Beams and Plates

Introduction

This document describes the implementation of beam and plate structural support elements in *RS3*.

Structural elements such as beams and plates are 3D elements which are relatively thin along certain axes. Hence, beams are formulated as one-dimensional elements (line), while plates are represented as two-dimensional (surface) elements.

Beams

1.1. 3D beam components

3D beam components are line structures that exhibit flexural rigidity about the two axes of the cross-sectional area of the beam. Therefore, in simple beam elements which are formulated as lines, two dimensions for the cross section should be defined.

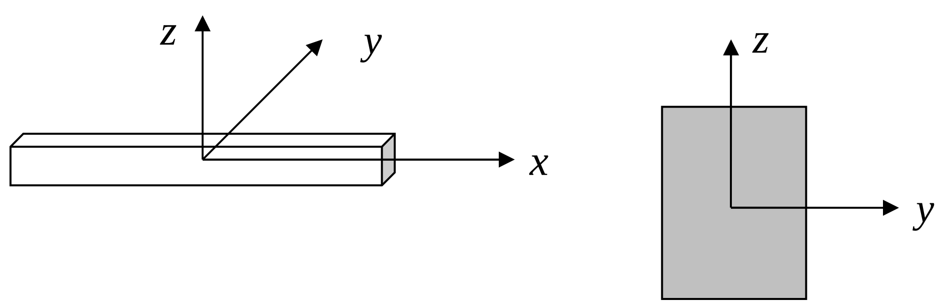

Figure 1 - Geometry of a 3D beam element.

These geometric properties are sometimes expressed in terms of moment of inertia about the two axes, I_{yy} and I_{zz}. Examples of 3D beams in underground excavation are the support beam elements in *RS3*.

> NOTE: the XYZ axes used to define the beam in Figure 1 are LOCAL axes used for the beam calculations, and should not be confused with the global XYZ axes of your *RS3* model.

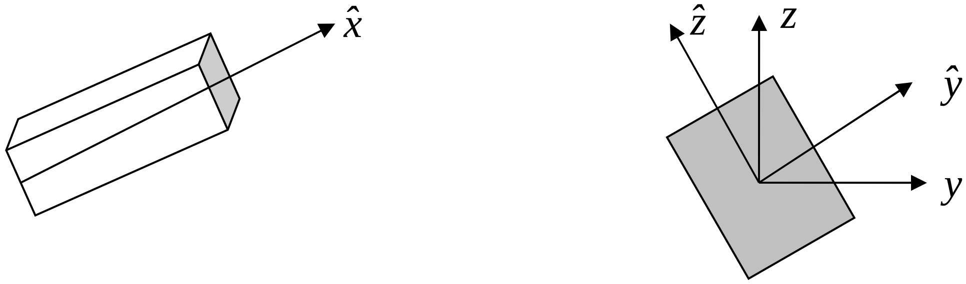

Figure 2 - Geometry of a 3D beam element.

1.1.1. Finite Element Formulation of 3D Beam Element in RS3

Beam elements are formulated as line components. Beam formulation can be based on the assumption of Bernoulli or Timoshenko beams (Cook et al, 2002). The former disregards the contribution of shear stresses, while the latter considers shear stresses on the cross-sectional area of a beam.

Linear (L2) and Quadrilateral (L3) beam elements

Formulation

These elements represent the extensional behavior along the x axis as well as the bending about x2 and x3. The relevant stress and moment components are represented in Figure 3.

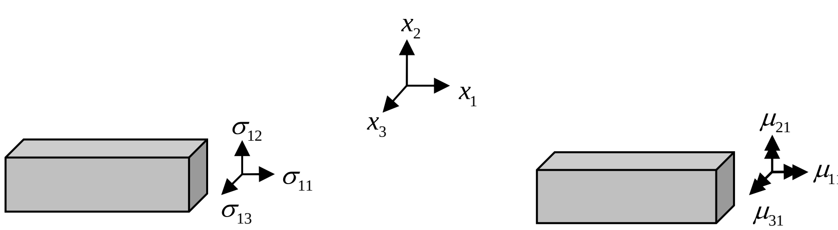

Figure 3 - 3D representation of stress and couple-stress in a beam component.

In *RS3*, moments index 1=x, index2=y and index 3=z.

The beam components are therefore, bending moments about x and y, which in the current input file are myx and mzx, torsional moment mxx, two transverse shear loads sxz and sxy and one membrane force sxx.

1.1.2. Beam Plasticity

Beam plasticity is conceptually based on the layered approach. But in beams interaction of two bending moments in creation of tension and compression zones must be considered.

It is assumed that the beam is divided into a number of regions as depicted in the Figure 4.

