

15 Simply Supported Rectangular Plate

15.1 Problem Description

This verification example verifies the moment and the displacement of a rectangular plate under uniform pressure $p = 100 \text{ MPa}$ (Figure 15-1).

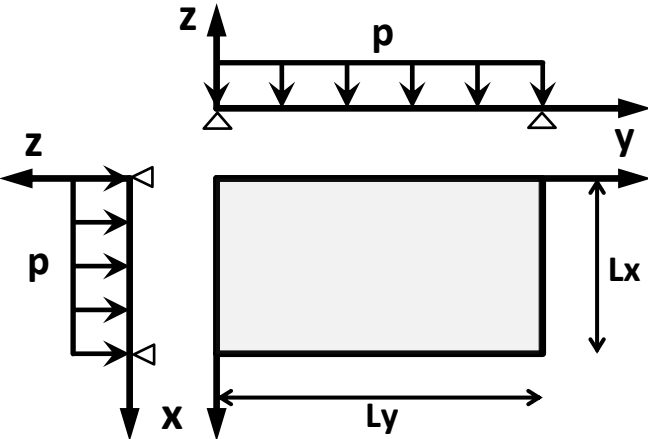


Figure 15-1: Simply supported rectangular plate under uniform pressure

The plate is isotropic and elastic with the following properties:

- Young's modulus = 200 GPa
- Poisson's ratio = 0.3
- $L_x = 10\text{m}$
- $L_y = 10\text{m}$

The results are compared to the analytical solution provided by Timoshenko and Woinowsky-Krieger [1].

15.2 Closed Form Solution

The classical solution can be used to find the displacement and moment distribution at any point in the plate [1].

The displacement w and the moment M_x and M_y at any point (x,y) are given by

$$w(x, y) = \frac{16 p}{\pi^6 D} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{\sin\left(\frac{m \pi x}{L_x}\right) \sin\left(\frac{n \pi y}{L_y}\right)}{m n \left[\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2 \right]^2}$$

$$M_x(x, y) = \frac{16p}{\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \left[\left(\frac{m}{L_x} \right)^2 + \nu \left(\frac{n}{L_y} \right)^2 \right] \frac{\sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right)}{mn \left[\left(\frac{m}{L_x} \right)^2 + \left(\frac{n}{L_y} \right)^2 \right]^2}$$

$$M_y(x, y) = \frac{16p}{\pi^4} \sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \left[\nu \left(\frac{m}{L_x} \right)^2 + \left(\frac{n}{L_y} \right)^2 \right] \frac{\sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right)}{mn \left[\left(\frac{m}{L_x} \right)^2 + \left(\frac{n}{L_y} \right)^2 \right]^2}$$

15.3 Model Information

The model is built in **RS3** with a uniform mesh of 6-noded triangular plate elements as shown in Figure 15-2. The plate has a thickness of 0.1m. The model is restrained in all edges in x , y and z direction. The model geometry was devised to ensure there would be plate elements aligned along the major areas of interest for better comparison to the analytical solution. This does not cause plate elements to be disconnected at the plate boundaries as shown in the results.

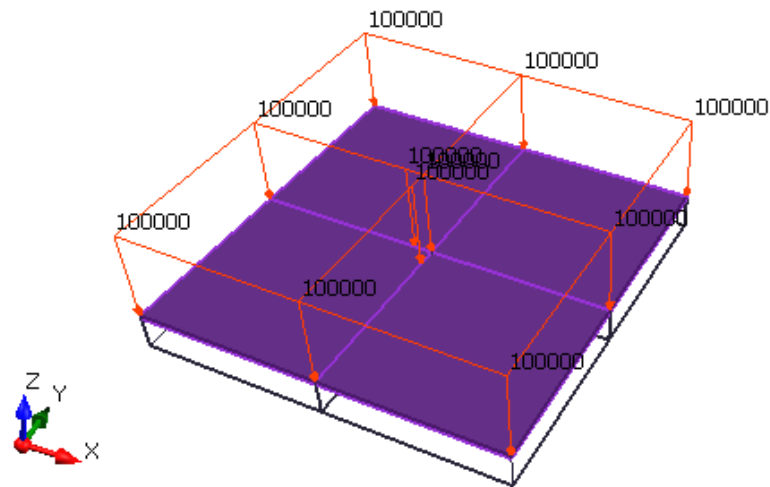


Figure 15-2: Model of a simply supported rectangular plate under uniform pressure in RS3

15.4 Results and Discussions

Figure 15-3, Figure 15-4 show the displacement and the moment distribution along a center line of the model (either the x - or y -axis). The **RS3** results are in very close agreement with the analytical solutions.

