2 Boussinesq Problem

2.1 Problem Description

In the Boussinesq problem we subject a half space to a point load, as shown in Figure 2-1. The medium has some Young's modulus, *E,* and Poisson's ratio, ν. The magnitude of the point load is *P*. The associated material has the following properties:

Young's modulus (*E*) = 20,000 MPa
Poisson's ratio (*v*) = 0.3

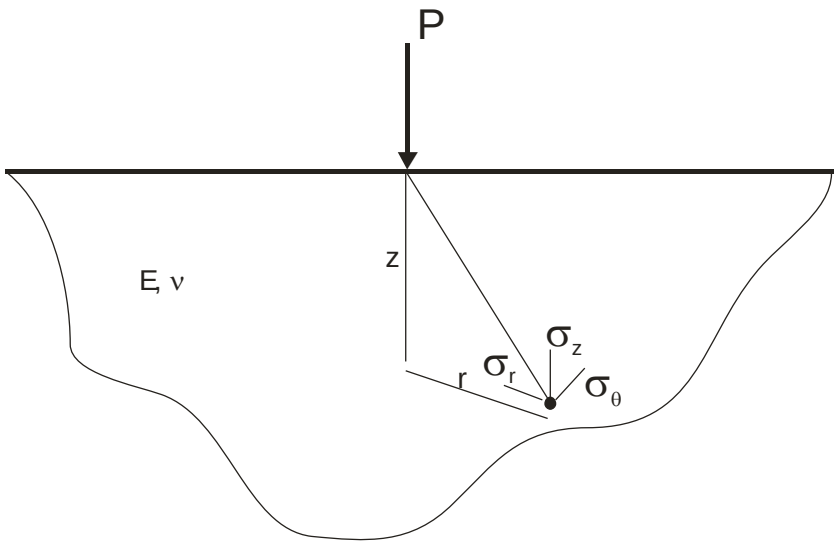

Figure 2-1: Boussinesq problem

2.2 Closed Form Solution

The analytical solution [Reference 1] for the problem of a point load on a half space gives the vertical displacement as:

$$w = \frac{P}{2\pi E}\left[\frac{(1+\nu)z^2}{(r^2+z^2)^{3/2}} + \frac{2(1-\nu^2)}{(r^2+z^2)^{1/2}}\right]$$

where *r* and *z* are radial and vertical distances from the point load, respectively. This equation clearly shows the 1/*r* singularity at the point of application of the load (*r* =0).

The stress components at a point are given by:

$$\sigma_z = \frac{3P}{2\pi}\left[\frac{z^3}{(r^2+z^2)^{5/2}}\right]$$

$$\sigma_r = \frac{P}{2\pi}\left[\frac{3r^2 z}{(r^2+z^2)^{5/2}} - \frac{1-2\nu}{(r^2+z^2)+z(r^2+z^2)^{1/2}}\right]$$

$$\sigma_\theta = -\frac{P}{2\pi}(1-2\nu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{(r^2+z^2)+z(r^2+z^2)^{1/2}}\right]$$

