8 Circular Tunnel Reinforced by Rock Bolts

8.1 Problem Description

This problem considers a circular tunnel in an elastic, isotropic rock mass reinforced with a circular array of rockbolts. Both end-anchored and grouted elastic rockbolts are considered; the former is assumed to interact with the rockmass only at the bolt ends and the latter is fully bonded to the rock along its entire length. The tunnel is exposed to an in-situ hydrostatic compression field of 10 MPa.

Figure 8-1 shows the problem as constructed in ***RS3***. The model uses a graded mesh of 4-noded tetrahedron elements. Fixed boundary conditions are used on the outer boundary, which is located 20 m from the hole centre.

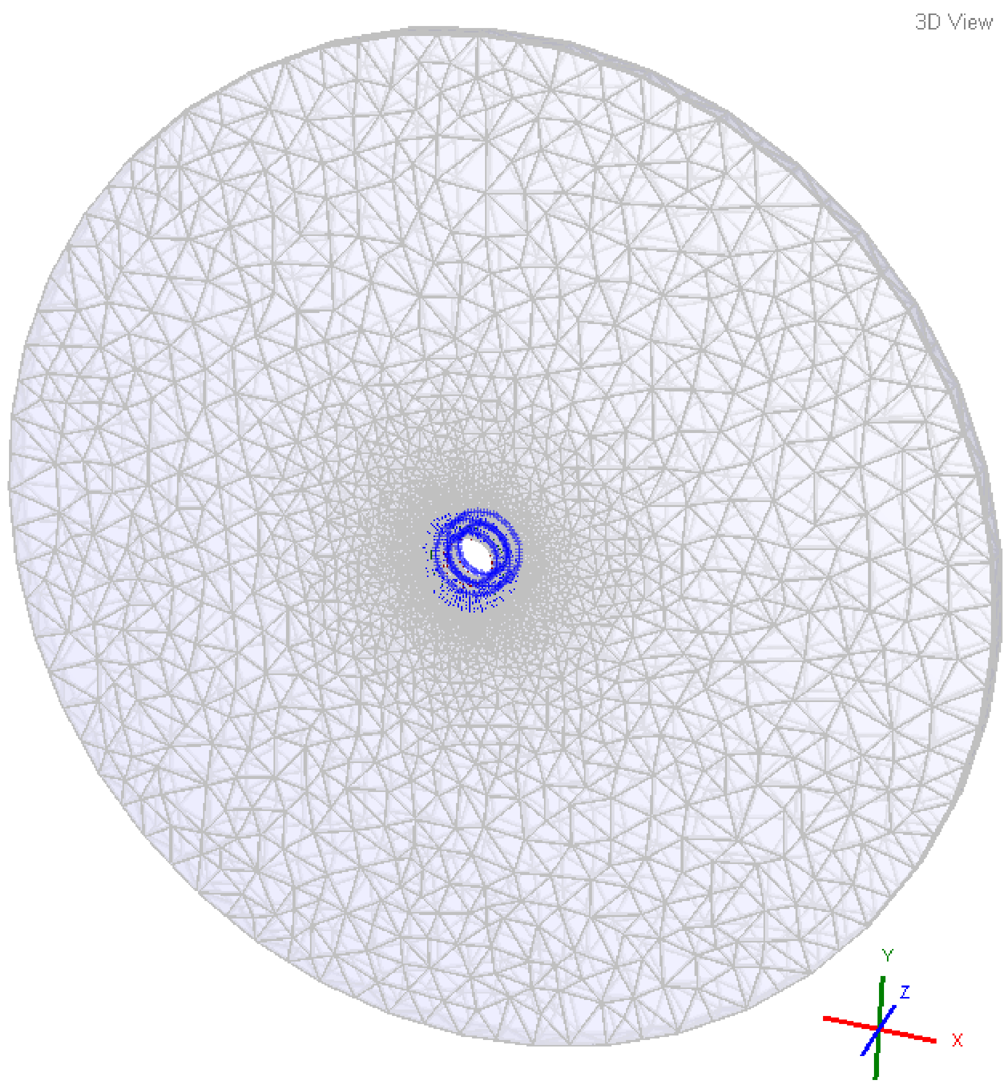

Figure 8-1: Tunnel in elastic medium supported by rockbolts as constructed in *RS3*

Table 8-1 and Table 8-2 summarize material and bolt properties.

