

**RocFall2**

# **Lump Mass Analysis**

Verification Manual

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# 1. *RocFall2* Lump Mass Analysis Verification

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This document presents several lump mass rockfall examples, which have been used as verification problems for *RocFall2*. *RocFall2* is a 2D statistical analysis program designed to assist with assessment of slopes at risk for rockfalls.

The results produced by *RocFall2* agree very well with the documented examples and confirm the reliability of *RocFall2* results.

## 1.1. RocFall2 Verification Problem #1 – Projectile

[RocFall2 Build 8.007]

### 1.1.1. Problem Description

The purpose of this verification is to confirm that the projectile algorithm used by the program is working correctly. The projectile algorithm calculates the motion of the rocks while they are travelling through the air, bouncing from one point on the slope to another. The vast majority of the simulation time in *RocFall2* takes place in the projectile algorithm.

Any errors in the projectile algorithm would surely produce incorrect results. Therefore, it is essential that the projectile algorithm work correctly.

The example consists of a slope with two benches and a single rock that begins its travel at the crest of the slope. The rock was given an initial velocity and bounced a number of times before coming to rest at the base of the slope. The initial velocity for the rock was chosen so that the rock would follow a distinct path (high and clearly above the slope) and so that the rock would have enough energy so that each of the impacts would occur on a different segment of the slope. This velocity does not necessarily reflect typical initial velocities that are used in rockfall analyses.

The slope was created by making minor modifications to the geometry of an actual slope profile. The geometry was modified so that the impacts would occur on slope segments with a positive slope, a negative slope, and a horizontal segment. This was done in order to verify that the projectile algorithm handles sign changes correctly. This verification also serves as a good example of the sign conventions that are used in the program.

The slope geometry and the input parameters were configured so that no sliding would occur. No statistics were incorporated into this verification (i.e. only mean values were used, and all standard deviations were set to 0). Although rock trajectories in an actual simulation typically have dozens of steps, only the first four steps are followed here. This was done in the interest of brevity.

The minimum velocity ( $V_{MIN}$ ) was set to 1 m/s. This minimum velocity was selected so that the simulation did not end before the four steps were complete. Other numbers used in this example (e.g. the mass of the rock) were selected primarily for their ease in manual calculations.

## 1.1.2. RocFall2 Analysis

### Slope Geometry and Material Properties

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

Table 1.1-1: Slope Geometry and Materials

	X-Coordinate	Y-Coordinate	Normal Coefficient of Restitution $R_N$	Tangential Coefficient of Restitution $R_T$
Vertex 1	0	60		
Segment 1			0.5	0.8
Vertex 2	7	39		
Segment 2			0.5	0.8
Vertex 3	19	40		
Segment 3			0.5	0.8
Vertex 4	26	22		
Segment 4			0.6	0.9
Vertex 5	38	20		
Segment 5			0.6	0.9
Vertex 6	46	0		
Segment 6			0.4	0.6
Vertex 7	89	0		

### Initial Conditions

The rock starts at location  $X_0 = 0$  m,  $Y_0 = 60$  m (which coincides with the first slope vertex). The rock was given an initial velocity of  $V_{X0} = 7$  m/s,  $V_{Y0} = 2$  m/s and a mass of 10 kg.

Enter the seeder and slope geometry values from Table 1.1-1 into *RocFall2*.

Note: Ensure that **Consider rotational velocity** and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The RocFall2 model looks like this:

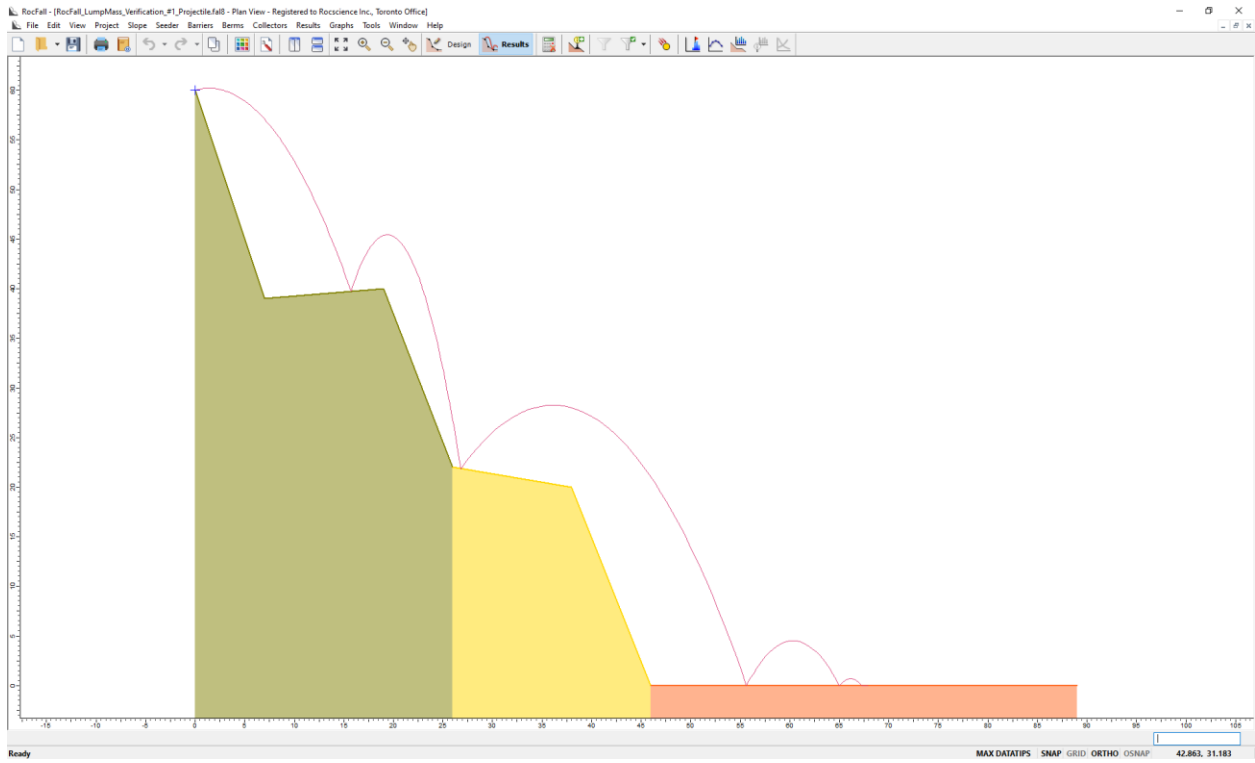


Figure 1.1-1: RocFall2 Rock Trajectory Model Results

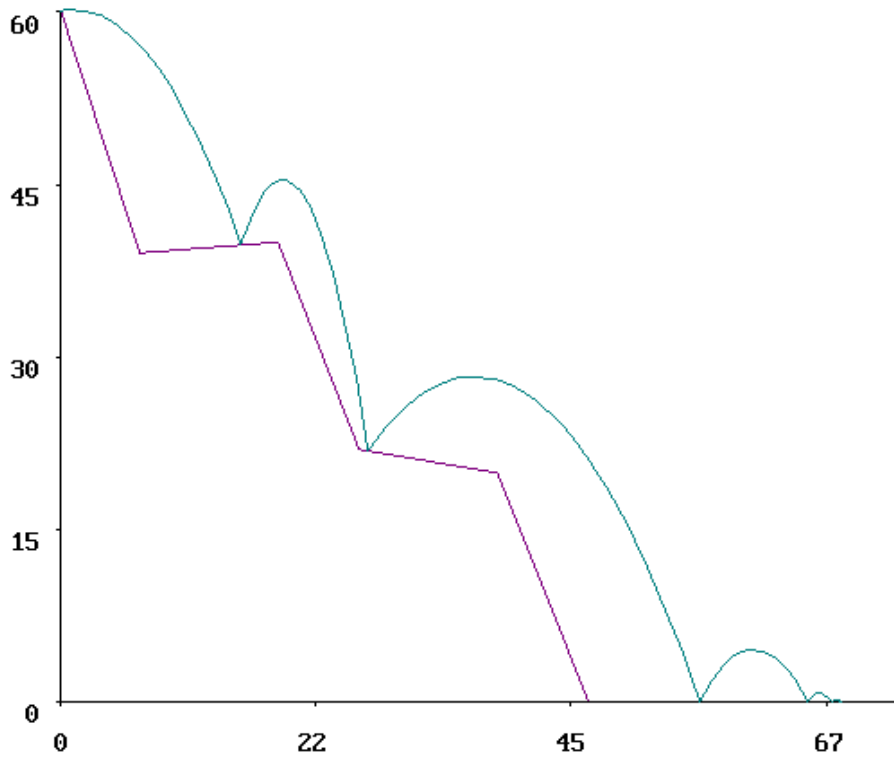


Figure 1.1-2: Rock Trajectory in Comparison Program

### 1.1.3. Analytical Solution

#### Equations and Sample Calculations

The projectile algorithm consists, mainly, of the process of determining the intersection between a parabola (the path the rock follows while it is in the air) and a line segment (one of the slope segments). The location of the parabola-line intersection is the roots of the quadratic equation:

$$\left[\frac{1}{2}g\right]t^2 + [V_{Y0} - qV_{X0}]t + [Y_0 - Y_1 + q(X_1 - X_0)] = 0 \quad (1)$$

Each step consists of determining the necessary parameters and solving the quadratic equation to find the intersection point. Once the intersection point is found, the impact is calculated. If the rock has enough velocity after the impact, as determined by a comparison to the minimum velocity ( $V_{MIN}$ ), another step is initiated.

In the interest of brevity, the process of searching for the slope segment where the impact occurs has been left out of the verification.

#### Step 1:

The rock starts at location  $X_0 = 0$  m,  $Y_0 = 60$  m (which coincides with the first slope vertex). The rock was given an initial velocity of  $V_{X0} = 7$  m/s,  $V_{Y0} = 2$  m/s. The necessary parameters are determined, and the quadratic equation is solved to find the time of intersection with the second slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1.417) \pm \sqrt{(1.417)^2 - 4(-4.90)(21.58)}}{2(-4.90)} \cong -1.959 \text{ or } 2.25 \text{ s} \quad (2)$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(40 - 39)}{(19 - 7)} \cong 0.0833$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 2 - (0.0833)(7) \cong 1.417$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 60 - 39 + (0.0833)(7 - 0) \cong 21.58$$

$t = -1.959$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ .

The intersection point and pre-intersection velocity are found by substituting  $t$  back into the following kinematic equations:

$$X_I = V_{X0}t + X_0 = (7)(2.25) + 0 = 15.732 \text{ m} \quad (3)$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(2.25)^2 + (2)(2.25) + 60 = 39.728 \text{ m} \quad (4)$$



$$V_{XB} = V_{X0} = 7 \frac{\text{m}}{\text{s}} \quad (5)$$

$$V_{YB} = V_{Y0} + gt = 2 + (-9.81)(2.25) = -20.0 \frac{\text{m}}{\text{s}} \quad (6)$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = \tan^{-1}(0.0833) = 4.77^\circ \quad (7)$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-20.0) \cos(4.77) - (7) \sin(4.77) = -20.5 \frac{\text{m}}{\text{s}} \quad (8)$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-20.0) \sin(4.77) + (7) \cos(4.77) = 5.31 \frac{\text{m}}{\text{s}} \quad (9)$$

The impact velocities are calculated by multiplying by the normal and tangential velocities with its corresponding coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.5(-20.5) = -10.28 \frac{\text{m}}{\text{s}} \quad (10)$$

$$V_{TA} = R_T V_{TB} = 0.8(5.31) = 4.25 \frac{\text{m}}{\text{s}} \quad (11)$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-10.28) \sin(4.77) + (4.25) \cos(4.77) = 3.38 \frac{\text{m}}{\text{s}} \quad (12)$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (4.25) \sin(4.77) - (-10.28) \cos(4.77) = 10.59 \frac{\text{m}}{\text{s}} \quad (13)$$

Step 1 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(3.38)^2 + (10.59)^2} = 11.12 \frac{\text{m}}{\text{s}} \quad (14)$$

Since the velocity of the rock,  $V_{CHECK}$  (= 11.12 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 2:

The final rock conditions for Step 1 are used as the initial conditions for Step 2. That is:

$$X_{0(step2)} = X_{I(step1)}$$

$$Y_{0(step2)} = Y_{I(step1)}$$

$$V_{X0(step2)} = V_{XA(step1)}$$

$$V_{Y0(step2)} = V_{YA(step1)}$$

The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the fourth slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(11.16) \pm \sqrt{(11.16)^2 - 4(-4.90)(16.02)}}{2(-4.90)} \cong -0.998 \text{ or } 3.27 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(20 - 22)}{(38 - 26)} \cong -0.1667$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 10.59 - (-0.1667)(3.38) \cong 11.16$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 39.73 - 22 + (-0.1667)(26 - 15.7) \cong 16.02$$

$t = -0.998$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ . The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = (3.38)(3.27) + 15.73 = 26.800 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(3.27)^2 + (11.16)(3.27) + 39.7 = 21.867 \text{ m}$$

$$V_{XB} = V_{X0} = 3.38 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 11.16 + (-9.81)(3.27) = -21.5 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = -9.46^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-21.5) \cos(-9.46) - (3.38) \sin(-9.46) = -20.6 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-21.5) \sin(-9.46) + (3.38) \cos(-9.46) = 6.87 \frac{\text{m}}{\text{s}}$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.6(-20.6) = -12.4 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = R_T V_{TB} = 0.8(6.87) = 6.18 \frac{\text{m}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-12.4) \sin(9.46) + (6.18) \cos(9.46) = 8.14 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta + V_{NA} \cos \theta = (6.18) \sin(9.46) - (-12.4) \cos(9.46) = 11.21 \frac{\text{m}}{\text{s}}$$

Step 2 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(8.14)^2 + (11.21)^2} = 13.85 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (= 13.85 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

### Step 3:

In a similar fashion to the previous step, the final rock conditions for Step 2 are used as the initial conditions for Step 3. The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the sixth slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(11.21) \pm \sqrt{(11.21)^2 - 4(-4.90)(21.87)}}{2(-4.90)} \cong -1.229 \text{ or } 3.54 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0 - 0)}{(89 - 46)} = 0$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 11.21 - (0)(8.12) \cong 11.21$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 21.9 - 0 + (0)(46 - 26.8) \cong 21.9$$

$t = -1.229$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ .

The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = (8.12)(3.54) + 26.8 = 55.642 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(3.54)^2 + (11.21)(3.54) + 21.9 = 0.000 \text{ m}$$

$$V_{XB} = V_{X0} = 8.12 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 11.21 + (-9.81)(3.54) = -23.55 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = 0.0^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-23.55) \cos(0) - (8.12) \sin(0) = -23.5 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-23.55) \sin(0) + (8.12) \cos(0) = 8.14 \frac{\text{m}}{\text{s}}$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.4(-23.5) = -9.42 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = R_T V_{TB} = 0.6(8.14) = 4.88 \frac{\text{m}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-9.42) \sin(0) + (4.88) \cos(0) = 4.88 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (4.88) \sin(0) - (-9.42) \cos(0) = 9.42 \frac{\text{m}}{\text{s}}$$

Step 3 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(4.88)^2 + (9.42)^2} = 10.6 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (= 10.6 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 4:

The final rock conditions for Step 3 are used as the initial conditions for Step 4. The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the sixth slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(9.42) \pm \sqrt{(9.42)^2 - 4(-4.90)(0)}}{2(-4.90)} \cong 0 \text{ or } 1.921 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0 - 0)}{(89 - 46)} = 0$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 9.42 - (0)(4.88) \cong 9.42$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 0 - 0 + (0)(46 - 55.6) = 0$$

$t = 0$  s is rejected because this is the starting point of the trajectory (we are already at *that* root).

The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = (4.88)(1.921) + 55.6 = 65.021 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(1.921)^2 + (9.42)(1.921) + 0 = 0.000 \text{ m}$$

$$V_{XB} = V_{X0} = 4.88 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 9.42 + (-9.81)(1.921) = -9.42 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = 0.0^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-9.42) \cos(0) - (4.88) \sin(0) = -9.42 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-9.42) \sin(0) + (4.88) \cos(0) = 4.88 \frac{\text{m}}{\text{s}}$$

The impact is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 0.4(-9.42) = -3.77 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = R_T V_{TB} = 0.6(4.88) = 2.93 \frac{\text{m}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-3.77) \sin(0) + (2.93) \cos(0) = 2.93 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (2.93) \sin(0) - (-3.77) \cos(0) = 3.77 \frac{\text{m}}{\text{s}}$$

Step 4 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(2.93)^2 + (3.77)^2} = 4.77 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (= 4.77 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

However, the hand calculations will not continue because they are very similar to Step 4, and will not provide much further verification, only repetition.

#### 1.1.4. Results

The same geometry and parameters were input into *RocFall2* and a simulation was performed. The results from *RocFall2* were compared to the manual calculations. The results from the sample calculations were identical to the *RocFall2* results for all practical purposes. The impact locations calculated by hand agreed with the program results up to the third decimal place in all cases (i.e. less than 0.5 mm difference, everywhere). Therefore, the projectile algorithm seems to be working correctly. Although coordinate output was not available from the comparison program, the graphical output matches that of *RocFall2* (as can be seen by comparing Figure 1.1-1 and Figure 1.1-2). This correlation is a good indication that the programs are performing the calculations as desired. Since the comparison program produces results that are very similar to the results produced by *RocFall2*, and the theoretical basis (the equations used) for the two programs are the same, it is reasonable to conclude that both programs are working correctly. The comparison of the results produced by these two programs does not prove the validity of the equations; however, it does provide greater confidence that the equations were properly coded into the programs.

#### 1.1.5. Input Files

*RocFall\_LumpMass\_Verification\_#1\_Projectile.fal8*

## 1.2. RocFall2 Verification Problem #2 – Sliding

[RocFall2 Build 8.007]

### 1.2.1. Problem Description

The purpose of this verification is to confirm that the sliding algorithm used by the program is working correctly. The sliding algorithm is used to calculate the motion of the rocks while they are in contact with the slope surface. The sliding algorithm is executed every time the rock stops moving. This stopping occurs, and thus the sliding algorithm is executed, at least once every simulation. Along with the projectile algorithm, the sliding algorithm determines the position and velocity of the rock. Any errors in the sliding algorithm would produce erroneous locations and velocities in the output. Therefore, it is essential that the sliding algorithm work correctly.

The sliding verification consists of four very similar examples that were designed to test every potential situation that the sliding algorithm might encounter. No statistics were used in this verification (i.e. only mean values were used, and all standard deviations were set to 0).

In order to guard against numerical instability; at the beginning of each simulation the program offsets the rock slightly into the analysis area (i.e. slightly into the “air”) and allows it to fall under the influence of gravity. The offset generated by the program, for the geometry used in this verification, was less than 0.02 mm. This “offsetting” requires the program to execute the projectile algorithm, at least once, before entering the sliding algorithm. This causes the program's results to differ slightly when compared to the manual calculations.

It should be noted that this “offsetting” will have a minimal effect on the outcome of a typical simulation. Since the offset is usually very small (0.02 mm in this case) and it is only applied once, at the beginning of the simulation, it will have a negligible effect on most simulations.

### 1.2.2. RocFall2 Analysis

#### Slope Geometry and Material Properties

The coefficient of tangential restitution ( $R_T$ ) was set to 1 and the coefficient of normal restitution ( $R_N$ ) was set to 0 in all cases. The choice of  $R_N = 0$  was made in order to force the program to initiate the sliding algorithm, immediately after the first pass through the projectile algorithm. The choice of  $R_T = 1$  was made in order to minimise the change in tangential velocity during the pass through the projectile algorithm.

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

Table 1.2-1: Slope Geometry and Materials

	X-Coordinate	Y-Coordinate	Normal Coefficient of Restitution $R_N$	Tangential Coefficient of Restitution $R_T$
Vertex 1	0	1		
Segment 1			0	1
Vertex 2	1	1		
Segment 2			0	1
Vertex 3	3	5		

Segment 3			0	1
Vertex 4	8	6.5		
Segment 4			0	1
Vertex 5	12	0		

### Initial Conditions

The rocks were started at  $X_0 = 6.5$  m,  $Y_0 = 6.05$  m (which lie on Segment 3) in all four cases. Because the rocks were placed directly on Segment 3 and were given a velocity that was tangential to the surface, all sliding occurred on Segment 3.

It will be useful to note that the slope of the segment on which sliding occurs (Segment 3) is:

$$\theta = \tan^{-1} \left( \frac{6.5 - 5}{8 - 3} \right) \cong 16.7^\circ$$

The initial velocity of the rocks and the friction angle of the slope are the only parameters that changed, depending on the case being considered. The difference between the four cases are summarized in a table:

Table 1.2-2: Difference Between Cases

Case	Initial Horizontal Velocity $V_{x0}$	Initial Vertical Velocity $V_{y0}$	Friction Angle of Slope $\phi$	Description
1	-1.0	-0.3	10°	Sliding downhill and off of the segment
2	-1.0	-0.3	18°	Sliding downhill and stopping
3	3.7	1.11	10°	Sliding uphill and off end of segment
4	3.7	1.11	18°	Sliding uphill and stopping

Enter the seeder and slope geometry values from Table 1.1-1 into *RocFall2*.

Note: Ensure that **Consider rotational velocity** and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The *RocFall2* model looks like this:

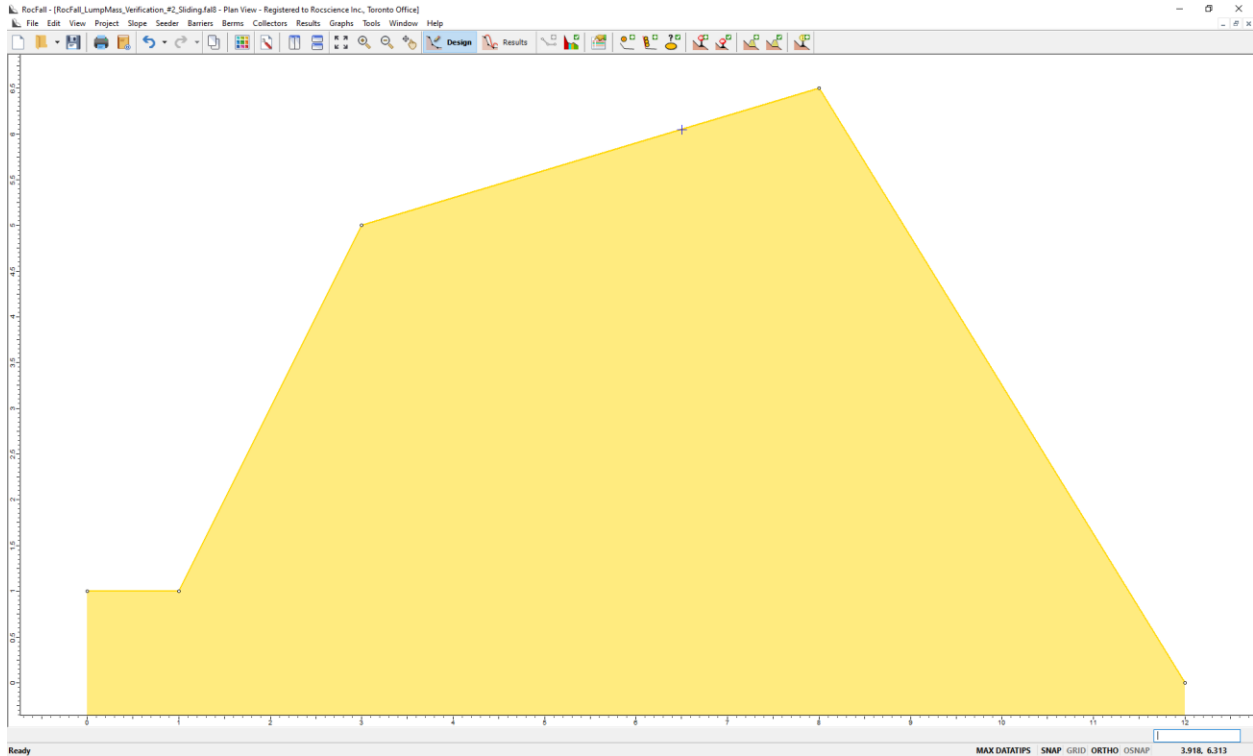


Figure 1.2-1: RocFall2 Model Geometry

### 1.2.3. Analytical Solution

#### Case 1: Sliding Downhill and Off of the Segment

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the downslope direction, and the conditions were such that the rock would slide off the downslope end of the segment. The rock was given an initial velocity of  $V_{X0} = -1$  m/s,  $V_{Y0} = -0.3$  m/s. The friction angle of the slope was set at  $10^\circ$ . Since the slope angle is greater than the friction angle, the rock will slide off the end of the segment. The exit velocity was calculated as follows:

$$V_{EXIT} = \sqrt{V_0^2 - 2sgk} = \sqrt{(1.044)^2 - 2(3.654)(-9.81)(0.1185)} \cong 3.095 \frac{\text{m}}{\text{s}} \quad (15)$$

Where:

$$V_0 = \sqrt{V_{X0}^2 + V_{Y0}^2} = \sqrt{(-1)^2 + (-0.3)^2} \cong 1.044 \frac{\text{m}}{\text{s}}$$

$$s = \sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2} = \sqrt{(3 - 6.5)^2 + (5 - 6.05)^2} \cong 3.654 \text{ m}$$

$$k = \sin \theta - \cos \theta \tan \phi = \sin(16.67) - \cos(16.67) \tan(10) = 0.1185$$



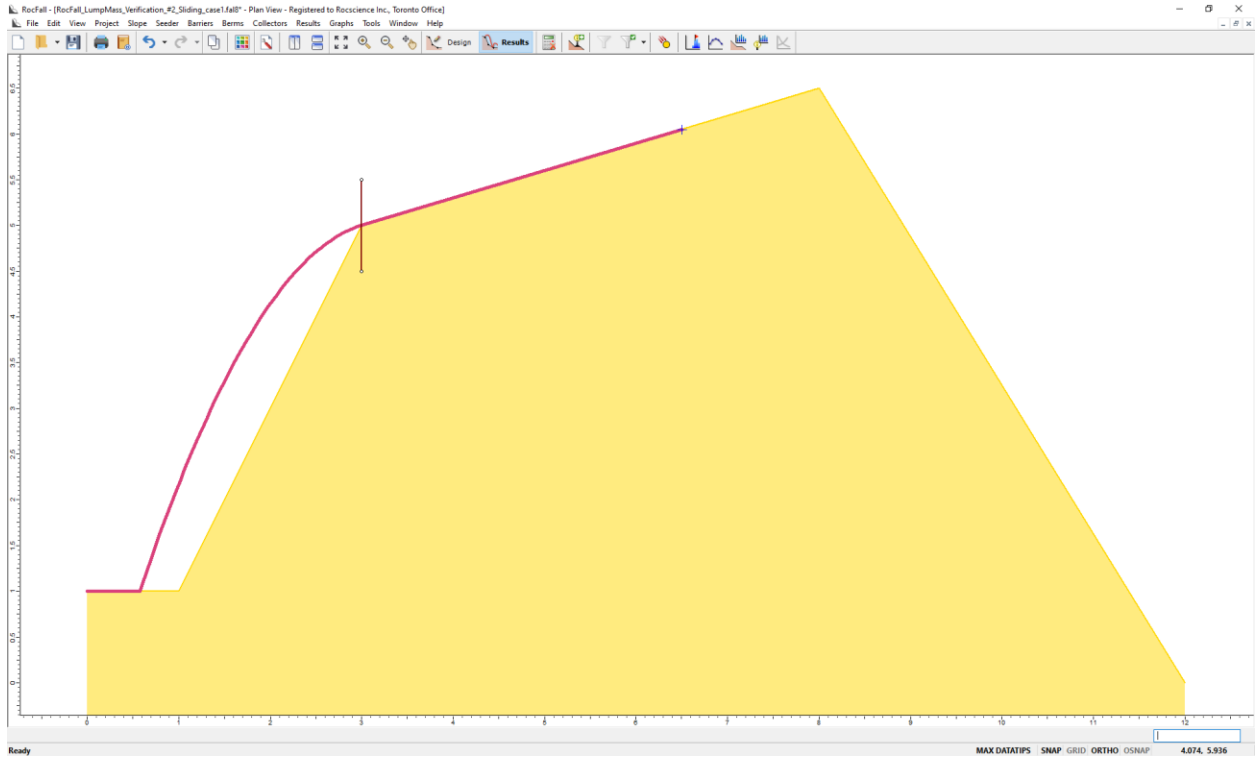


Figure 1.2-2: RocFall2 Rock Trajectory Model Results (Case 1)