$$\tau_{rz} = \frac{3P}{2\pi}\left[\frac{rz^2}{(r^2+z^2)^{5/2}}\right]$$

2.3 Model Information

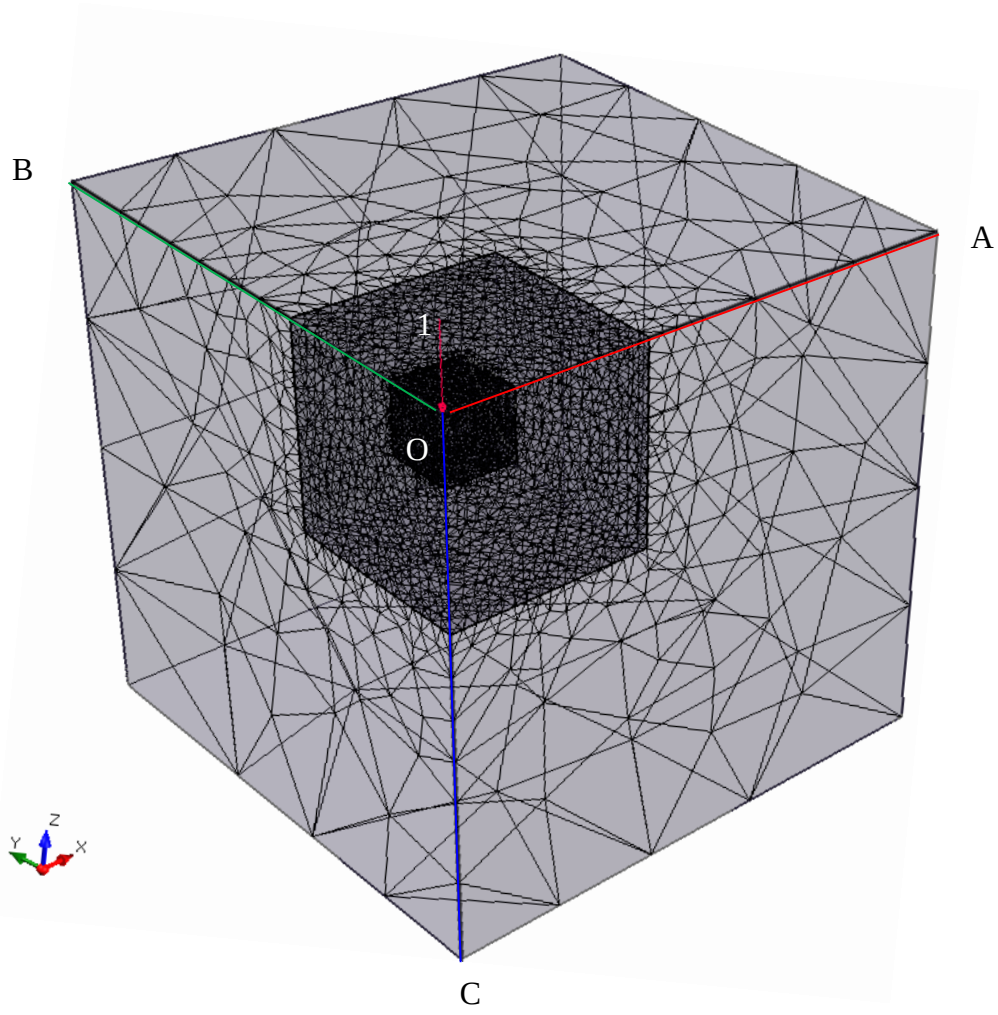

Figure 2-2: *RS3* model with mesh and query line segments

Exploiting symmetry, we analyze only a quarter of the half space close to the action point. The ***RS3*** model for this problem has the following specifications:

- Dimensions of 2m × 2m × 2m
- 10-noded tetrahedron elements
- Boundary conditions and discretization density as depicted in Figure 2-2

2.4 Results and Discussions

The Boussinesq problem is a true 3D problem because we can have different results in all 3 directions. Vertical displacement, stress in the x direction and stress in the y direction are analyzed along line segments OA and OB. They are presented in Figure 2-3 through Figure 2-8. See Figure 2-2 for visualization of line segments. Figure 2-10 through Figure 2-11 present the vertical displacement and stresses along line segment OC.

Note: For all the graphs presented here, the initial few points from ***RS3*** results are omitted. The problem is ill-posed because the point load induces an asymptotic stress.

For all of these graphs, the ***RS3*** results are very similar or identical in both the x-direction and y-direction. The vertical displacement and stress results are very close to the analytical solution.

