rocscience

RSPile

Helical Pile Capacity

Theory Manual

November 6, 2024

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Notation

A_B: Area of the bottom helix.

A_n: Area of helix n.

A_T: Area of the top helix.

c: Cohesion/undrained shear strength.

d: Diameter of the shaft.

$D = B = L$: Diameter of the helix (Perko 2009), assuming the helix is always circular.

D_T: Diameter of the top helix

d_c, d_q, d_γ: Shape factors refined by Hansen (1970) and Vesic (1973).

f_s: Unit skin friction.

H: Embedment depth of the top helix.

H_{eff}: Effective shaft length.

h_{depth}: Depth at which the effective stress is being calculated.

h_{gwt}: Depth of the groundwater table.

h_w: Height of the water table above depth z.

K: Scaling factor.

n: number of segments

m: Number of helical plates.

N_c, N_q, N_γ: Bearing capacity factors.

N_c', N_q', N_γ': Adjusted bearing capacity factors.

P_u: Ultimate capacity.

$Q_{helices}$: Capacity of the helices.

$Q_{s\,(segment)}$: Skin resistance of the segment

q': Effective overburden stress.

q_{ult}: Ultimate bearing pressure of the soil

s: Spacing between two helical plates.

s_c, s_q, s_γ: Shape factors refined by Hansen (1970) and Vesic (1973).

$f_{s\,ult}$: Ultimate shear stress acting on the cylinder of soil between the helices

t: Thickness of a soil segment

z: Current elevation

z_H: Elevation of the helix.

z_{GS}: Elevation of the ground surface

z_{WT}: Elevation of the water table

α: Adhesion factor

γ_w: Unit weight of water.

γ: Unit weight of the soil.

μ: Height reduction factor.

ϕ: Internal friction angle of the soil

All the above and other notations are defined within the text.

1. Introduction

1.1. Calculated Capacities

Ultimate compressive capacity: Calculated using limit state analysis, where the capacity is calculated using both the individual bearing method and the cylindrical shear method are computed for the pile under compression, and the limiting (lowest) capacity is the ultimate calculated capacity of the pile.

Ultimate uplift capacity: Calculated using limit state analysis, where the capacity is calculated using both the individual bearing method and the cylindrical shear method are computed for the pile in uplift, and the limiting (lowest) capacity is the ultimate calculated capacity of the pile.

2. Capacity Calculations

2.1. Helical Pile Capacity

2.1.1. Individual Plates Bearing Method

2.1.1.1. Compression

The compressive capacity calculated using the Individual Bearing Method is as follows: RSPile sums all individual helix plate's capacities and the skin friction along the shaft in the following equation:

$$Q_{ult} = Q_{sult\ (shaft)} + Q_{bult\ \ (helices)} = \sum_{i=1}^{n} f_{sulti} \pi D_i t_i + \sum_{j=1}^{m} q_{bult\,j} A_j$$

The term f_{sulti} for shaft friction is discussed in section 2.2.

The second summation term defines the bearing capacity provided by the helical plates:

$$Q_{bult\ (helices)} = \sum_{j}^{m} q_{bult\,j} A_j$$

Where:

A_j: Area of each helix j

n: number of segments of the embedded part of the shaft until the first helix

D_i: The diameter of the shaft segment i (Instead of πD_i use perimeter for squares)

$q_{bult\,j}$: The soil ultimate bearing pressure, is calculated using Meyerhof's (1951) modified version of Terzaghi's (1943) formula:

$$q_{bult} = cN_c{}' + q_H'\left(N_q{}' - 1\right) + 0.5\gamma D N_\gamma{}'$$

Where:

c: Cohesion (undrained shear strength if ϕ = 0).

q'_H: Effective overburden pressure, calculated by:

$$q'_H = \left[\sum_{i=1}^{n_H} \gamma_i t_i\right] - \gamma_w \max(z_{WT} - z_H, 0)$$

Where:

n_H: Number of segments from ground surface until the helix.

i: The segment number being calculated, starting from ground surface.

γ_i: The bulk unit weight of the soil in segment i.

t_i: The thickness of segment i.

