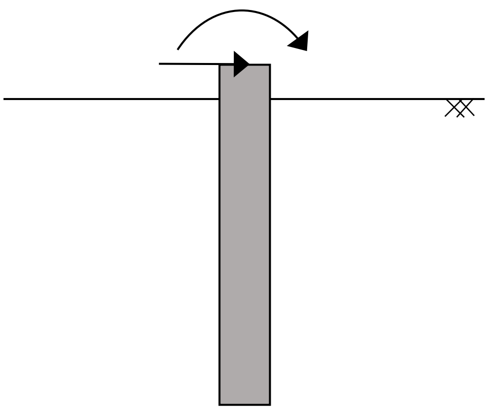
RSPile

Axial and Lateral Pile Analysis



Verification 5 Laterally Loaded Long Pile – Analytical Solution

Keywords: Laterally Loaded Pile, Elastic Pile, Analytical Solution, Applied Head Loading, Subgrade Reaction

5 Laterally Loaded Long Pile – Analytical Solution

5.1 Problem Description

This problem examines three cases of laterally loaded long elastic piles in soil with stiffness linearly increasing with depth. These analyses simulate a typical pile support in soil with loading at the head. The problem will analyze both deflections and bending moments for free and fixed head piles.

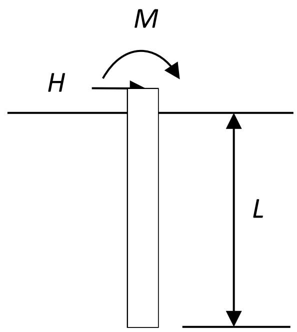


Figure 5-1: Pile embedded in soil with loading at the head

Case 1:
The pile properties are presented in Table 5-1 and the soil properties are presented in Table 5-2. The pile is subjected to shear force of 4.9 kN as shown by *H* in Figure 5-1.

Table 5-1: Pile Properties – Case 1

Parameter	Value
Bending Stiffness (E_pI_p)	320 kNm^2
Diameter	0.1 m
Embedment Length	5.25 m

Table 5-2: Soil Properties – Case 1

Parameter	Value
Constant of Horizontal Subgrade Reaction (n_h)	3.57 MN/m^3

Case 2:
The pile properties are presented in Table 5-3 and the soil properties are presented in Table 5-4. The pile is loaded with a shear force (*H*) of 155.68 kN and a bending moment (*M*) of 395.43 kN as shown in Figure 5-1. The same pile with a fixed head is loaded with a shear force of 155.68 kN.

Table 5-3: Pile Properties – Case 2

Parameter	Value
Bending Stiffness (E_pI_p)	1434836 kNm^2
Diameter	1 m
Embedment Length	24.38 m

Table 5-4: Soil Properties – Case 2

Parameter	Value
Constant of Horizontal Subgrade Reaction (n_h)	1357.17 kN/m^3

Case 3:
The pile properties are presented in Table 5-5 and the soil properties are presented in Table 5-6. The pile is loaded with a number of different shear forces as show in Table 5-7. A moment is also introduced due to the loading point of the shear force occurring 0.305m above the soil surface.

Table 5-5: Pile Properties – Case 3

Parameter	Value
Bending Stiffness (E_pI_p)	163000 kNm^2
Diameter	0.61 m
Embedment Length	21 m

Table 5-6: Soil Properties – Case 3

Parameter	Value
Constant of Horizontal Subgrade Reaction (n_h)	15000 kN/m^3

Table 5-7: Loading Cases– Case 3

Parameter	Value
Load 1	32 kN
Load 2	54 kN
Load 3	77 kN
Load 4	98 kN
Load 5	123 kN
Load 6	146 kN
Load 7	169 kN

5.2 Analytical Solution

Starting with Equation 1, the differential equation for a beam, Fayun Liang et al propose an analytical solution based on the Fourier-Laplace integral method, which recovers power series solutions for small depths and Wentzel-Kramers-Brillouin (WKB) asymptotic solutions for large depth.

$$E_p I_p \frac{d^4 y}{dz^4} + Ky = 0 \qquad \text{Equation 1}$$

where	z	=	pile depth
	y	=	pile deflection
	$E_p I_p$	=	pile bending stiffness
	K	=	modulus of subgrade reaction

As Terzaghi proposed, the modulus of subgrade reaction increases linearly with depth from a value of zero at the ground surface for sand. This value can be written as shown in Equation 2:

$$K = n_h z \qquad \text{Equation 2}$$

where	n_h	=	constant of horizontal subgrade reaction (kPa/m)

In RSPile, the input for elastic soil is the horizontal modulus of subgrade reaction k_h, also with units kPa/m. The modulus of subgrade reaction is calculated as the product of k_h and the diameter of the pile. Linearly varying properties were selected in RSPile and correction was made to account for this difference in achieving results.