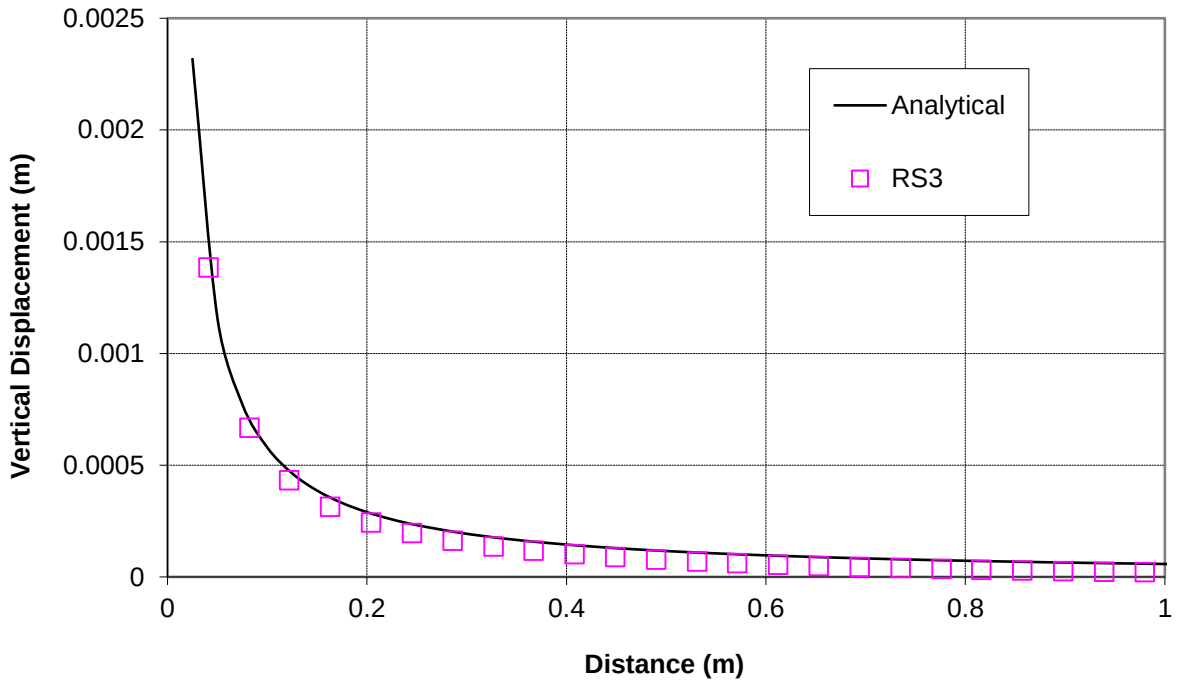

Figure 2-3: Comparison of vertical displacement along line segment OA

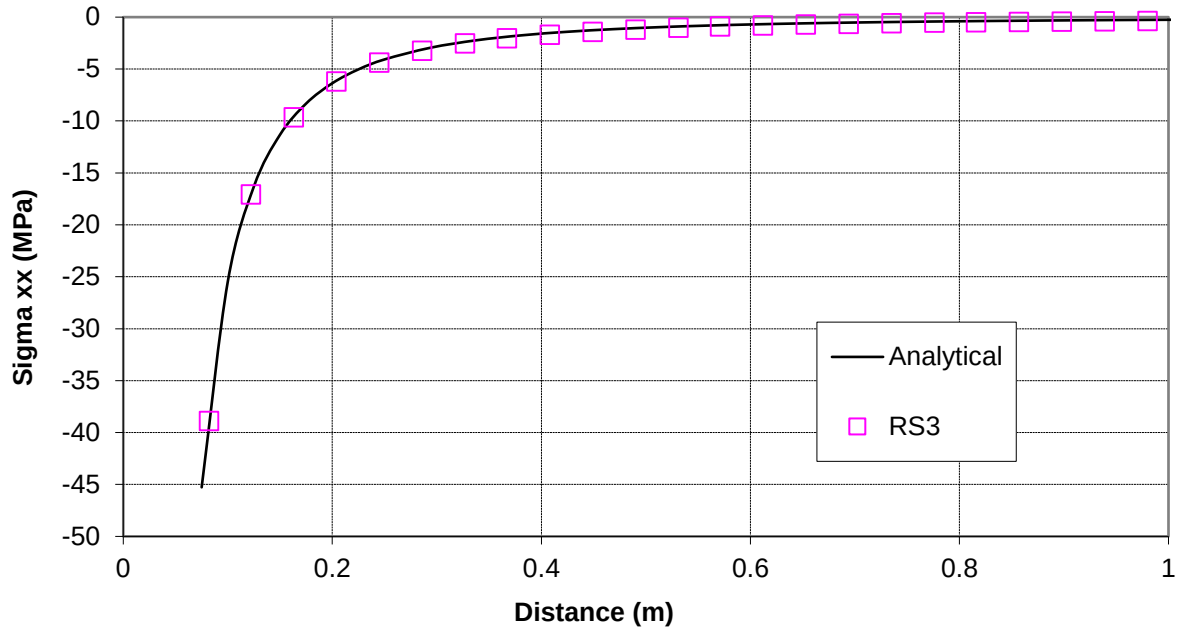

Figure 2-4: Comparison of stress in the x direction along line segment OA

