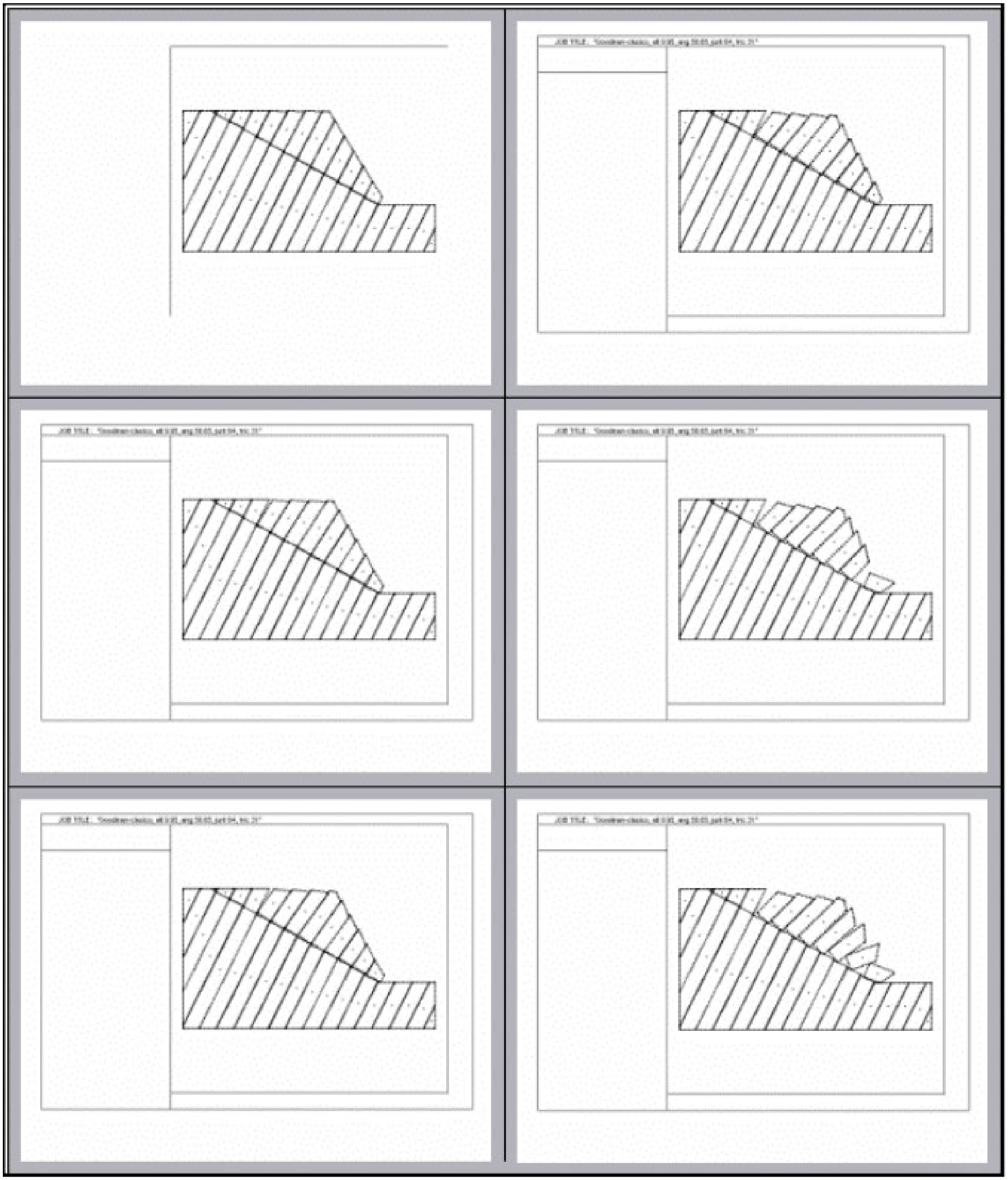


Figure 7-6 Evolution of the toppling captured from 2D DEM [1]

7.3. References

1. Alejano, L. R., & Alonso, E. (2005). Application of the 'Shear and Tensile Strength Reduction Technique' to Obtain Factors of Safety of Toppling and Footwall Rock slopes. Eurock: Impact of Human Activity on the Geological Environment.

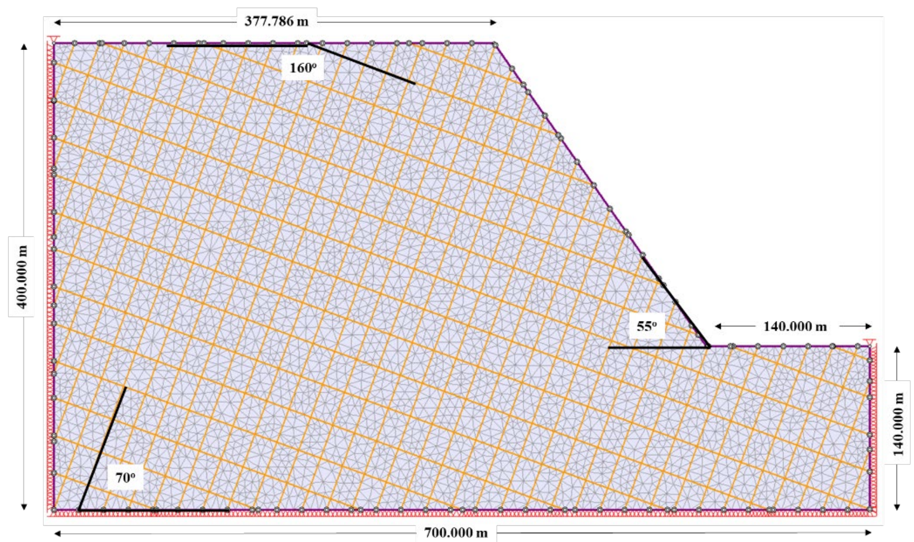
7.4. Data Files

The input data file **JointVerification-07.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

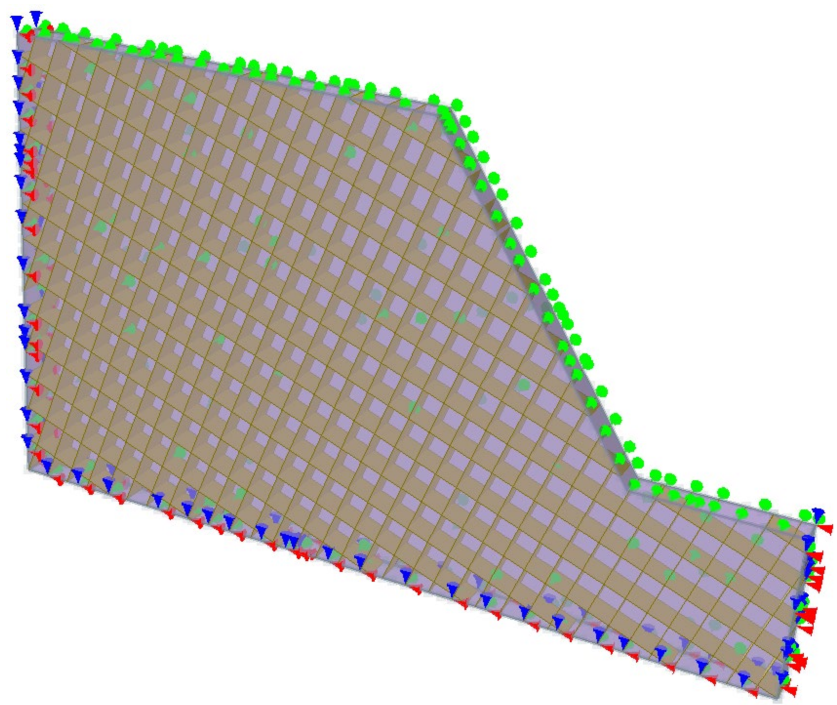
8. Lorig and Varona Forward Block Toppling

8.1. Problem Description

The work by Lorig and Varona (2004) [1] presents the slope stability analysis with the presence of two intersecting joint sets to simulate forward block toppling failure mode using 2D DEM. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the forward block toppling failure mechanism at the safety factor specified by Lorig and Varona (2004) [1]. Figure 8-1 shows the constructed RS2 and RS3 models.



(a)



(b)
Figure 8-1 Problem Geometry (a) RS2 (6 noded) and in (b) RS3 (10 noded)

This problem focuses on the failure solely induced by joints. Hence, the failure of solid material is strictly restricted by implementing perfectly elastic material model. Considered geometrical and mechanical parameters are provided in Table 8-1.

Table 8-1: Model Parameters

Parameter	***Value***
Slope height	260 m
Slope angle	55 degrees
Joint angle	70 and 160 degrees
Peak friction angle (ϕ') (joint)	40 degrees
Peak tensile strength (σ_t) (rock)	0 MPa
Unit weight (γ)	26.1 kN/m^3

8.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 1.15 and 1.14, respectively (critical SRF). As shown in Figure 8-2 and Figure 8-3, the mechanical behaviours captured from the two models are in alignment to one another but also with the numerical exercise by Lorig and Varona (2004) [1] (Figure 8-4).

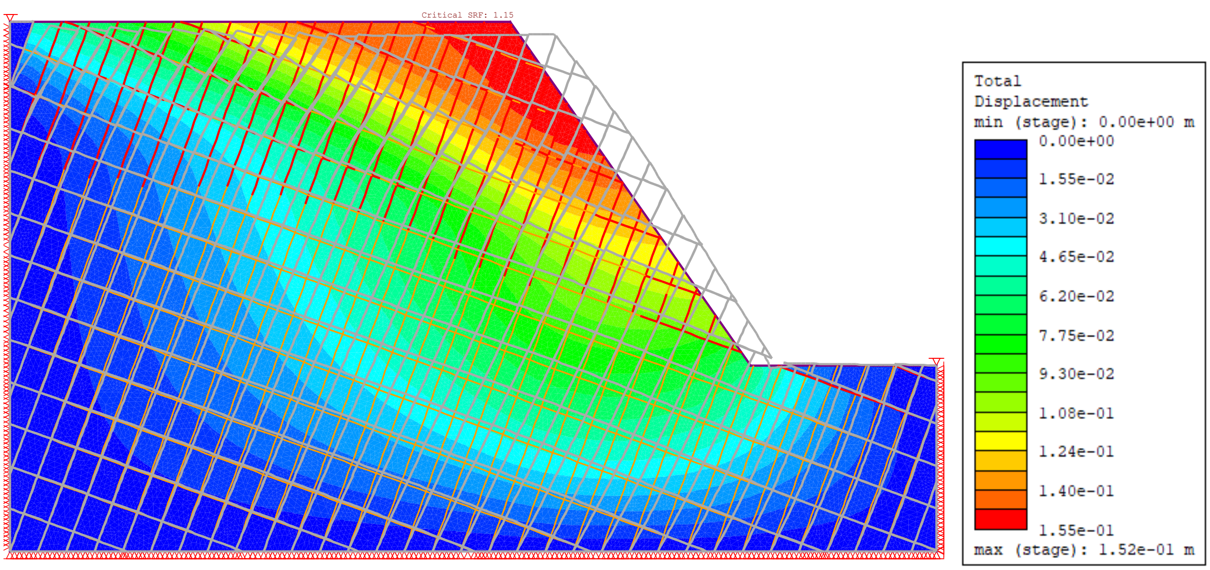
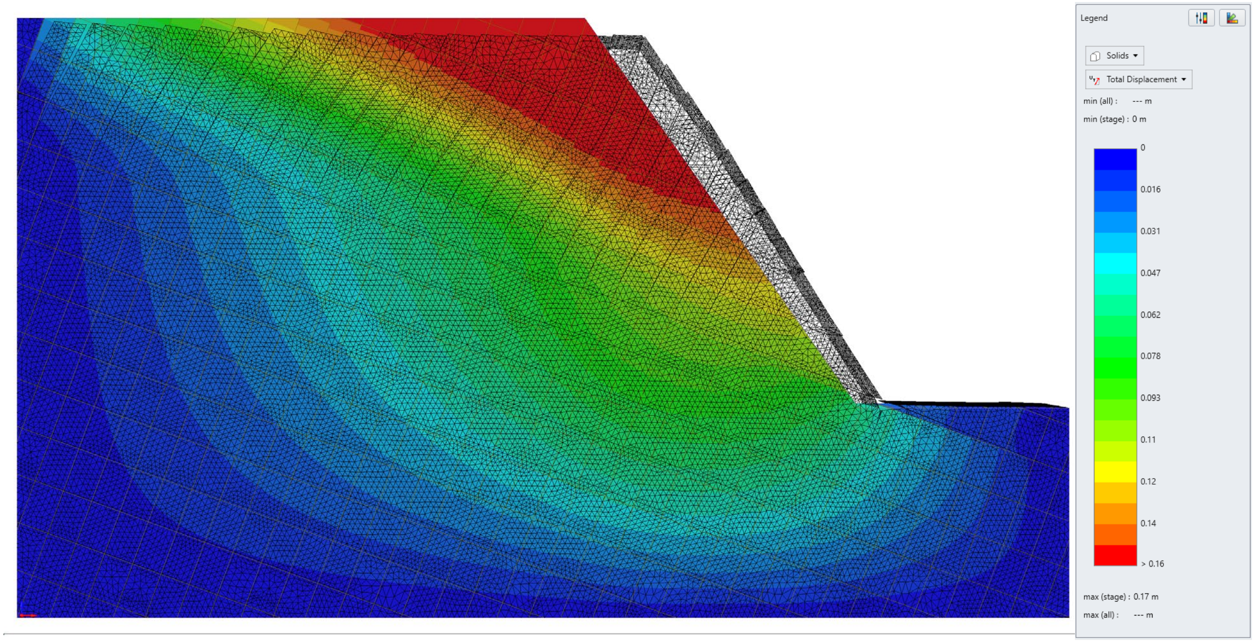
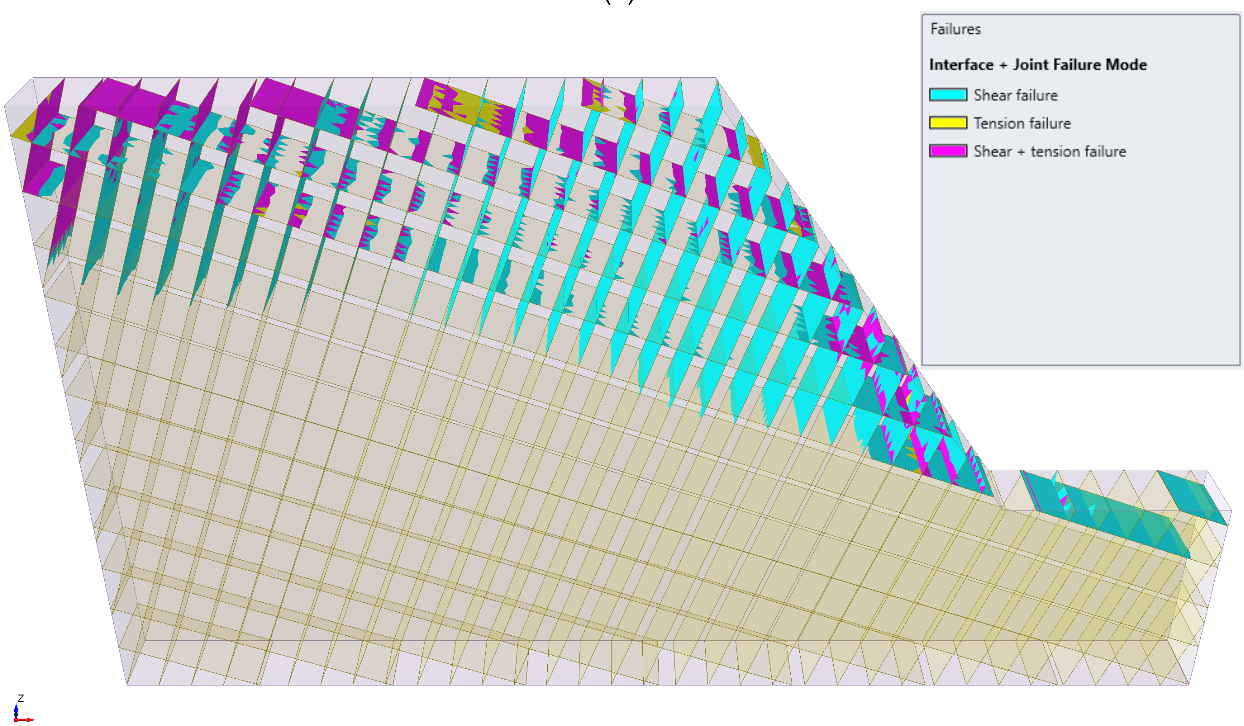


