# 3 Cylindrical Hole in an Infinite Mohr-Coulomb Medium

## 3.1 Problem Description

This problem verifies stresses and displacements for the case of a cylindrical hole in an infinite elastic-plastic medium subjected to a constant in-situ (compression +) stress field of:

$P_0$ = 30 MPa

The material is assumed to be linearly elastic and perfectly plastic with a failure surface defined by the Mohr-Coulomb criterion. Both the associated (dilatancy = friction angle) and non-associated (dilatancy = 0) flow rules are used. The following material properties are assumed:

Young's modulus (E) = 6778 *MPa*
Poisson's ratio (ν) = 0.21
Cohesion ($c$) = 3.45 *MPa*
Friction angle (φ) = 30°
Dilation angle ($\psi$) = 0° and 30°

The results can be directly compared to the 2D analytical solution. The radius of the hole is 1 m and is assumed to be small compared to the length of the cylinder; 2D plane strain conditions are in effect.

## 3.2 Closed Form Solution

The yield zone radius $R_0$ is given analytically by a theoretical model based on the solution of Salencon (1969):

$$R_0 = a\left(\frac{2}{K_p+1}\frac{P_0+\frac{q}{K_p-1}}{P_i+\frac{q}{K_p-1}}\right)^{1/(K_p-1)}$$

where

$$K_p = \frac{1+\sin\phi}{1-\sin\phi}$$

$$q = 2\cdot c\cdot\tan(45+\phi/2)$$

$P_0$ = initial in-situ stress

$P_i$ = internal pressure

The radial stress at the elastic-plastic interface is

$$\sigma_{re} = \frac{1}{K_p+1}(2P_0-q)$$

The stresses and radial displacement in the elastic zone are

$$\sigma_r = P_0 - (P_0 - \sigma_{re})\left(\frac{R_0}{r}\right)^2$$

$$\sigma_\theta = P_0 + (P_0 - \sigma_{re})\left(\frac{R_0}{r}\right)^2$$

$$u_r = \frac{R_0^2}{2G}\left(P_0 - \frac{2P_0 - q}{K_p + 1}\right)\frac{1}{r}$$

where r is the distance from the field point ($x$,$y$) to the center of the hole. The stresses and radial displacement in the plastic zone are

$$\sigma_r = -\frac{q}{K_p - 1} + \left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}$$

$$\sigma_\theta = -\frac{q}{K_p - 1} + K_p\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}$$

$$u_r = \frac{r}{2G}\left[(2\nu - 1)\left(P_0 + \frac{q}{K_p - 1}\right) + \frac{(1-\nu)(K_p^2 - 1)}{K_p + K_{ps}}\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{R_0}{a}\right)^{(K_p - 1)}\left(\frac{R_0}{r}\right)^{(K_{ps} + 1)} + \left(\frac{(1-\nu)(K_p K_{ps} + 1)}{K_p + K_{ps}} - \nu\right)\left(P_i + \frac{q}{K_p - 1}\right)\left(\frac{r}{a}\right)^{(K_p - 1)}\right]$$

where

$$K_{ps} = \frac{1 + \sin\psi}{1 - \sin\psi}$$

$\psi =$ Dilation angle

$\nu =$ Poisson’s Ratio

$G =$ Shear modulus

## 3.3 Model Information

The ***RS3*** model for this problem is built with 10-noded tetrahedron elements. As seen in Figure 3-1, the model uses:

- a graded mesh
- excavation radius $a$ = 1 m
- 1 m extrusion
- 40 segments (discretizations) around the circular opening
- fixed external boundary, located 21 m from the hole center