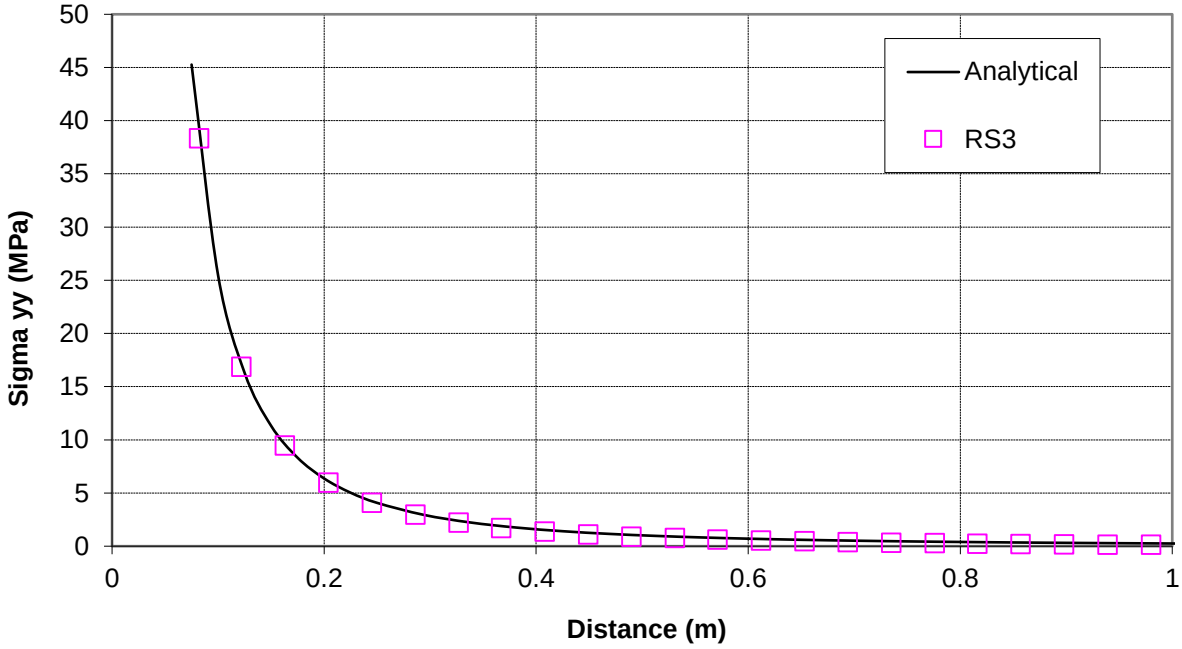

Figure 2-5: Comparison of stress in the y direction along line segment OA

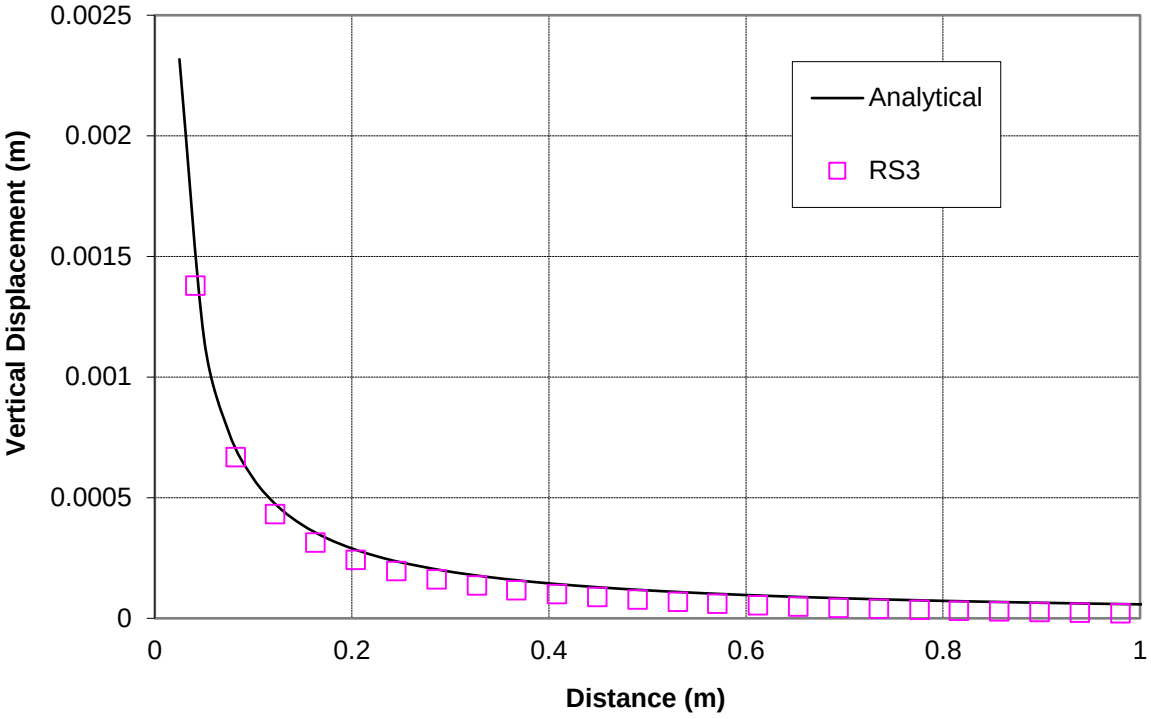