Figure 8-2 RS2 Total Displacement Contour Plot at SRF = 1.15



(a)



(b)

Figure 8-3 RS3 Modeling Result at SRF = 1.14 showing (a) Total displacement (b) Joint failure modes

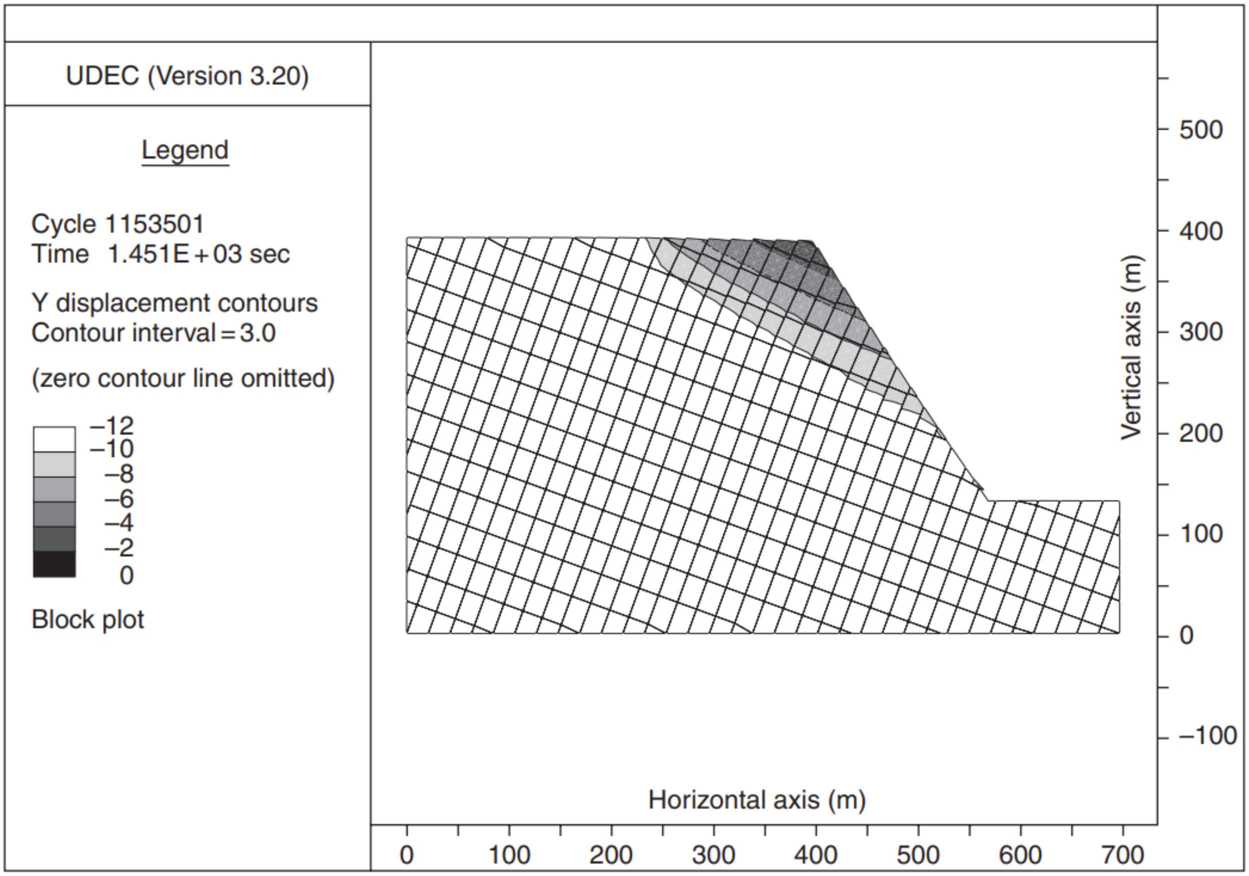


Figure 8-4 Y Displacement Contour Plot [1]

As presented in Table 8-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Lorig and Varona (2004) [1]. Moreover, it is important to note that the forward block toppling failure mechanism is successfully captured as driving mode of slope failure as it did with RS2 and [1].

Table 8-2: Problem Factor of Safety

	Factor of Safety
RS2	1.15
RS3	1.14
Lorig and Varona (2004) [1]	1.13

8.3. References

1. Lorig, L., & Varona, P. (2004). Toppling Failure - Block and Flexural. In D. C. Wyllie, & C. W. Mah, Rock Slope Engineering Civil and Mining 4th Edition (pp. 234-238). New York: Spon Press Taylor & Francis Group.

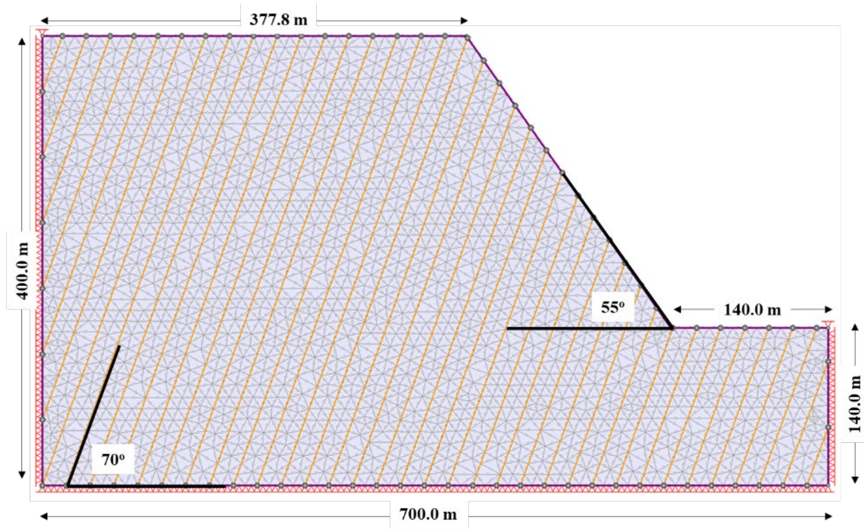
8.4. Data Files

The input data file **JointVerification-08.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

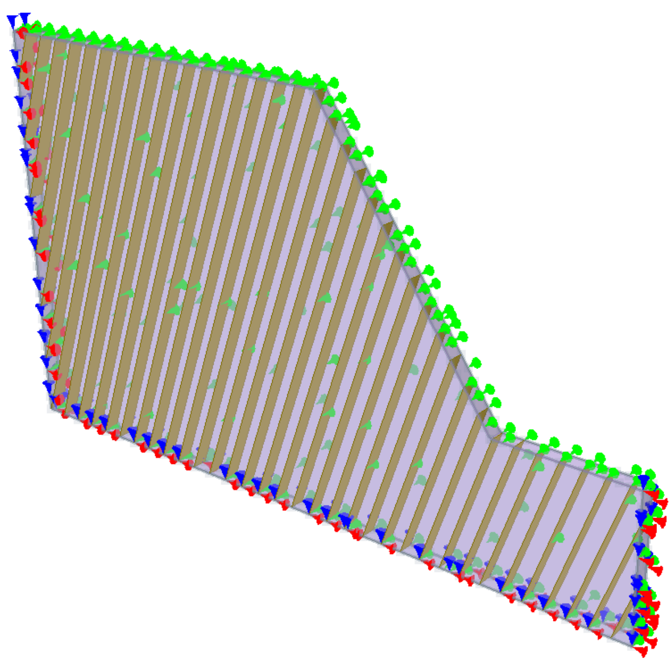
9. Lorig and Varona Flexural Toppling

9.1. Problem Description

The work by Lorig and Varona (2004) [1] presents the slope stability analysis with the presence of a steeply dipping joint set to simulate flexural toppling failure mode using 2D DEM. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the flexural toppling failure mechanism at the safety factor specified by Lorig and Varona (2004) [1]. Figure 9-1 shows the constructed RS2 and RS3 models.



(a)



(b)

Figure 9-1 Problem Geometry (a) RS2 and in (b) RS3

This problem involves simulating the failure of both rock and joints. Hence, elastic perfectly plastic material model is implemented to solid material to allow yielding to occur and peak strength is defined for joints. Considered geometrical and mechanical parameters are provided in Table 9-1.

Table 9-1: Model Parameters

Parameter	***Value***
Slope height	260 m
Slope angle	55 degrees
Joint angle	70 degrees
Joint peak cohesion (c)	0.1 MPa
Joint peak friction angle (ϕ')	40 degrees
Joint tensile strength (σ_t)	0 MPa
Rock peak cohesion (c)	0.675 MPa
Rock peak friction angle (ϕ')	43 degrees
Rock tensile strength (σ_t)	0 MPa
Unit weight (γ)	26.1 kN/m^3

9.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 1.33 and 1.30, respectively (critical SRF). The modeling outcomes reveal that the interaction between the shearing of the toppling joint and the deformation of the rock mass at depth leads to the bending of toppling blocks (Figure 9-2 and Figure 9-3). This indicates that the flexural toppling failure mechanism is the main driving factor of slope instability. The modeling results also show that the failure planes formed by RS2 and RS3 are in alignment with that captured from the work by Lorig and Varona (2004) [1] with 2D DEM (Figure 9-4).

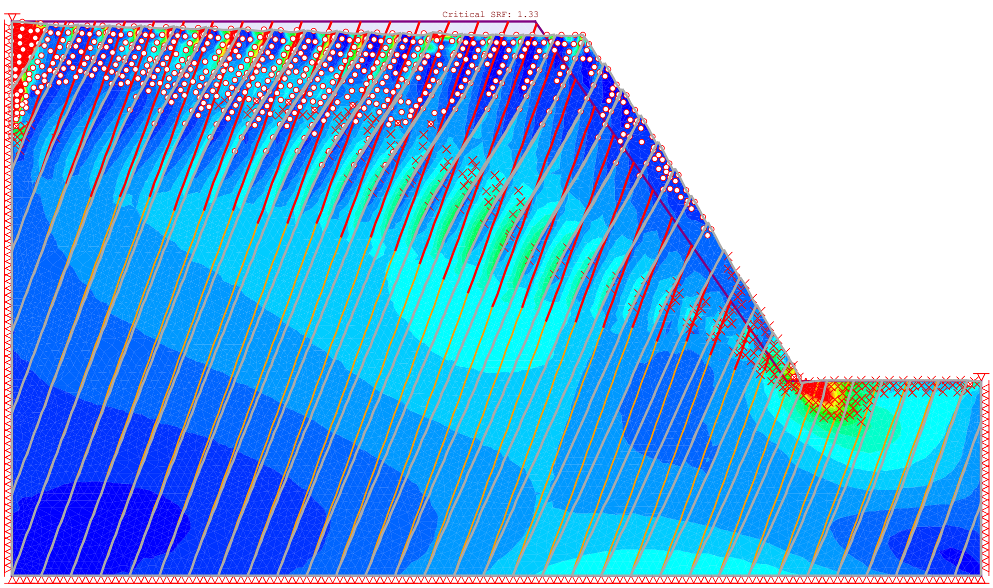
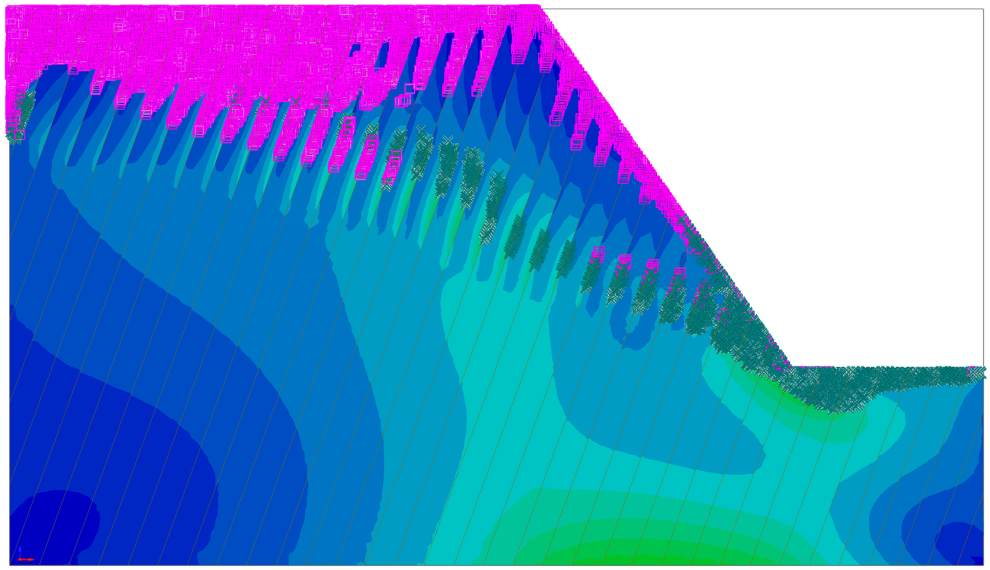
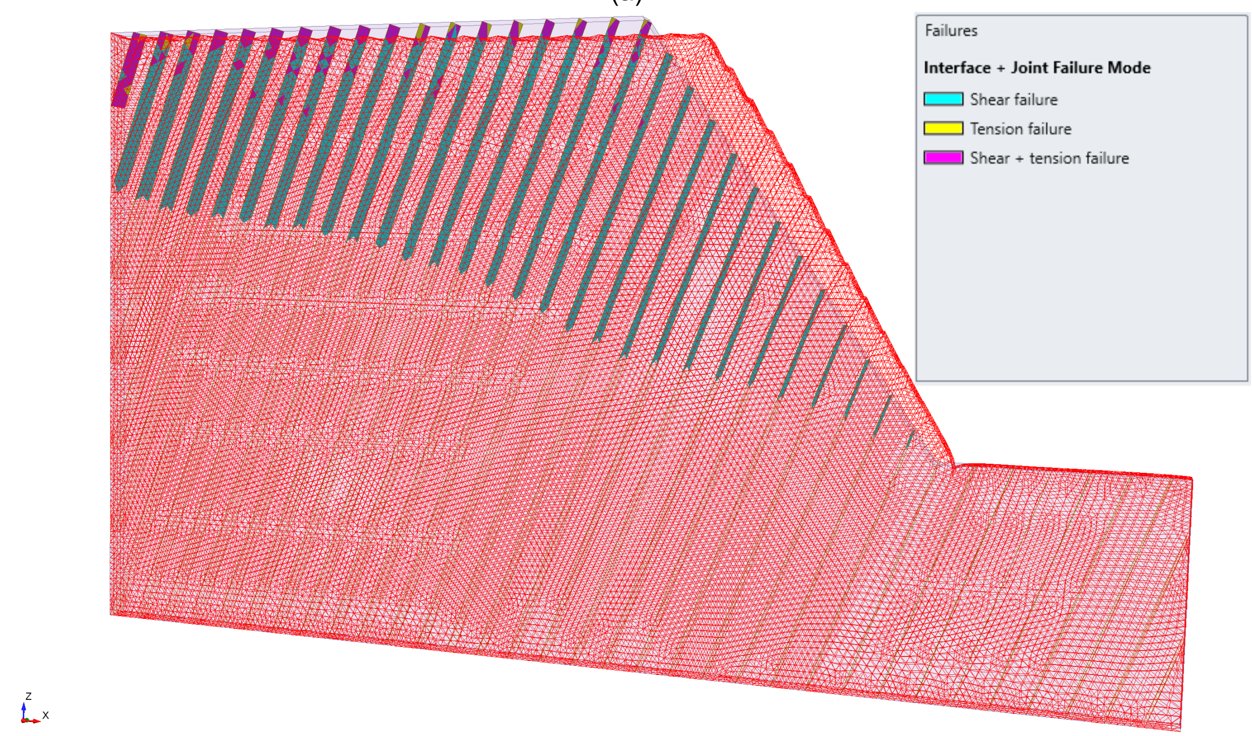


Figure 9-2 RS2 Maximum Shear Strain Contour Plot at SRF = 1.34 (Critical SRF = 1.33)



