



RSPile

Axially Loaded Piles

Theory Manual

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1. t-z Curve Method using Finite Element Analysis

The stress-strain relationship for an axially loaded pile can be described through three loading mechanisms: axial deformation in the pile, soil skin friction along the shaft, and soil end-bearing (Figure 1 1a). Using a spring-mass model in which springs represent material stiffness, numerical techniques can be employed to conduct the load-settlement analysis (Figure 1 1b).

As per the assumed sign convention, the x-axis typically corresponds to the distance along the pile, while the z-axis corresponds to the distance below the ground surface. However, since the pile length above the ground surface does not affect the stress distribution of the pile below ground, the z-axis has been traditionally used to denote the distance along the embedded pile length and will be used as such for this manual.

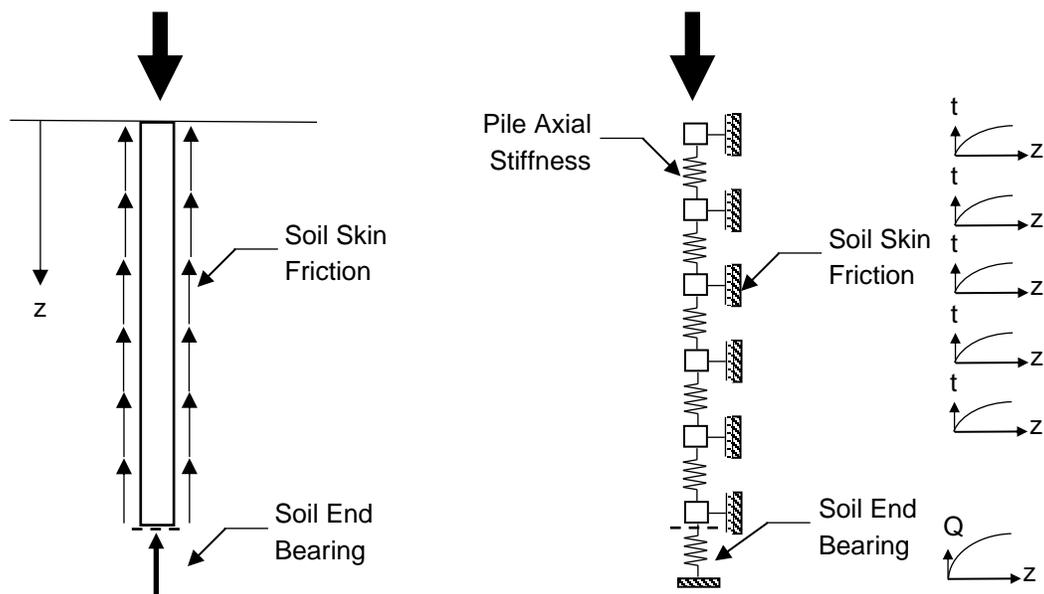


Figure 1-1: (a) Load transfer mechanisms in an axially loaded pile and (b) spring mass model

The t-z curve method using finite element analysis was employed to solve the governing differential equation. The t-z curve method allows for simulation of the non-linear stress-strain behavior in soil by employing non-linear stiffness curves denoted as t-z curves for soil skin friction and Q-z curve for the soil end bearing resistance. The stiffness is computed at each iteration based on the solved displacement values.

2. Governing Differential Equation

From force equilibrium of the free body diagram shown in Figure 2-1, we obtain an equation for the load transfer of the externally applied loads to skin friction and pile deformation.

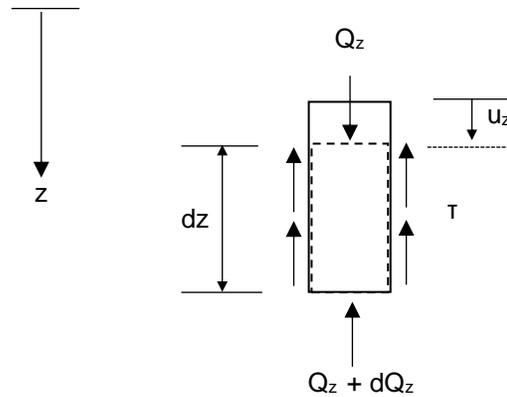


Figure 2-1: Free body diagram of pile segment

$$Q_z = Q_z + dQ_z + \tau \cdot C \cdot dz$$

$$-\frac{dQ_z}{dz} = \tau \cdot C \quad \text{Equation 1}$$

- where Q_z = internal pile force at depth z
 τ = soil unit skin friction at depth z
 C = circumference of pile segment at depth z