Table 8-1: Model parameters

Parameter	*Value*
Tunnel radius (*a*)	1 m
In-situ stresses (σ_1, σ_3, σ_z, σ_0)	10 MPa
Young's modulus (*E*)	250 MPa
Poisson's ratio (*v*)	0.3

Table 8-2: Bolt parameters

Parameter	*Value*
Diameter (d_b)	25 mm
Young's modulus (E_b)	116667 MPa
Length (L_b)	1 m
Number of bolts (N_b)	72
Bolt spacing along tunnel axis (*D*)	0.1 m

8.2 Analytical Solution

Carranza-Torres [1] presents analytical stress and displacement distributions for both end-anchored and fully grouted rockbolts in an elastic medium. This solution assumes that the effect of the support can be "smeared" circumferentially and along the tunnel axis to produce a single axisymmetric stress/displacement distribution. Figure 8-2 illustrates the tunnel schematically.

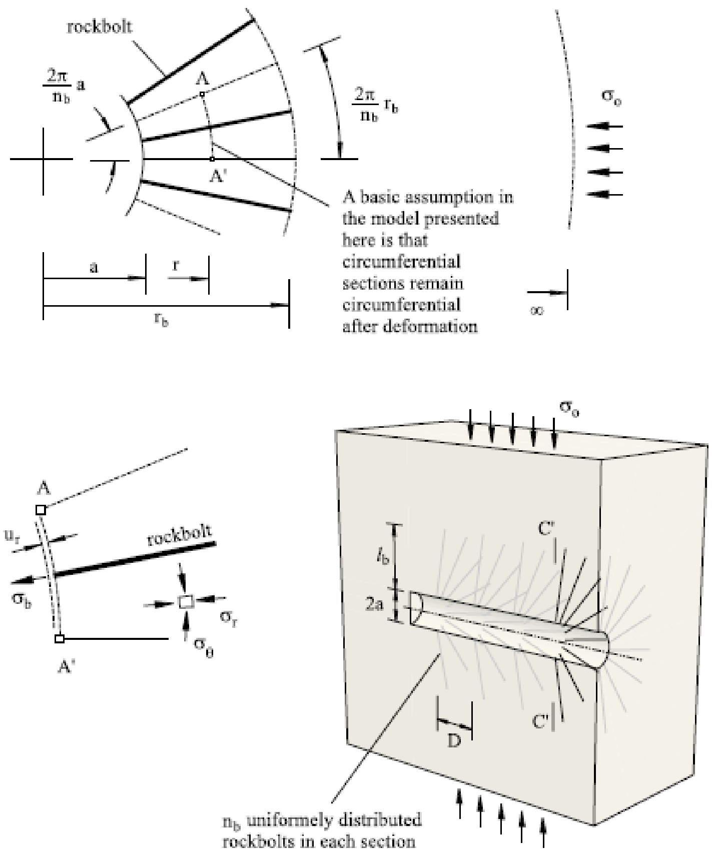

Figure 8-2: Reinforced circular tunnel [1]

Dimensionless parameters *β, α,* μ, and *ρ* are defined as follows:

$$\alpha = \frac{N_b A_b}{2\pi a D}$$

$$\beta = \frac{\alpha E_b}{2G}$$

$$\rho = \frac{a}{r}$$

$$\mu = \frac{v}{1-2v}$$

where *r* is the radial distance from the centre of the tunnel and *G* is the shear modulus of the rockmass. At the ends of the rockbolts, i.e. $r = r_b$, the non-dimensional parameter *ρ* has the value:

$$\rho_b = \frac{r_b}{a}$$

For the end-anchored case, the radial stress σ_r^b at $r = r_b$ is:

$$\frac{\sigma_r{}^b}{\sigma_0} = \frac{2\left(1-\rho_b{}^2\right)+2\mu(1-\rho_b)(1+\rho_b+\beta)+\beta\rho_b(1-\rho_b)(3+\rho_b)}{2+\beta\rho_b(3-\rho_b)+2\mu\left(1+\beta\rho_b-\beta\rho^2\right)}$$