(a)



(b)

Figure 9-3 RS3 Modeling Result at SRF = 1.31 showing (Critical SRF = 1.30) (a) Maximum Shear Strain and (b) Joint Failure Modes and solid deformation

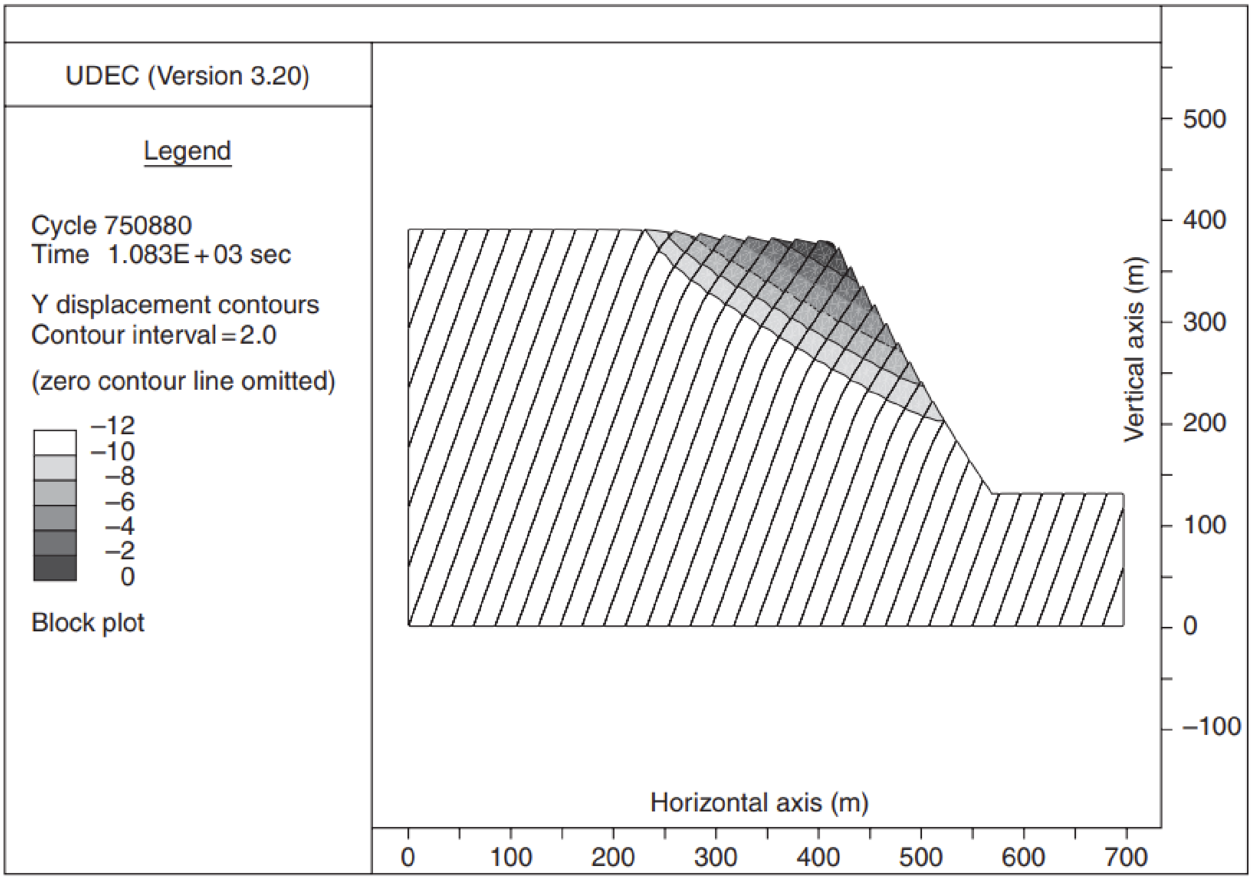


Figure 9-4 DEM modeling Results by [1]

As presented in Table 9-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Lorig and Varona (2004) [1]. Moreover, it is important to note that the flexural toppling failure mechanism is successfully captured as driving mode of slope failure as it did with RS2 and [1].

Table 9-2: Problem Factor of Safety

	Factor of Safety
RS2	1.33
RS3	1.30
Lorig and Varona (2004) [1]	1.30

9.3. References

1. Lorig, L., & Varona, P. (2004). Toppling Failure - Block and Flexural. In D. C. Wyllie, & C. W. Mah, Rock Slope Engineering Civil and Mining 4th Edition (pp. 234-238). New York: Spon Press Taylor & Francis Group.

9.4. Data Files

The data file **JointVerification-09.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

10. Lorig and Varona Backward Block Toppling

10.1. Problem Description

The work by Lorig and Varona (2004) [1] presents the slope stability analysis with the presence of two intersecting joint sets to simulate backward block toppling failure mode using 2D DEM. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the backward block toppling failure mechanism at the safety factor specified by Lorig and Varona (2004) [1]. Figure 10-1 shows the completed model in RS2 and RS3.

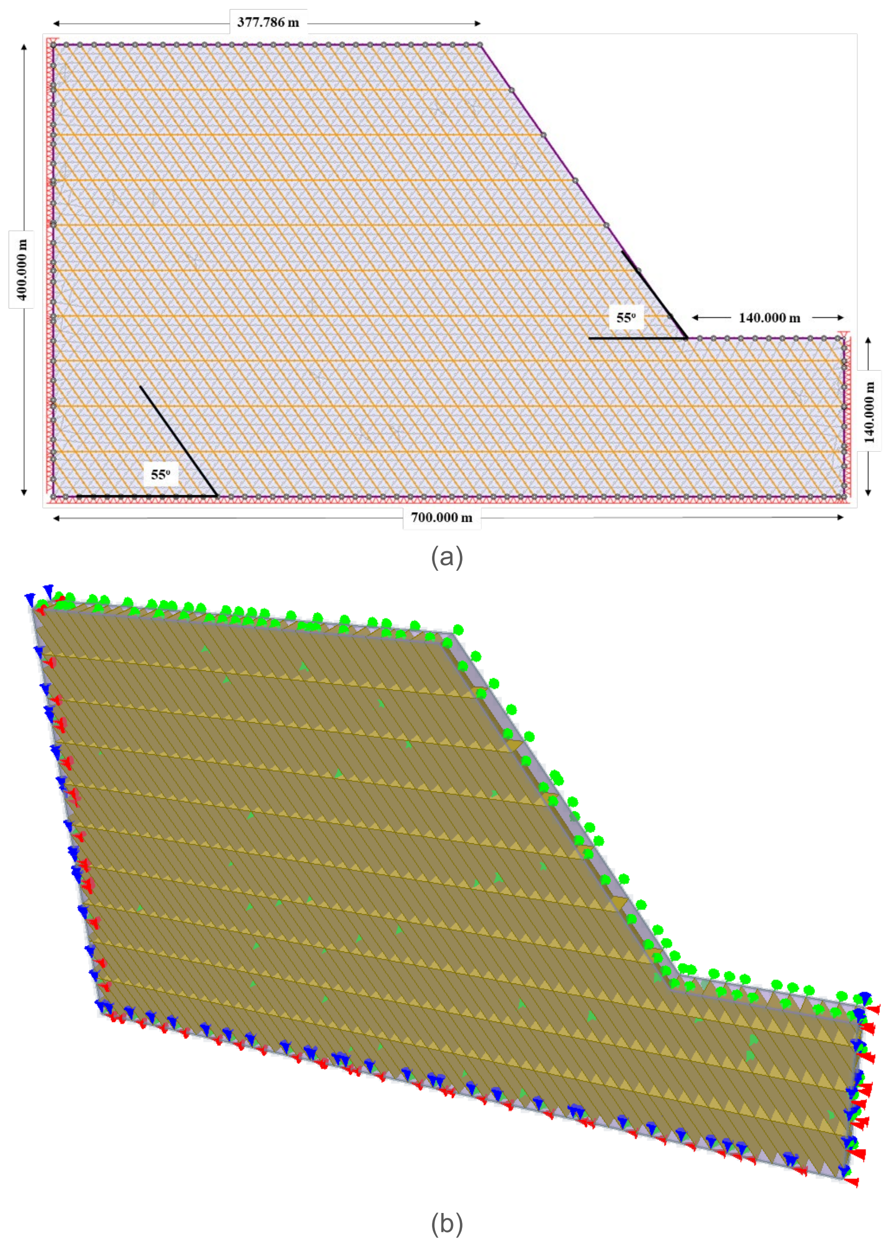


Figure 10-1 Problem Geometry (a) RS2 and in (b) RS3

This problem focuses on the failure solely induced by joints. Hence, the failure of solid material is strictly restricted by implementing perfectly elastic material model. Considered geometrical parameters of slope and joint parameters are provided in Table 10-1.

Table 10-1: Model Parameters

Parameter	***Value***
Slope height	260 m
Slope angle	55 degrees
Joint angle	55 and 0 degrees
Peak friction angle (ϕ') (joint)	40 degrees
Peak tensile strength (σ_t) (rock)	0 MPa
Unit weight (γ)	26.1 kN/m^3

10.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 1.87 and 1.90, respectively (critical SRF). As shown in Figure 10-2 and Figure 10-3, the mechanical behaviours captured from the two models are in alignment. Moreover, similar behaviour was captured from the work by Lorig and Varona (2004) [1] with 2D DEM (Figure 10-4).

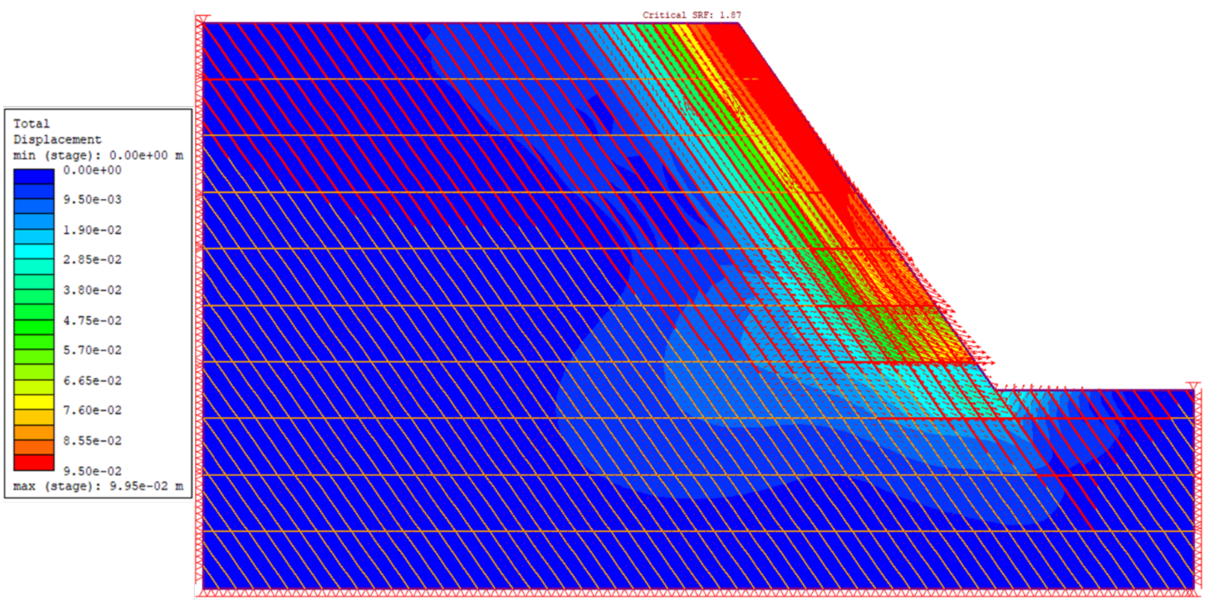
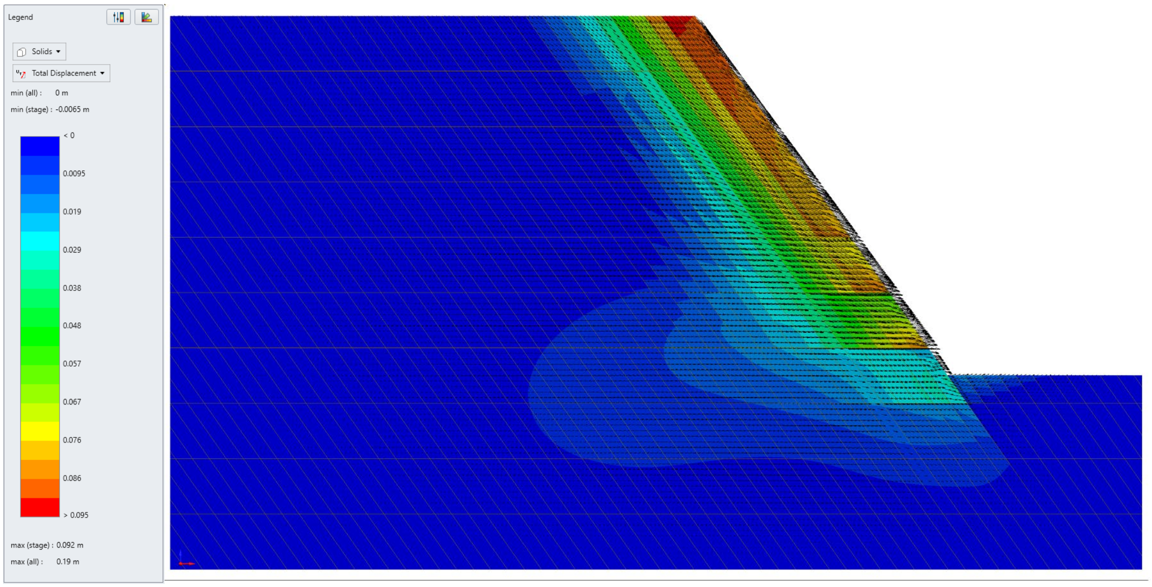
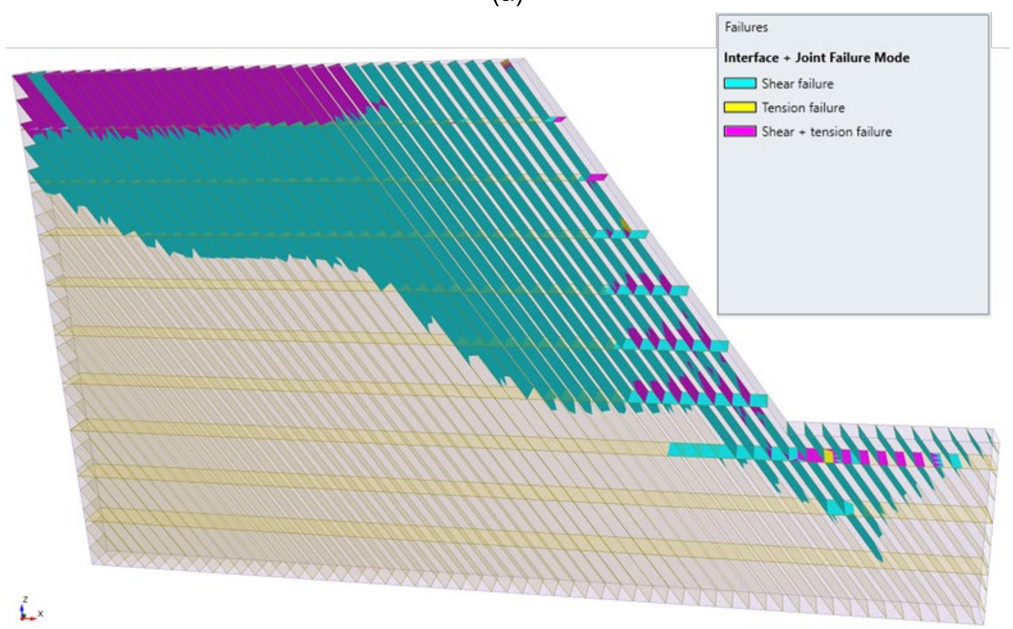


Figure 10-2 RS2 Total Displacement Contour Plot at SRF = 1.87



(a)



(b)

Figure 10-3 RS3 Modeling Result at SRF = 1.87 showing (a) Total displacement (b) Joint failure modes

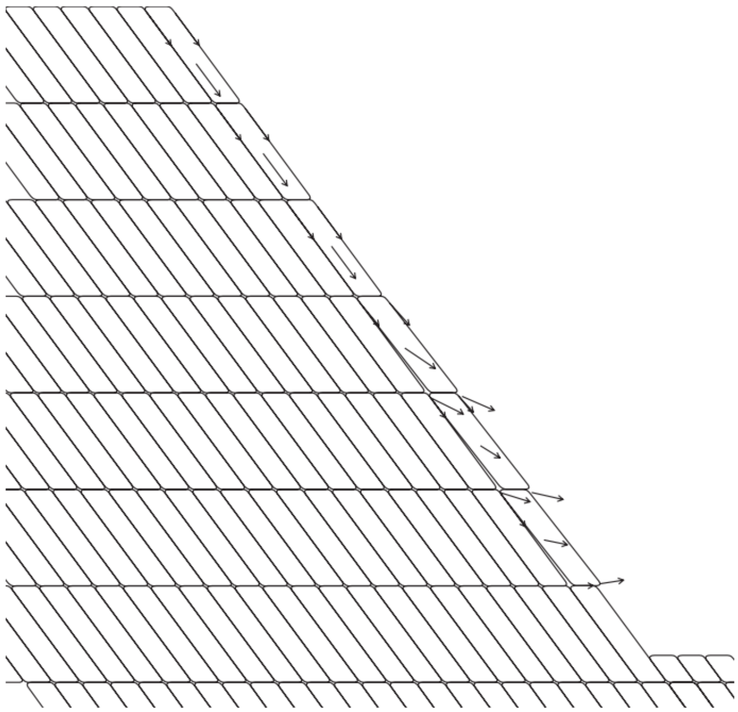


Figure 10-4 Block Displacement Vector Diagram [1]

As presented in Table 10-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Lorig and Varona (2004) [1]. Moreover, it is important to note that the backward block toppling failure mechanism is successfully captured as driving mode of slope failure as it did with RS2 and [1].