For axially loaded beams, the following equation can be used to describe the development of internal forces in the pile due to axial deformation of the pile segment.

$$Q_z = -EA \frac{du_z}{dz} \quad \text{Equation 2}$$

- Where E = pile segment modulus of elasticity at depth z
 A = pile segment cross sectional area at depth z
 u_z = pile segment displacement at depth z due to applied loads

Differentiating Equation 2 by z and substituting into Equation 1 yields the following governing differential equation for the pile and soil.

$$EA \frac{d^2 u_z}{dz^2} = \tau \cdot C$$

$$-EA \frac{d^2 u_z}{dz^2} + \tau \cdot C = 0 \quad \text{Equation 3}$$

For an axially loaded pile, the body force produced by the pile unit weight is negligible compared to the applied loads and is therefore neglected.

3. Finite Element Discretization

The pile was discretized into segments consisting of two pile elements and one soil shear element, as shown in Figure 3-1. Note the use of the term pile segment to describe the configuration of two pile elements and one soil shear element. Each soil shear element characterizes the effect of skin friction between the pile and soil. The element configuration was selected to ensure that the soil displacement used to calculate the skin friction is based on the midpoint of each pile segment. As such, the length of each pile element to calculate pile axial stiffness is half the length of the chosen segment length since there are two pile elements per segment. The length of each soil shear element is equal to the full length of each segment since there is one shear element per segment.

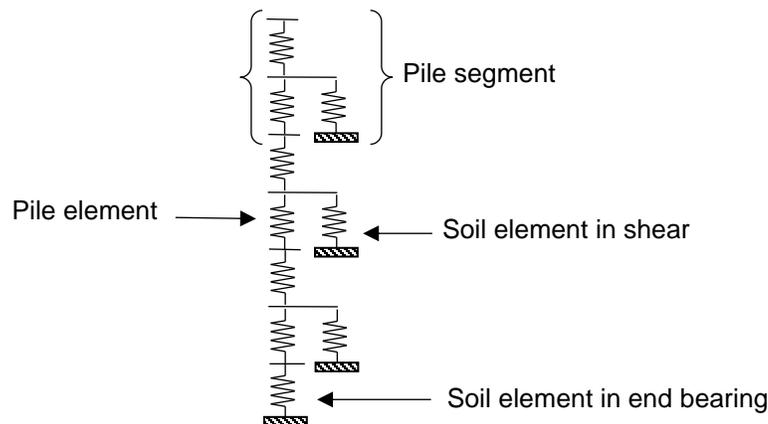


Figure 3-1: Pile segment discretization into pile elements and soil elements

The assumptions made for axial load analysis by solving the governing differential equation using the finite element method are as follows:

1. The pile is geometrically straight such that second order effects are not considered
2. Eccentric loads are not considered
3. The pile will not deform significantly throughout the simulation as to alter the original pile geometry
4. The pile material is isotropic

4. Pile Axial Stiffness

The pile is assumed to be linear elastic and perfectly plastic under axial compression. The stiffness of each pile element ($k_{pile,axial}$) in the elastic range is calculated using the following equation.

$$k_{pile,axial} = \frac{E_{pile}A_{pile}}{L_{pile}} \quad \text{Equation 4}$$

where E_{pile} = Pile modulus of elasticity
 A_{pile} = Cross sectional area of the pile element
 L_{pile} = Length of pile element

5. Soil Load Transfer Mechanisms

5.1. Skin Friction

The stiffness of each soil skin element $k_{soil, shear}$ is found using the t-z curve to obtain the unit skin friction corresponding to the soil displacement of the current iteration.

$$k_{soil, shear} = T/L_{segment} \quad \text{Equation 5}$$

Where $L_{segment}$ = length of each pile segment
 T = total skin friction of each pile segment in units of force