For the fully grouted case, the radial stress $\sigma_r{}^b$ at $r = r_b$ is:

$$\frac{\sigma_r{}^b}{\sigma_0} = \frac{\beta N+(1+\mu)(N_4-N_3)-2\beta\rho_b(N_2-N_1)+2(1+\mu)(N_2-N_1)\ln\left(\frac{1+\mu+\beta\rho_b}{1+\mu}\right)}{\beta N-N_3(1+\mu)+2\beta\rho_b N_1-2(1+\mu)N_1\ln\left(\frac{1+\mu+\beta\rho_b}{1+\mu}\right)}$$

where, in the fully grouted case:

$$N = -2\beta(1-\rho_b)(1+2\mu)^2+2(1+\mu)(1+\beta+2\mu)(1+2\mu+\beta\rho_b)\ln\left(\frac{1+\mu+\beta}{1+\mu+\beta\rho_b}\right)$$

$$N_1 = \beta(1+2\mu+\beta)$$

$$N_2 = \beta(1+2\mu+\beta\rho_b)$$

$$N_3 = 2\beta^2\frac{1+2\mu}{1+\mu}-2\beta(1+2\mu+\beta)\ln\left(\frac{1+\mu+\beta}{1+\mu}\right)$$

$$N_4 = 2\beta^2\rho_b\frac{1+2\mu}{1+\mu}-2\beta(1+2\mu+\beta\rho_b)\ln\left(\frac{1+\mu+\beta\rho_b}{1+\mu}\right)$$

$$C_1 = -\frac{N_1\left(1-\frac{\sigma_r{}^b}{\sigma_0}\right)-N_2}{N}$$

$$C_2 = -\frac{N_3\left(1-\frac{\sigma_r{}^b}{\sigma_0}\right)-N_4}{N}$$

In the end-anchored case:

$$C_1 = \frac{(1-\rho_b)(1+\rho_b+\beta\rho_b)-(1+\beta\rho_b)\frac{\sigma_r{}^b}{\sigma_0}}{2\beta\rho_b(1+\mu)(1-\rho_b)+(1+2\mu)\left(1-\rho_b{}^2\right)}$$

$$C_2 = -\frac{\beta(1-\rho_b)-(1+2\mu+\beta)\frac{\sigma_r{}^b}{\sigma_0}}{2\beta\rho_b(1+\mu)(1-\rho_b)+(1+2\mu)\left(1-\rho_b{}^2\right)}$$

For both cases, the stress and displacements in the unreinforced region $r \geq r_b$ are given by:

$$\frac{2G\mu_r}{\sigma_0 a} = \left(1 - \frac{\sigma_r{}^b}{\sigma_0}\right)\frac{\rho}{\rho_b{}^2}$$

$$\frac{\sigma_r}{\sigma_0} = 1 - \left(1 - \frac{\sigma_r{}^b}{\sigma_0}\right)\frac{\rho^2}{\rho_b{}^2}$$

$$\frac{\sigma_\theta}{\sigma_0} = 1 + \left(1 - \frac{\sigma_r{}^b}{\sigma_0}\right)\frac{\rho^2}{\rho_b{}^2}$$

In the reinforced region $r < r_b$, the solution for the end-anchored case is:

$$\mu_r = \frac{a\sigma_0}{2G}\left(\frac{C_1}{\rho} + C_2\rho\right)$$

$$\sigma_r = \sigma_0 + 2G\mu\rho\frac{\mu_r}{a} - 2G(1+\mu)\frac{\rho^2}{a}\frac{d\mu_r}{d\rho}$$

$$\sigma_\theta = \sigma_0 + 2G(1+\mu)\rho\frac{u_r}{a} - 2G\mu\frac{\rho^2}{a}\frac{d\mu_r}{d\rho}$$

$$\frac{d\mu_r}{d\rho} = -\frac{a\sigma_0}{2G}\left(\frac{C_1}{\rho^2} - C_2\right)$$