γ_w: The unit weight of water.

z_{WT}: Elevation of the water table.

z_H: Elevation of the helix.

γ (in the third term): Unit weight of the soil at the helix elevation.

A major assumption in RSPile helical pile capacity calculations is that the bearing soil is the same as the soil at the level of the helix.

$N_c{}'$, $N_q{}'$, $N_\gamma{}'$: Meyerhof's (1951) modified bearing capacity factors, given by:

$N_q{}' = N_q s_q d_q$, ***and in the equation, it is ($\boldsymbol{N'_q - 1}$) to change from gross to net end bearing.***

$N_c{}' = N_c s_c d_c$

$N_\gamma{}' = N_\gamma s_\gamma d_\gamma$

Where:

N_c, N_q, N_γ: Meyerhof's redefined bearing capacity factors, given by:

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \frac{\phi}{2})$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Where:

ϕ: The angle of internal friction of the soil, in degrees.

s_c, s_q, s_γ: Shape factors refined by Hansen (1970) and Vesic (1973)

$$s_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + \tan\phi$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 0.6$$

Where:

B the width of the footing, L the length of the footing, are both $=$ D: the diameter of the helix (Perko 2009), assuming the helix is always circular.

d_c, d_q, d_γ: Depth factors refined by Hansen (1970) and Vesic (1973).

$d_c = 1 + 0.4K$

$d_q = 1 + 2K\tan\phi(1 - \sin\phi)^2$

$d_\gamma = 1$

Where:

K: Scaling factor.

$$K = \begin{cases} \frac{d_H}{B}, & for \ \frac{d_H}{B} \leq 1 \\ \arctan\left(\frac{d_H}{B}\right), & for \ \frac{d_H}{B} > 1 \end{cases}$$

Where:

d_H: The depth from the ground surface to the helix = $z_{GS} - z_H$

The user shall choose a value for N_c'. The program default for will be 9.

For authors Hanson (1970) and Vesic (1973), if followed, a value of 10 is suggested.

2.1.1.2. Uplift

The uplift capacity calculated using the Individual Bearing Method is as follows: RSPile sums all individual helix plate's capacities and the skin friction along the shaft in the following equation:

$$T_{ult} = T_{sult\ (shaft)} + T_{bult\ \ (helices)} = \sum_{i=1}^{n} f_{sulti}\pi D_i t_i + \sum_{j=1}^{m} q_{bultj} A_m$$

The term f_{sulti} for shaft friction is discussed in section 2.2.

The bearing capacity provided by the helical plates for uplift is the same as for compression and is calculated through the summation of all individual helical plate's capacities:

$$T_{bult\ (helices)} = \sum_{j=1}^{m} q_{bultj} A_m$$

Where:

q_{bultj}: The soil bearing pressure, is calculated using Meyerhof's (1951) modified version of Terzaghi's (1943) formula:

$q_{bult} = cN_c{}' + q_H' N_q{}' + 0.5\gamma D N_\gamma{}'$

Where:

c: Cohesion (undrained shear strength if ϕ = 0).

q'_H: Effective overburden pressure at helix level, calculated by:

$$q'_H = \left[\sum_{i=1}^{n_H} \gamma_i t_i\right] - \gamma_w \max(z_{WT} - z_H, 0)$$

Notations are as explained previously

${N_c}'$, ${N_q}'$, ${N_\gamma}'$: Meyerhof's (1951) modified bearing capacity factors, given by:

${N_q}' = N_q s_q d_q$, ***in the equation there is no longer a deduction of 1 from*** $\boldsymbol{{N_q}'}$ ***because gross and net uplift capacity are assumed equal.***

$${N_c}' = N_c s_c d_c$$

$${N_\gamma}' = N_\gamma s_\gamma d_\gamma$$

Where:

N_c, N_q, N_γ: Meyerhof's redefined bearing capacity factors, given by:

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \frac{\phi}{2})$$

$$N_c = \left(N_q - 1\right)\cot\phi$$

$$N_\gamma = \left(N_q - 1\right)\tan(1.4\phi)$$

s_c, s_q, s_γ: Shape factors refined by Hansen (1970) and Vesic (1973)

$$s_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + \tan\phi$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 0.6$$