The total skin friction (T) for a pile segment is calculated from the following equation.

$$T = \tau A_{s,ext} \quad \text{Equation 6}$$

where τ = soil unit skin friction in units of force per area
 $A_{s,ext}$ = surface area of the pile segment exterior in contact with soil in shear

Recommended t-z curves are presented later in this report. Currently, the exterior surface area for pre-defined, standard sections, such as those found in the AISC shape database, are calculated based on the

equivalent diameter. The equivalent diameter is the diameter that would produce a circular area that is the same as the section's cross sectional area. The exterior surface area of a pile segment for a standard section is the length of the segment multiplied by the theoretical circumference based on the equivalent diameter. For a User Defined Section, the equivalent diameter defined by the user is used to compute the theoretical exterior surface area. For a future update, the exterior surface area will be based on the actual shape of the selected standard section.

5.2. End Bearing Resistance – Typical Cross Section

The end bearing resistance (Q) for a pile segment is calculated from the following equation.

$$Q = qA_{toe} \quad \text{Equation 7}$$

where q = unit end bearing resistance in units of force per area
 A_{toe} = cross sectional area of pile toe

5.3. End Bearing Resistance – Plugged and Unplugged Condition

For an open tube pile, the ultimate end bearing resistance is the minimum of the plugged and unplugged conditions. The plugged condition is the end bearing resistance when the open tube pile is compacted with soil such that the load transfer mechanism is the soil against the full cross sectional area ($A_{plugged}$), as shown in Figure 5-1a. The unplugged condition occurs if we consider the load transfer mechanism at the pile toe to consist of soil against the pile cross sectional area ($A_{unplugged}$) and internal skin friction ($\tau_{internal}$) from soil moving inside the pile shaft, as shown in Figure 5-1b. In *RSPile*, the total embedment length is considered in the calculation of internal skin friction. For cohesive soils, the remolded shear strength is used to calculate the internal skin friction since the soil inside the shaft has been disturbed from its in-situ condition.

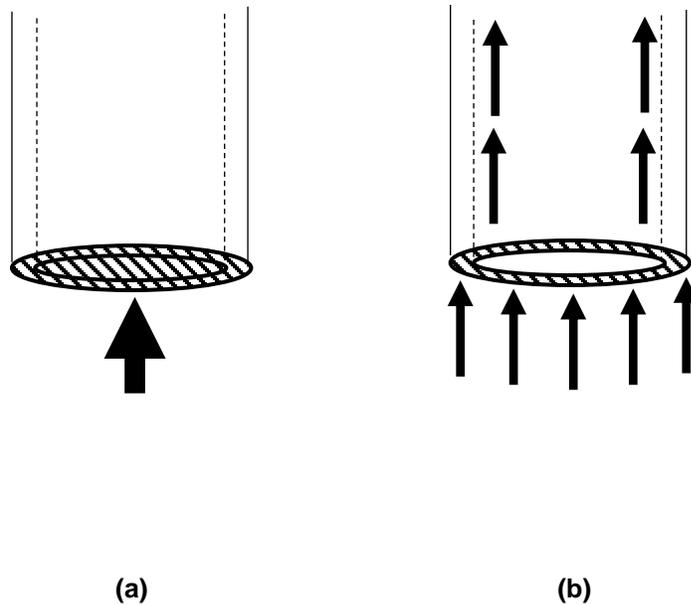


Figure 5-1: End bearing resistance considering (a) plugged condition and (b) unplugged condition

The following equation is used to calculate the end bearing resistance considering plugged condition ($Q_{plugged}$).

$$Q_{plugged} = qA_{plugged} \quad \text{Equation 8}$$

where q = unit end bearing resistance in units of force per area
 $A_{plugged}$ = full or plugged cross sectional area at the pile toe

The following equation is used to calculate the end bearing resistance considering unplugged condition ($Q_{unplugged}$).

$$Q_{unplugged} = qA_{unplugged} + \tau_{internal}A_{s,int} \quad \text{Equation 9}$$

where q = unit end bearing resistance in units of force per area
 $\tau_{internal}$ = internal skin friction in units of force per area
 $A_{unplugged}$ = unplugged cross sectional area at the pile toe
 $A_{s,int}$ = surface area of the pile segment inside the shaft interior in contact with soil in shear

Plugged and unplugged conditions are considered for any cross section that may potentially accumulate compacted soil, such as hollow shapes, and thus change the cross sectional area considered for end bearing resistance. Typical piles are governed by plugged condition for end bearing resistance since the embedment length is much larger than the cross-sectional area.