Table 10-2: Problem Factor of Safety

	Factor of Safety
RS2	1.87
RS3	1.9
Lorig and Varona (2004) [1]	1.7

10.3. References

1. Lorig, L., & Varona, P. (2004). Toppling Failure - Block and Flexural. In D. C. Wyllie, & C. W. Mah, Rock Slope Engineering Civil and Mining 4th Edition (pp. 234-238). New York: Spon Press Taylor & Francis Group.

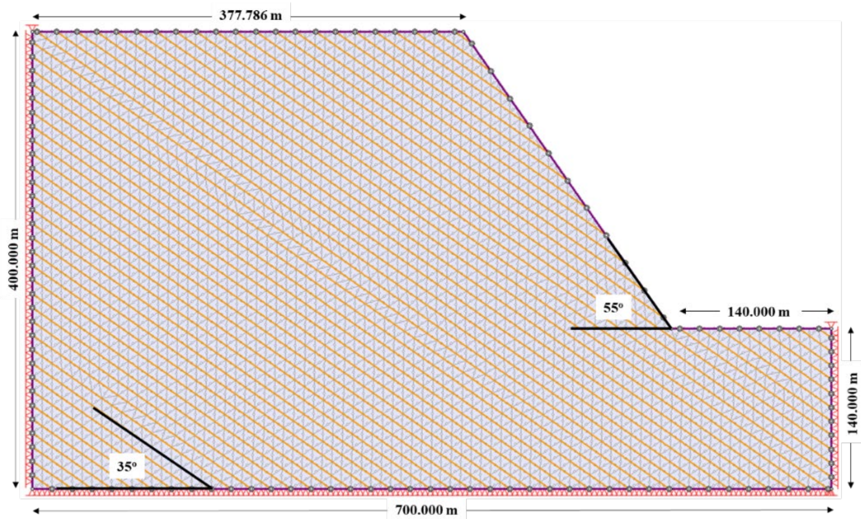
10.4. Data Files

The input data file **JointVerification-10.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

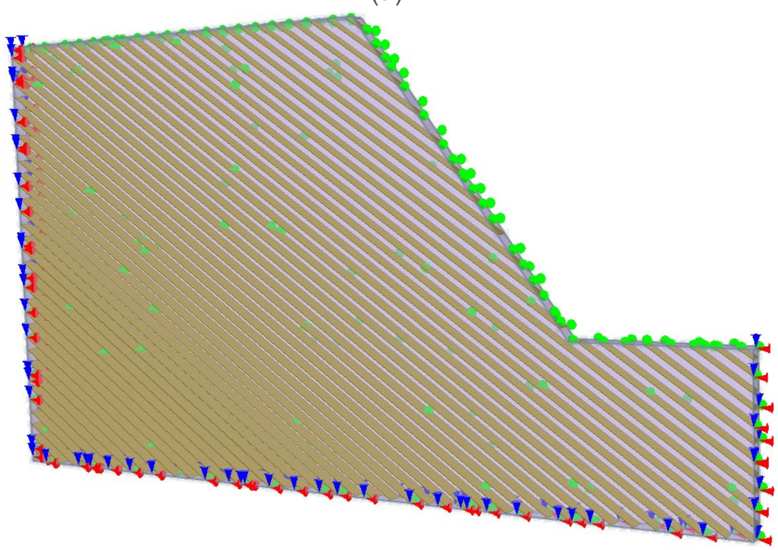
11. Lorig and Varona Plane Failure with Daylighting Discontinuities

11.1. Problem Description

The work by Lorig and Varona (2004) [1] presents the slope stability analysis with the presence of a joint set subparallel (but at lower inclination) to the slope to simulate daylighting plane failure mode using 2D DEM. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the daylighting plane failure mechanism at the safety factor specified by Lorig and Varona (2004) [1]. Constructed models are presented in Figure 11-1.



(a)



(b)

Figure 11-1 Problem Geometry (a) RS2 and in (b) RS3

This problem involves simulating the failure of both rock and joints. Hence, elastic perfectly plastic material model is implemented to solid material to allow yielding to occur and peak strength is defined for joints. Considered geometrical and mechanical parameters are provided in Table 11-1.

Table 11-1: Model Parameters

Parameter	**Value**
Slope height	260 m
Slope angle	55 degrees
Joint angle	70 degrees
Joint peak cohesion (c)	0.1 MPa
Joint peak friction angle (ϕ')	40 degrees
Joint tensile strength (σ_t)	0 MPa
Rock peak cohesion (c)	0.675 MPa
Rock peak friction angle (ϕ')	43 degrees
Rock tensile strength (σ_t)	0 MPa
Unit weight (γ)	26.1 kN/m^3

11.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 1.32 and 1.31, respectively (critical SRF). As presented by shear strain contour plots (Figure 11-2 and Figure 11-3), the planar failure is driven by shearing of solid elements in upper portion and continues with joints that intersects the slope surface. The modeling results also show that the failure planes formed by RS3 is in alignment with that captured by RS2 and the work by Lorig and Varona (2004) [1] with 2D DEM.

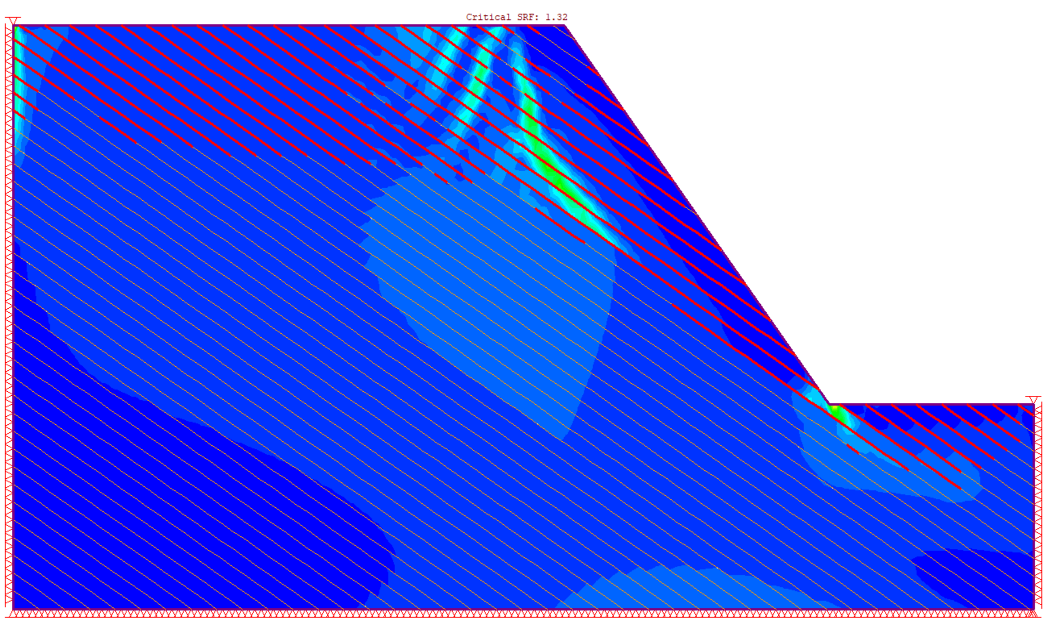
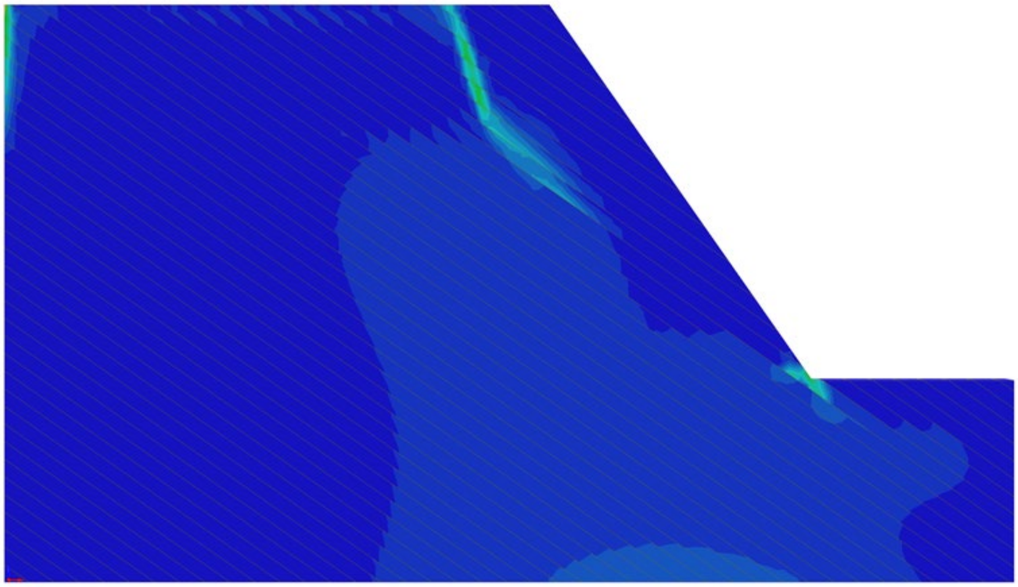
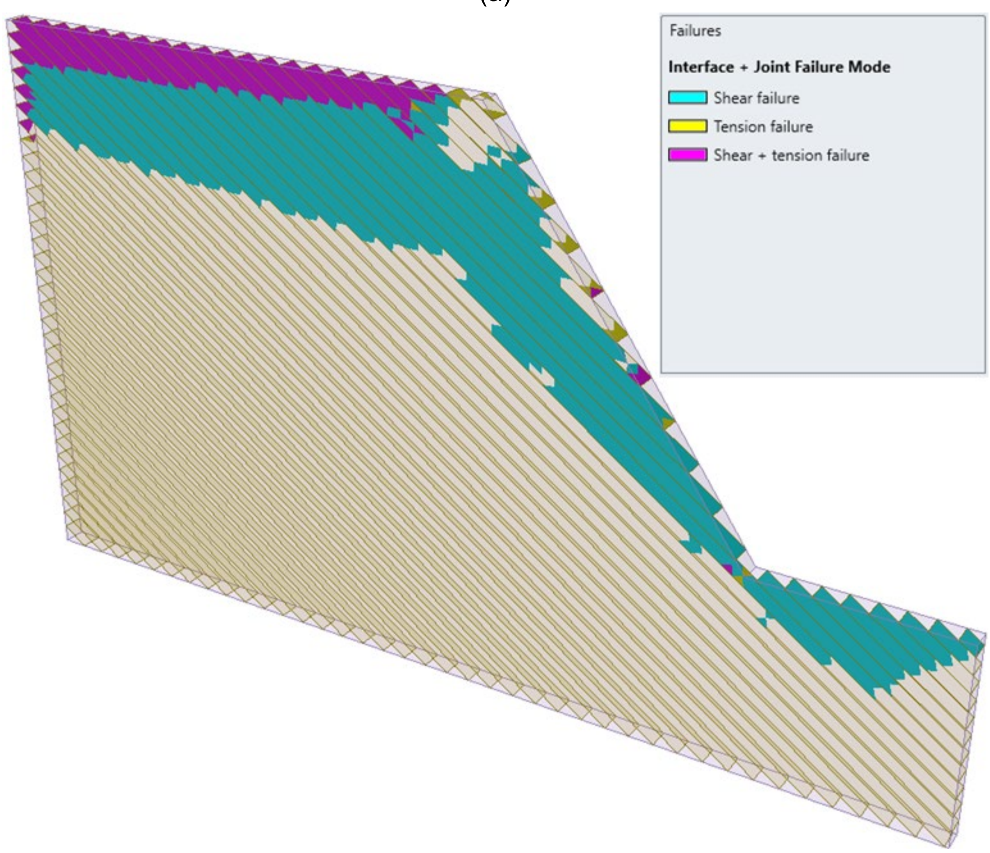


Figure 11-2 RS2 Maximum Shear Strain Contour Plot at SRF = 1.33 (Critical SRF = 1.32), yielded joint represented by thick red lines



(a)



(b)

Figure 11-3 RS3 Modeling Result at SRF = 1.32 showing (Critical SRF = 1.31) (a) Maximum Shear Strain (b) Joint Failure Modes

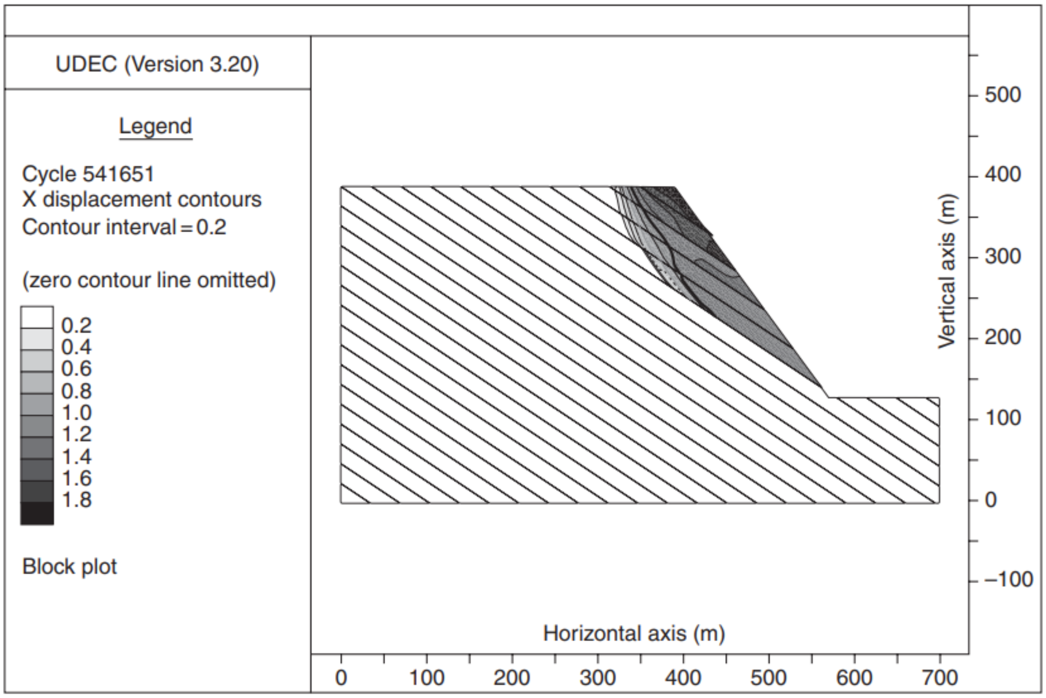


Figure 11-4 DEM modeling Results by [1]

As presented in Table 11-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Lorig and Varona (2004) [1]. Moreover, it is important to note that the forward daylighting planar failure mechanism is successfully captured as driving mode of slope failure as it did with RS2 and [1].