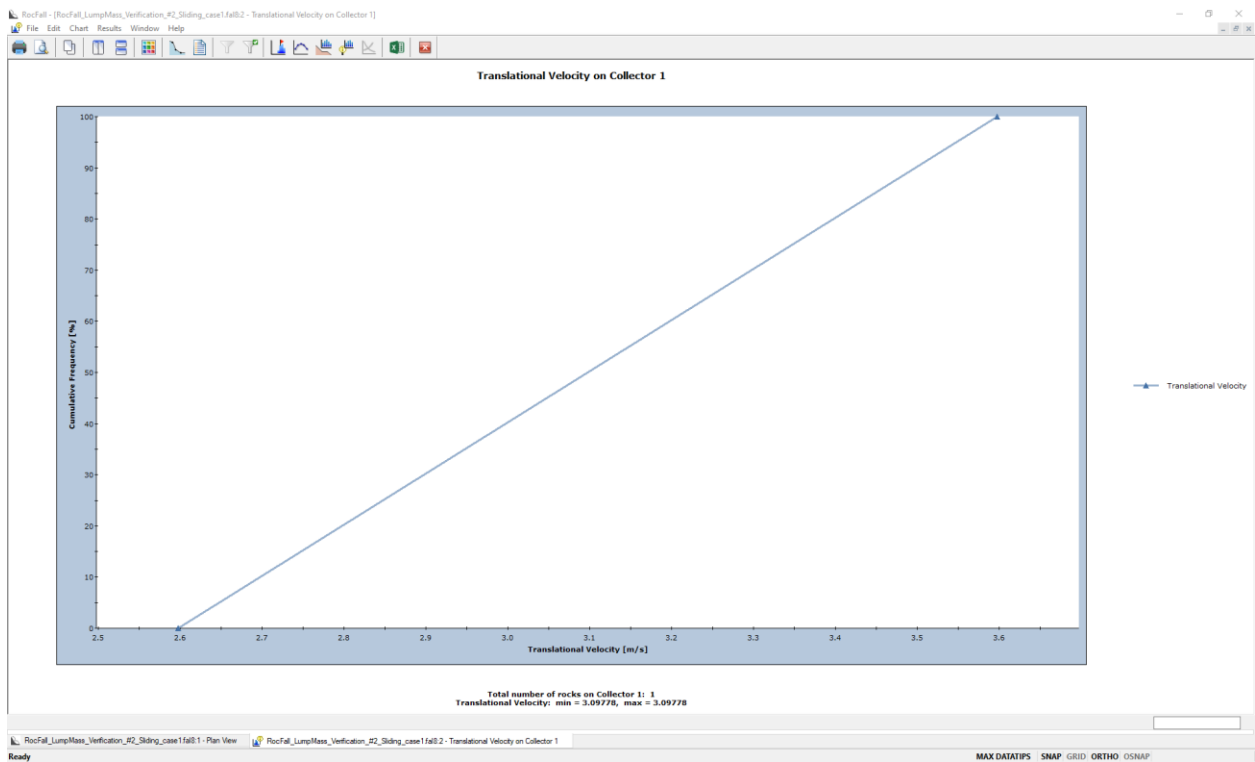


Figure 1.2-3: RocFall2 Data Collector Translational Velocity (Case 1)

In order to check the program's results, a **Data Collector** was added in *RocFall2*. The **Data Collector** was added from coordinates (3, 4.5) to coordinates (3, 5.5). The results were graphed using the **Graph Collector Data** option and selecting **Translational Velocity** in *RocFall2*. The exit velocity was obtained from the graph.

The results are presented in the following table:

Table 1.2-3: Comparison of Results (Case 1)

	Hand Calculation	<i>RocFall2</i>	Difference
Exit Velocity	3.095	3.098	0.1%

The results are very similar. The reason for the difference is that, in the program the rock gains some velocity falling from its offset position and starts with a velocity *slightly* greater than the value that was used in the hand calculations.

### Case 2: Sliding Downhill and Stopping

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the downslope direction, and the conditions were such that the rock would be slowed by friction and stop before reaching the downslope end of the segment. The rock was given an initial velocity of  $V_{X0} = -1$  m/s,  $V_{Y0} = -0.3$  m/s. The friction angle of the slope was set at  $18^\circ$ . Since the angle of the slope is less than the friction angle the rock will slow down, and depending on the length of the segment, stop before reaching the end of the segment. A calculation is made to see how far the rock will slide before stopping:

$$s = \frac{V_0^2}{2gk} = \frac{(1.044)^2}{2(-9.81)(-0.0239)} = 2.328 \text{ m} \quad (16)$$

Where:

$$V_0 = \sqrt{V_{X0}^2 + V_{Y0}^2} = \sqrt{(-1)^2 + (-0.3)^2} \cong 1.044 \frac{\text{m}}{\text{s}}$$

$$k = \sin \theta - \cos \theta \tan \phi = \sin(16.67) - \cos(16.67) \tan(18) = -0.0239$$

The rock will stop moving at a point 2.328 m downslope of where it began sliding. A check is made to see if the rock will reach the end of the segment before it stops sliding. The distance to the end of the segment is calculated:

$$s_D = \sqrt{(X_0 - X_1)^2 + (Y_0 - Y_1)^2} = \sqrt{(6.5 - 3)^2 + (6.05 - 5)^2} \cong 3.65 \text{ m} \quad (17)$$

Since the stopping distance,  $s$  ( $= 2.328$  m) is less than the distance to the end of the segment,  $s_D$  ( $= 3.65$  m) the rock will stop before sliding off of the end of the segment. The location where the rock stopped is calculated:

$$X = X_0 - s \cos \theta = 6.5 - (2.328) \cos(16.67) = 4.27 \text{ m} \quad (18)$$

$$Y = Y_0 - s \sin \theta = 6.05 - (2.328) \sin(16.67) = 5.38 \text{ m} \quad (19)$$

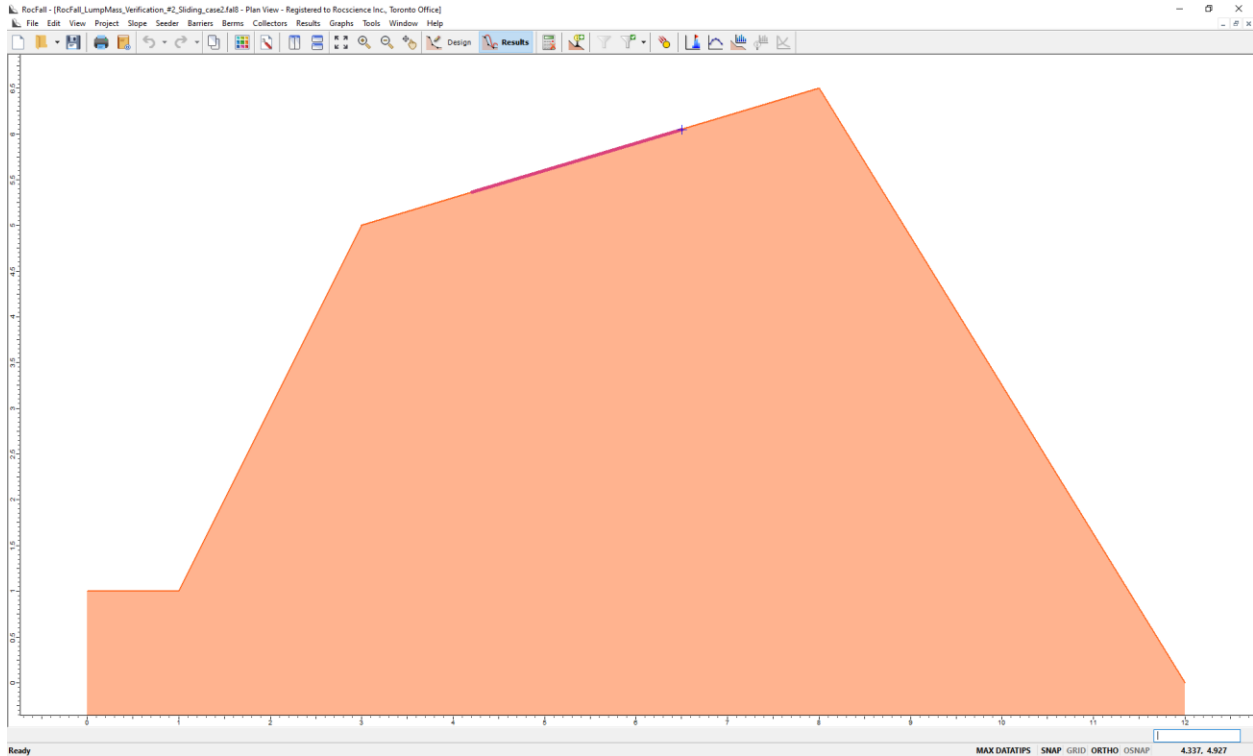


Figure 1.2-4: *RocFall2* Rock Trajectory Model Results (Case 2)

In order to check the program's results the simulation was performed in *RocFall2*. The results were graphed using the **Export Path Details** option in *RocFall2*. The location of the rock endpoint was obtained from the text file that was generated from **Impact and Ground Events Details**.

The results are presented in the following table:

Table 1.2-4: Comparison of Results (Case 2)

	Hand Calculation	<i>RocFall2</i>	Difference
X-Coordinate of Endpoint	4.270	4.207	1.5%
Y-Coordinate of Endpoint	5.381	5.362	0.4%

The results are very similar. The reason for the difference was found by stepping through the program as it executed. All values were identical to the manual calculations except for the value of  $V_0$ . The value of  $V_0$  in the program was 1.118 m/s. The value calculated by hand was 1.044 m/s. The higher value of  $V_0$  in the program was caused by the rock being offset into the analysis area at the beginning of the simulation and then gaining some velocity by falling under the influence of gravity.

### Case 3: Sliding Uphill and Off of the Segment

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the upslope direction and the conditions were such that the rock would slide off the upslope end of the segment. The rock was given an initial velocity of  $V_{X0} = 3.7$  m/s,  $V_{Y0} = 1.11$  m/s. The friction angle of the slope was set at  $10^\circ$ . The exit velocity is calculated:

$$V_{EXIT} = \sqrt{V_0^2 - 2sgk} = \sqrt{(3.863)^2 - 2(1.566)(-9.81)(-0.4562)} \cong 0.953 \frac{\text{m}}{\text{s}}$$

Where:

$$V_0 = \sqrt{V_{X_0}^2 + V_{Y_0}^2} = \sqrt{(3.7)^2 + (1.11)^2} \cong 3.863 \frac{\text{m}}{\text{s}}$$

$$s = \sqrt{(X_2 - X_0)^2 + (Y_2 - Y_0)^2} = \sqrt{(8 - 6.5)^2 + (6.5 - 6.05)^2} \cong 1.566 \text{ m}$$

$$k = -\sin \theta - \cos \theta \tan \phi = -\sin(16.67) - \cos(16.67) \tan(10) = -0.4562$$

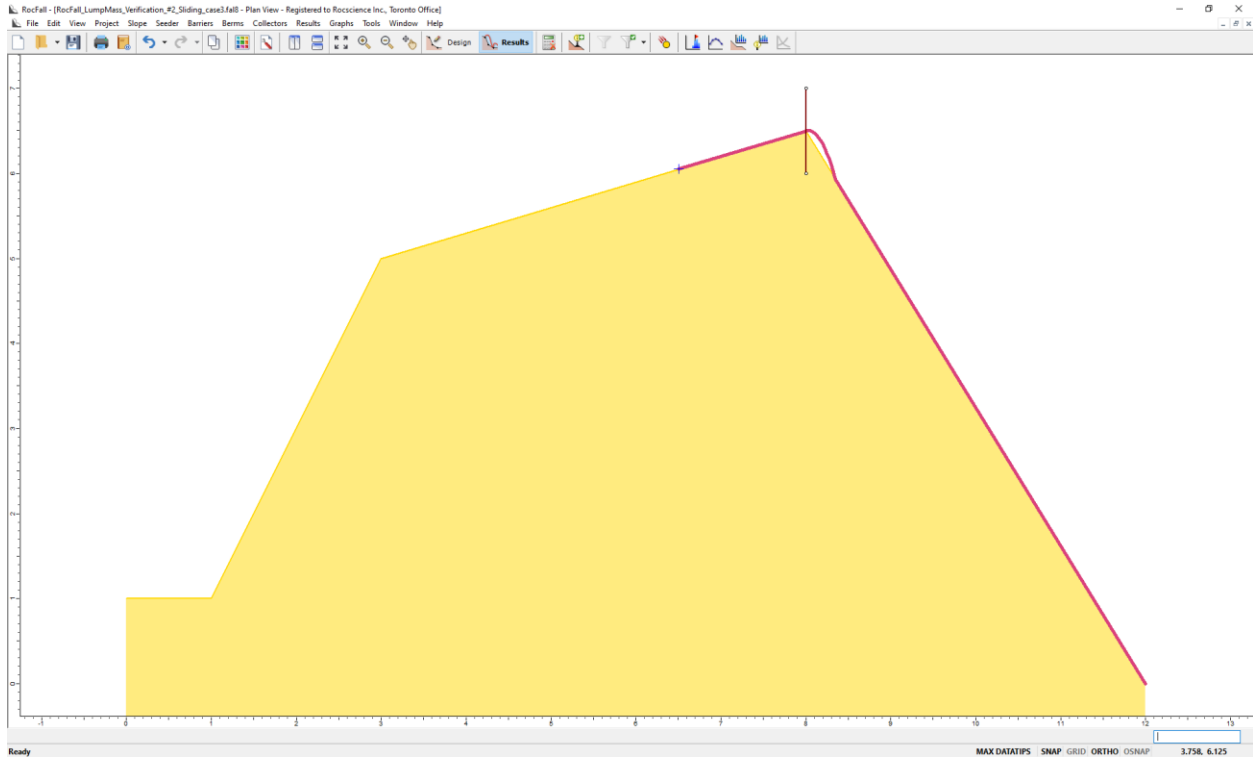


Figure 1.2-5: RocFall2 Rock Trajectory Model Results (Case 3)

In order to check the results from the program a **Data Collector** was added in RocFall2. The **Data Collector** was added from coordinates (8, 6) to coordinates (8, 7). The results were graphed using the **Graph Collector Data** option and selecting **Translational Velocity** in RocFall2. The exit velocity was obtained from the graph.