For H piles in stiff clays, it is possible that soil becomes trapped and compacted between flange and web causing a plugged condition. Currently, *RSPile* does not consider plugged and unplugged conditions for pre-defined, standard sections, such as those found in the AISC shape database. The standard cross sectional area is used instead. This also applies to User Defined Sections.

6. Soil Models

6.1. American Petroleum Institute (API)

API (2002) [1] provides recommendations for calculating ultimate skin friction, ultimate end bearing resistance, skin friction (t-z) load transfer curves and end bearing (Q-z) load transfer curves for driven piles in sand and clay.

For driven piles in clay, the ultimate unit skin friction (τ_{ult}) in units of force per area is calculated from the following equation.

$$\tau_{ult} = \alpha c_u \quad \text{Equation 10}$$

where α = dimensionless factor
 c_u = undrained shear strength of soil at calculation point

The dimensionless factor α is calculated from the following equation.

$$\begin{aligned}\alpha &= 0.5\psi^{-0.5}, & \psi &\leq 1.0, & \alpha &\leq 1.0 \\ \alpha &= 0.5\psi^{-0.25}, & \psi &> 1.0, & \alpha &\leq 1.0\end{aligned}\tag{Equation 11}$$

where $\psi = \frac{c_u}{\sigma_v}$
 σ_v = effective overburden pressure at calculation point

The ultimate unit end bearing resistance for clay (q) in units of force per area is calculated from the following equation.

$$q = 9c_u\tag{Equation 12}$$

where c_u = undrained shear strength at the pile tip

For driven piles in sand, the ultimate unit skin friction (τ_{ult}) in units of force per area is calculated from the following equation.

$$\tau_{ult} = K\sigma_v \tan \delta\tag{Equation 13}$$

where K = coefficient of lateral earth pressure
 σ_v = effective overburden pressure at calculation point
 δ = friction angle between pile and soil interaction defined as $(\varphi - 5^\circ)$, where φ is the soil friction angle

The ultimate end bearing resistance (q) in units of force per area is calculated from the following equation.

$$q = \sigma_v N_q\tag{Equation 14}$$

where σ_v = effective overburden pressure at the pile toe
 N_q = dimensionless bearing capacity factor

API (2002) [1] provides recommendations for estimating the bearing capacity factor (N_q), pile-soil friction angle (δ), maximum unit skin friction (τ_{max}) and maximum end bearing resistance (q_{max}).

API (2002) [1] recommends a linear elastic perfectly plastic skin friction transfer curve for sand as shown in Table 6-1 and Figure 6-1.

Table 6-1: API Sand Skin Friction (t-z) Load Transfer Curve

Soil Displacement (z) (in)	Unit Skin Friction/ Ultimate Unit Skin Friction (τ/τ_{ult})
0	0
0.1	1
∞	1

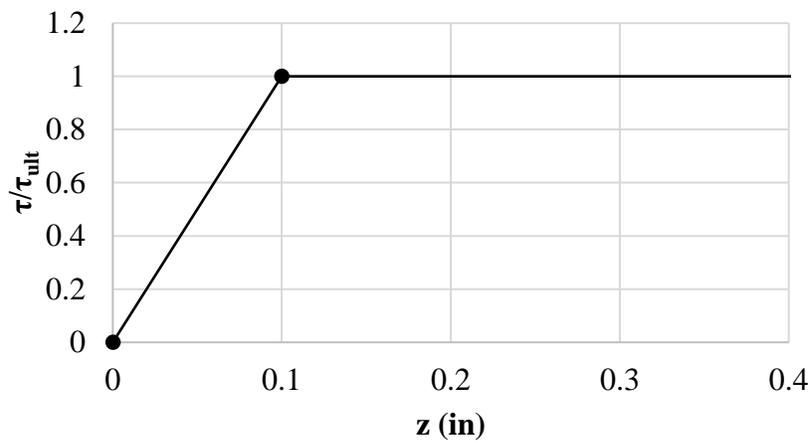


Figure 6-1: API Sand Skin Friction (t-z) Load Transfer Curve

API (2002) [1] recommends the following skin friction transfer curve for clay as shown in Table 6-2 and Figure 6-2.