A contour plot of the displacement is also and presented in Figure 15-5.

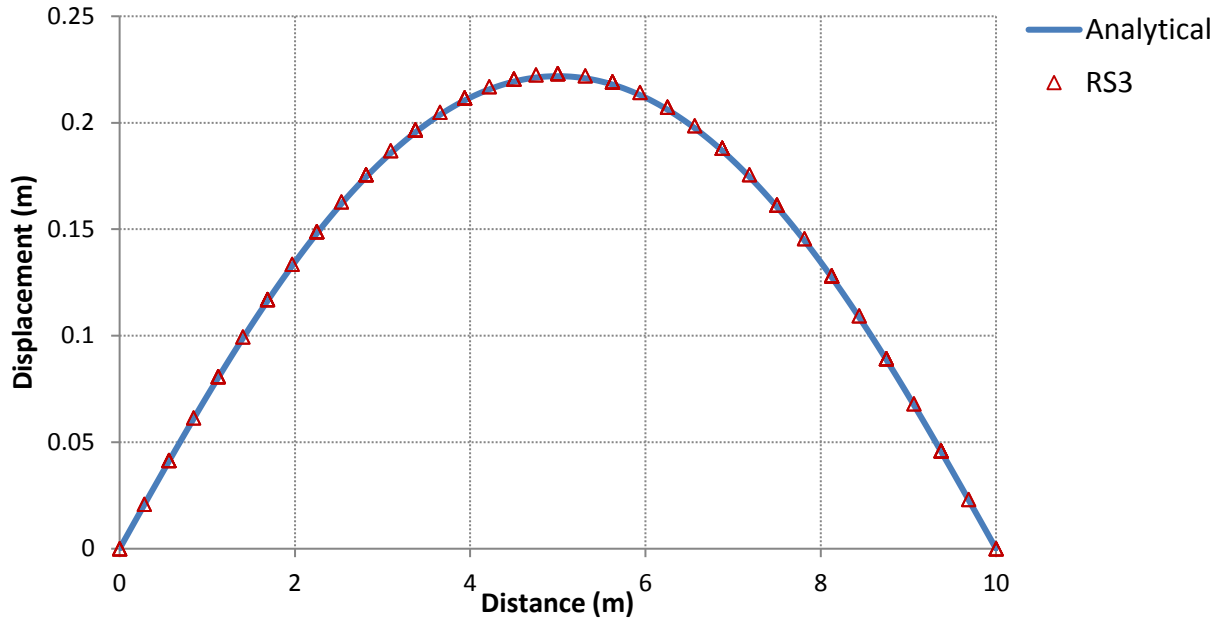


Figure 15-3: Comparison of displacement along the line $x = 5\text{m}$

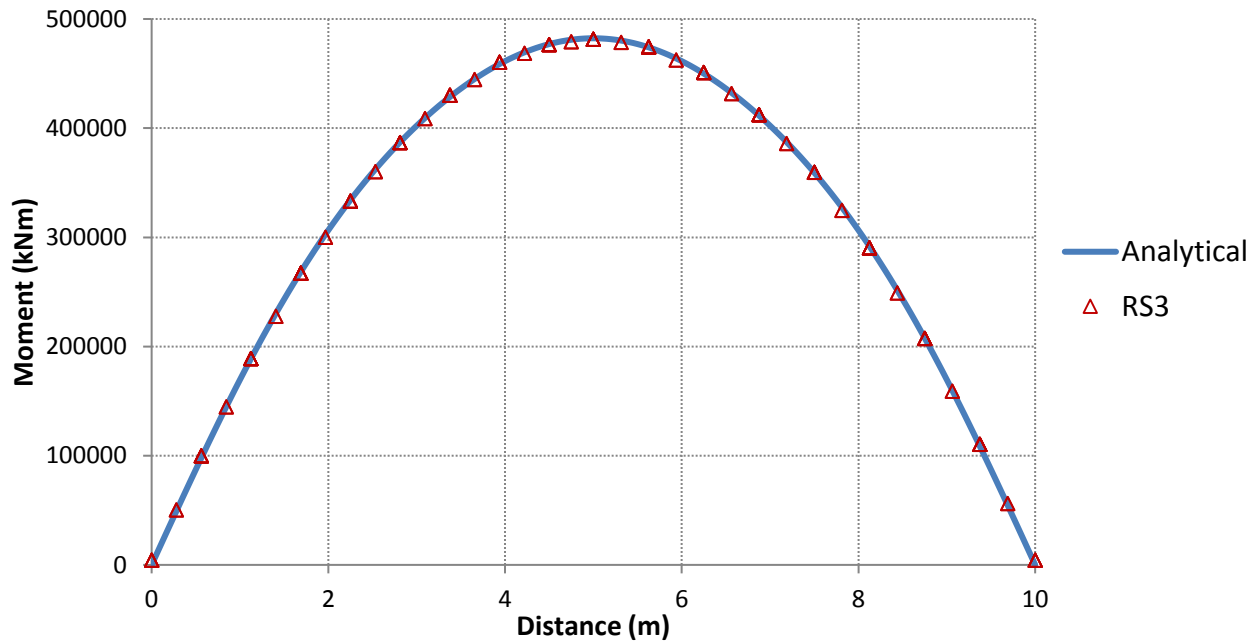


Figure 15-4: Comparison of moment along the line $x = 5\text{m}$