Figure 2-6: Comparison of vertical displacement along line segment OB

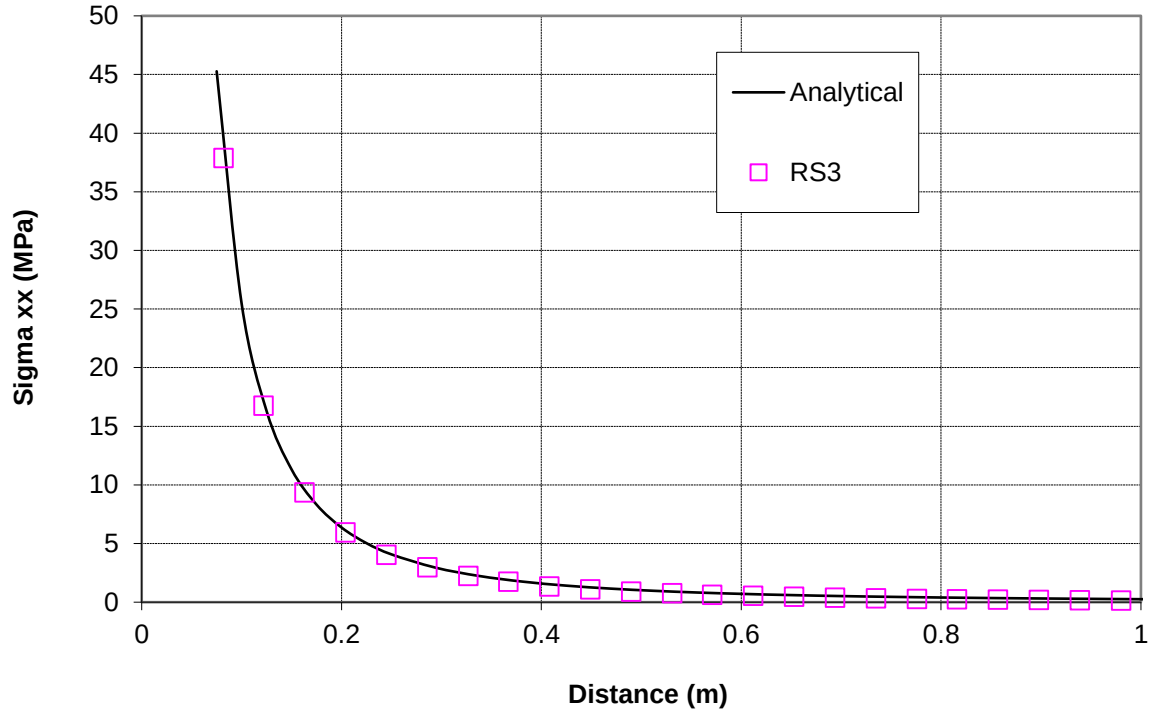

Figure 2-7: Comparison of stress in the x direction along line segment OB

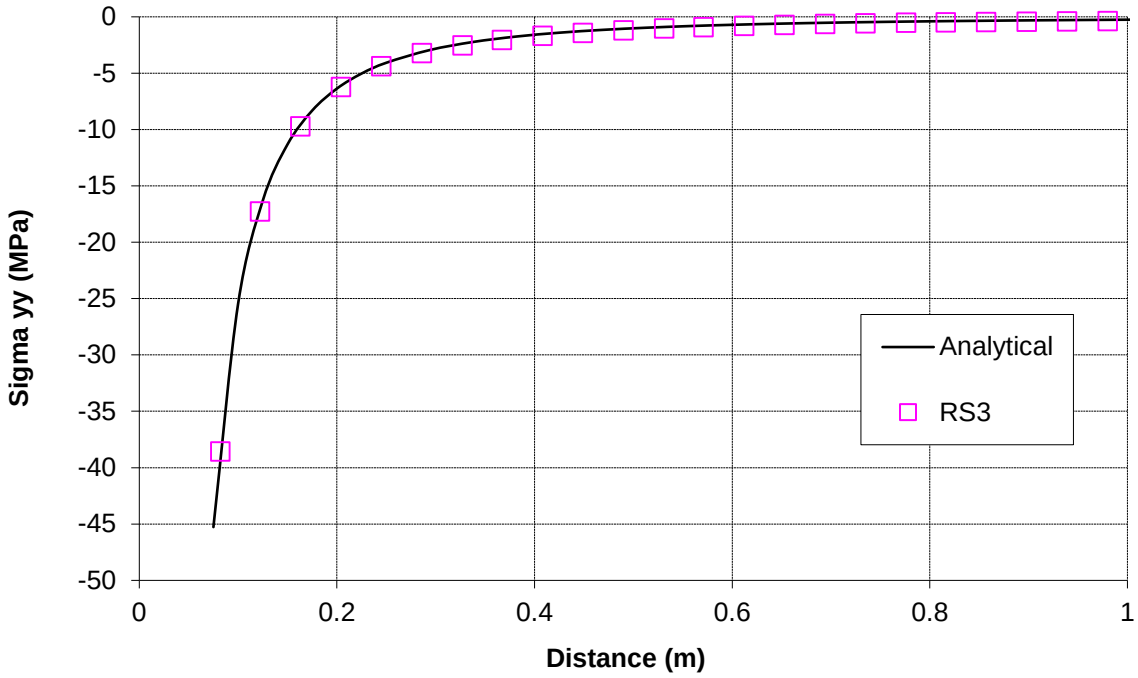