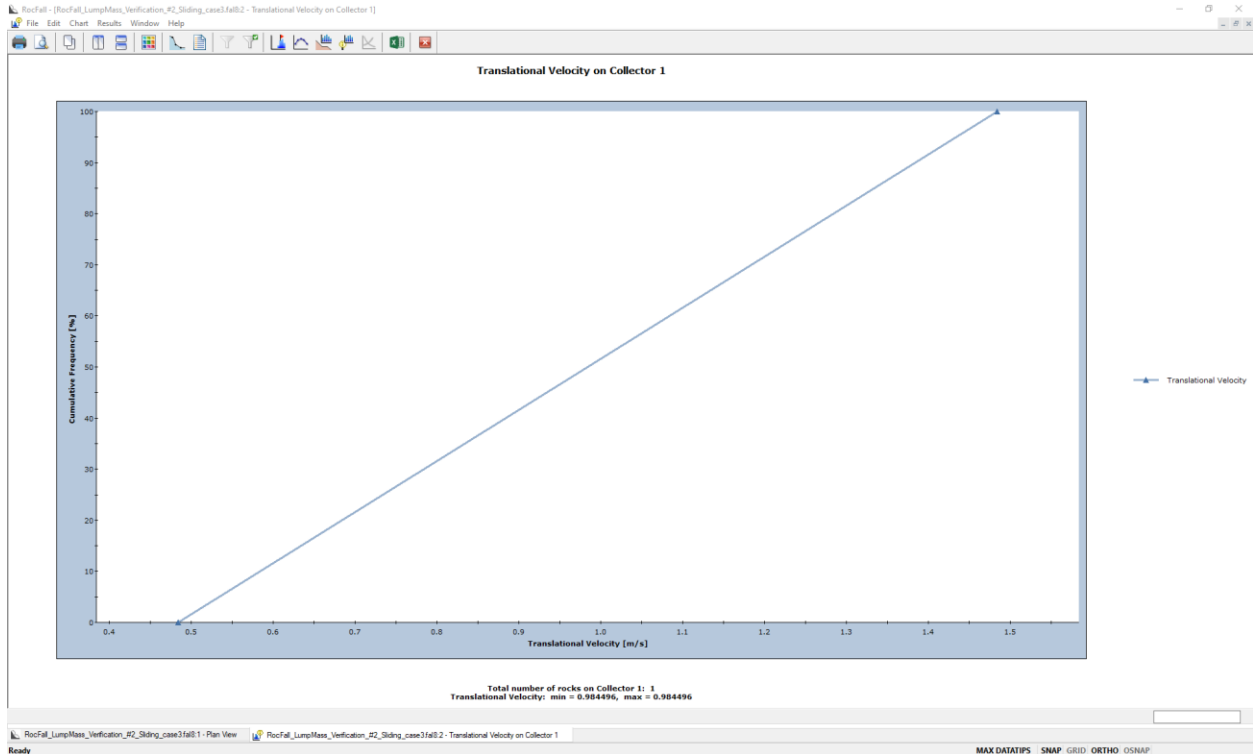


Figure 1.2-6: RocFall2 Data Collector Translational Velocity (Case 3)

The results are presented in the following table:

Table 1.2-5: Comparison of Results (Case 3: Sliding Uphill and Off of the Segment)

	Hand Calculation	RocFall2	Difference
Exit Velocity	0.953	0.984	3.1%

The results are similar. The reason for the difference was found by stepping through the program as it executed. All of the values in the program were identical to the manually calculated values except for  $V_0$  and  $s$ . The value of  $s$  in the program was 1.548 m (vs. 1.566 m for the hand calculation). The value of  $V_0$  in the program was 3.849 m/s (vs. 3.863 m/s for the hand calculation). The lower values of  $V_0$  and  $s$  in the program were caused by the rock being offset into the analysis area and then gaining some velocity and changing position during the fall from the offset position.

#### Case 4: Sliding Uphill and Stopping

This case was designed to test the behaviour of the program when the initial velocity of the rock was in the upslope direction, and the conditions were such that the rock would slow down and stop before it reached the upslope end of the segment. The rock was given an initial velocity of  $V_{X0} = 3.7$  m/s,  $V_{Y0} = 1.11$  m/s. The friction angle of the slope was set at  $18^\circ$ . Since the angle of the slope is less than the friction angle the rock will slow down, and depending on the length of the segment, stop before reaching the end of the segment. A calculation is made to see how far the rock will slide before stopping:

$$s = \frac{V_0^2}{2gk} = \frac{3.863^2}{2(-9.80665)(-0.599)} = 1.271 \text{ m}$$

Where:

$$V_0 = \sqrt{(3.7)^2 + (1.11)^2} \cong 3.863 \frac{\text{m}}{\text{s}}$$

$$k = -\sin \theta - \cos \theta \tan \phi = -\sin(16.67) - \cos(16.67) \tan(18) = -0.599$$

The rock will stop moving at a point 1.271 m upslope of where it began sliding. A check is made to see if the rock will reach the end of the segment before it stops sliding. The distance to the end of the segment is calculated:

$$s_D = \sqrt{(X_0 - X_2)^2 + (Y_0 - Y_2)^2} = \sqrt{(6.5 - 8)^2 + (6.05 - 6.5)^2} \cong 1.570 \text{ m}$$

Since the stopping distance,  $s$  ( $= 1.271 \text{ m}$ ) is less than the distance to the end of the segment,  $s_D$  ( $= 1.570 \text{ m}$ ) the rock will stop before reaching the end of the segment. The location where the rock stops is calculated:

$$X = X_0 + s \cos \theta = 6.5 + (1.271) \cos(16.67) = 7.717 \text{ m}$$

$$Y = Y_0 - s \sin \theta = 6.05 + (1.271) \sin(16.67) = 6.415 \text{ m}$$

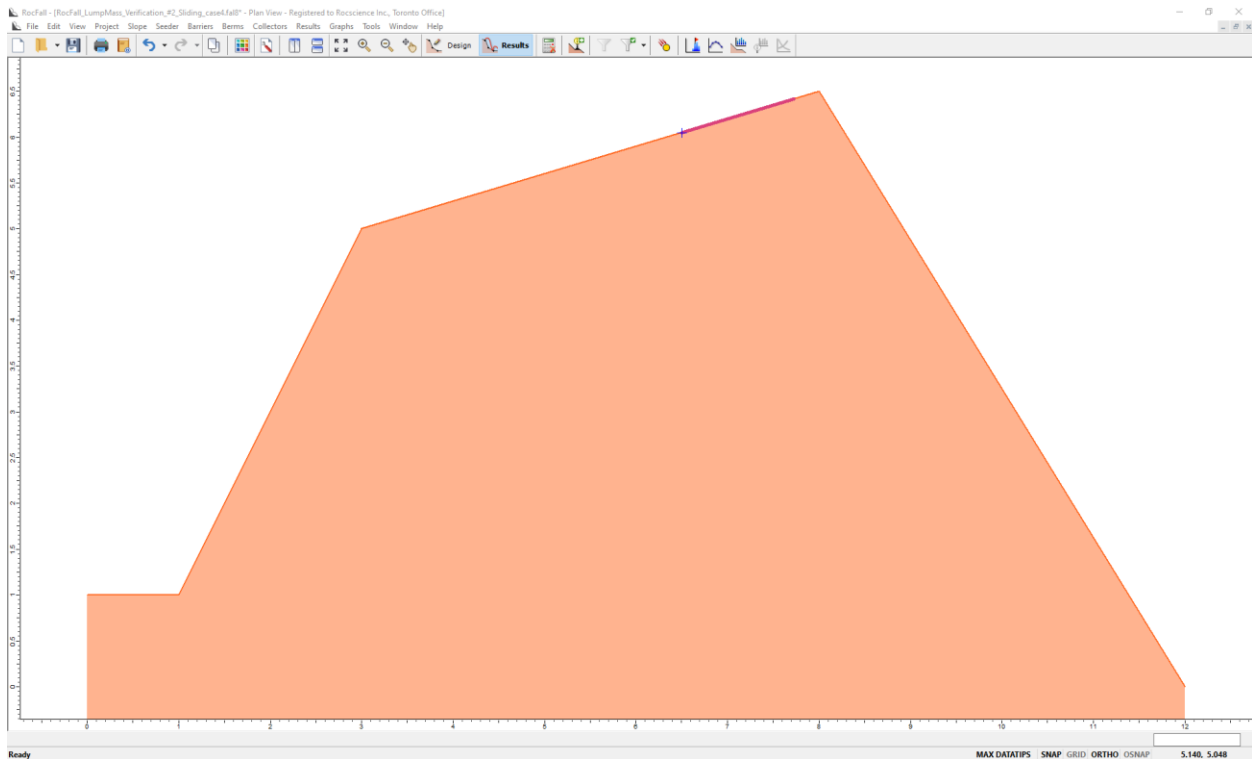


Figure 1.2-7: *RocFall2* Rock Trajectory Model Results (Case 4)

In order to check the program's results the simulation was performed in *RocFall2*. The results were graphed using the **Export Path Details** option in *RocFall2*. The location of the rock endpoint was obtained from the text file that was generated from **Impact and Ground Events Details**.

The results are presented in the following table:

Table 1.2-6: Comparison of Results (Case 4)

	Hand Calculation	<i>RocFall2</i>	Difference
X-Coordinate of Endpoint	7.717	7.727	1.3%
Y-Coordinate of Endpoint	6.415	6.418	0.05%

The results are very similar. The reason for the difference was found by stepping through the program as it executed. All of the values in the program were identical to the manual calculations except for the value of  $V_0$ . The value of  $V_0$  in the program was 3.850 m/s (vs. 3.863 m/s for hand calculation). This was caused by the rock being “offset into the analysis area” and then gaining some velocity by falling under the influence of gravity

#### 1.2.4. Results

There is one additional case that has not been can be presented: the case where the rock slides uphill, stops, and then slides back down and off the downslope end of the segment. This was not presented as a separate case because it is dealt with in the program by treating it as two separate cases that have been considered (Case 4 followed by Case 1).

The sliding algorithm seems to be working correctly. In each of the four cases the difference between the manual calculations and the results produced by *RocFall2* were explained by the “offsetting”, and not by errors in the sliding algorithm.

The results are presented as they are (with the slight difference caused by the offsetting), so that anyone using *RocFall2* could duplicate the verification cases. Although the results from the manual calculations could have been duplicated exactly (by inserting the values directly into the sliding algorithm) this was not done, because this option is only available to the program developer and not to someone using *RocFall2*.

Considering how poorly defined many of the significant quantities (such as  $R_T$ ) are, the consequences of moving the initial rock position a few hundredths of a millimeter at the beginning of each simulation can be ignored in the majority of simulations.

#### 1.2.5. Input Files

RocFall\_LumpMass\_Verification\_#2\_Sliding\_case1.fal8  
 RocFall\_LumpMass\_Verification\_#2\_Sliding\_case2.fal8  
 RocFall\_LumpMass\_Verification\_#2\_Sliding\_case3.fal8  
 RocFall\_LumpMass\_Verification\_#2\_Sliding\_case4.fal8

## 1.3. RocFall2 Verification Problem #3 – Probability

[RocFall2 Build 8.007]

### 1.3.1. Problem Description

The purpose of this verification is to confirm that the program is generating random numbers properly and applying statistics correctly. Random numbers are generated and used many different times during each simulation. If the random numbers were not being generated and applied correctly the analysis would not be statistically valid, and rational decisions could not be made based on the output. Almost all analyses performed with *RocFall2* rely on the probabilistic nature of the program. Therefore, it is important to verify that these components are working correctly.

It is difficult to perform the verification of a random system. By definition, a random system should not be replicable; however, replication is the basis for most verification. Since the results from the program could not be replicated when the initial conditions were specified by a random distribution, all of the other verification models performed did not include a random element. This made it difficult to test the coupling of the random number generation with the algorithms that use the random numbers.

This example was designed to verify that the random number generation is being performed correctly, and that the coupling of the random number generation with the other parts of the program (e.g. the projectile algorithm) was executed correctly. Since it would be very difficult to duplicate a random procedure by hand (generating enough random samples by hand to create a statistically valid data set would be extremely time consuming), this was not done. The example was constructed so as to generate a result that could be duplicated by hand. This duplication was not achieved by reproducing each individual result, as was done with the other verification models, but rather by reproducing the result in the collective form of a random distribution.

### 1.3.2. RocFall2 Analysis

#### Random Number Generation

It is important to remember that *RocFall2*, like most computer programs, only generates pseudo-random numbers (pseudo-random implies that there is *some sort* of pattern to the numbers).

The number generator is “seeded” at the beginning of each simulation. This “seeding” provides a starting point from which to begin the pseudo-random generation. A **Default Seed** or **Custom Seed** may be specified for **Random Number Generation**. This ensures that with the same seed, results of the probabilistic rockfall simulations are reproducible.

All random numbers generated in the program are sampled either from:

- Normal distribution,
- Uniform distribution,
- Triangular distribution,
- Beta distribution,
- Exponential distribution,
- Lognormal distribution, or
- Gamma distribution.

The samples are generated using either the Monte Carlo or Latin Hypercube sampling method.



## Slope Geometry and Material Properties

The example consists of a slope with two horizontal segments.

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

Table 1.3-1: Slope Geometry and Materials

	X-Coordinate	Y-Coordinate	Normal Coefficient of Restitution $R_N$		Tangential Coefficient of Restitution $R_T$		
			Mean $\mu$	Std. Dev. $\sigma$	Mean $\mu$	Std. Dev. $\sigma$	Range
Vertex 1	-1	0					
Segment 1			1	0	0.5	0.1	0 to 1
Vertex 2	6	0					
Segment 2			0	0	0	0	-
Vertex 3	19	0					

### Initial Conditions

The rocks begin by falling from a location that is slightly above the middle of the first segment. The parameters were chosen so that the location of the rock endpoints would form a normal distribution with statistical properties that could easily be determined by hand calculations.