The solution for the fully grouted case is:

$$\frac{2G}{\sigma_0}\frac{\mu_r}{a} = -C_2\frac{1}{\rho}\frac{1+\mu}{\beta} + 2C_1\left(1 - \frac{1}{\rho}\frac{1+\mu}{\beta}\ln\left(\frac{1+\mu+\beta\rho}{1+\mu}\right)\right)$$

$$\sigma_r = \sigma_0 + 2G\mu\rho\frac{u_r}{a} - 2G(1+\mu)\frac{\rho^2}{a}\frac{du_r}{d\rho}$$

$$\sigma_\theta = \sigma_0 + 2G(1+\mu)\rho\frac{u_r}{a} - 2G\mu\frac{\rho^2}{a}\frac{du_r}{d\rho}$$

$$\frac{du_r}{d\rho} = C_2\frac{1}{\rho^2}\frac{1+\mu}{\beta} - 2C_1\left(\frac{1+\mu}{\rho(1+\mu+\beta\rho)} - \frac{1}{\rho^2}\frac{1+\mu}{\beta}\ln\left(\frac{1+\mu+\beta\rho}{1+\mu}\right)\right)$$

8.3 Results

Figure 8-3 to Figure 8-6 shows the analytical stress and displacement distributions as determined analytically and using ***RS3***. Both sets of results are very similar.

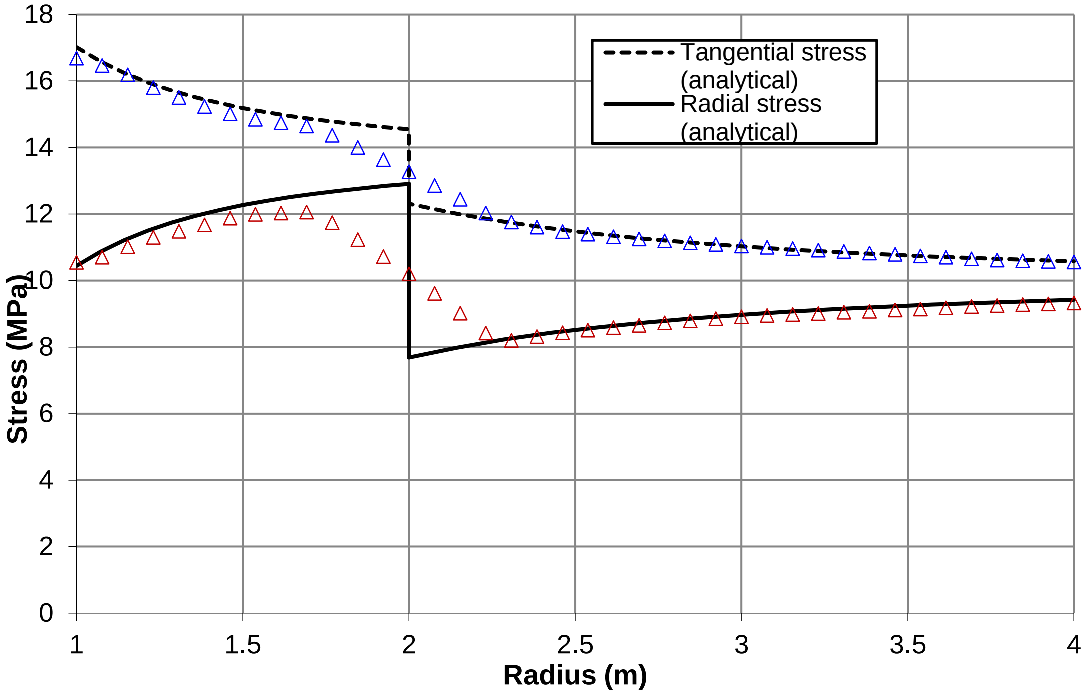

Figure 8-3: Radial and tangential stress distributions surrounding the tunnel reinforced with end-anchored bolts