Where:

$B = L = D$: The diameter of the helix (Perko 2009), assuming the helix is always circular.

d_c, d_q, d_γ: Depth factors refined by Hansen (1970) and Vesic (1973).

$$d_c = 1 + 0.4K$$

$$d_q = 1 + 2K\tan\phi(1 - \sin\phi)^2$$

$$d_\gamma = 1$$

Where:

K: Scaling factor.

$$K = \begin{cases} \frac{d_H}{B}, & for \ \frac{d_H}{B} \leq 1 \\ \arctan\left(\frac{d_H}{B}\right), & for \ \frac{d_H}{B} > 1 \end{cases}$$

Where:

d_H: The depth from the ground surface to the helix = $z_{GS} - z_H$

2.1.2. Cylindrical Shear Method

2.1.2.1. Compression

The capacity in compression calculated using the Cylindrical Shear Method is the bearing capacity of the bottom helix and the skin friction along the soil cylinder between the top and the bottom helices, in addition to the shaft friction:

$$Q_{ult} = Q_{sult\ (shaft)} + Q_{sult\ (soil\ cylinder)} + Q_{bult\ \ (bottom\ helix)} = \left[\sum_{i=1}^{n} f_{sulti}\pi D_i t_i\right] + \left[\sum_{j=1}^{m} f_{sultj}\pi D_j t_j\right] + q_{bult\ B}A_B$$

The first term $\sum_1^n f_{sulti}\pi D_i t_i$ and the second term $\sum_1^m f_{sultj}\pi D_j t_j$ which are the shaft friction and the soil cylinder friction, respectively, are discussed in sections 2.2. and 2.3.

The ultimate bearing capacity provided by the bottom helix is calculated using:

$$Q_{bult\ (bottom\ helix)} = q_{bult\ B}A_B$$

Where:

A_B: The area of the bottom helix.

q_{bultj}: The soil ultimate bearing pressure, is calculated using Meyerhof's (1951) modified version of Terzaghi's (1943) formula:

$$q_{bult} = cN_c{}' + q_H'\left(N_q{}' - 1\right) + 0.5\gamma DN_\gamma{}'$$

Where:

c: Cohesion (undrained shear strength if ϕ = 0).

q_H': Effective overburden pressure, calculated by:

$$q_H' = \left[\sum_{i=1}^{n_H} \gamma_i t_i\right] - \gamma_w \max(z_{WT} - z_H, 0)$$

Where:

n_H: Number of segments from ground surface until the helix.

i: The segment number being calculated, starting from ground surface.

γ_i: The bulk unit weight of the soil in segment i.

t_i: The thickness of segment i.

γ_w: The unit weight of water.

z_{WT}: Elevation of the water table.

z_H: Elevation of the helix.

γ (in the third term): Unit weight of the soil at the helix elevation.

A major assumption in RSPile helical pile capacity calculations is that the bearing soil is the same as the soil at the level of the helix.

$N_c{}'$, $N_q{}'$, $N_\gamma{}'$: Meyerhof's (1951) modified bearing capacity factors, given by:

$N_q{}' = N_q s_q d_q$, ***and in the equation, it is (*** $\boldsymbol{N_q' - 1}$ ***) to change from gross to net end bearing.***

$N_c{}' = N_c s_c d_c$

$N_\gamma{}' = N_\gamma s_\gamma d_\gamma$

Where:

N_c, N_q, N_γ: Meyerhof's redefined bearing capacity factors, given by:

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \frac{\phi}{2})$$

$$N_c = \left(N_q - 1\right)\cot\phi$$

$$N_\gamma = \left(N_q - 1\right)\tan\,(1.4\phi)$$

Where:

ϕ: The angle of internal friction of the soil, in degrees.

s_c, s_q, s_γ: Shape factors refined by Hansen (1970) and Vesic (1973)

$$s_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L}\tan\phi \;\; = \; 1 \; + \; \tan\phi$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 0.6$$

Where:

B the width of the footing, L the length of the footing, are both = D: The diameter of the helix (Perko 2009), assuming the helix is always circular.

d_c, d_q, d_γ: Depth factors refined by Hansen (1970) and Vesic (1973).

$$d_c = 1 + \; 0.4K$$

$$d_q = 1 + 2K\tan\phi(1 - \sin\phi)^2$$

$$d_\gamma = 1$$

Where:

K: Scaling factor.

$$K = \begin{cases} \dfrac{d_H}{B}\,, & for \;\; \dfrac{d_H}{B} \leq 1 \\ \arctan\left(\dfrac{d_H}{B}\right), & for \;\; \dfrac{d_H}{B} > 1 \end{cases}$$

Where:

d_H: The depth from the ground surface to the helix = $z_{GS} - z_H$

$N_c{}'$ is taken instead as 10 according to Perko 2009.

2.1.2.2. Uplift

The ultimate capacity in uplift is calculated using the Cylindrical Shear Method, which sums the bearing capacity of the cylinder of soil based on the top helix diameter and the skin friction along the soil cylinder, and along the shaft is the following equation:

$$T_{ult} = T_{sult\ (shaft)} + T_{sult\ (soil\ cylinder)} + T_{bult\ \ (top\ helix)} = \left[\sum_{i=1}^{n} f_{sulti}\pi D_i t_i\right] + \left[\sum_{i}^{n} f_{sulti}\pi D_i t_i\right] + q_{bult\ T}A_T$$

Therefore, the uplift capacity provided by the soil cylinder is calculated using:

$$T_{bult\ \ (top\ helix)} = q_{ult\ T}A_{\mathrm{T}}$$

Where:

A_T: The area of the top helix.

$q_{bult\ T}$: The soil bearing pressure, is calculated using Meyerhof's (1951) modified version of Terzaghi's (1943) formula:

$$q_{bult\ T} = cN_c{}' + q'_H N_q{}' + 0.5\gamma D N_\gamma{}'$$

Where:

c: Cohesion (undrained shear strength if ϕ = 0).

q'_H: Effective overburden pressure, calculated by:

$$q'_H = \left[\sum_{i=1}^{n_H} \gamma_i t_i\right] - \gamma_w \max(z_{WT} - z_H, 0)$$

Notation is as previously mentioned.

γ (in the third term): Unit weight of the soil at the helix elevation.

A major assumption in RSPile helical pile capacity calculations is that the bearing soil is the same as the soil at the level of the helix.

$N_c{}'$, $N_q{}'$, $N_\gamma{}'$: Meyerhof's (1951) modified bearing capacity factors, given by:

$N_q{}' = N_q s_q d_q$, ***in the equation there is no longer a deduction of 1 from*** $N_q{}'$ ***because gross and net uplift capacity are assumed equal.***

$N_c{}' = N_c s_c d_c$

$N_\gamma{}' = N_\gamma s_\gamma d_\gamma$

Where:

N_c, N_q, N_γ: Meyerhof's redefined bearing capacity factors, given by:

$$N_q = e^{\pi \tan\phi} \tan^2(45 + \frac{\phi}{2})$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

s_c, s_q, s_γ: Shape factors refined by Hansen (1970) and Vesic (1973)

$$s_c = 1 + \frac{N_q}{N_c}\frac{B}{L} = 1 + \frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L}\tan\phi = 1 + \tan\phi$$

$$s_\gamma = 1 - 0.4\frac{B}{L} = 0.6$$

Where:

B = L = D: The diameter of the helix (Perko 2009), assuming the helix is always circular.

d_c, d_q, d_γ: Depth factors refined by Hansen (1970) and Vesic (1973).

$$d_c = 1 + 0.4K$$

$$d_q = 1 + 2K\tan\phi(1 - \sin\phi)^2$$

$$d_\gamma = 1$$

Where:

K: Scaling factor.

$$K = \begin{cases} \frac{d_H}{B}, & for\ \frac{d_H}{B} \leq 1 \\ \arctan\left(\frac{d_H}{B}\right), & for\ \frac{d_H}{B} > 1 \end{cases}$$

