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RS2 & RS3 THEORY

Soil Behaviors in Unsaturated Zones

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In unsaturated zones, soil strength increases with suction, as demonstrated by the self-supporting stability of vertical clay cuts. Unsaturated soil mechanics has been extensively studied (Lewis, 1998; Borja, 2006), particularly in the areas of shear strength and consolidation, with various formulations developed and applied as a result, as mentioned by Fredlund et al. (2006).

Terzaghi's Effective Stress Equation

Let's first examine the behavior of fully saturated soils. Terzaghi's effective stress equation is a well-known expression used to describe stresses in saturated soils, including sands, silts, and clays. It explains how these stresses contribute to the deformation of the solid-fluid mixture, affecting compaction, distortion, and shear resistance. The equation is given by:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + u\boldsymbol{I} \tag{1}$$

where $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$ are the effective and total stress tensors, respectively, $\boldsymbol{I}$ is the identity tensor, and u is the pore-fluid pressure. The sign convention assumes positive normal stress in tension and positive pore-fluid pressure.

Effect of Biot's Coefficient

Typically, the soil skeleton is more compressible than the individual grains within a soil mass, making the deformation of individual grains negligible. However, in deep soil layers under high pressure or in porous rocks, the stiffness of the soil or rock matrix becomes comparable to that of the solid grains. In such cases, the compressibility of the solid material must be considered, as it affects the distribution of total stress between effective stress and pore pressure. For compressible solid materials such as porous rocks, Terzaghi's effective stress equation can be modified as follows:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \alpha u\boldsymbol{I} \tag{2}$$

where α is Biot's pore pressure coefficient, given by:

$$\alpha = 1 - \frac{K_t}{K_s} \tag{3}$$

where K_t is the effective bulk modulus of the soil matrix, given by $K_t = E/[3(1-2\nu)]$ where E and ν are the soil's Young's modulus and Poisson's ratio, respectively, and K_s is the bulk modulus of the solid grains. Notably, when the solid grains are incompressible ($K_s \to \infty$), Terzaghi's original effective stress equation remains valid. Lower values of α indicate that, for a given total stress and pore water pressure, the resulting effective stress is higher than that of an incompressible solid material.

For most soil types, α is approximately 1, but for hard rocks, it can vary. Typical values of α for different rocks are reported by Luo et al. (2015) and are also presented in Table 3.1 of the RS2 theory manual on the undrained analysis.

In RS2 and RS3, Biot's pore-fluid pressure coefficient α is automatically calculated using Equation (3), but the user can manually adjust its value if necessary. It should be noted that when calculating α automatically, RS2 and RS3 assume a constant value of 69 GPa for K_s, as stated in the RS2 theory manual *"Coupled Consolidation".*

Stress State Variables of Unsaturated Zones

In unsaturated soils, where the pores are partially filled with water, two separate variables, formed as combinations of total stress ($\boldsymbol{\sigma}$), pore-water pressure (u), and pore-air pressure (u^a), must be used to define the stress state and response of the soil. Examples of these variable pairs include: $(\boldsymbol{\sigma} + u^a\boldsymbol{I})$ and $(u - u^a)$, $(\boldsymbol{\sigma} + u\boldsymbol{I})$ and $(u - u^a)$, or $(\boldsymbol{\sigma} + u^a\boldsymbol{I})$ and $(\boldsymbol{\sigma} + u\boldsymbol{I})$ (Fredlund et al., 2012). When dealing with different methods, one should consider that the use of total stress in constitutive models can cause difficulties in development. Additionally, the transition between saturated and unsaturated states should be smooth. The material parameters required for selecting the appropriate method should be conveniently accessible.

RS2 and RS3 employ the first pair (Bishop, 1959), assuming that the pore-air pressure is equal to atmospheric pressure. This approach in RS2 and RS3 is known as the single effective stress and is described in the following section.

Single Effective Stress

The single effective stress method, using the stress variables $(\boldsymbol{\sigma} + u^a\boldsymbol{I})$ and $(u^a - u)$, is available for all failure criteria models in RS2 and RS3 except for the Barcelona model. This expression is given by:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + u^a\boldsymbol{I} + \chi(\alpha u - u^a)\boldsymbol{I} \tag{4}$$

where χ is the surface fraction coefficient, commonly a function of the degree of saturation. By considering the pore-air pressure u^a as the atmospheric and reference pressure, the pore-fluid pressure and stresses can be expressed relative to it, resulting in:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \alpha\chi u\boldsymbol{I} \tag{5}$$

RS2 and RS3 use nine different methods to calculate χ (Fredlund et al., 2006), including:

1) Bishop (1959):

$$\chi = S_w \tag{6}$$

Where S_w is the degree of saturation, defined as $S_w = \frac{\theta}{n}$, θ is the water content, and n is the soil porosity.

2) The user can define a table of χ versus matric suction values.
3) The user can define a table of χ versus degree of saturation (S_w) values.

4) The user can define a table of χ versus effective degree of saturation (S_e) values. The effective degree of saturation can be calculated as $S_e = \frac{S_w - S_{we}}{S_{sat} - S_{we}}$, where S_w is the degree of saturation, S_{we} is the residual degree of saturation, and S_{sat} is the maximum degree of saturation.