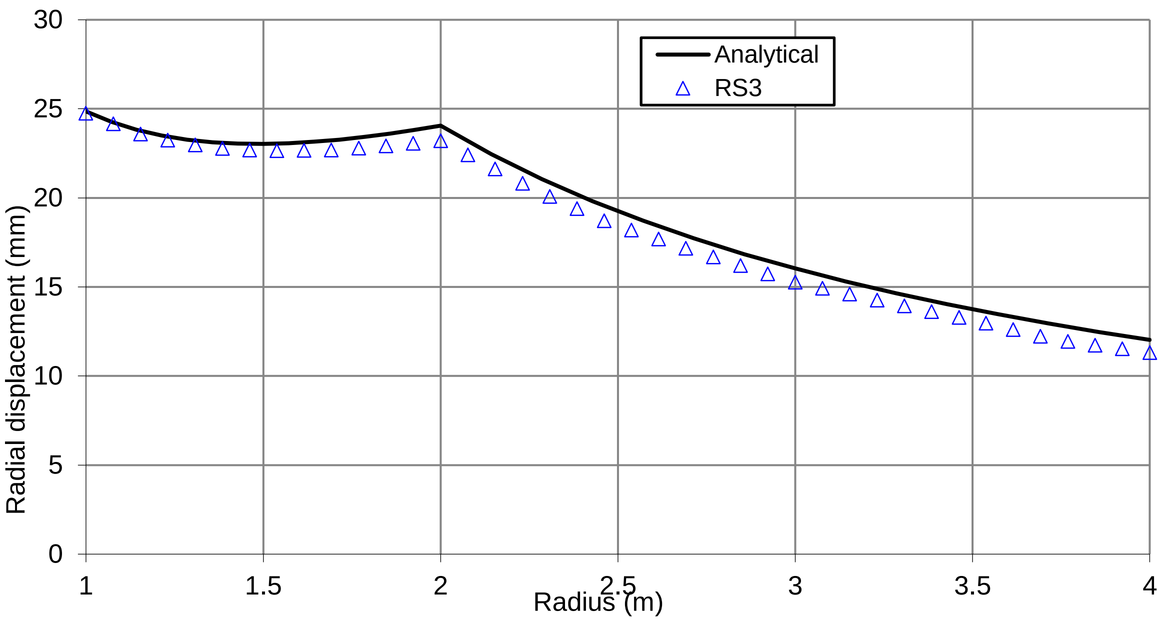

Figure 8-4: Radial displacement distributions surrounding the tunnel reinforced with end-anchored bolts