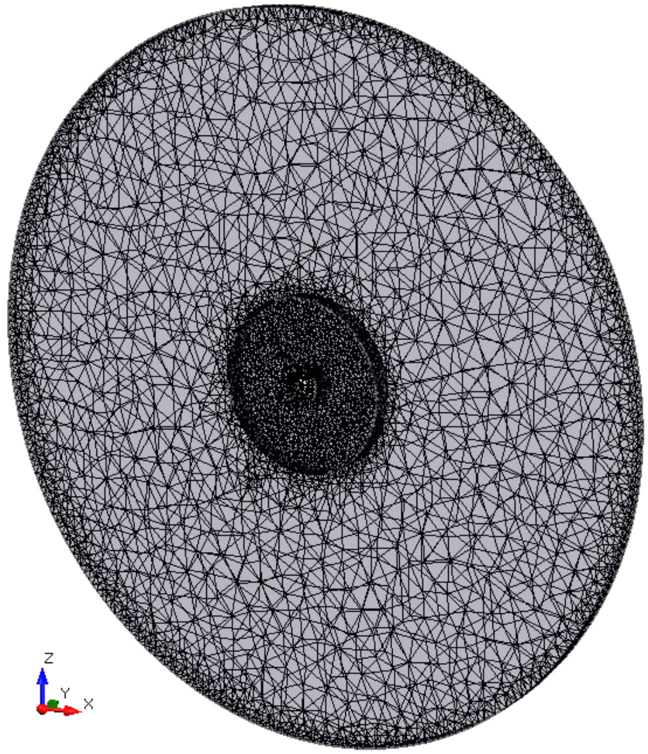


**Figure 3-1: Cylindrical hole in Mohr-Coulomb medium**

## 3.4 Results and Discussions

For associated flow (dilation angle $\psi = 30^0$), Figure 3-2 and Figure 3-3 show a direct comparison between ***RS3*** results and the analytical solution along the x-axis. Tangential stress ($\sigma_\theta$) is plotted against radial distance in Figure 3-2, while radial displacement ($u_r$) is plotted against radial distance in Figure 3-3. The comparable results of stresses and displacement for non-associated flow with dilation angle $\psi = 0^0$ are shown in Figure 3-4 and Figure 3-5. These plots indicate a close agreement between ***RS3*** results and the analytical solution.