Figure 2-8: Comparison of stress in the y direction along line segment OB

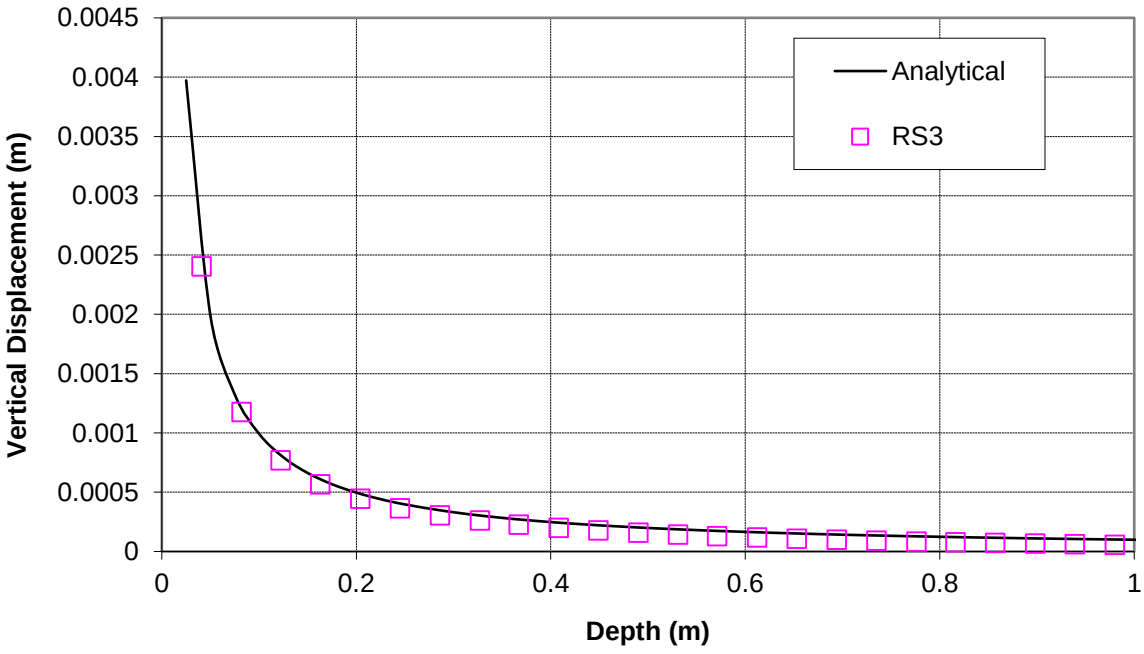

Figure 2-9: Comparison of vertical displacement along line segment OC