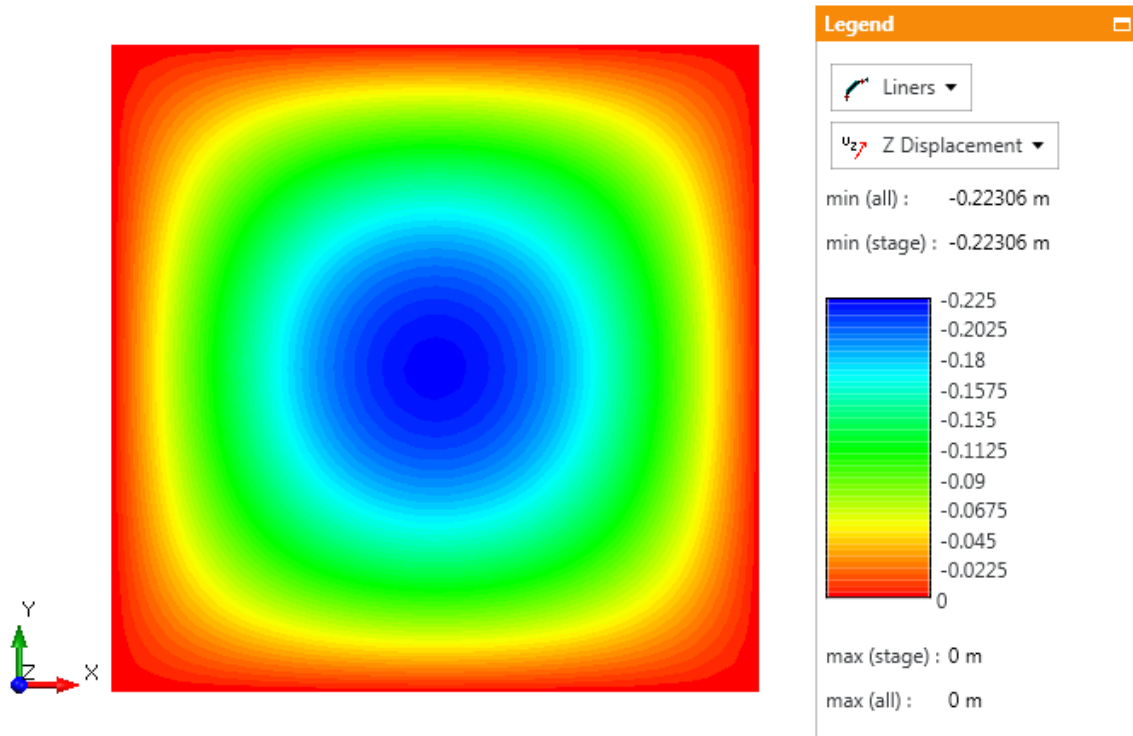


Figure 15-5 Vertical displacement distribution of RS3 results

15.5 References

1. Timoshenko, S. and Woinowsky-Krieger, S. "Theory of plates and shells". McGraw-Hill New York, 1959.

15.6 Data Files

The input data file **V015.rs3model** can be found in the **RS3** installation folder.