5) Gudehus (1995):

$$\chi = S_w(2 - S_w) \tag{7}$$

6) Khalili (2004):

$$\chi = \begin{cases} \left(\frac{s}{s_e}\right)^{-0.55} & if\ s > s_e \\ 1 & if\ s \le s_e \end{cases} \tag{8}$$

where s is the matric suction, defined as $s = u_a - u_w$, and s_e is the air entry suction.

7) Bolzon (1996):

$$\chi = S_e \tag{9}$$

where S_e is the effective degree of saturation (recall that $S_e = \frac{S_w - S_{we}}{S_{sat} - S_{we}}$).

8) Aitchison (1961):

$$\chi = \begin{cases} 1 & if\ S_w = 1 \\ ({}^{\alpha}/_{s})_{s_e} & if\ S_w < 1 \end{cases} \tag{10}$$

where α is a unitless material parameter.

9) Based on Kohgo et al (1993), the effective stress is defined alternatively in the following form:

$$\sigma' = \sigma - u_{eq} \tag{11}$$

where u_{eq} is the equivalent pore-fluid pressure. This pressure is intended to average the effects of all fluid pressures within the pores and is designed to recover Terzaghi's effective stress in saturated states. Consequently, the authors expressed the equivalent pore pressure in terms of the air entry suction value (s_e), a critical suction (s_c), and a material parameter (a_e):

$$u_{eq} = u_a - s \quad if\ s \le s_e \tag{12}$$

$$u_{eq} = u_a - \left(s_e + \frac{s_c - s_e}{(s - s_e) + a_e}(s - s_e)\right) \quad if\ s > s_e \tag{13}$$

This formulation is equivalent to using Equation (5) with:

$$\chi = a_e\,(s_c - s_e)/(s - s_e + a_e)^2 \tag{14}$$

Note that in RS2, the **Use Effective Stress Analysis** option must be selected to enable the **Single Effective Stress method** for unsaturated materials. This option is activated by default unless the user changes it. Note that RS3 does not have this option because RS3 always uses the effective stress concept for deformation analyses. For details on RS2 and RS3, refer to the relevant modeling procedures.

Unsaturated Shear Strength

This approach, based on the Mohr-Coulomb criterion, has been widely used by introducing an additional friction angle for the unsaturated zone. In RS2 and RS3, it can only be applied to the Mohr-Coulomb model. Two methods are introduced in this approach: one by Fredlund et al. (1978) and another by Vanapalli et al. (1996).

According to Fredlund et al (1978), the shear strength is written as follows:

$$\tau = c' + (\sigma_n - u_a)\tan\Phi' + (u_a - u_w)\tan\Phi^b \qquad (15)$$

where:

- c' is the effective cohesion,
- σ_n is the total normal compressive stress,
- u_a is the pore-air pressure,
- u_w is the pore-water pressure,
- $(\sigma_n - u_a)$ is the net normal compressive stress on the plane of failure at failure,
- $(u_a - u_w)$ is the matric suction,
- Φ' is the effective angle of friction, and
- Φ^b is the angle of friction accounting for the contribution of matric suction to shear strength.

In order to account for the approach correctly in RS2 and RS3, the **Maximum Negative PWP for Unsaturated Strength** option needs to be **ON** with an input value of 0, to avoid double-counting of suction effects.

Another form of the shear strength was proposed by Vanapalli et al (1996), where it can be calculated in terms of either the degree of saturation or the water content.

With degree of saturation:

$$\tau = c' + (\sigma_n - u_a)\tan\Phi' + (u_a - u_w)\left[(\tan\Phi')\left(\frac{S_w - S_{we}}{100 - S_{we}}\right)\right] \qquad (16)$$

where, as mentioned in the previous section, S_w is degree of saturation and S_{we} is the residual degree of saturation.

With water content:

$$\tau = c' + (\sigma_n - u_a)\tan\Phi' + (u_a - u_w)\left[(\tan\Phi')\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)\right] \qquad (17)$$

where, as mentioned in the previous section, θ is the water content, θ_r is the residual water content, and θ_s is the saturated water content.

Constitutive Models that account for Unsaturated Behaviors

To account for the effect of suction on mechanical behaviors, an alternative to the single effective stress approach is to use constitutive models that incorporate suction and/or the degree of saturation (e.g., the Barcelona Basic Model).

In such cases:

- If the analysis involves steady-state conditions or no consolidation, the coefficient of permeability (k^{rw}) governs the unsaturated behavior (refer to RS2 and RS3 user guides on the hydraulic models).
- If consolidation is included in the transient analysis, the unsaturated behavior will be considered differently depending on the type of consolidation (coupled or uncoupled).

  For coupled consolidation, refer to the **Coupled Consolidation** theory manual for the governing equilibrium equations. Equation (21) in the manual will be applied in this context.

  For uncoupled consolidation, instead of the compressibility matrix $\boldsymbol{S}$ (used in coupled consolidation; refer to the RS2 theory manual *"Coupled Consolidation"*), the coefficient of water volume change (m_2^w) is used to account for the influence of solid deformation on the fluid. For details on calculating m_2^w in both saturated and unsaturated soils, refer to the **Transient Fluid Flow** section of **Groundwater Seepage** theory manual in RS2.

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