5.3 Results

Figure 5-2 through Figure 5-4 show the results obtained in comparing RSPile to the analytical solution proposed by F Liang et al.

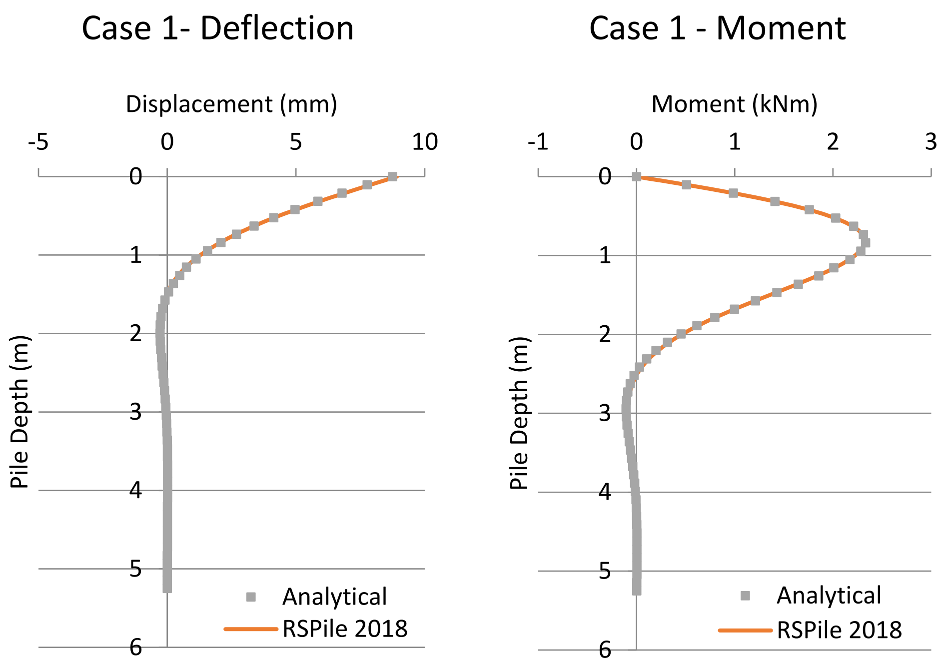


Figure 5-2: Case 1 – Pile Deflection and Moment

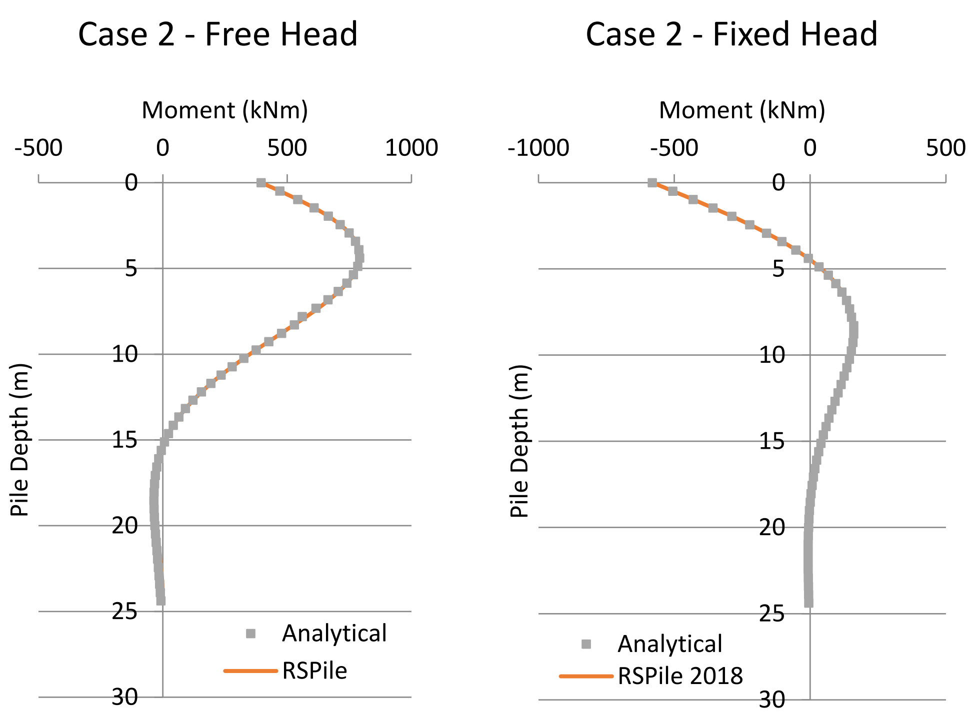


Figure 5-3: Case 2 – Pile Moment for Free and Fixed Head

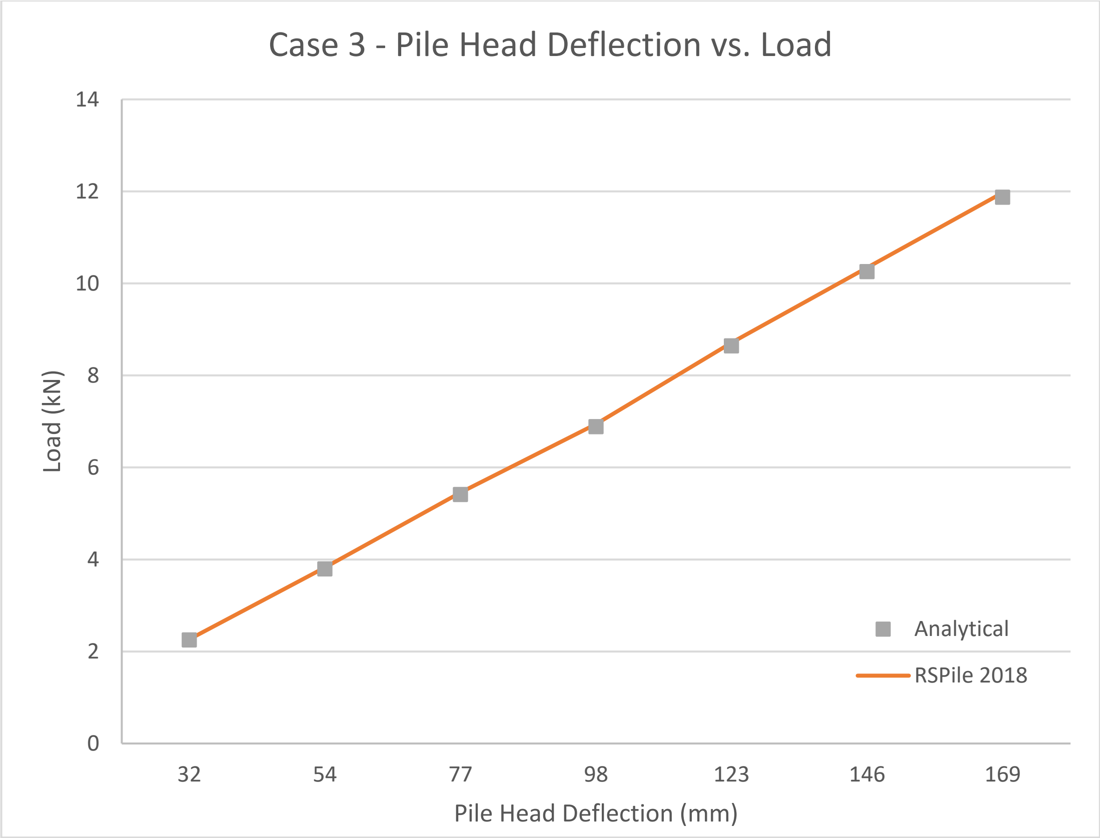


Figure 5-4: Case 3 – Pile Head Deflection for Different Load Cases

5.4 References

1. Liang, F., Y. Li, L. Li & J. Wang. (2014). "Analytical solution for laterally loaded long piles based on Fourier-Laplace integral." *Applied Mathematical Modelling*, Volume 38. Issues 21-22. pp. 5198-5216.

5.5 Data Files

The input data files below can be found in the installation folder.
Verification 005 (Case 1 Analytical Solution - Liang et al).rspile
Verification 005 (Case 2 Fixed Head Analytical Solution - Liang et al).rspile
Verification 005 (Case 2 Free Head Analytical Solution - Liang et al).rspile
Verification 005 (Case 3 Analytical Solution - Liang et al).rspile