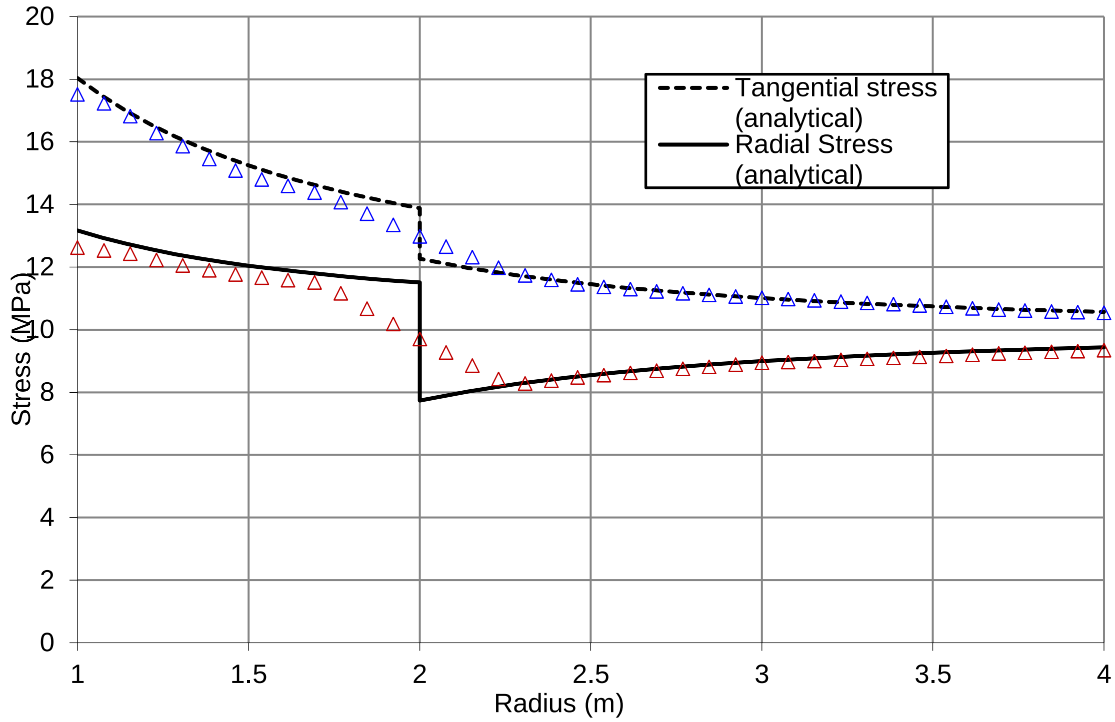

Figure 8-5: Radial and tangential stress distributions surrounding the tunnel reinforced with fully grouted bolts

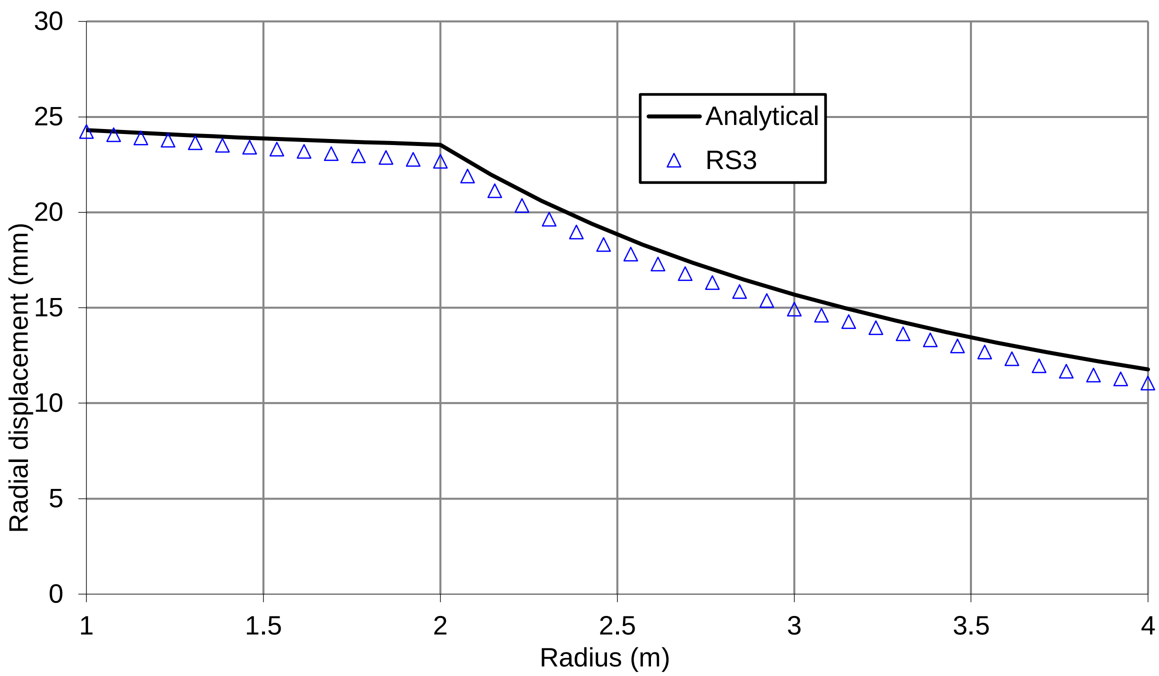

Figure 8-6: Radial displacement distributions surrounding the tunnel reinforced with fully grouted bolts

8.4 References

1. Carranza-Torres, C. (2002). "Elastic solution for the problem of excavating a circular tunnel reinforced by i) *anchored* or ii) *fully grouted* rockbolts in a medium subject to uniform far-fields stresses". Note for the International Canada/US/Japan joint cooperation on rockbolt analysis.

8.5 Data Files

The input data files **V008 End Anchored.rs3model** and **V008 Fully Bonded).rs3model** can be found in the *RS3* installation folder.