Where:

d_H: The depth from the ground surface to the helix = $z_{GS} - z_H$

$N_c{}'$ ***is taken as input as discussed above.***

2.2. Shaft Skin Friction

The shaft friction is calculated through an effective depth H_{eff}. For compression, H_{eff} = the depth from the minimum of (the pile head elevation and the ground surface elevation) to the embedment depth of the top helix. For uplift, H_{eff} = the depth from the minimum of (the pile head elevation and the ground surface elevation) to a distance = μ * D_T above the top helix, where the height reduction factor μ is between 1.4 and 2.3. This height reduction factor will be obtained through user input.

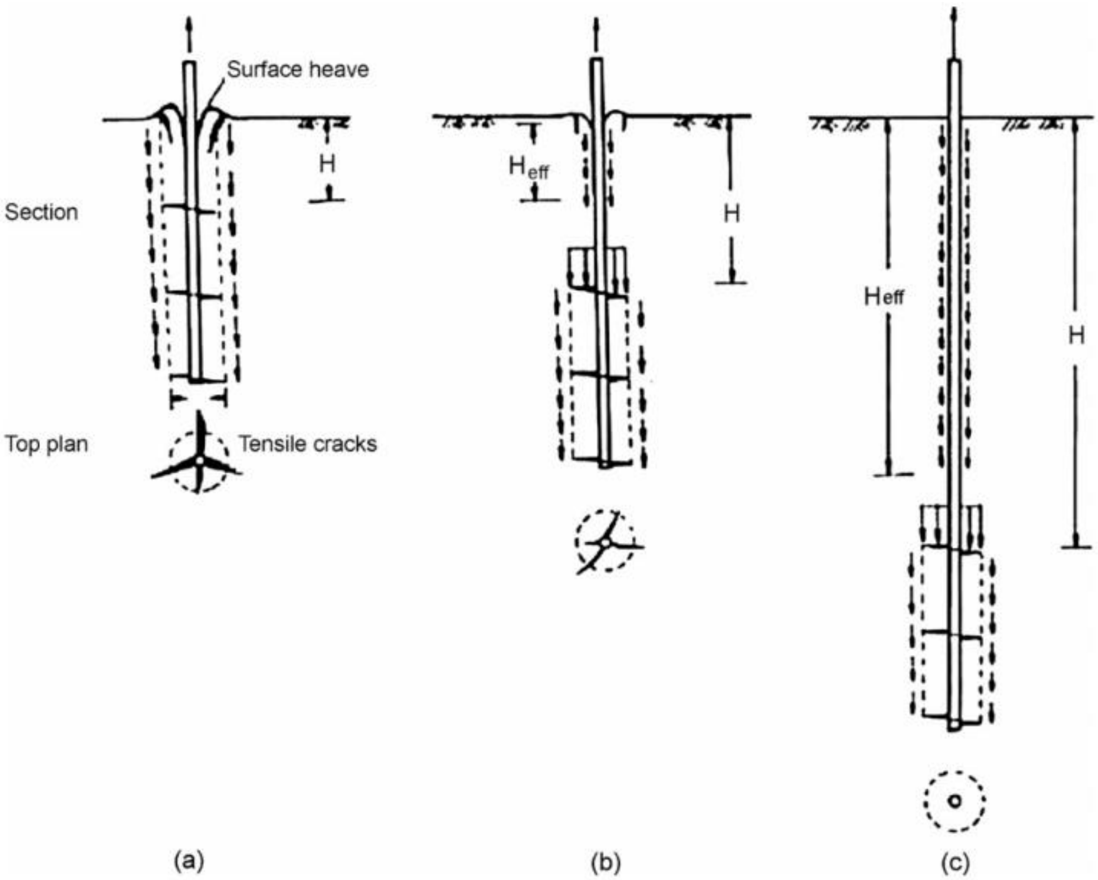

Figure 1: This illustrates the full shaft length to the top helix compared to the effective shaft length.

Total skin friction is the summation of the skin friction of all shaft segments, each of which is computed by calculating the unit skin friction and multiplying it by the surface area of the segment.