Table 6-2: API Clay Skin Friction (t-z) Load Transfer Curve

Soil Displacement/Pile Diameter (z/D)	Unit Skin Friction/ Ultimate Unit Skin Friction (τ/τ_{ult})
0	0
0.0016	0.3
0.0031	0.5
0.0057	0.75
0.0080	0.9
0.0100	1
0.0200	0.7 to 0.9*
∞	0.7 to 0.9*

*The residual strength of API Clay is assumed to be 0.9 times the ultimate unit skin friction

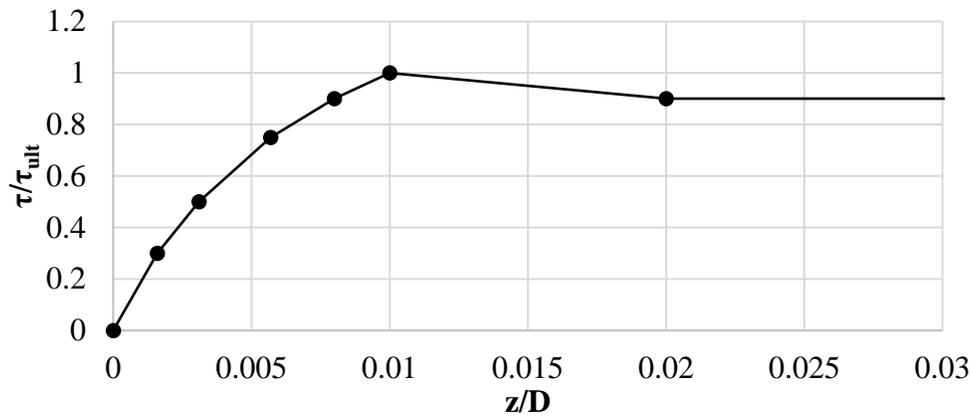


Figure 6-2: API Clay Skin Friction (t-z) Load Transfer Curve

For non-circular pile cross sections, **an equivalent diameter** is used by equating the cross-sectional area with the area of a circle and solving for the diameter.

API (2002) [1] recommends the following end bearing transfer curve for sand and clay as shown in Table 6-3 and Figure 6-3.

Table 6-3: API Sand and Clay End Bearing (Q-z) Load Transfer Curve

Soil Displacement/Pile Diameter (z/D)	Unit Skin Friction/ Ultimate Unit Skin Friction (τ/τ_{ult})
0	0
0.002	0.25
0.013	0.5
0.042	0.75
0.073	0.9
0.1	1
∞	1