Table 11-2 Problem Factor of Safety

	Factor of Safety
RS2	1.32
RS3	1.31
Lorig and Varona (2004) [1]	1.27

11.3. References

1. Lorig, L., & Varona, P. (2004). Plane Failure – Daylighting and Non-Daylihgting. In D. C. Wyllie, & C. W. Mah, Rock Slope Engineering Civil and Mining 4th Edition (pp. 233-235). New York: Spon Press Taylor & Francis Group.

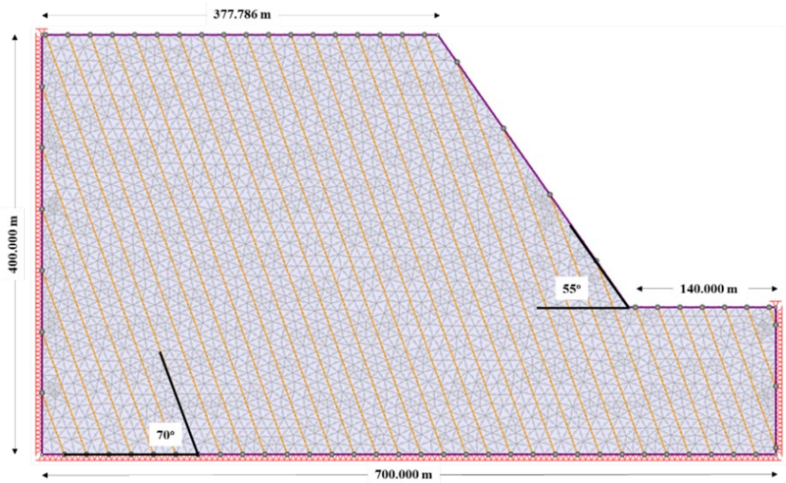
11.4. Data Files

The input data file **JointVerification-11.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

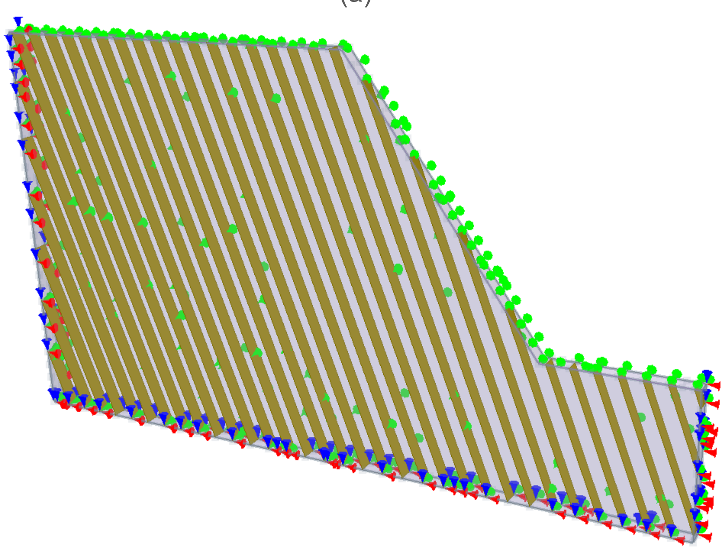
12. Lorig and Varona Plane Failure with Non-Daylighting Discontinuities

12.1. Problem Description

The work by Lorig and Varona (2004) [1] presents the slope stability analysis with the presence of a joint set subparallel to the slope (but at higher inclination) to simulate non-daylighting plane failure mode using 2D DEM. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the non-daylighting plane failure mechanism at the safety factor specified by Lorig and Varona (2004) [1]. Constructed models are presented in Figure 12-1.



(a)



(b)

Figure 12-1 Problem Geometry (a) RS2 and in (b) RS3

This problem involves simulating the failure of both rock and joints. Hence, elastic perfectly plastic material model is implemented to solid material to allow yielding to occur and peak strength is defined for joints. Considered geometrical and mechanical parameters are provided in Table 12-1.

Table 12-1 Model Parameters

Parameter	***Value***
Slope height	260 m
Slope angle	55 degrees
Joint angle	70 degrees
Joint peak cohesion (c)	0.1 MPa
Joint peak friction angle (ϕ')	40 degrees
Joint tensile strength (σ_t)	0 MPa
Rock peak cohesion (c)	0.675 MPa
Rock peak friction angle (ϕ')	43 degrees
Rock tensile strength (σ_t)	0 MPa
Unit weight (γ)	26.1 kN/m^3

12.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 1.60 and 1.59, respectively (critical SRF). As presented by shear strain contour plots (Figure 12-2 and Figure 12-3), the planar failure is driven by failure of joints at upper portion and the failure plane continues with shearing of solid elements. The modeling results also show that the failure planes formed by RS2 and RS3 are in alignment with that captured from the work by Lorig and Varona (2004) [1] with 2D DEM.

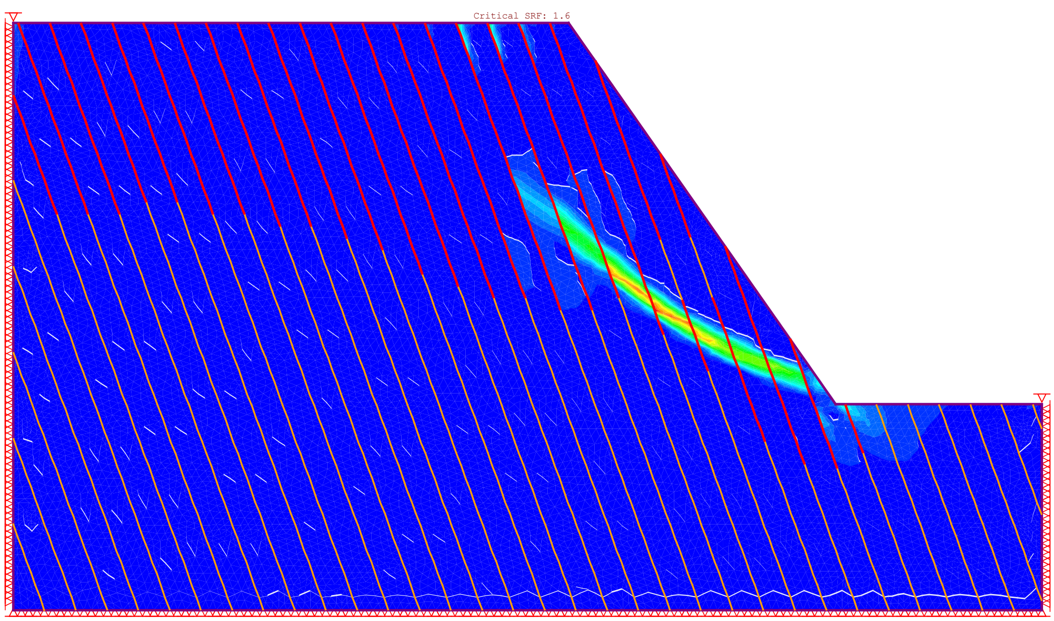
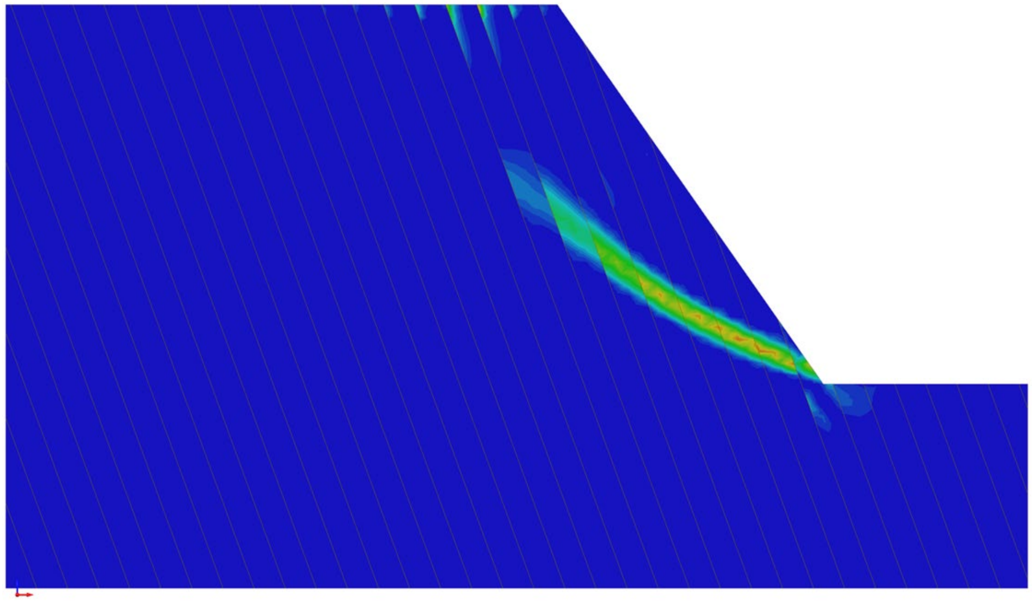
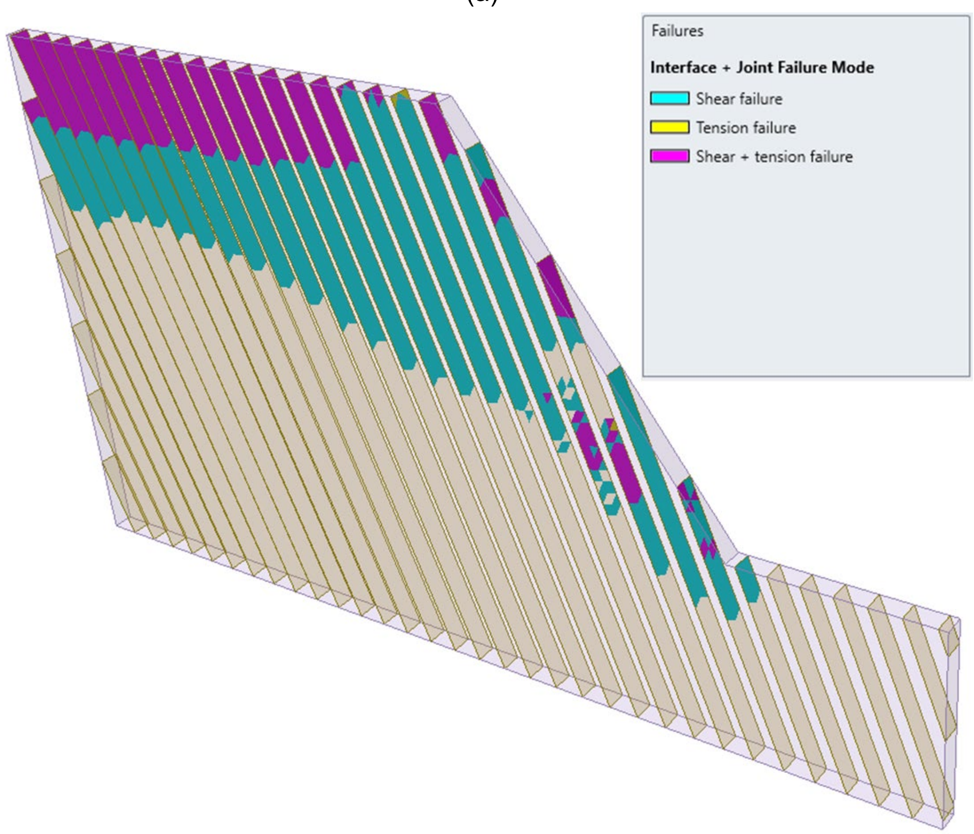


Figure 12-2 RS2 Maximum Shear Strain Contour Plot at SRF = 1.60 (Critical SRF = 1.61), yielded joint represented by thick red lines



(a)



(b)

Figure 12-3 RS3 Modeling Result at SRF = 1.59 showing (Critical SRF = 1.60) (a) Maximum Shear Strain (b) Joint Failure Modes

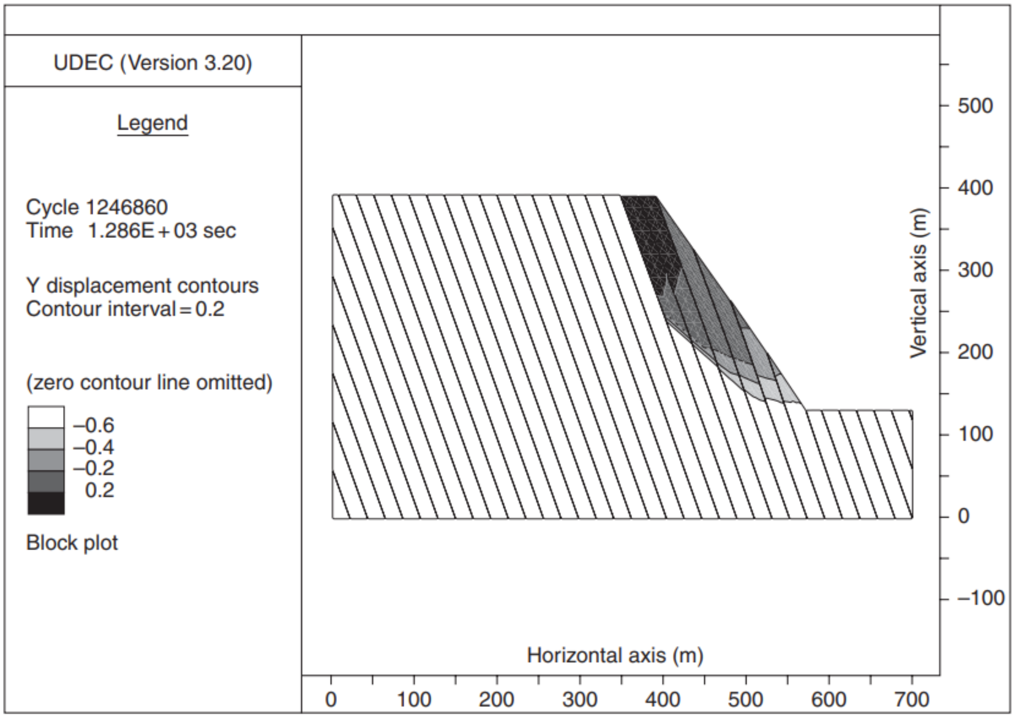


Figure 12-4 DEM modeling Results by [1]

As presented in Table 12-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Lorig and Varona (2004) [1]. Moreover, it is important to note that the non-daylighting planar failure mode is successfully captured as driving mode of slope failure as it did with RS2 and [1].

Table 12-2: Problem Factor of Safety

	Factor of Safety
RS2	1.60
RS3	1.59
Lorig and Varona (2004) [1]	1.5

12.3. References

1. Lorig, L., & Varona, P. (2004). Plane Failure – Daylighting and Non-Daylihgting. In D. C. Wyllie, & C. W. Mah, Rock Slope Engineering Civil and Mining 4th Edition (pp. 233-235). New York: Spon Press Taylor & Francis Group.

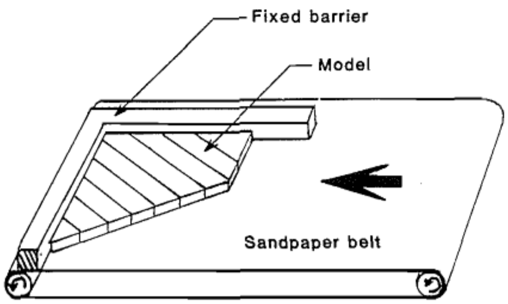
12.4. Data Files

The input data file **JointVerification-12.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

13. Flexural Toppling in Base Friction Model

13.1. Problem Description

The work by Pritchard and Savigny (1990) [1] presents a 2D DEM model that reproduces small-scale base friction model of flexural toppling reported by Hittinger (1978) [2] (Figure 13-1). When the belt is in motion, friction between the model and the sandpaper simulates body forces in the model. The model consists of joint set dipping at 65 degrees that successfully develops a well-defined flexural-toppling failure surface in a slope of uniform geometry, structure, and composition. This numerical investigation, which was successfully reproduced with RS2, is re-conducted with RS3 to verify the validity of its numerical solution. The result verification is evaluated based on the ability to accurately capturing the flexural toppling failure mechanism at the safety factor specified by Pritchard and Savigny (1990) [1]. Constructed models are presented in Figure 13-2.