In cohesive soils the skin friction on the shaft can be calculated using the equation:

$$Q_{sult\ (shaft)} = \sum_{i=1}^{n} f_{sulti} \pi D_i t_i$$

Where:

n: is the number of segments from ground surface to a depth equal to Heff.

$f_{sulti\ (shaft)} = \alpha * c$: The unit skin friction.

Where:

α: The adhesion factor.

c: The cohesion or undrained shear strength of the soil.

t_i: The thickness of the shaft segment i.

D_i: The diameter of the shaft segment i (Use perimeter if square).

In cohesionless soil the skin friction on the shaft can be calculated using the equation:

$$Q_{sult\ (shaft)} = \sum_{i=1}^{n} f_{sulti} \pi D_i t_i$$

Where:

n: is the number of segments from minimum of pile head elevation and the ground surface elevation to an elevation for the depth of Heff.

$f_{sulti\ (shaft)}$: The unit skin friction is given by:

$$f_{sulti\ (shaft)} = K_i \left\{ \left[\sum_{i=1}^{n_s} \gamma_i t_i \right] - \gamma_w \max(z_{WT} - z_i, 0) \right\} \tan\delta_i$$

Where:

K_i: $K_0 = 1 - \sin\phi_i$

δ_i: The angle of friction between the soil and the shaft 2/3 ϕ_i to 3/4 ϕ_i

But K_i and δ_i are user input for the soil types.

ϕ: The angle of internal friction of the soil, in degrees.

γ_i: The unit weight of the soil of segment i.

t_i: The thickness of segment i.

z_i: The depth of segment i.

γ_w: The unit weight of water.

z_{WT}: Elevation of the ground water table.

n_s: Number of segments from ground surface until that point.

t_i: The thickness/height of the segment of the shaft.

D_i: The diameter of the shaft segment.

2.3. Shear Stress along the Soil Cylinder

2.3.1. Compression

In cohesive soil the shear strength along the soil cylinder can be calculated using the equation:

$$Q_{ult} = Q_{sult\ (shaft)} + Q_{sult\ (soil\ cylinder)} + Q_{bult\ \ (bottom\ helix)}$$

The third term was previously defined in section 2.1, and the first term is defined in section 2.2.

For the second term,

$$Q_{sult\ (soil\ cylinder)} = \sum_{i=1}^{n} f_{sulti} \pi D_i t_i$$

$f_{sulti\ (soil\ cylinder)} = \ c$: the undrained shear strength of the soil if ϕ = 0.

t_i: The thickness of segment i.

D_i: The interpolated diameter of the soil cylinder at the respective depth

n: number of segments of the soil cylinder

In cohesionless soil the shear strength along the soil cylinder can be calculated using the equation:

$f_{sulti\ (soil\ cylinder)} = \ T_i$: Unit skin friction, calculated using the equation cited in Perko 2009:

$$T_i = \left(0.09e^{0.08\phi_i}\right)\left\{\left[\sum_{j=1}^{i} \gamma_j t_j\right] - \gamma_w \max(z_{WT} - z_i, 0)\right\} \tan\phi_i$$

Where:

$(0.09e^{0.08\phi})$: The equation obtained from the regression using Mitsch and Clemence recommended values, as shown in Figure 2:

Angle of Internal Friction, Φ	25	30	35	40	45
Coefficient of Lateral Pressure, K_h	0.7	0.9	1.5	2.35	3.2