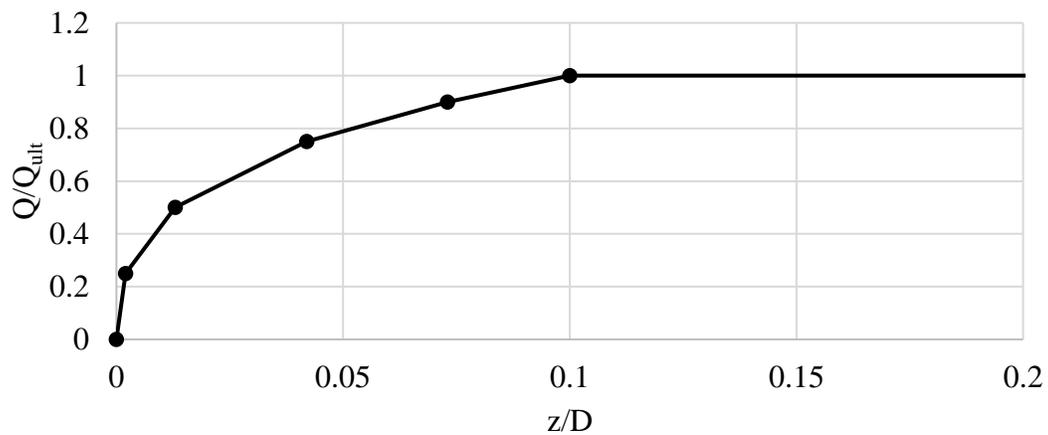


Figure 6-3: API Sand and Clay End Bearing (Q-z) Load Transfer Curve

API (2002) [1] provides the following recommendations in

Table 6-4 for cohesionless soil properties.

Density	Soil Description	Soil-Pile Friction Angle (δ)	Max Skin Friction Values in kPa (kips/ft ²)	Bearing Capacity Factor, N_q	Max Unit End Bearing Values in MPa (kips/ft ²)
Very Loose	Sand	15	47.8 (1.0)	8	1.9 (40)
Loose	Sand-Silt**				
Medium	Silt				
Loose	Sand	20	67.0 (1.4)	12	2.9 (60)
Medium	Sand-Silt**				
Dense	Silt				
Medium	Sand	25	81.3 (1.7)	20	4.8 (100)
Dense	Sand-Silt**				
Dense	Sand	30	95.7 (2.0)	40	9.6 (200)
Very Dense	Sand-Silt**				
Dense	Gravel	35	114.8 (2.4)	50	12.0 (250)
Very Dense	Sand				

Density	Soil Description	Soil-Pile Friction Angle (δ)	Max Skin Friction Values in kPa (kips/ft ²)	Bearing Capacity Factor, N_q	Max Unit End Bearing Values in MPa (kips/ft ²)
Very Loose	Sand	15	47.8 (1.0)	8	1.9 (40)
Loose	Sand-Silt**				
Medium	Silt				
Loose	Sand	20	67.0 (1.4)	12	2.9 (60)
Medium	Sand-Silt**				
Dense	Silt				
Medium	Sand	25	81.3 (1.7)	20	4.8 (100)
Dense	Sand-Silt**				
Dense	Sand	30	95.7 (2.0)	40	9.6 (200)

Very Dense	Sand-Silt**				
Dense	Gravel	35	114.8 (2.4)	50	12.0 (250)
Very Dense	Sand				

Table 6-4: Design Parameters for Cohesionless Siliceous Soil* (API, 2002)

*The parameters listed in this table are intended as guidelines only. Where detailed information such as in situ cone tests, strength tests on high quality samples, model tests, or pile driving performance is available, other values may be justified.

**Sand-Silt includes those soils with significant fractions of both sand and silt. Strength values generally increase with increasing sand fractions and decrease with increasing silt fractions.

6.2. Elastic Soil Model

An elastic soil material has infinite strength and can easily be defined using a constant stiffness. For elastic soil, the user must define the following parameters:

- Unit Skin Friction Stiffness - The change in unit skin friction per unit of soil displacement at each depth.
- Unit End Bearing Stiffness - The change in unit end bearing resistance per unit of soil displacement at the pile toe.

The unit skin friction stiffness is multiplied by the surface area of the pile segment to obtain the skin friction per unit of soil displacement. The unit end bearing stiffness is multiplied by the plugged cross sectional area at the pile toe to obtain the end bearing resistance per unit of soil displacement.

6.3. User Defined Soil Model

A user defined soil model allows the user to input the t-z and q-z curves based on the ultimate skin friction or end bearing resistance and the curve shape. The curve shape is defined by entering stress to max stress ratios for various soil displacement. The stress to max stress ratio is multiplied by the ultimate skin friction or end bearing resistance for a depth to calculate the assumed soil response due to soil displacement. The stress to max stress ratios for each displacement value allows the user to vary the ultimate resistance value along the depth while maintaining the same t-z or q-z curve shape for the material. When the soil displacement exceeds the last (maximum) entered displacement value, the soil resistance is assumed to be the last entered resistance.