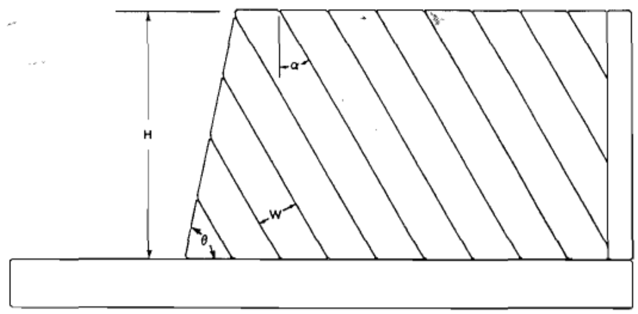
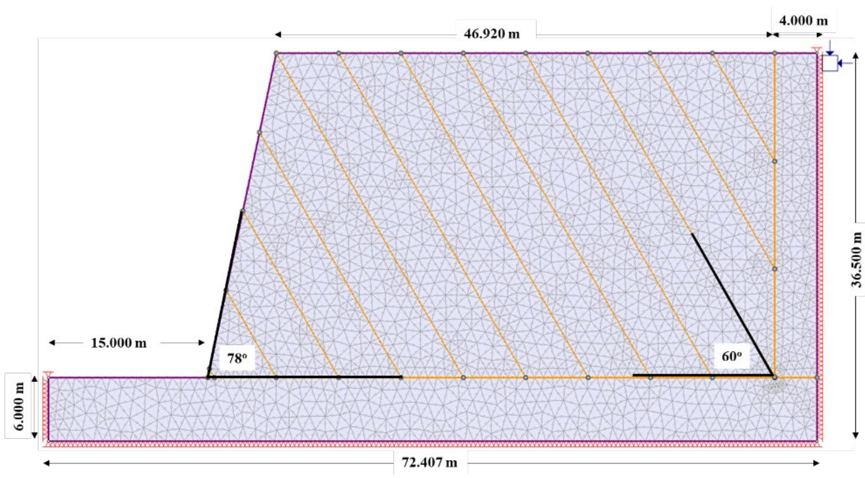
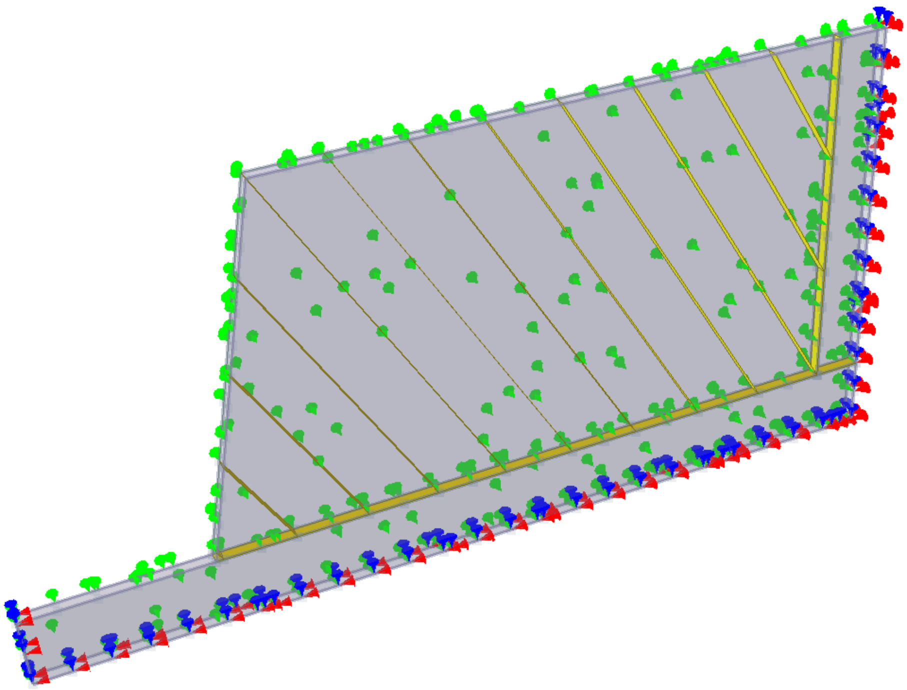


Figure 13-1 Illustration of a base friction model and test sample (left) and the geometry of the representing numerical model (right)



(a)



(b)

Figure 13-2 Problem Geometry (a) RS2 and in (b) RS3

This problem involves simulating the failure of both rock and joints. Hence, elastic perfectly plastic material model is implemented to solid material to allow yielding to occur and peak strength is defined for joints. Considered geometrical and mechanical parameters are provided in Table 13-1.

Table 13-1: Model Parameters

Parameter	***Value***
Slope height	30.5 m
Slope angle	78 degrees
Joint angle	60 degrees
Joint peak cohesion (c)	0 MPa
Joint peak friction angle (ϕ')	39 degrees
Joint tensile strength (σ_t)	0 MPa
Rock peak cohesion (c)	0.06 MPa
Rock peak friction angle (ϕ')	39 degrees
Rock tensile strength (σ_t)	0.075 MPa
Unit weight (γ)	25.506 kN/m^3

13.2. Results

The slope stability analysis can be conducted using Shear Strength Reduction (SSR) analysis with RS3. This method iteratively computes stress analysis with different Strength Reduction Factor (SRF) to determine the point of which the convergence failure occurs. For this example, RS2 and RS3 models reach critical state at the SRF of 0.76 and 0.74, respectively (critical SRF). As shown in Figure 13-3 and Figure 13-4, the mechanical behaviours captured from the two models are in alignment. Moreover, similar pattern was captured from the work by Pritchard and Savigny (1990) [1] with 2D DEM (Figure 13-5).

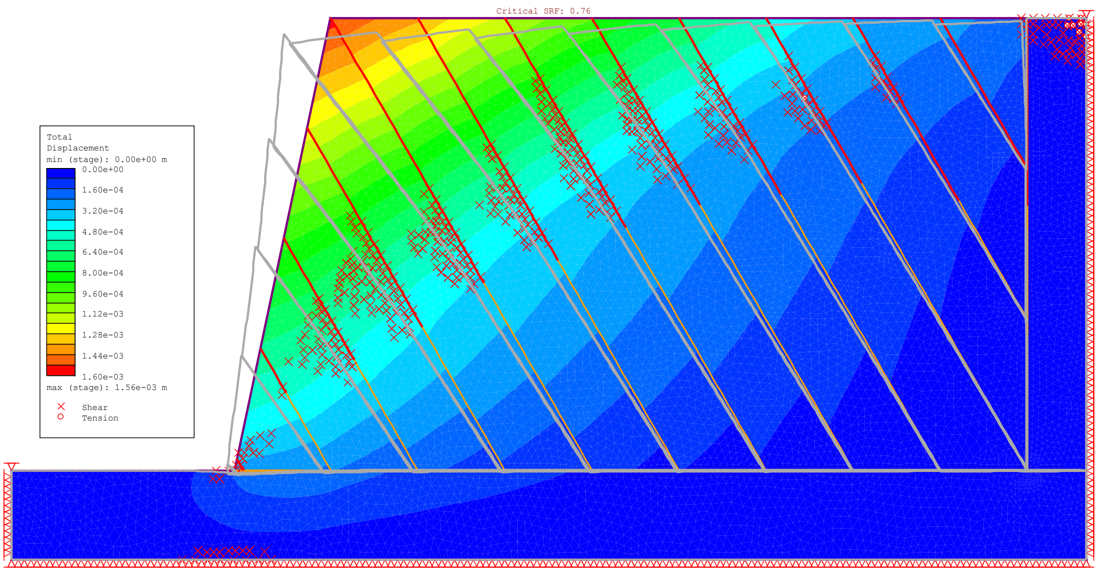
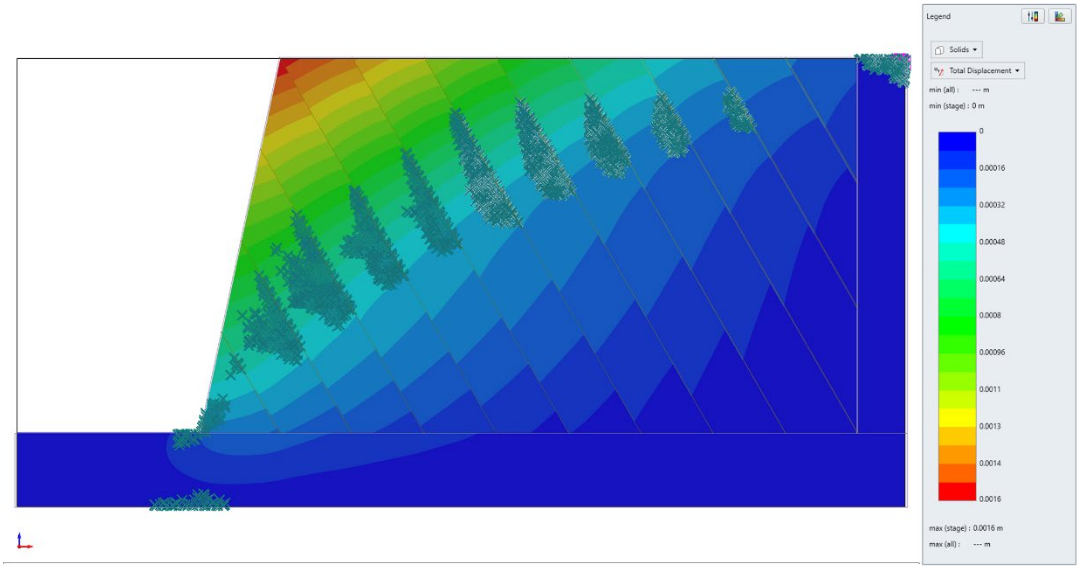
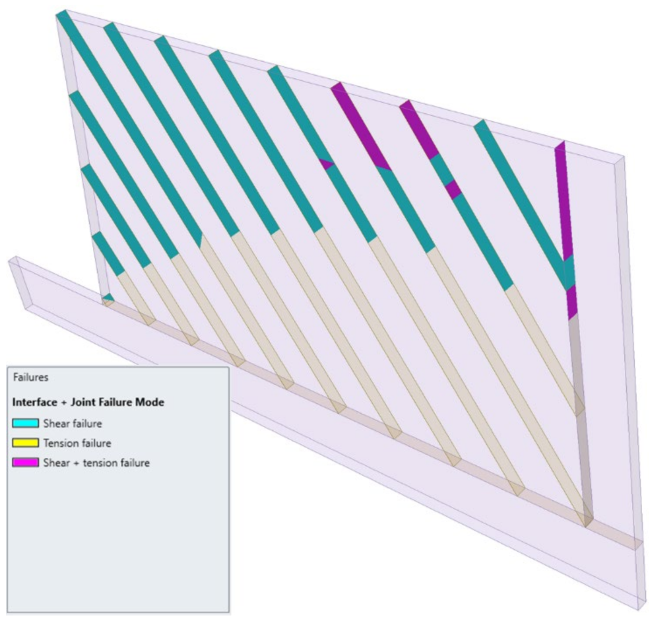


Figure 13-3 RS2 Total Displacement Contour Plot at SRF = 0.74

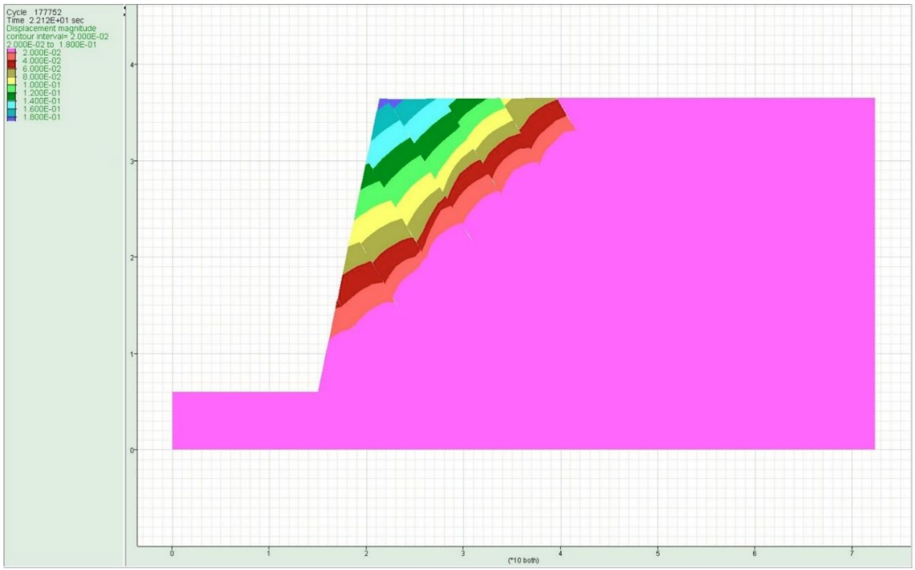


(a)

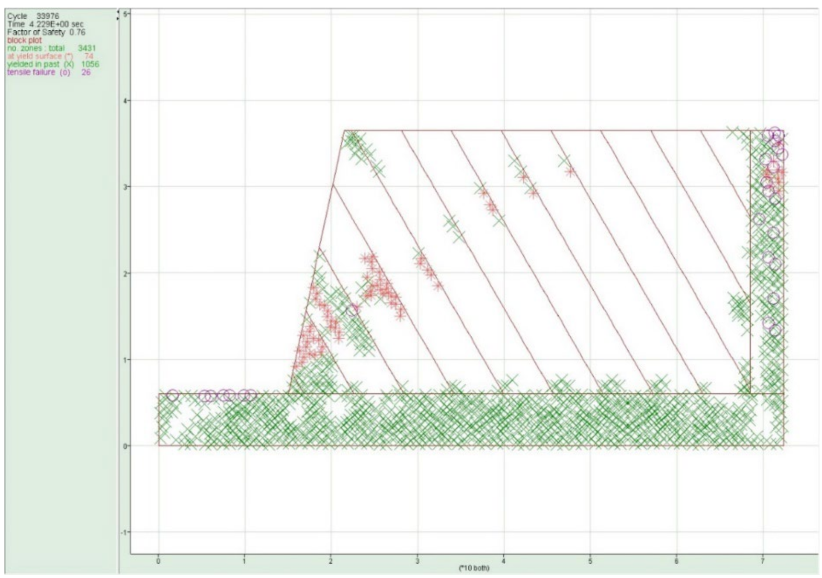


(b)

Figure 13-4 RS3 Modeling Result at SRF = 0.74 showing (a) Total Displacement and (b) Joint Failure Modes



(a)



(b)

Figure 13-5 DEM modeling Results showing (a) Displacement Plot and (b) Failure Zones [1]

As presented in Table 13-2, the factor of safety computed by RS3 (Critical SRF) is in agreement with that of RS2 and the work by Pritchard and Savigny (1990) [1]. Moreover, it is important to note that the agreeing flexural toppling failure mode is successfully captured as driving mode of slope failure in RS3 as captured by RS2 and [1].

Table 13-2: Problem Factor of Safety

	Factor of Safety
RS2	0.76
RS3	0.74
Pritchard and Savigny (1990) [1]	0.76

13.3. References

1. Pritchard, M. A., & Savigny, K. W. (1990). Numerical Modelling of Toppling. Canadian Geotechnical Journal, 823-834.

2. Hittinger, M. 1978. Numerical analysis of toppling failures in jointed rock. Ph.D. thesis, University of California, Berkeley.