The rocks were started at  $X_0 = 0$  m,  $Y_0 = 4.903325$  m. The rocks were given an initial velocity of  $V_{X0} = 5$  m/s,  $V_{Y0} = 0$  m/s.

### Deterministic Analysis

The calculations were performed once, using the constant values, in order to ascertain the deterministic value for the location of the rock endpoints. The effect of the random variable ( $R_T$ ) was then applied, in order to determine the expected value and standard deviation of the location of rock endpoints. The deterministic results (without the random variable) are presented first.

Enter the seeder and mean slope geometry values from Table 1.1-1 into *RocFall2*.

Note: Ensure that **Consider rotational velocity** and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The *RocFall2* model looks like this:

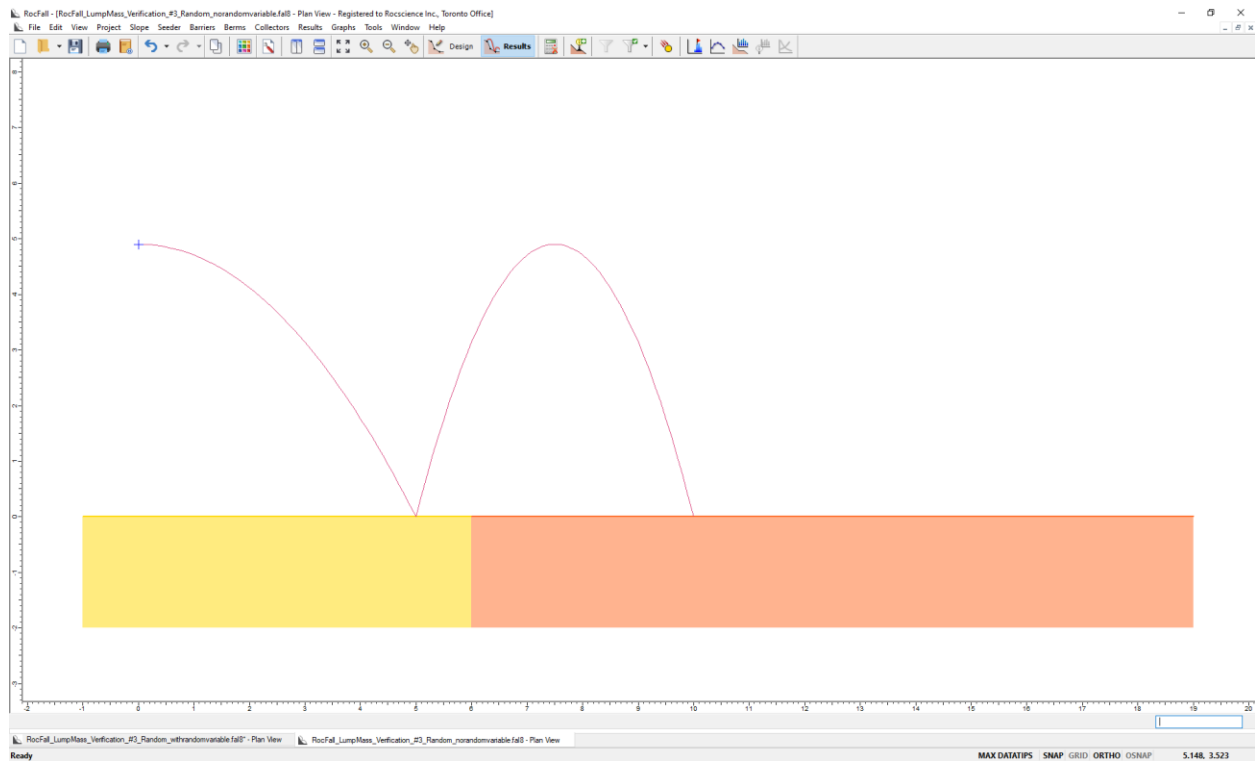


Figure 1.3-1: *RocFall2* Rock Trajectory Model Results without Random Variable

### Probabilistic Analysis

Enter the seeder and slope geometry values from Table 1.1-1 into *RocFall2*.

Set the **Number of Rocks** to 500 under **Seeder Properties**.

The coefficient of tangential restitution ( $R_T$ ) along the first segment is the only parameter in the example that has any statistical variation; all other parameters are constant.  $R_T$  was given a standard deviation of 0.1 (equivalent to a variance of 0.01). The choice of  $R_N = 0$  and  $R_T = 0$  for the second segment was done to ensure that the rocks stopped at their respective points of impact on the second segment.  $R_T$  was chosen as the random variable because it is in the “middle” of the projectile algorithm. This was thought to be preferable to varying, say, the initial velocity of the rocks, which is only used at the beginning of the simulation. It was thought that because  $R_T$  is in the “middle” of the calculations it may be more prone to error.

Set the tangential coefficient of restitution to a **Normal** distribution and the **Rel. Min.** and **Rel. Max** to 0.5, to allow the values of the random variable to vary between 0 to 1.

The sampling method is set to Monte-Carlo. The *RocFall2* model looks like this:

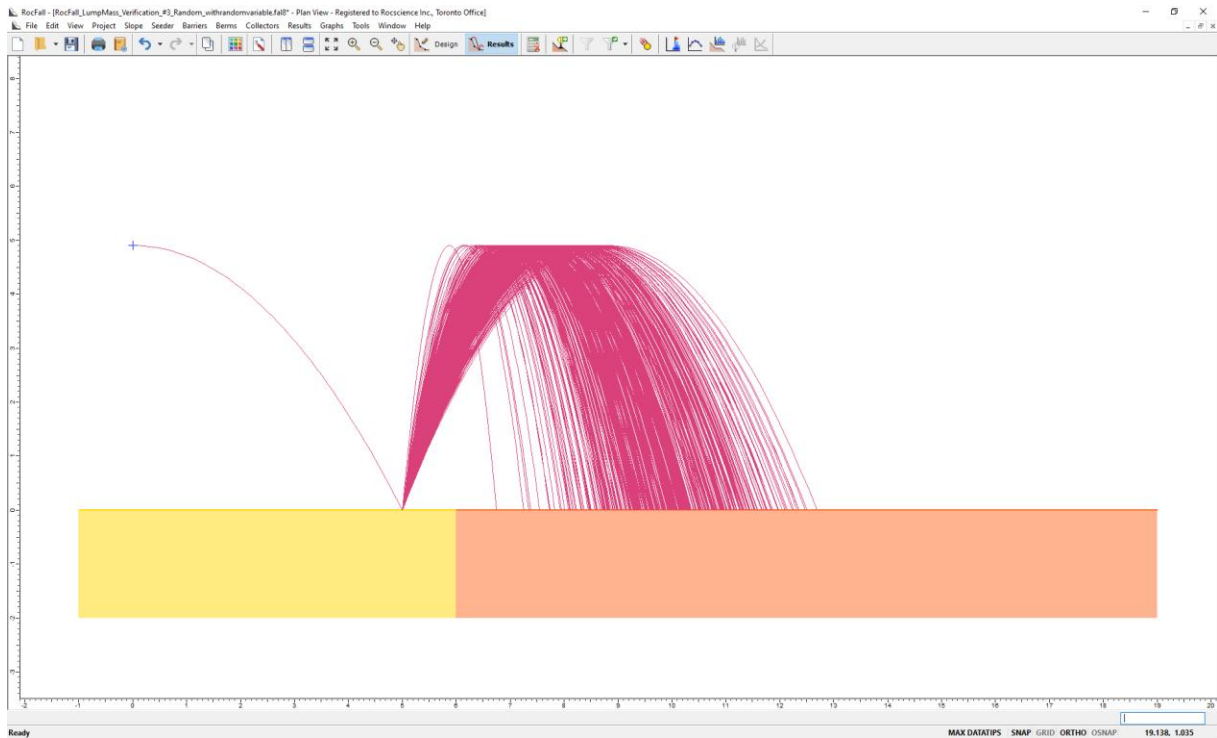


Figure 1.3-2: *RocFall2* Rock Trajectory Model Results with Random Variable

Using the **Graph Endpoints** option in *RocFall2*, a distribution of rock path end locations is generated. The **Number of Bins** is set to 100.

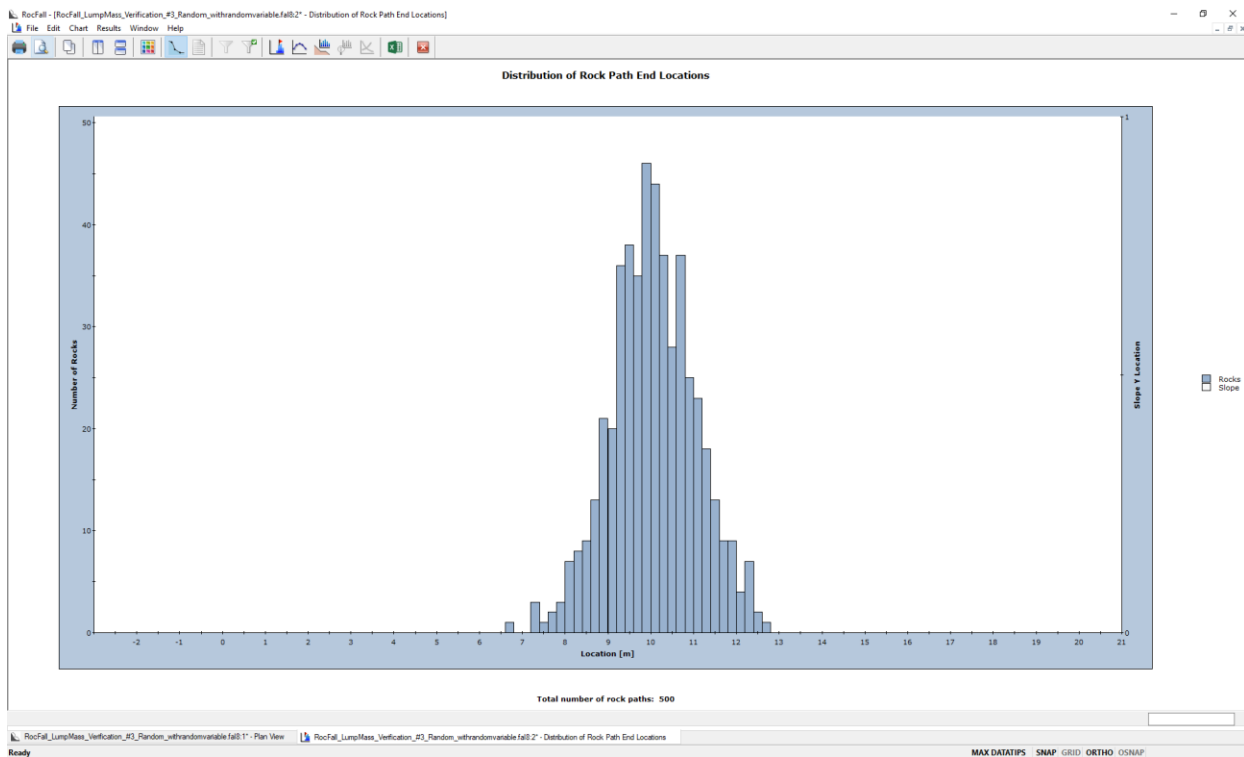


Figure 1.3-3: *RocFall2* Endpoints Distribution with Random Variable

### 1.3.3. Analytical Solution

#### Sample Calculations without Random Variable

The rocks were started at  $X_0 = 0$  m,  $Y_0 = 4.903325$  m. The rocks were given an initial velocity of  $V_{X0} = 5$  m/s,  $V_{Y0} = 0$  m/s. They will fall onto the first slope segment according to:

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0$$

Noting that  $V_{Y0} = 0$  and intersection with the first slope segment implies  $Y = 0$ , the above equation can be solved for  $t$ :

$$t = \sqrt{\frac{-2Y_0}{g}} = \sqrt{\frac{-2(4.903325)}{-9.80665}} = 1 \text{ s}$$

The intersection location and the velocity just before impact are calculated:

$$X_I = V_{X0}t + X_0 = 5(1) + 0 = 5 \text{ m}$$

$$V_{XB} = V_{X0} = 5 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 0 + (-9.80665)(1) = -9.80665 \frac{\text{m}}{\text{s}}$$

The pre-impact velocity is transformed into components normal and tangential to the slope segment:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0 - 0)}{(7 - (-1))} = 0$$

$$\theta = \tan^{-1} q = \tan^{-1}(0) = 0^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-9.80665) \cos(0) - (5) \sin(0) = -9.80665 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-9.80665) \sin(0) + (5) \cos(0) = 5 \frac{\text{m}}{\text{s}}$$

The post-impact velocity is calculated by multiplying by the coefficients of restitution:

$$V_{NA} = R_N V_{NB} = 1.0(-9.80665) = -9.80665 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = R_T V_{TB} = 0.5(5) = 2.5 \frac{\text{m}}{\text{s}} \quad (*)$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-9.81) \sin(0) + (2.5) \cos(0) = 2.5 \frac{\text{m}}{\text{s}} \quad (**)$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (2.5) \sin(0) - (-9.80665) \cos(0) = 9.80665 \frac{\text{m}}{\text{s}}$$

The rock's intersection with the second slope segment is calculated. Noting that  $Y_0 = 0$  and intersection with the second slope segment implies  $Y = 0$ , solve for  $t$ :

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0$$

$$t = \frac{-2V_{Y0}}{g} = \frac{-2(9.80665)}{-9.80665} = 2 \text{ s}$$

The intersection location and the velocity of the rock, just prior to impact, are calculated:

$$V_{XB} = V_{X0} = 2.5 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 9.80665 + (-9.80665)(2) = -9.80665 \frac{\text{m}}{\text{s}}$$

$$X = V_{X0}t + X_0 = 2.5(2) + 5 = 10 \text{ m} \quad (***)$$

### Sample Calculations with Random Variable

The deterministic result ( $X = 10 \text{ m}$ ) has been calculated above, using the constant values. The expected value and standard deviation of the rock endpoints will be calculated by applying statistical identities to the deterministic calculations.

All of the parameters remain unchanged except the value of  $R_T$  along the first slope segment.  $R_T$  is changed from a constant of 0.5 to a normally distributed random variable with a mean of 0.5 and a standard deviation of 0.1.