7. Ground Slope and Pile Batter

RSPile allows the input of a ground slope and a pile batter angle. For axially loaded piles, it is assumed that the soil layers are horizontal. Therefore, the slope angle is neglected in the analysis of axially loaded piles. However, the pile batter angle is considered in the computation of overburden pressure. A modified

lateral earth pressure is introduced for cohesionless soil formulations that assume the lateral earth pressure is perpendicular to the pile axis.

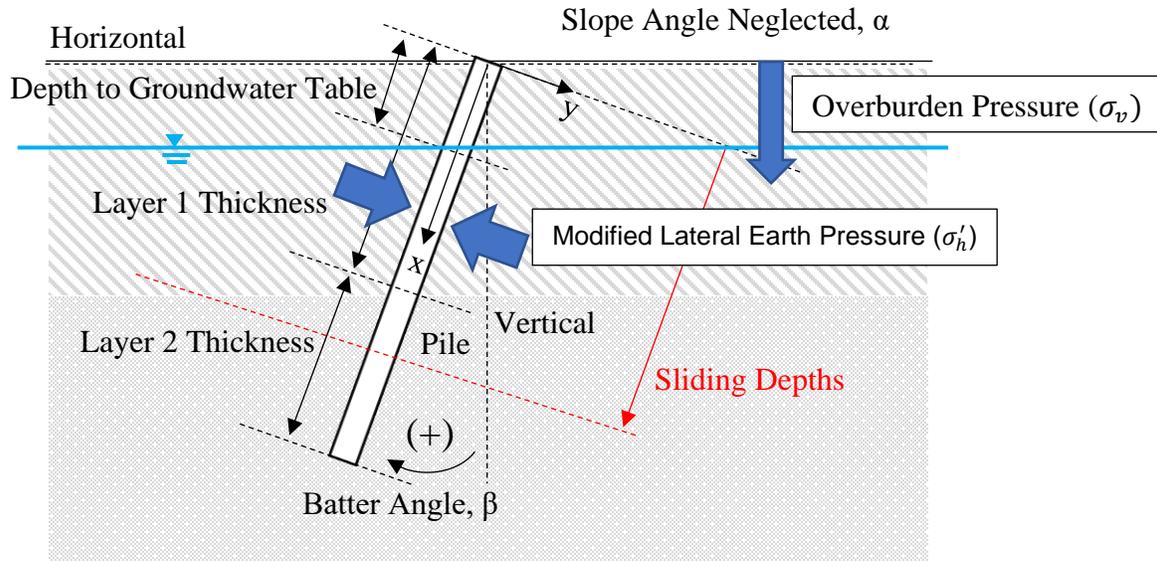


Figure 7-1: Effect of Batter Angle on Axially Loaded Pile Analysis

Note that sign convention is clockwise positive for inputted angle values as shown in the “Sign Convention” document in the Help menu.

For cohesionless materials, such as API [1] recommendations for driven piles in sand, the modified lateral earth pressure (σ'_h) can be computed as the combination of effective overburden pressure (σ_v) and the lateral earth pressure (σ_h).

$$\sigma'_h = \sigma_v \sin(\beta) + \sigma_h \cos(\beta) \quad \text{Equation 15}$$

where $\sigma_h = K\sigma_v$
 $K =$ coefficient of lateral earth pressure
 $\sigma_v =$ effective overburden pressure at calculation point

For API [1] driven piles in sand, this would modify the ultimate unit skin friction (τ_{ult}) to the following equation.

$$\tau_{\text{ult}} = \sigma'_h \tan \delta$$

Equation 16

where δ = friction angle between pile and soil interaction

See section above on API[1] recommendations for more details. A similar approach is applied to other formulations that rely on lateral earth pressure.

8. References

1. American Petroleum Institute (2002). "API Recommended Practice 2A-WSD - Planning, Designing, and Constructing Fixed Offshore Platforms – Working Stress Design". 21st ed. American Petroleum Institute. 2003.
2. Loehr, E.J. and Brown, D.A. (2008). "A Method for Predicting Mobilization Resistance for Micropiles Used in Slope Stabilization Applications", A Report Prepared for the Joint ADSC/DFI Micropile Committee.