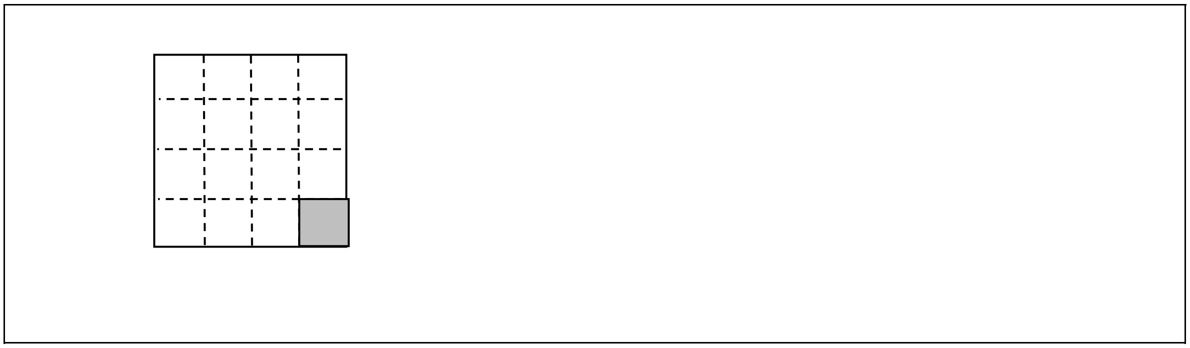

Figure 4 - Division of a beam profile into a number of regions.

The axial force is the result of membrane stress, axial stress caused by moment about y and axial stress caused by moment about z in each region. The algorithm loop will go through the whole profile.

Plates and Shells

Plate and shell components are two-dimensional elements that exhibit flexural rigidity about their local x and y axis. The only reduced dimension of a plate or shell is its thickness; therefore, the additional input geometry that may be defined (depending on the formulation) is the thickness of the component. Clearly, moments would be expressed as N.m/m (which are moment per unit length).

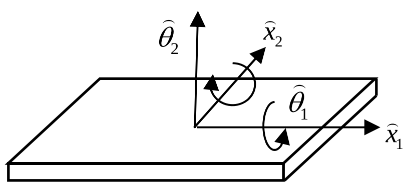

Figure 5 - geometric representation of a plate component.

Figure 5 shows a plate component. It must be noted that in *RS3* formulation $\hat{\theta}_1$ is defined as the rotation about $\hat{x}_1$ axis, and $\hat{\theta}_2$ represents rotation about $\hat{x}_2$ axis. The notation is therefore different from some references (Owen and Hinton).

In *RS3*, plate elements formulate both membrane and bending behavior. Examples of shell and plate structures are liners and shotcrete in underground excavations.

Bending Stresses

Bending stresses correspond to two bending moments about the local x and y axis and a twisting moment. Also, shear deformation is considered which results in shear stress through the thickness of the plate. In plate and beam formulation, moments are expressed per unit length, while stresses are expressed in conventional dimensions (force per unit area).

Membrane Stresses

Membrane stresses correspond to stresses in the in-plane direction. They act tangent to the mid-surface, and produce mid-surface-tangent forces. There are two membrane stresses σ_{xx} and σ_{yy} associated with shells.

2.1.1. Finite Element Formulation of Plate Elements

Formulation of *RS3* plate elements is based on the Mindlin-Reisnner theory (Cook et al. 2002)

In their local coordinate system, these elements are 2 dimensional iso-parametric elements. Bending mechanism is formulated base on Reissner-Mindlin Plate theory. For the mechanics and simple form of the formulation refer to Cook et al. (2002). In Reissner - Mindlin plate elements, each node is associated with one translational (deflection) degree of freedom and two additional rotational degrees of freedom.

The *RS3* plate elements, however, are modified to take into account the following effects:

1- The membrane stresses are considered. Therefore, each node has all three translation degrees of freedom.

2- Shear locking effects on the linear plate element is eliminated based on Tessler formulation (Tessler, 1991)

2.1.2. Plasticity formulation of plate elements

The plasticity formulation is based on the assumption of maximum tension or compression exceeding the yield strength in the two axis directions of the plate. Therefore, yield due to shear stresses is disregarded. The procedure is described in Owen and Hinton (1980).

From an implementation point of view, a layered approach is adopted, which considers a linear variation of axial stress through the thickness of the plate.

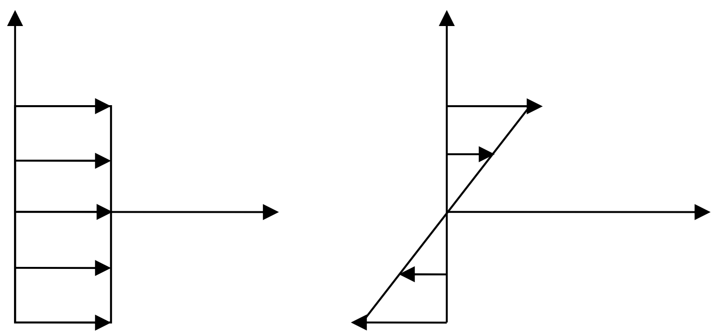

Figure 6 - Distribution of axial stress distribution due to (a) membrane mechanism, (b) bending mechanism through the thickness of plate.

References

1. Owen, D. R. J. and Hinton., E. (1980). "Finite Elements in Plasticity: theory and practice".
2. Cook, R.D, Malkus, D.S., Plesha, M.E. and Witt, R. J. (2002). "Concepts and Application of Finite Element Analysis", 4th edition.
3. Tessler A. (1991). "A two-node beam element including transverse shear and transverse normal deformations". International journal for numerical methods in engineering. 32: 1027-1039