Before continuing, it will be useful to recall some statistical identities concerning expected value and variances. Proofs for these identities can be found in Ross (1987).

$$\sigma(x) = \sqrt{\text{var}(x)} \quad (2.5.0 \text{ of Ross})$$

$$E[mx + n] = mE[x] + n \quad (2.5.2 \text{ of Ross})$$

$$\text{Var}(mx + n) = m^2\text{Var}(x) \quad (2.6.2 \text{ of Ross})$$

Where:

- $m$  and  $n$  are constants
- $\sigma$  is standard deviation
- $x$  is a random variable,
- $E[ \ ]$  denotes expected value
- $\text{Var}( \ )$  denotes variance

The trajectory of the rock will be re-calculated, incorporating the effect of the random variable. Since  $R_T$  is the only parameter that has changed, the only equations that need to be recalculated are equations (\*), (\*\*), and (\*\*\*)

Equation (\*) will be re-calculated, incorporating the random variable.

Substituting  $m = V_{TB}$ ,  $n = 0$ , and  $x = R_T$  (expected value = 0.5, variance = 0.01) into the second and third statistical identities yields:

$$E[V_{TB}R_T + 0] = V_{TB}E[R_T] + 0$$

$$Var(V_{TB}R_T + 0) = V_{TB}^2 Var(R_T)$$

Substituting  $V_{TA} = R_T V_{TB}$ ,  $V_{TB} = 5$ ,  $E[R_T] = 0.5$  and  $Var(R_T) = \sigma^2 = (0.1)^2 = 0.01$  yields:

$$E[V_{TA}] = V_{TB}E[R_T] + 0 = 5[0.5] + 0 = 2.5$$

$$Var(V_{TA}) = V_{TB}^2 Var(R_T) = 5^2(0.01) = 0.25$$

$V_{TA}$  is now a random variable with an expected value of 2.5 and variance of 0.25.

Equation (\*\*) will now be recalculated incorporating  $V_{TA}$  (which is now a random variable). Substituting  $m = \cos(0)$ ,  $n = V_{NA} \sin \theta$ , and  $x = V_{TA}$  into the second and third statistical identities yields:

$$E[\cos(0) V_{TA} + V_{NA} \sin \theta] = \cos(0) E[V_{TA}] + V_{NA} \sin \theta$$

$$Var(\cos(0) V_{TA} + V_{NA} \sin \theta) = (\cos(0))^2 Var[V_{TA}]$$

Substituting  $V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta$ ,  $Var(V_{TA}) = 0.25$  and  $V_{NA} \sin \theta = 0$  (Therefore,  $\theta = 0$ ):

$$E[V_{XA}] = E[V_{TA}] + 0 = 2.5$$

$$Var(V_{XA}) = Var(V_{TA}) = 0.25$$

$V_{XA}$  is now a random variable with an expected value of 2.5 and variance of 0.25. Since the post-impact velocity of the first trajectory ( $V_{XA}$ ) is equal to the initial velocity of the next trajectory ( $V_{X0}$ ),  $V_{X0}$  is also a random variable with an expected value of 2.5 and variance of 0.25.

Equation (\*\*\*) will now be re-calculated incorporating  $V_{X0}$  (which is now a random variable). Substituting  $m = t$ ,  $n = X_0$  and  $x = V_{X0}$  yields:

$$E[tV_{X0} + X_0] = tE[V_{X0}] + X_0$$

$$Var(tV_{X0} + X_0) = t^2 Var(V_{X0})$$

Substituting  $t = 2$ ,  $E[V_{X0}] = 2.5$ ,  $Var(V_{X0}) = 0.25$ ,  $X_0 = 5$ , and  $X = V_{X0}t + X_0$  yields:

$$E[x] = (2)(2.5) + 5 = 10$$

$$Var(x) = (2)^2(0.25) = 1$$

The standard deviation is calculated from the variance by substituting  $x = X$  into the first statistical identity:

$$\sigma(x) = \sqrt{var(x)} = 1$$

Therefore, the expected value of  $X$  (the horizontal coordinate of the endpoints) is 10 m and the variance and standard deviation of the endpoints is 1 m. Since  $X$  is a normally distributed random variable, the distribution of the rock endpoints should take the shape of a typical normal distribution (a bell curve) with a center at 10 m. As can be seen by inspection of Figure 1.3-3, the distribution is of the correct shape.

### 1.3.4. Results

The same geometry and parameters were entered into *RocFall2* and five thousand simulations were performed. The results were graphed using the **Graph Endpoints** option in *RocFall2*. The data was extracted from the graph using the **Copy Data** option and pasted into a spreadsheet, where the statistical analysis was performed. The results from the program were compared to the manually calculated values. The results are summarized in a table:

Table 1.3-2: Comparison of Distribution of Rock Endpoints

	Hand Calculation	<i>RocFall2</i> Result	Difference
Number of Samples	-	500	-
Mean	10	10.0564	-0.56%
Standard Deviation	1	0.9896	1.04%
Variance	1	0.9948	0.52%

Note: Since the data is grouped, grouped methods of calculating mean, standard deviation, and variance must be applied.

$$\mu = \frac{Mf}{n} \quad (20)$$

$$s^2 = \frac{\sum M^2 f - \frac{(\sum Mf)^2}{n}}{n - 1} \quad (21)$$

Where:

- $\mu$  is the grouped mean
- $s$  is the grouped sample standard deviation
- $M$  is the midpoint of the data group
- $f$  is the frequency of the data group
- $n$  is the number of samples

The results from the program were very similar to the manual calculations. Given that this is a random process and *exact* answers cannot be expected, the program appears to be performing correctly; that is, the random numbers are being generated correctly and the calculations are using the random variables properly.

### 1.3.5. Input Files

*RocFall\_LumpMass\_Verification\_#3\_Random\_norandomvariable.fal8*  
*RocFall\_LumpMass\_Verification\_#3\_Random\_withrandomvariable.fal8*

## 1.4. RocFall2 Verification Problem #4 – Envelopes

[RocFall2 Build 8.007]

### 1.4.1. Problem Description

The purpose of this verification is to confirm that the envelopes produced by the program are correct and that the graphs based on these envelopes are being generated properly. There are three envelopes in the program: the kinetic energy envelope, the velocity envelope, and the bounce-height envelope. Each envelope is defined by the maximum value (e.g. maximum velocity) at a number of evenly spaced horizontal locations along the slope profile. The kinetic energy envelope measures the highest kinetic energy that any rock attained while passing each horizontal location. The velocity envelope measures the highest velocity that any rock attained while passing each horizontal location. The bounce-height graph measures the maximum height that any rock reached, minus the slope height, at each horizontal location (i.e. the maximum height above the slope). These envelopes are often used to determine where remedial measures should be placed, so it is very important that they operate correctly.

### 1.4.2. RocFall2 Analysis

#### Slope Geometry and Material Properties

In the process of completing this verification, we will take advantage of the fact that the projectile verification (RocFall2 Verification Problem #1 – Projectile) has already been performed.

#### Slope Geometry and Material Properties

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

Table 1.4-1: Slope Geometry and Materials

	X-Coordinate	Y-Coordinate	Normal Coefficient of Restitution $R_N$	Tangential Coefficient of Restitution $R_T$
Vertex 1	0	60		
Segment 1			0.5	0.8
Vertex 2	7	39		
Segment 2			0.5	0.8
Vertex 3	19	40		
Segment 3			0.5	0.8
Vertex 4	26	22		
Segment 4			0.6	0.9
Vertex 5	38	20		
Segment 5			0.6	0.9
Vertex 6	46	0		
Segment 6			0.4	0.6
Vertex 7	89	0		



## Initial Conditions

The rock starts at location  $X_0 = 0$  m,  $Y_0 = 60$  m (which coincides with the first slope vertex). The rock was given an initial velocity of  $V_{X0} = 7$  m/s,  $V_{Y0} = 2$  m/s and a mass of 10 kg.

Enter the seeder and slope geometry values from Table 1.1-1 into *RocFall2*.

Note: Ensure that **Consider rotational velocity** and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The *RocFall2* model looks like this:

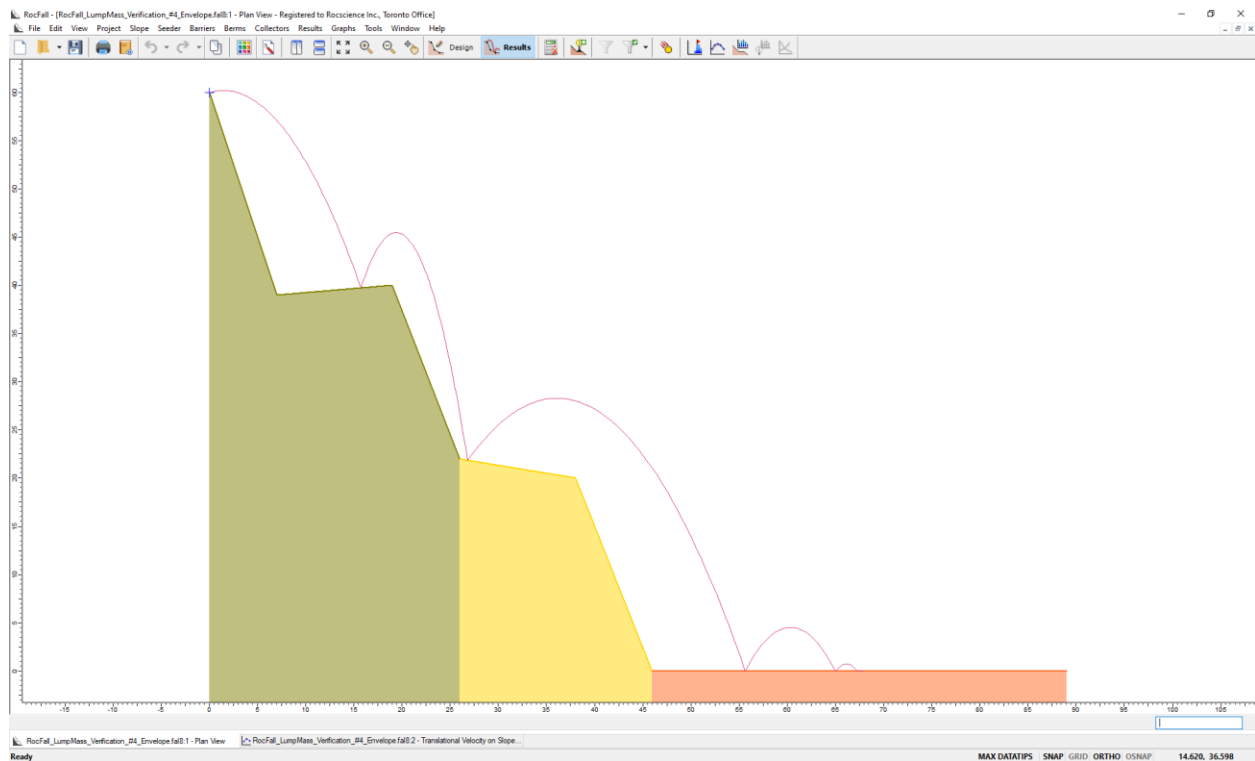


Figure 1.4-1 *RocFall2* Rock Trajectory Model Results

## Velocity and Kinetic Energy Envelopes

Using the **Graph Data on Slope** option in *RocFall2*, the following **Translational Velocity** and **Translational Kinetic Energy** graphs were generated:

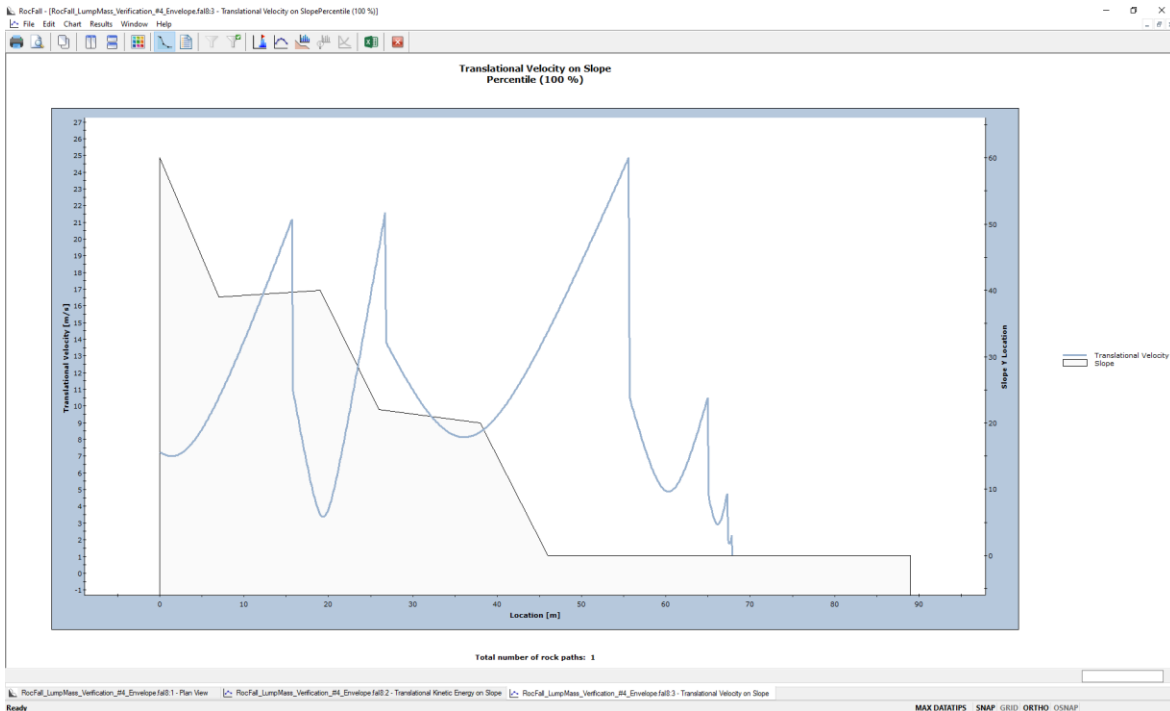


Figure 1.4-2: RocFall2 Translational Velocity Graph

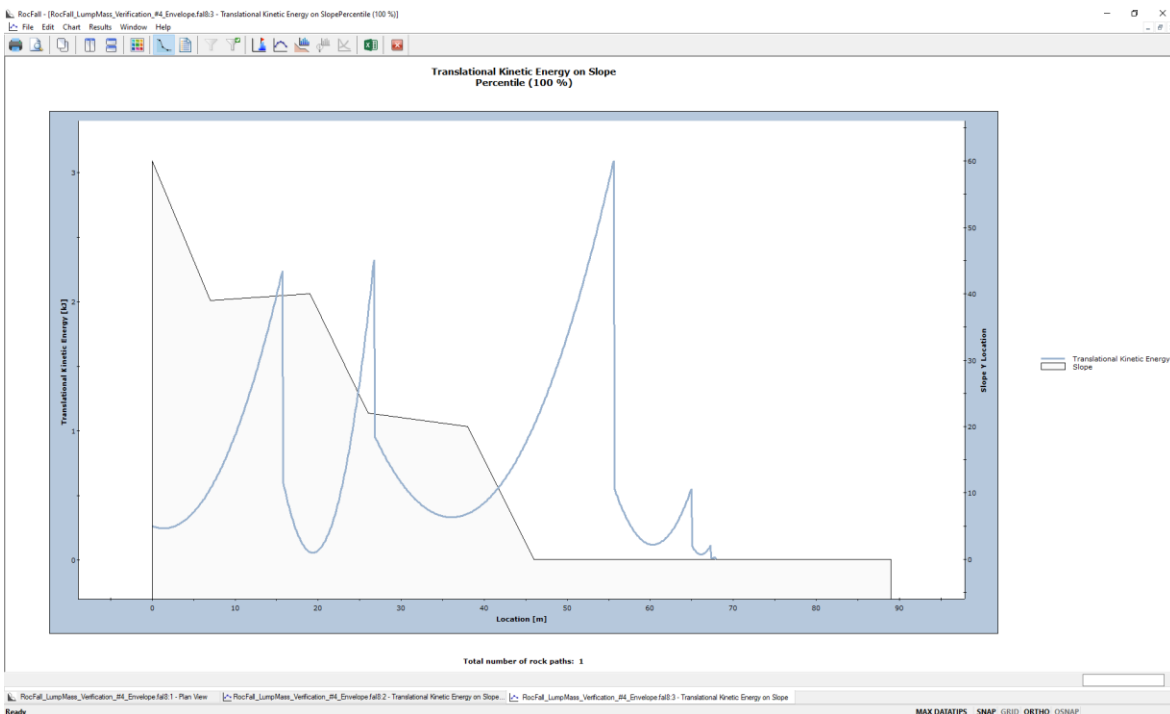


Figure 1.4-3: RocFall2 Translational Kinetic Energy Graph

The **Horizontal Locations** was set to 1000 when plotting using the **Distribution Graph** option in *RocFall2*. This value was chosen so that the program would collect data for the envelopes at numerous locations, especially those locations that are close to the points of interest.

We can see from inspection of Figure 1.4-2 and Figure 1.4-3 that the velocity and kinetic energy envelopes are of the correct shape.

The velocity and kinetic energy envelopes have peaks and troughs at the correct locations and the curvature of each section appears to be correct. The discontinuities occur at the correct locations (where the rock impacts the slope and loses energy). Since the shape of the graphs appeared to be correct, the graphs were verified by checking the value of each graph at significant locations along the slope profile. The values were checked at locations just before, and just after, each discontinuity. The values were also checked at the peak of each rock trajectory (the troughs on the velocity and kinetic energy graphs).

It is useful to note that at the peak of the rock's trajectory (the top of the parabola) the vertical velocity is zero. Since the horizontal velocity does not change while the rock is in the air, the only velocity that the rock possesses at the peak of the trajectory is its initial horizontal velocity. Therefore, the velocity at the peak of each parabolic path (the troughs of the velocity and kinetic energy envelopes) is equal to the post-impact velocity ( $V_{xA}$ ) of the previous impact.

### 1.4.3. Analytical Solution

The calculations that were performed in the projectile verification will provide the velocity of the rock at all of the locations that will be of interest in this verification.

Step 1:

The velocity and kinetic energy just before the first impact (at  $X \cong 15.7$  m) are calculated:

(The velocities are taken from the corresponding step of the projectile verification problem, *RocFall2* Verification Problem #1 – Projectile).

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(7)^2 + (-20)^2} = 21.19 \frac{\text{m}}{\text{s}} \quad (22)$$

$$KE_B = 0.5mV_B^2 = 0.5(10)(21.19)^2 = 2245 \text{ J} \quad (23)$$

The velocity and kinetic energy just after the first impact are calculated:

$$V_A = \sqrt{V_{xA}^2 + V_{yA}^2} = \sqrt{(3.38)^2 + (10.59)^2} = 11.12 \frac{\text{m}}{\text{s}} \quad (24)$$

$$KE_A = 0.5mV_A^2 = 0.5(10)(11.12)^2 = 618 \text{ J} \quad (25)$$

The velocity and kinetic energy at the second peak of the rock trajectory (at  $X \cong 19.4$  m) are calculated:

$$V_{PEAK} = V_{XA} = 3.38 \frac{\text{m}}{\text{s}} \quad (26)$$

$$KE_{PEAK} = 0.5mV_{PEAK}^2 = 0.5(10)(3.38)^2 = 57.1 \text{ J} \quad (27)$$

A comparison of the results produced by manual calculation and by the program are presented in the following table:

Table 1.4-2: Comparison of Velocity and Kinetic Energy Results for Step 1

	Hand Calculation	RocFall2	Difference
$V_B$	21.19	21.14	0.24%
$V_A$	11.12	11.06	0.54%
$V_{PEAK}$	3.38	3.38	-
$KE_B$	2245	2234	0.49%
$KE_A$	618	612	0.97%
$KE_{PEAK}$	57.1	57.2	0.17%

Step 2:

The velocity and kinetic energy just before the second impact (at  $X \cong 26.8$  m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(3.38)^2 + (-21.5)^2} = 21.76 \frac{\text{m}}{\text{s}}$$

$$KE_B = 0.5mV_B^2 = 0.5(10)(21.76)^2 = 2367 \text{ J}$$

The velocity and kinetic energy just after the second impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(8.14)^2 + (11.21)^2} = 13.85 \frac{\text{m}}{\text{s}}$$

$$KE_A = 0.5mV_A^2 = 0.5(10)(13.85)^2 = 959 \text{ J}$$

The velocity and kinetic energy at the third peak of the rock trajectory (at  $X \cong 36.1$  m) are calculated:

$$V_{PEAK} = V_{XA} = 8.14 \frac{\text{m}}{\text{s}}$$

$$KE_{PEAK} = 0.5mV_{PEAK}^2 = 0.5(10)(8.14)^2 = 331 \text{ J}$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

Table 1.4-3: Comparison of Velocity and Kinetic Energy Results for Step 2

	Hand Calculation	RocFall2	Difference
$V_B$	21.76	21.74	0.1%
$V_A$	13.85	13.78	0.5%
$V_{PEAK}$	8.14	8.14	-
$KE_B$	2367	2363	0.17%
$KE_A$	959	949	1.0%
$KE_{PEAK}$	331	331	-

### Step 3

The velocity and kinetic energy just before the third impact (at  $X \cong 55.6$  m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(8.12)^2 + (-23.55)^2} = 24.91 \frac{\text{m}}{\text{s}}$$

$$KE_B = 0.5mV_B^2 = 0.5(10)(24.91)^2 = 3103 \text{ J}$$

The velocity and kinetic energy just after the third impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(4.88)^2 + (9.42)^2} = 10.61 \frac{\text{m}}{\text{s}}$$

$$KE_A = 0.5mV_A^2 = 0.5(10)(10.61)^2 = 563 \text{ J}$$

The velocity and kinetic energy at the fourth peak of the rock trajectory (at  $X \cong 60.3$  m) are calculated:

$$V_{PEAK} = V_{XA} = 4.88 \frac{\text{m}}{\text{s}}$$

$$KE_{PEAK} = 0.5mV_{PEAK}^2 = 0.5(10)(4.88)^2 = 119.1 \text{ J}$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

Table 1.4-4: Comparison of Velocity and Kinetic Energy Results for Step 3

	Hand Calculation	RocFall2	Difference
$V_B$	24.91	24.90	0.04%
$V_A$	10.61	10.48	1.23%
$V_{PEAK}$	4.88	4.88	-
$KE_B$	3103	3099	0.13%
$KE_A$	563	549	2.49%
$KE_{PEAK}$	119.1	119.2	0.08%

### Step 4:

The velocity and kinetic energy just before the fourth impact (at  $x \cong 55.6$  m) are calculated:

$$V_B = \sqrt{V_{XB}^2 + V_{YB}^2} = \sqrt{(4.88)^2 + (-9.42)^2} = 10.61 \frac{\text{m}}{\text{s}}$$

$$KE_B = 0.5mV_B^2 = 0.5(10)(10.61)^2 = 563 \text{ J}$$

The velocity and kinetic energy just after the fourth impact are calculated:

$$V_A = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(2.93)^2 + (3.77)^2} = 4.77 \frac{\text{m}}{\text{s}}$$

$$KE_A = 0.5mV_A^2 = 0.5(10)(4.77)^2 = 111 \text{ J}$$

The velocity and kinetic energy at the fifth peak of the rock trajectory (at  $x \cong 60.3 \text{ m}$ ) are calculated:

$$V_{PEAK} = V_{XA} = 2.93 \text{ m/s}$$

$$KE_{PEAK} = 0.5mV_{PEAK}^2 = 0.5(10)(2.93)^2 = 42.9 \text{ J}$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

Table 1.4-5: Comparison of Velocity and Kinetic Energy Results for Step 4

	Hand Calculation	RocFall2	Difference
$V_B$	10.61	10.52	0.85%
$V_A$	4.77	4.67	2.1%
$V_{PEAK}$	2.93	2.93	-
$KE_B$	563	553	1.78%
$KE_A$	111	109	1.80%
$KE_{PEAK}$	42.9	42.9	-

### Bounce-Height Envelope

We can see by inspection of Figure 1.4-4 that the bounce-height envelope is of the correct shape. The curvature of each section appears to be correct and the peaks occur at the correct locations. The bounce-height is also zero at the correct locations (where the rock impacts the slope). Since the shape of the bounce-height envelope appeared to be correct, the graph was verified by checking the value (the bounce-height) of the graph at significant locations along the slope profile. The value was checked at each of the slope vertices. The vertices offered a good place to check the values on the graph because they correspond to either an abrupt change in slope of the graph or a peak on the bounce-height envelope.

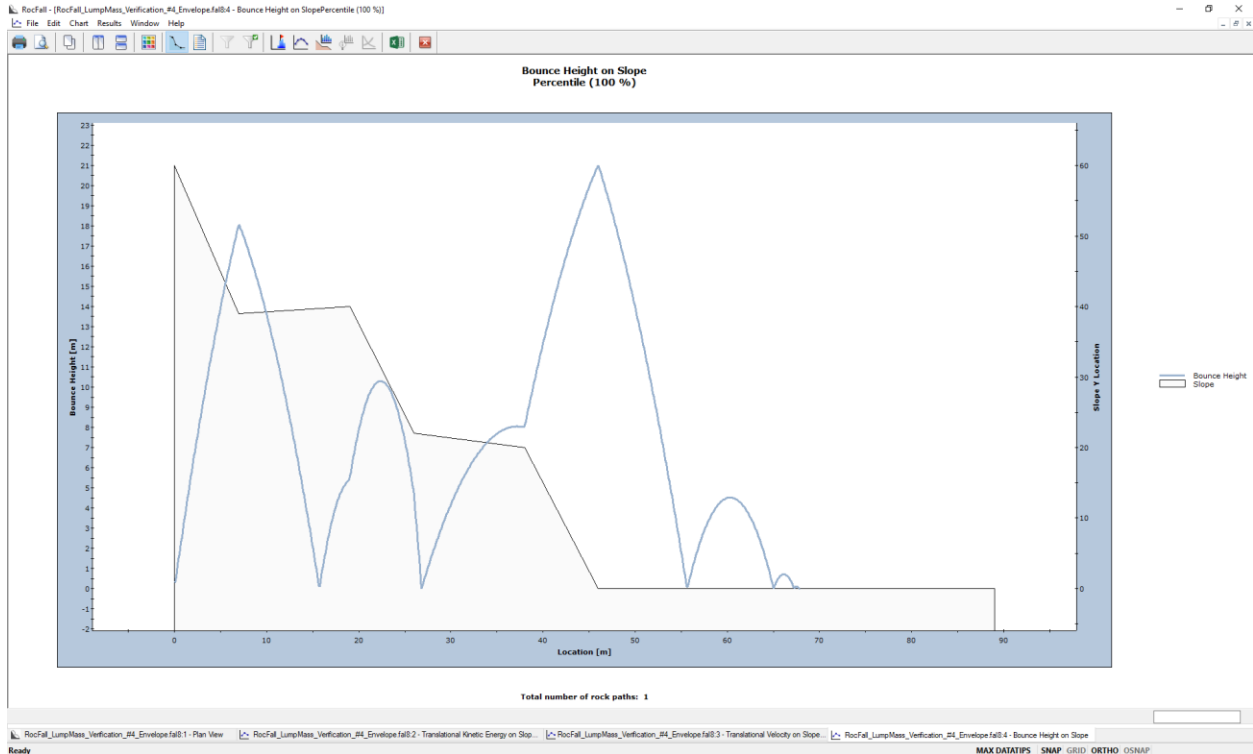


Figure 1.4-4: RocFall2 Bounce Height Graph

The bounce-height at the second slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_x} = \frac{(7 - 0)}{7} = 1 \text{ s}$$

$$Y_I = \frac{1}{2}gt^2 + V_yt + Y_0 = \frac{1}{2}(-9.81)(1)^2 + 2(1) + 60 = 57.1 \text{ m}$$

$$\Delta h = Y_I - H_s = 57.1 - 39 = 18.100 \text{ m}$$

The bounce-height at the third slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_x} = \frac{(19 - 15.732)}{3.38} = 0.967 \text{ s}$$

$$Y_I = \frac{1}{2}gt^2 + V_yt + Y_0 = \frac{1}{2}(-9.81)(0.967)^2 + 10.59(0.967) + 39.728 = 45.380 \text{ m}$$

$$\Delta h = Y_I - H_s = 45.38 - 40 = 5.380 \text{ m}$$

The bounce-height at the fourth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_x} = \frac{(26 - 15.732)}{3.38} = 3.04 \text{ s}$$

$$Y_I = \frac{1}{2}gt^2 + V_yt + Y_0 = 0.5(-9.81)(3.04)^2 + 10.59(3.04) + 39.728 = 26.648 \text{ m}$$

$$\Delta h = Y_I - H_S = 26.648 - 22 = 4.648 \text{ m}$$

The bounce-height at the fifth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_x} = \frac{(38 - 26.8)}{8.14} = 1.38 \text{ s}$$

$$Y_I = \frac{1}{2}gt^2 + V_yt + Y_0 = \frac{1}{2}(-9.81)(1.38)^2 + 11.21(1.38) + 21.867 = 28.008 \text{ m}$$

$$\Delta h = Y_I - H_S = 28.008 - 20 = 8.008 \text{ m}$$

The bounce-height at the sixth slope vertex is calculated:

$$t = \frac{(X_2 - X_1)}{V_x} = \frac{(46 - 26.8)}{8.14} = 2.36 \text{ s}$$

$$Y_I = \frac{1}{2}gt^2 + V_yt + Y_0 = \frac{1}{2}(-9.81)(2.36)^2 + 11.21(2.36) + 21.867 = 21.028 \frac{\text{m}}{\text{s}}$$

$$\Delta h = Y_I - H_S = 21.028 - 0 = 21.028 \text{ m}$$

A comparison of the results produced by hand calculation and by the program are presented in the following table:

Table 1.4-6: Comparison of Bounce Height Results

Vertex	Hand Calculation	<i>RocFall2</i>	Difference
2	18.1	18.06	0.22%
3	5.38	5.37	0.19%
4	4.648	4.719	1.50%
5	8.008	8.010	0.03%
6	21.028	20.999	0.14%



#### 1.4.4. Results

The results were very similar in all cases. The difference between the manual calculations and the values produced by *RocFall2* were less than 2.5% in all cases, and typically much less. Therefore, it seems that the velocity envelope, the kinetic energy envelope, and the bounce-height envelope are all being produced correctly.