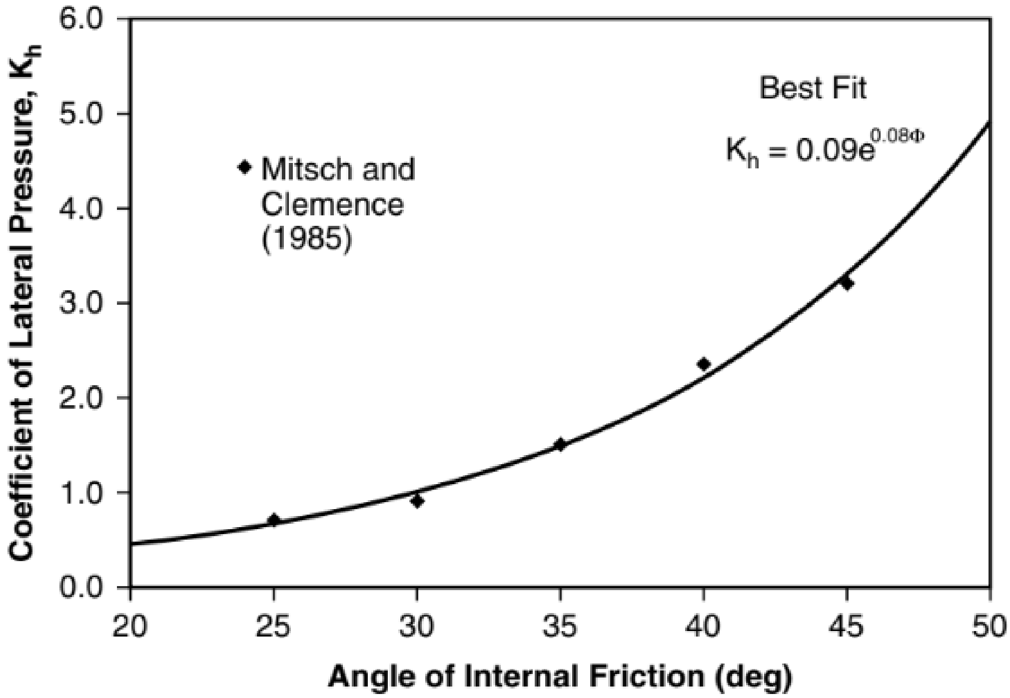

Figure 2: The factor representing the lateral earth pressure coefficient from recommended values of Mitsch and Clemence (1985).

i: The segment number being calculated, starting from ground surface.

ϕ_i: The angle of internal friction of the soil, in degrees at the elevation of segment i.

γ_j: The unit weight of the soil of segment j.

t_j: The thickness of segment j.

z_i: The elevation of segment i.

γ_w: The unit weight of water.

z_{WT}: Elevation of the ground water table.

2.3.2. Uplift

In cohesive soil the shear strength along the soil cylinder can be calculated using the equation:

$$T_{ult} = T_{sult\ (shaft)} + T_{sult\ (soil\ cylinder)} + T_{bult\ \ (top\ helix)} = \left[\sum_{i}^{n} f_{sulti}\pi D_i t_i\right] + \left[\sum_{i}^{n} f_{sulti}\pi D_i t_i\right] + q_{bult}A_T$$

The third term was previously defined in section 2.1, and the first term is defined in section 2.2.

So, for the second term,

$f_{sulti\ (soil\ cylinder)} = c$: the undrained shear strength of the soil.

t_i: The thickness of segment i.

D_i: The interpolated diameter of the soil cylinder at the respective depth

In cohesionless soil the shear strength along the soil cylinder can be calculated using the equation:

$$Q_{sult\ (soil\ cylinder)} = \sum_{i}^{n} f_{sulti} \pi D_i t_i$$

Where:

t_i: The thickness of segment i.

D_i: The interpolated diameter of the soil cylinder at the respective depth

$f_{sulti\ (shaft)} = T_i$: Unit skin friction, calculated using the (Perko 2009) equation:

$$T_i = \left(0.09e^{0.08\phi_i}\right)\left\{\left[\sum_{i=1}^{n_H} \gamma_i t_i\right] - \gamma_w \max(z_{WT} - z_i, 0)\right\}\tan\phi_i$$

Where:

$(0.09e^{0.08\phi})$: The equation obtained from the regression using Mitsch and Clemence recommended values, refer to Figure 2.

i: The segment number being calculated, starting from ground surface.

ϕ_i: The angle of internal friction of the soil, in degrees at the elevation of segment i.

γ_i: The unit weight of the soil of segment i.

t_i: The thickness of segment i.

z_i: The elevation of segment i.

γ_w: The unit weight of water.

z_{WT}: Elevation of the water table.

n_H: Number of segments from ground surface until the helix.

3. References

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