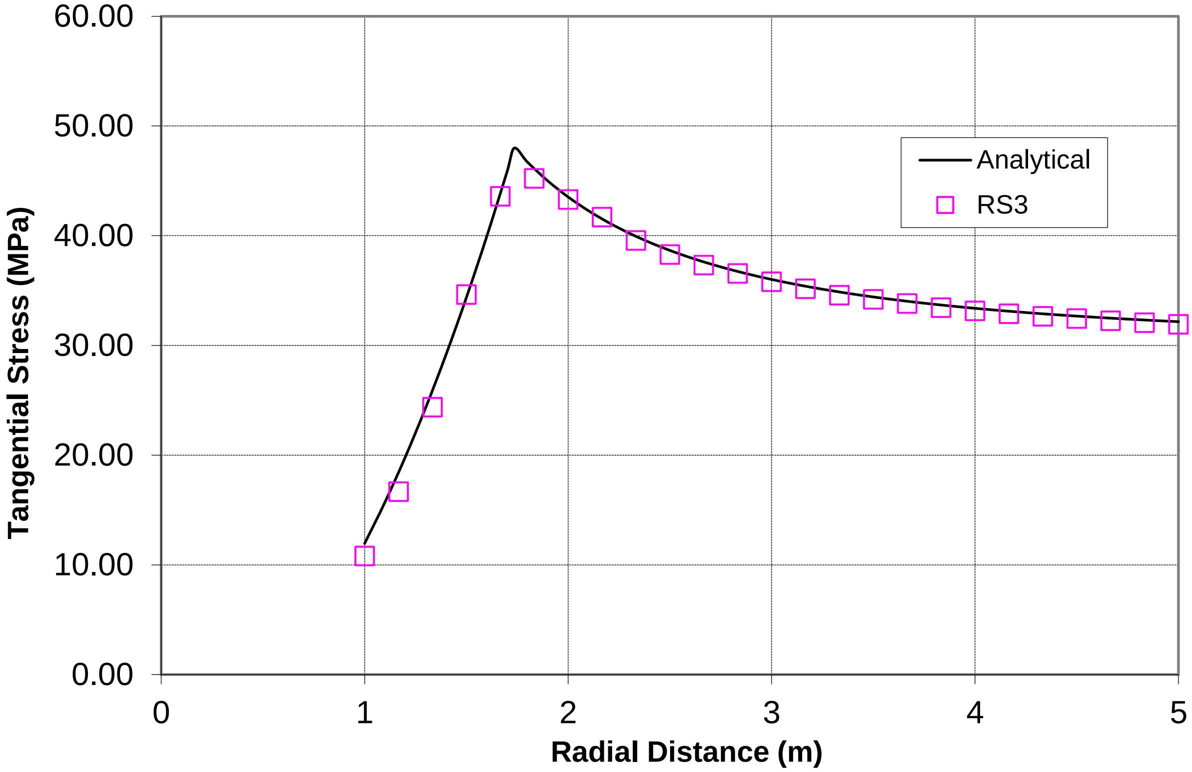


**Figure 3-2: Comparison of tangential stress for associated flow**

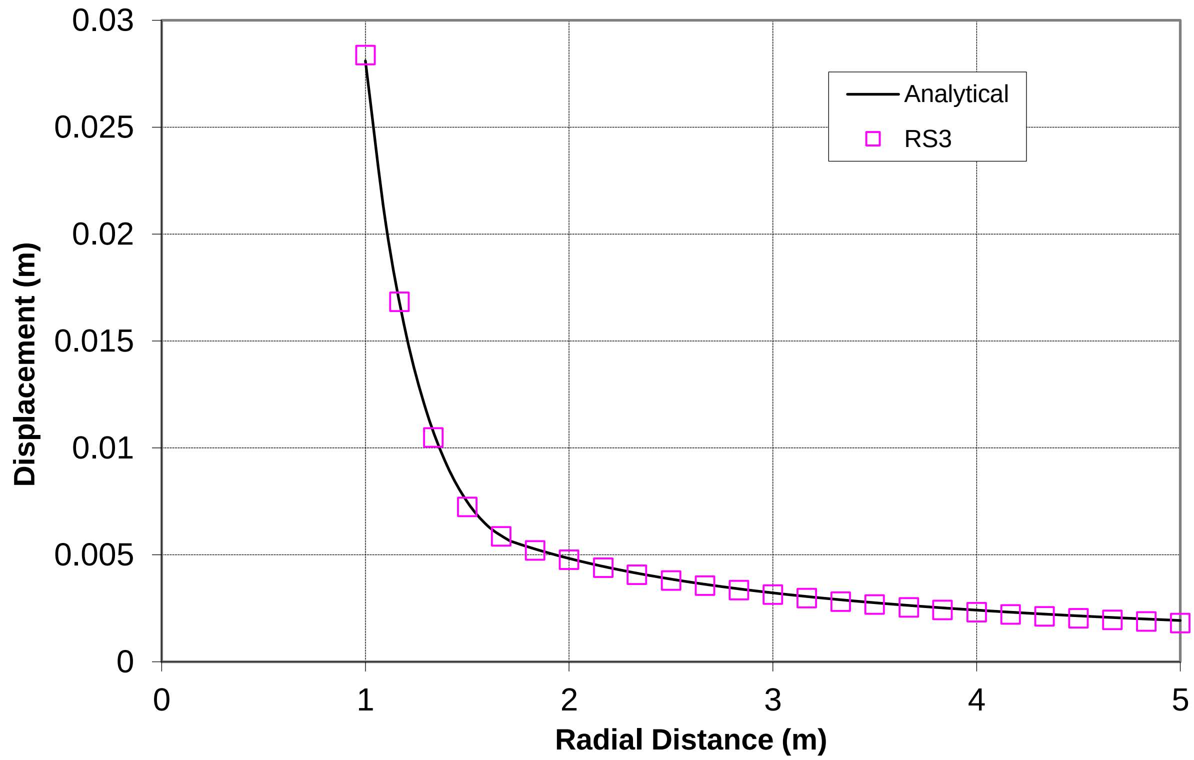


**Figure 3-3: Comparison of radial displacement for associated flow**

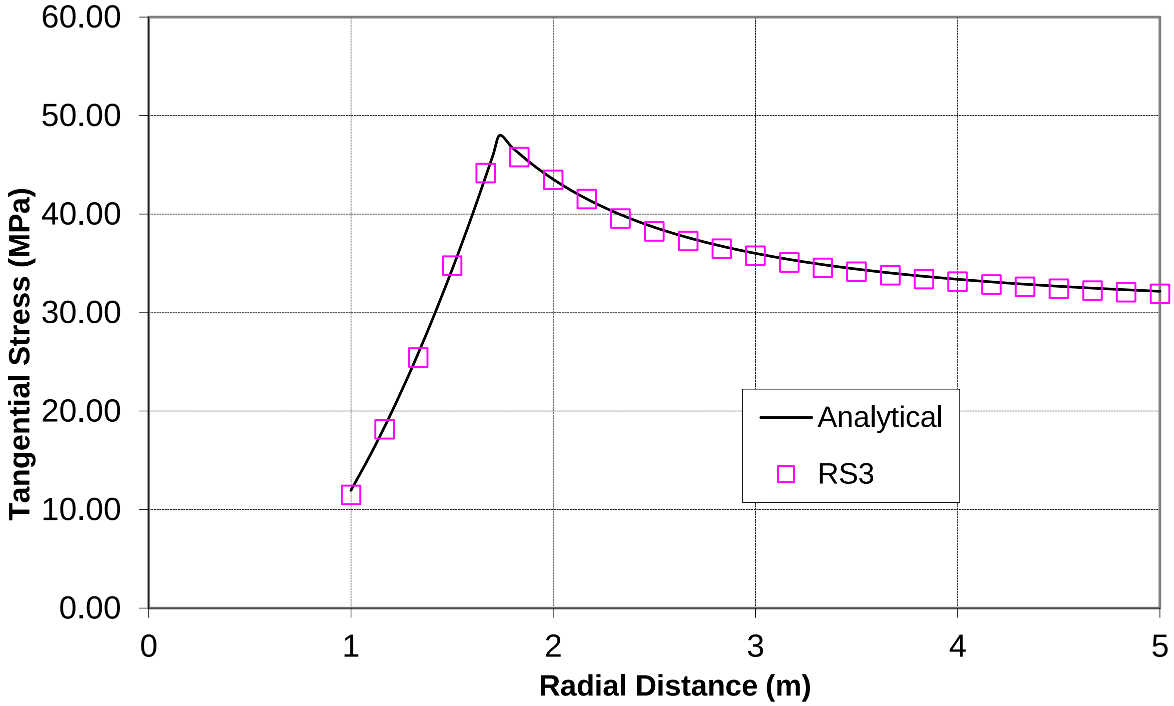


**Figure 3-4: Comparison of tangential stress for non-associated flow**

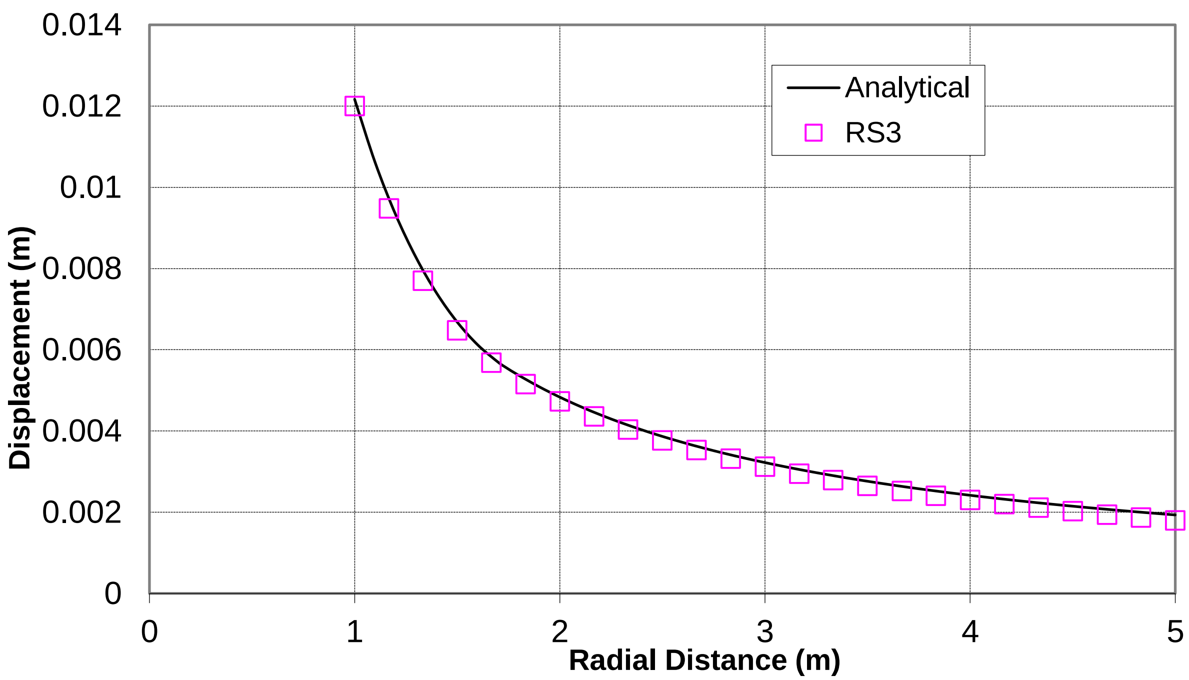


**Figure 3-5: Comparison of radial displacement for non-associated flow**

### 3.5 References

1. Salencon, J. (1969), « Contraction Quasi-Statique D’une Cavite a Symetrie Spherique Ou Cylindrique Dans Un Milieu Elasto-Plastique », *Annales Des Ports Et Chaussees*, Vol. 4, pp. 231-236.

### 3.6 Data Files

The input data files can be found in the ***RS3*** installation folder:

- **V003 Associated Flow.rs3model**
- **V003 Non-Associated Flow.rs3model**