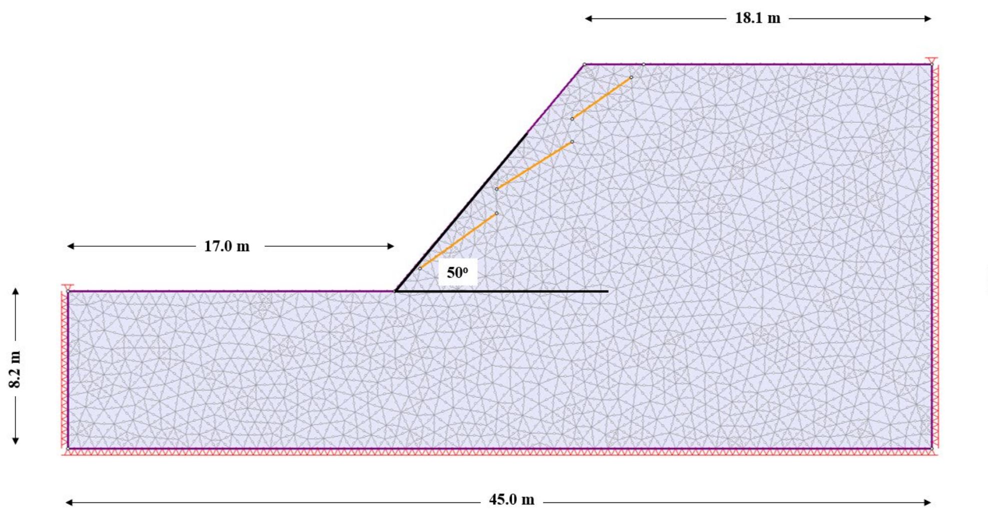
13.4. Data Files

The input data file **JointVerification-13.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

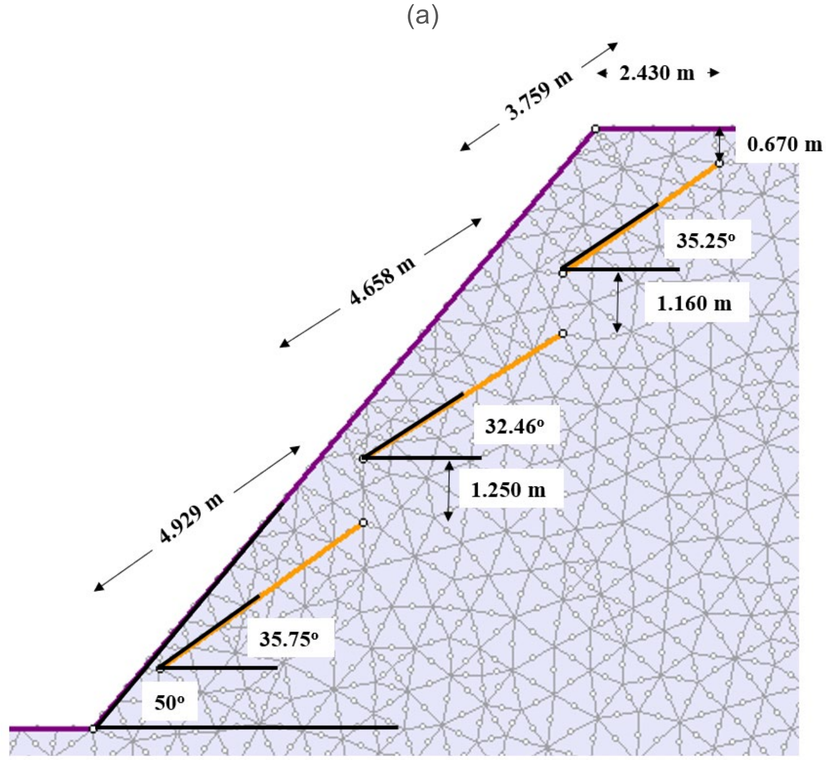
14. Step-Path Failure with En-Echelon Joints

14.1. Problem Description

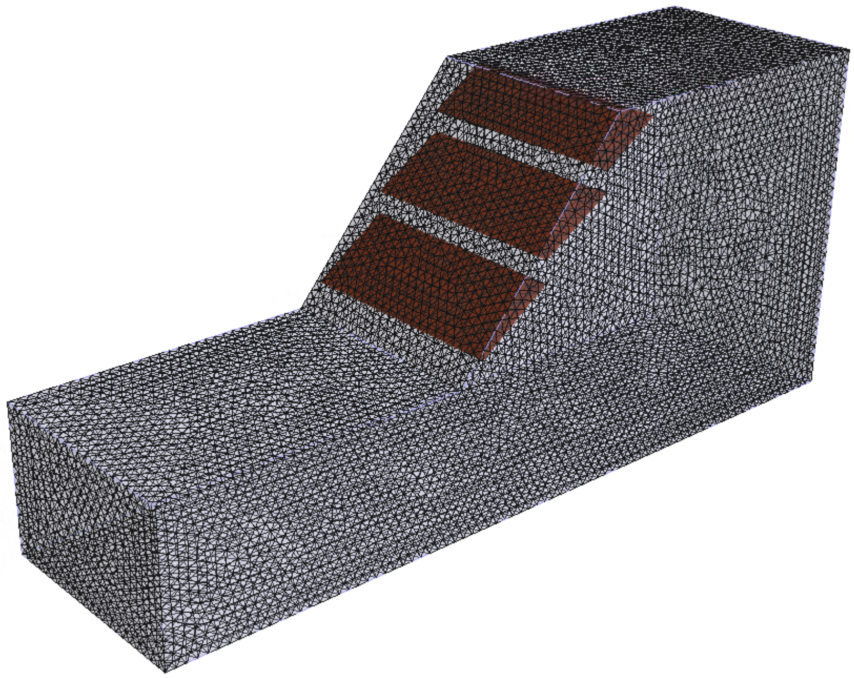
RS3 was used to analyze the step-path failure in a rock slope containing three non-continuous en-echelon joints. Step-path failure occurs when shear failure along joints combines with shear and tensile failure in the intact rock bridging between joints. RS3 results are compared to the 2D DEM analysis results provided in the reference.



(a)



(b)



(c)

Figure 14-1 Problem Geometry (a) RS2 (b) RS2 Detailed (c) RS3

The problem is using the same parameters that are listed in Table 14-1.

Table 14-1: Model Parameters

Parameter	***Value***
Slope height	11.8 m
Slope angle	50 degrees
Peak friction angle (ϕ’)	35 degrees
Unit weight (γ)	19.62 kN/m^3

14.2. Results

Table 14-2 shows the factors of safety obtained by RS2, RS3, and the UDEC.

Table 14-2: Problem Factor of Safety

	Factor of Safety
RS2	1.20
RS3	1.17
UDEC	1.29

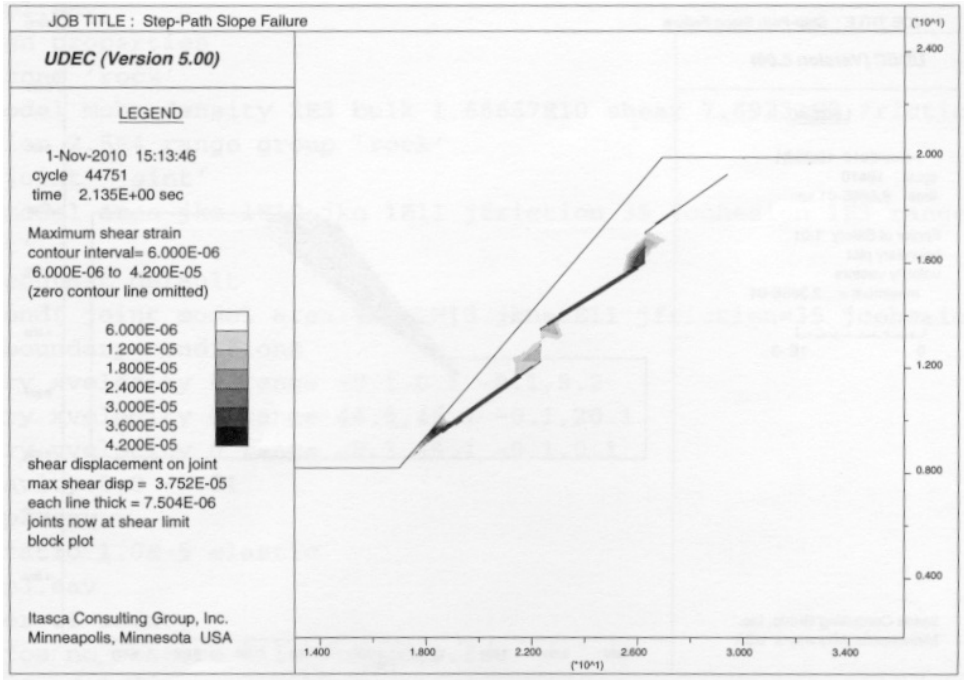


Figure 14-2 Joint Maximum Shear Displacement and Rock Maximum Shear Strain [1]

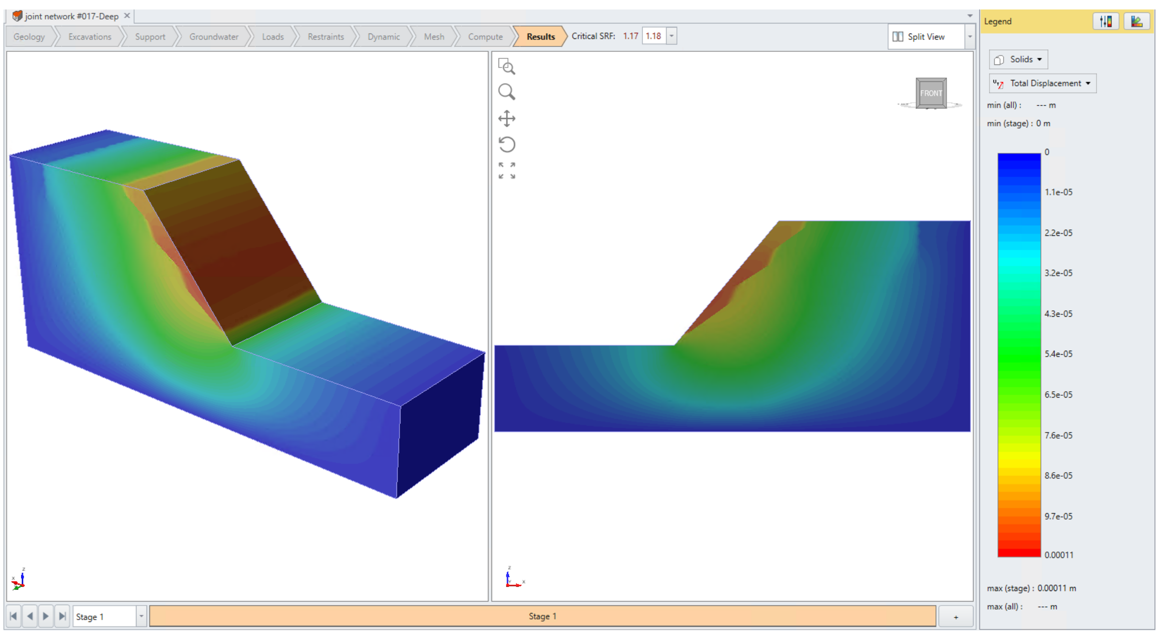
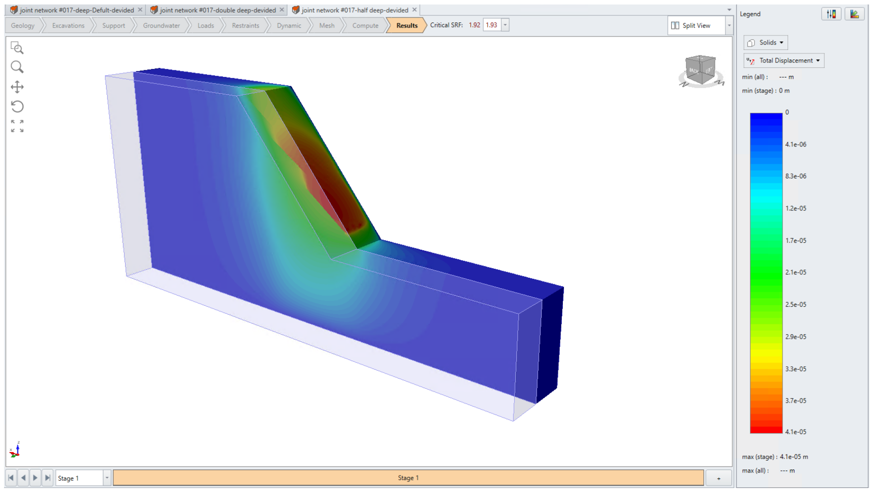
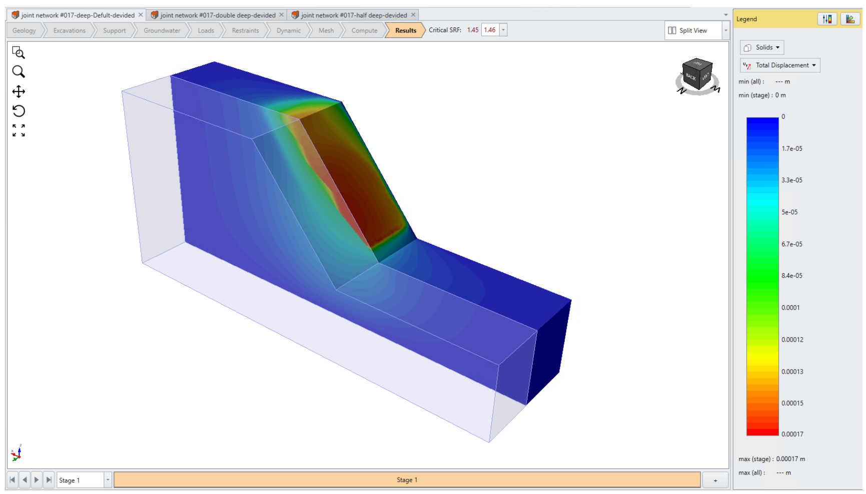


Figure 14-3 Total Displacement (RS3)

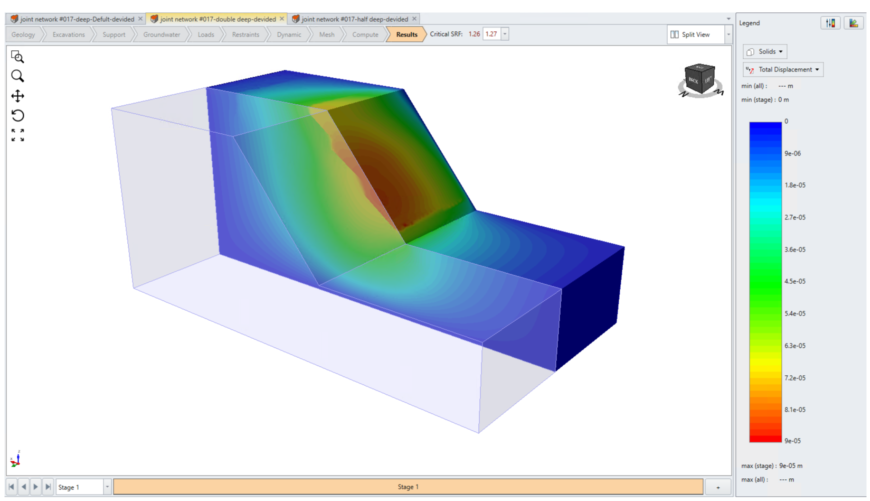
The model described above is an extruded version of a 2D plane strain example. To investigate the behavior of similar model in actual 3D, all boundaries of the slope is fixed in x,y,z directions and the factor of safety is calculated for three different depth (6.25m, 12.5m and 25m). The results are presented in Figure 14-4:



(a)



(b)



(c)

Figure 14-4 Total Displacement (a) Thickness=6.25m (b) Thickness=12.5m (c) Thickness= 25m

As it can be seen, by increasing the depth of the model, the results would be approaching the plane strain case. However, as the boundaries are fixed, the failure of the models with less thickness would happen at higher SRF.

14.3. References

1. Itasca Consulting Group Inc. (2011). Step-Path Failure of Rock Slopes. In I. C. Inc., UDEC Version 5.0 Example Applications (pp. 13-1 to 13-9). Minneapolis.