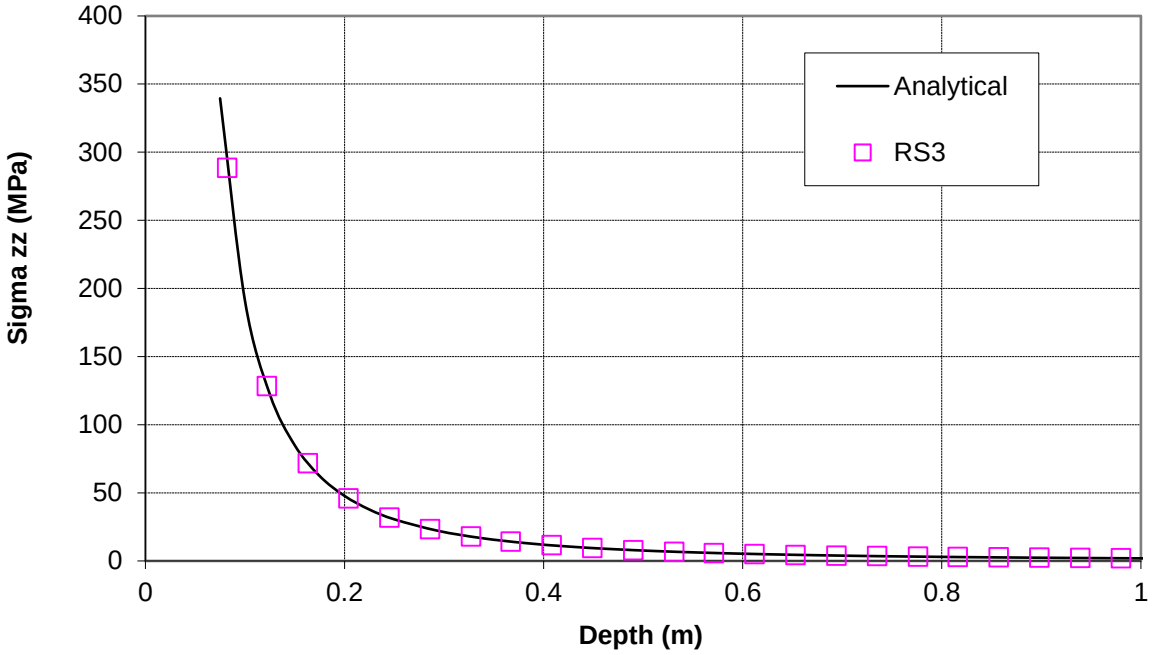

Figure 2-10: Comparison of vertical stress along line segment OC

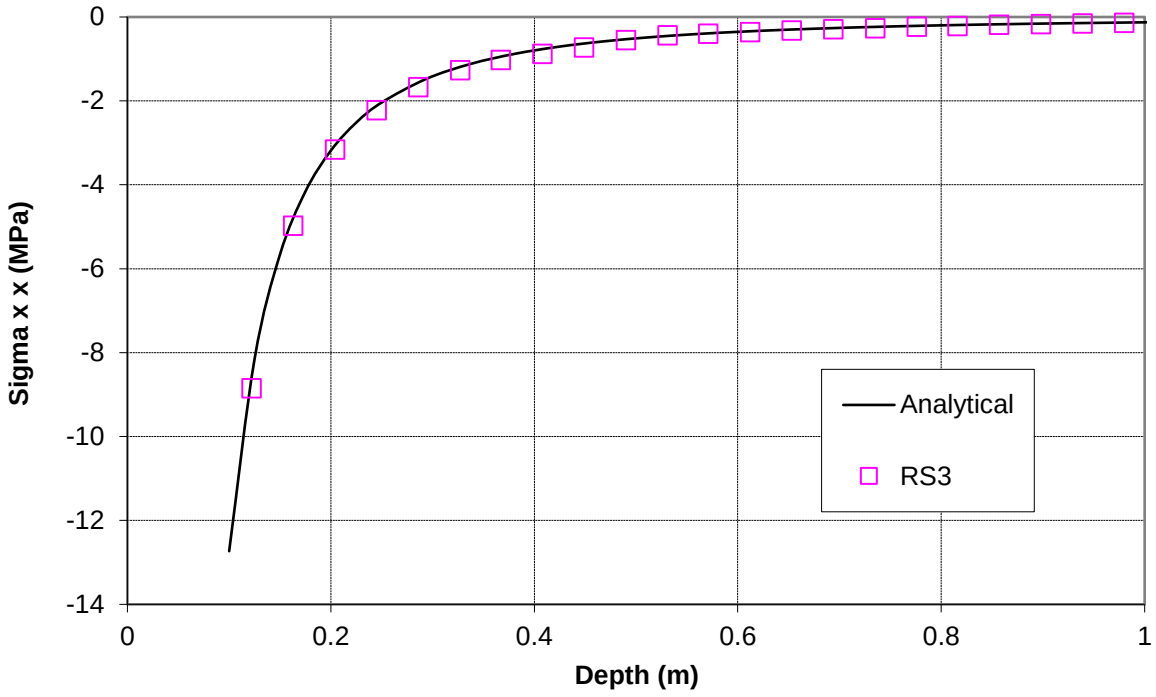

Figure 2-11: Comparison of stress in the x direction along line segment OC

2.5 References

1. Craig, R.F. (1997). *Soil Mechanics 6th Edition.* New York: Spon Press.

2.6 Data Files

The input data file **V002.rs3model** can be found in the ***RS3*** installation folder.