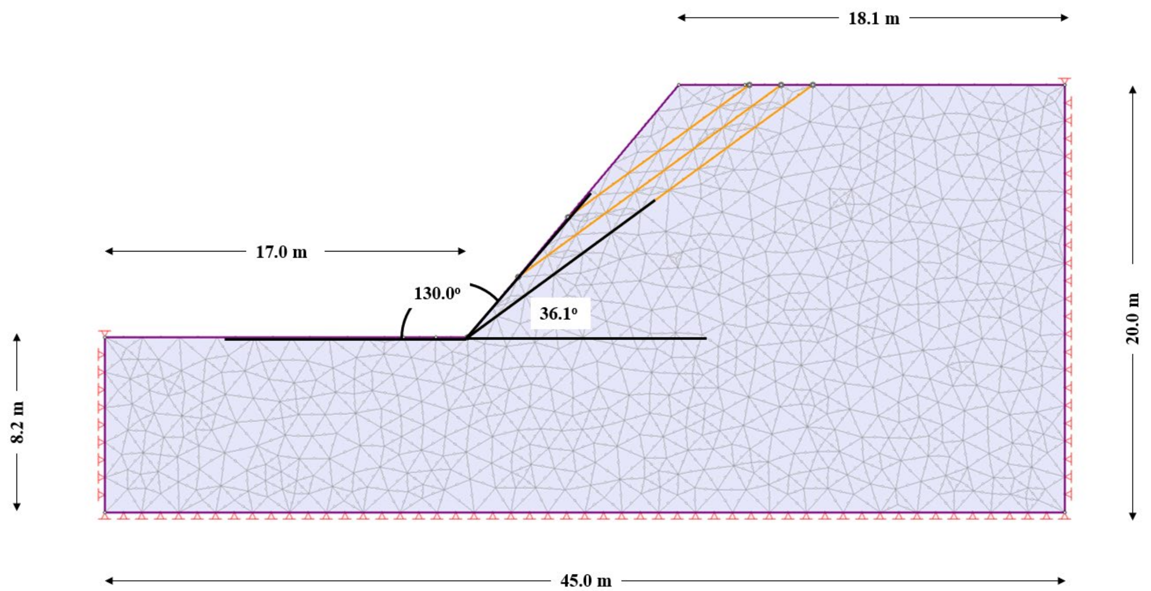
14.4. Data Files

The input data file **JointVerification-14.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

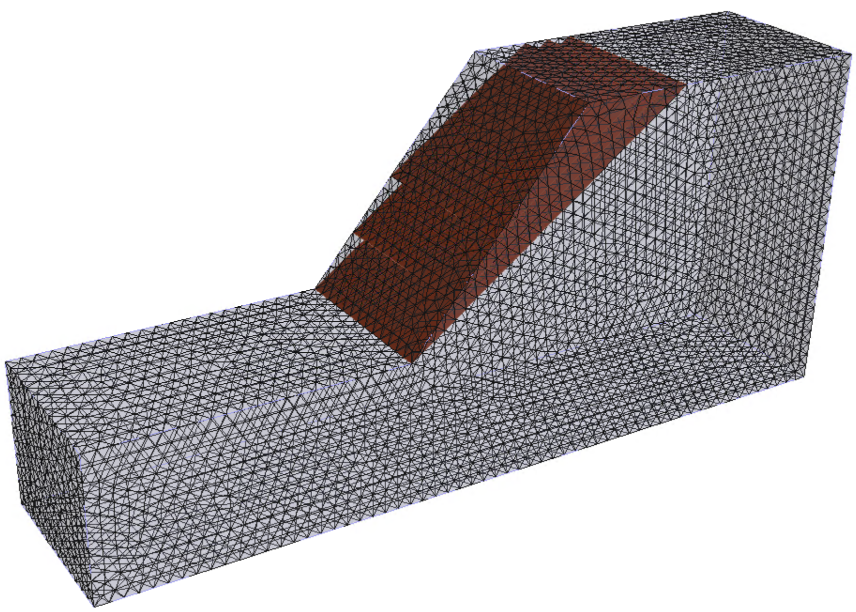
15. Step-path Failure with Continuous Joints

15.1. Problem Description

RS3 was used to analyze the step-path failure in a rock slope containing three continuous joints. Step-path failure occurs when shear failure along joints combines with shear and tensile failure in the intact rock bridging between joints. RS3 results are compared to the UDEC results provided in the reference. Figure 15-1 shows the completed model in RS3.



(a)



(b)

Figure 15-1 Problem Geometry (a) RS2 and in (b) RS3

The problem is using the same parameters that are listed in Table 15-1.

Table 15-1: Model Parameters

Parameter	***Value***
Slope height	11.8 m
Slope angle	50 degrees
Joint angle	36.1 degrees
Peak friction angle (ϕ')	35 degrees
Joint spacing	0.883 m
Unit weight (γ)	19.62 kN/m^3

15.2. Results

Table 15-2 shows the factors of safety obtained by RS2, RS3, and the UDEC method.

Table 15-2: Problem Factor of Safety

	Factor of Safety
RS2	1.00
RS3	1.00
UDEC	1.01

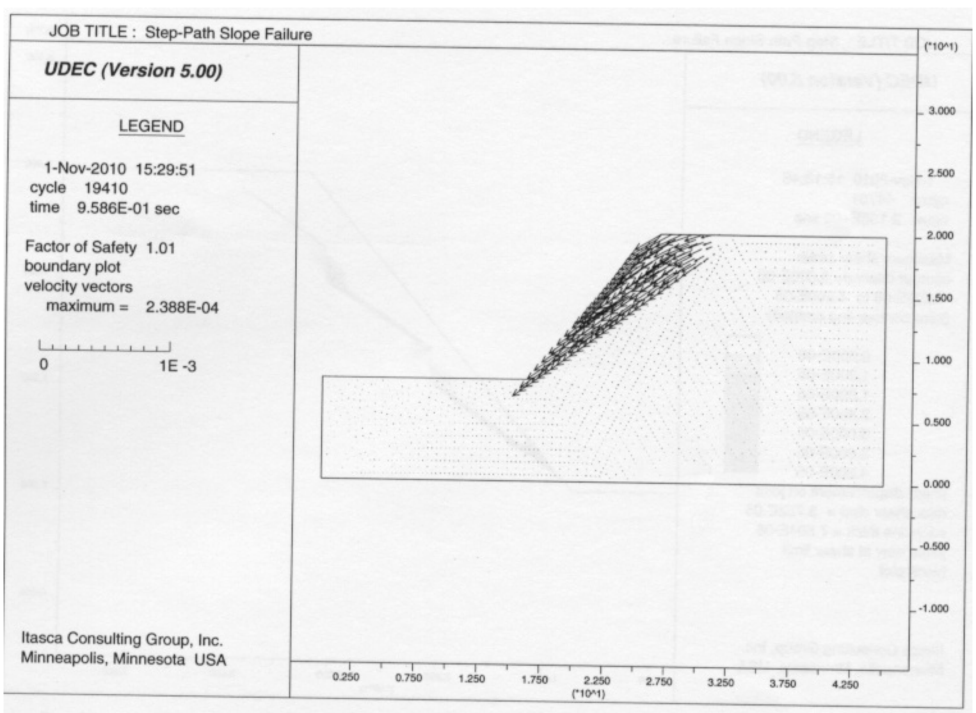


Figure 15-2 Velocity Vectors [1]

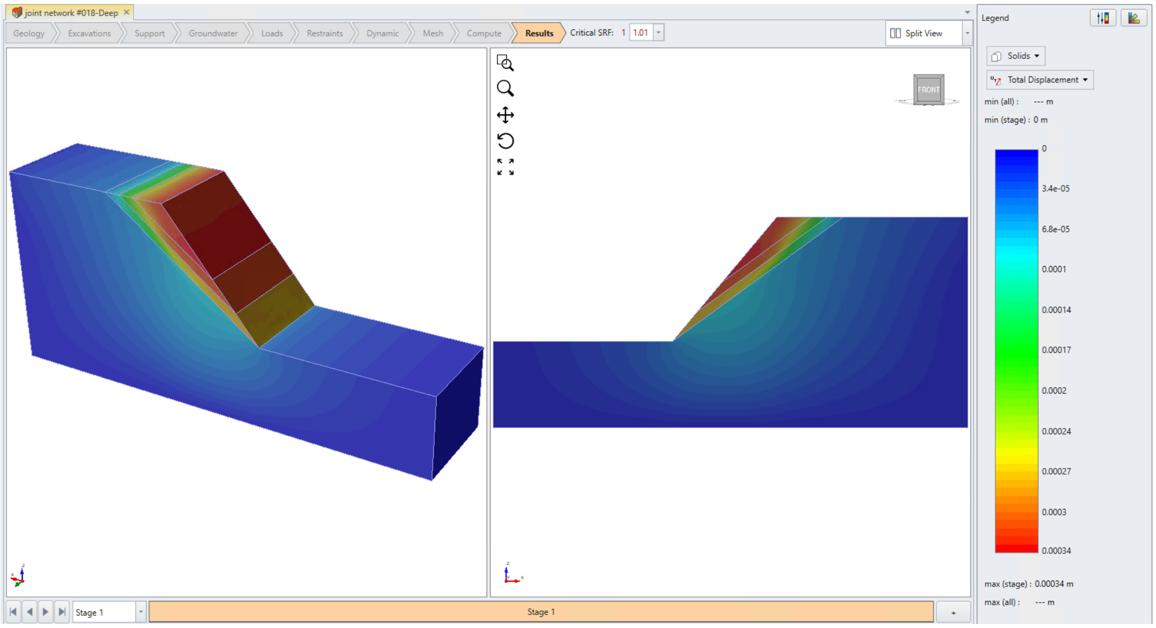


Figure 15-3 Total Displacement (RS3)

15.3. References

1. Itasca Consulting Group Inc. (2011). Step-Path Failure of Rock Slopes. In I. C. Inc., UDEC Version 5.0 Example Applications (pp. 13-1 to 13-9). Minneapolis.

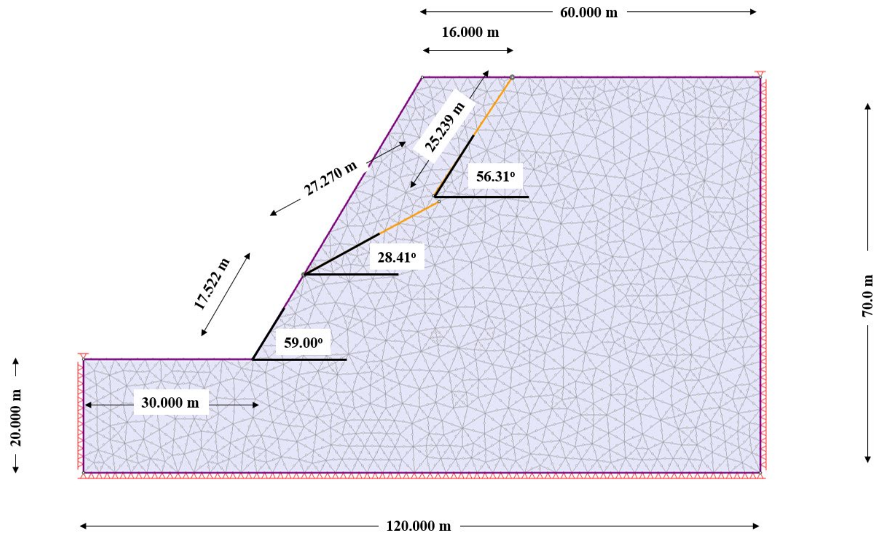
15.4. Data Files

The input data file **JointVerification-15.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.

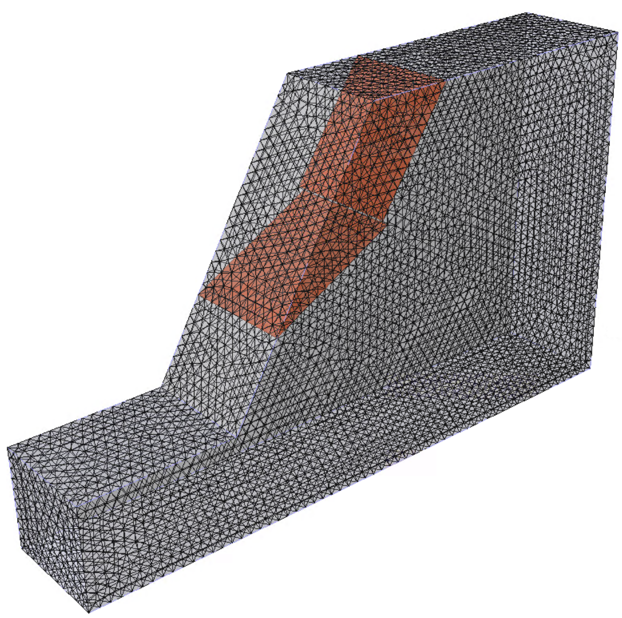
16. Bi-planar step-path failure

16.1. Problem description

RS3 was used to analyze the step-path failure in a rock slope containing two discontinuous joints. Step-path failure occurs when shear failure along joints combines with shear and tensile failure in the intact rock bridging between joints. Given the same material properties, RS3 results are compared to RS2 and 2D DEM analysis results. Figure 16-1 shows the completed model in RS3.



(a)



(b)

Figure 16-1 Problem Geometry (a) RS2 and in (b) RS3

The problem is using the same parameters that are listed in Table 16-1.

Table 16-1: Model Parameters

Parameter	***Value***
Slope height	50 m
Slope angle	59 degrees
Peak friction angle (ϕ')	40 degrees
Unit weight (γ)	27 kN/m^3

16.2. Results

Table 16-2 shows the factors of safety obtained by RS2, RS3, and the 2D DEM analysis.

Table 16-2: Problem Factor of Safety

	Factor of Safety
RS2	1.40
RS3	1.44
Yan et al. (2007) [1]	1.46

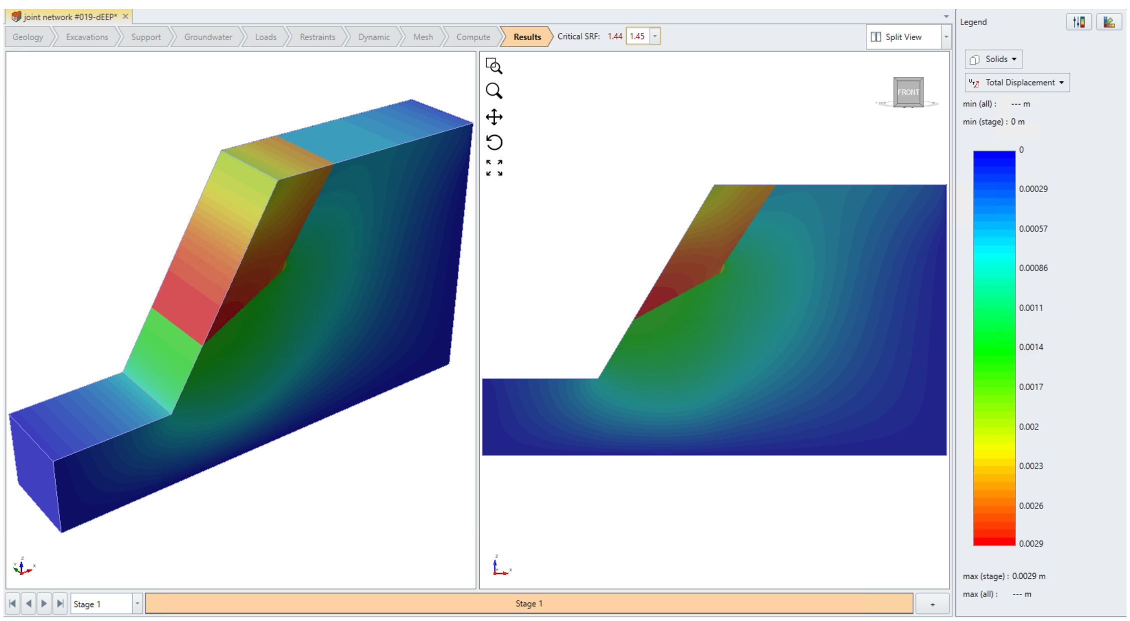


Figure 16-2 Total Displacement (RS3)

16.3. References

1. Yan, M., Elmo, D., & Stead, D. (2007). Characterization of Step-Path Failure Mechanisms: A Combined Field-Based Numerial Modelling Study. In E. Eberhardt, D. Stead, & T. Morrison, Rock Mechanics Meeting Society's Challenges and Demands Volume 1: Fundamentals, New Technologies and New Ideas (p. 499). London, U.K.: Taylor and Francis Group.

16.4. Data Files

The input data file **JointVerification-16.rs3v3** can be downloaded from the RS3 Online Help page for Verification Manuals.