The reason the results produced by *RocFall2* do not correspond *exactly* to the manual calculations is because the points on the envelope are not at exactly the same locations as the points used for the manual calculations. The program collects data for the envelopes at a number of locations evenly spaced along the slope profile (the number of locations used in this example was 1000). Since the program only collects data at these points there will be many locations where there is no data (i.e. in the spaces between the envelope points). In these cases, the envelope value at the closest horizontal location was used.

For example: The value for the bounce-height, by manual calculation, at  $X = 46.000$  m was 21.028 m. The program produced results at  $X = 45.924$  m (bounce-height = 20.929 m) and  $X = 46.013$  m (bounce-height = 20.999 m). Since the peak value is at 46.000 m and the two locations produced by the program border this location, they will be slightly lower, and the results will not be exact.

Increasing the number of locations used to collect data would have decreased the difference between the program results and the manual calculations. However, given the lack of certainty in much of the input data (e.g.  $R_T$ ), the additional precision is of questionable value.

#### 1.4.5. Input Files

*RocFall\_LumpMass\_Verification\_#4\_Envelope.fal8*

## 1.5. RocFall2 Verification Problem #5 – Angular Velocity

[RocFall2 Build 8.007]

### 1.5.1. Problem Description

The purpose of this verification is to confirm that the angular velocity algorithm used by the program is working correctly.

The example consists of a slope with two benches and a single rock that begins its travel at the crest of the slope. This example is identical to RocFall2 Verification Problem #1 – Projectile except that angular velocity has been considered in the equations. The rock was given an initial velocity and bounced a number of times before coming to rest at the base of the slope.

The slope was created by making minor modifications to the geometry of an actual slope profile. The geometry was modified so that the impacts would occur on slope segments with a positive slope, a negative slope and a horizontal segment.

The slope geometry and the input parameters were configured so that no sliding would occur. No statistics were incorporated into this verification (i.e. only mean values were used; all standard deviations were set to 0). Although rock trajectories in an actual simulation typically have dozens of steps, only the first four steps are followed here. This was done in the interest of brevity.

The minimum velocity ( $V_{MIN}$ ) was set to 1 m/s. This minimum velocity was selected so that the simulation did not end before the four steps were complete. Other numbers used in this example (e.g. the mass of the rock) were selected primarily for their ease in manual calculations.

### 1.5.2. RocFall2 Analysis

The same geometry and parameters from RocFall2 Verification Problem #1 – Projectile were entered into RocFall2.

#### Slope Geometry and Material Properties

The location of the slope vertices and the coefficients of restitution for each slope segment are presented in the following table:

Table 1.5-1: Slope Geometry and Materials

	X-Coordinate	Y-Coordinate	Normal Coefficient of Restitution $R_N$	Tangential Coefficient of Restitution $R_T$
Vertex 1	0	60		
Segment 1			0.5	0.8
Vertex 2	7	39		
Segment 2			0.5	0.8
Vertex 3	19	40		
Segment 3			0.5	0.8
Vertex 4	26	22		
Segment 4			0.6	0.9
Vertex 5	38	20		
Segment 5			0.6	0.9

Vertex 6	46	0		
Segment 6			0.4	0.6
Vertex 7	89	0		

### Initial Conditions

The rock starts at location  $X_0 = 0$  m,  $Y_0 = 60$  m (which coincides with the first slope vertex). The rock was given an initial velocity of  $V_{X0} = 7$  m/s,  $V_{Y0} = 2$  m/s and a mass of 10 kg.

The rock was given an angular velocity  $\omega_0 = 0$  m/s. The rock is assumed to be a sphere with a density of 2100 kg/m<sup>3</sup> for purposes of calculating a radius and moment of inertia from the mass:

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}} = \sqrt[3]{\frac{3 \cdot 10}{4 \cdot \pi \cdot 2100}} = 0.104 \text{ m} \quad (28)$$

$$I = \frac{2mr^2}{5} = \frac{2 \cdot 10 \cdot 0.104^2}{5} = 0.0436 \text{ kg} \cdot \text{m}^2 \quad (29)$$

As a basis for comparison, the same model is computed once with the consideration of rotational velocity and once without.

Enter the seeder and slope geometry values from Table 1.1-1 into *RocFall2*.

Note: Ensure that **Consider rotational velocity** and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The *RocFall2* model looks like this:

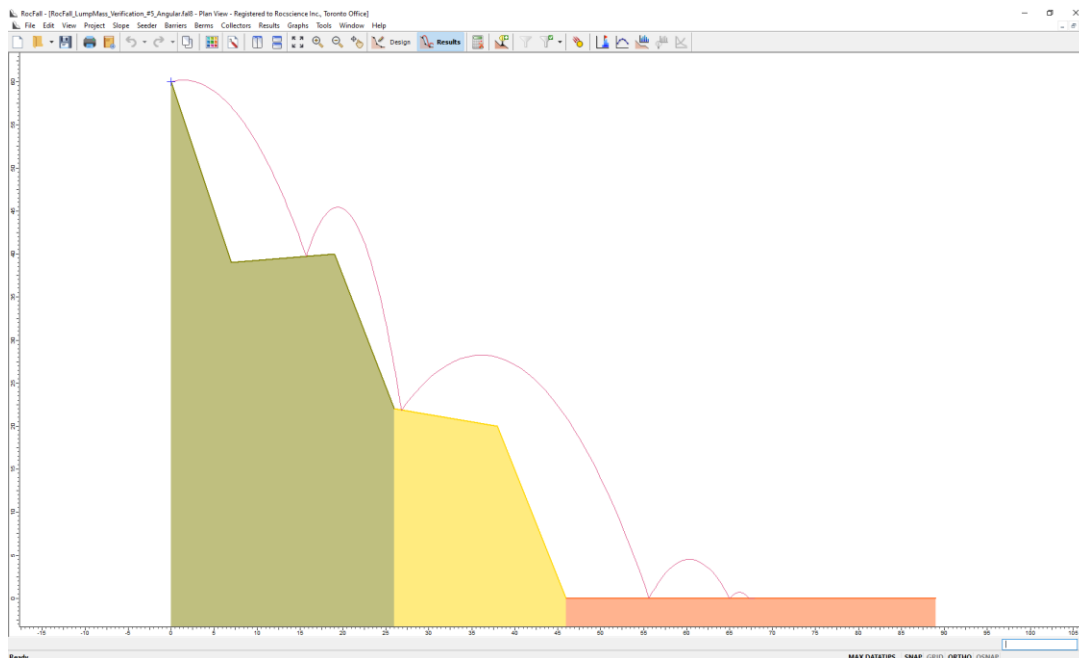


Figure 1.5-1: RocFall2 Rock Trajectory Model Results (Angular Velocity Not Considered)

Now, ensure that rotation velocity is considered.

Note: Ensure that **Consider rotational velocity** is checked and both **Scale Rn by Velocity** and **Scale Rn by Mass** are unchecked under **Project Settings**.

The RocFall2 model looks like this:

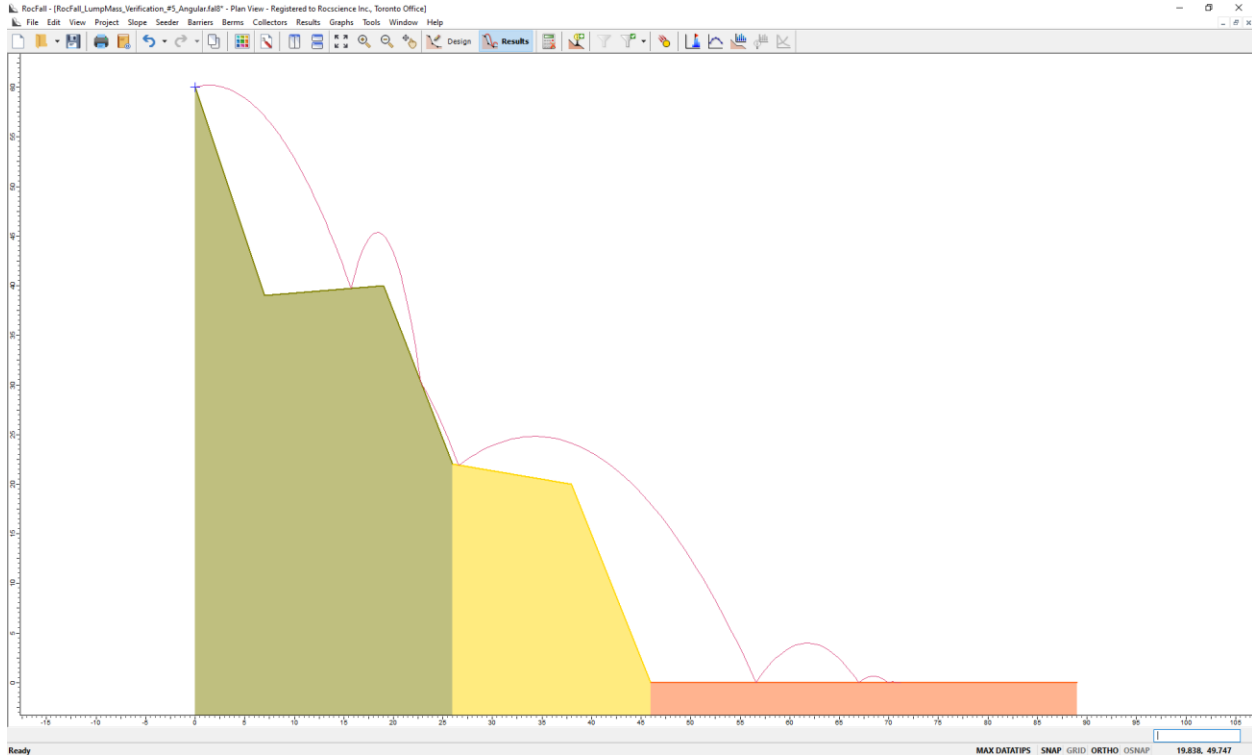


Figure 1.5-2: RocFall2 Rock Trajectory Model Results (Angular Velocity Considered)

### 1.5.3. Analytical Solution

The angular velocity calculations are very similar to the projectile calculations. The difference between the two is between the steps where the velocity is transformed from horizontal and vertical components into normal and tangential components; and when it is transformed back into horizontal and vertical components. For a more detailed explanation of the steps involved in the projectile algorithm, please consult RocFall2 Verification Problem #1 – Projectile.

#### Equations and Sample Calculations

##### Step 1:

The rock starts at location  $X_0 = 0$  m,  $Y_0 = 60$  m (which coincides with the first slope vertex). The rock was given an initial velocity of  $V_{X0} = 7$  m/s,  $V_{Y0} = 2$  m/s. The necessary parameters are determined, and the quadratic equation is solved to find the time of intersection with the second slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1.417) \pm \sqrt{(1.417)^2 - 4(-4.90)(21.58)}}{2(-4.90)} \cong -1.958 \text{ or } 2.25 \text{ s}$$



Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(40 - 39)}{(19 - 7)} \cong 0.0833$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 2 - (0.0833)7 \cong 1.417$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 60 - 39 + 0.0833(7 - 0) \cong 21.58$$

$t = -1.958$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ . The intersection point and pre-intersection velocity are found by substituting  $t$  back into the following equations:

$$X_I = V_{X0}t + X_0 = 7(2.25) + 0 = 15.729 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(2.25)^2 + 2(2.25) + 60 = 39.727 \text{ m}$$

$$V_{XB} = V_{X0} = 7 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 2 + (-9.81)2.25 = -20.04 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = 4.77^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-20.04) \cos(4.77) - (7) \sin(4.77) = -20.6 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-20.04) \sin(4.77) + (7) \cos(4.77) = 5.31 \frac{\text{m}}{\text{s}}$$

The friction function ( $F_1$ ) and scaling function ( $F_2$ ) are calculated (the empirical constants  $C_{F1} = 6.096$  m/s (20 ft/s) and  $C_{F2} = 76.2$  m/s (250 ft/s) are used in these equations):

$$F_1 = R_T + \frac{(1 - R_T)}{\left[\frac{V_{TB} - \omega_B r}{C_{F1}}\right]^2 + 1.2} = 0.8 + \frac{(1 - 0.8)}{\left[\frac{5.31 - (0)(0.104)}{6.096}\right]^2 + 1.2} = 0.9021 \quad (30)$$

$$F_2 = \frac{R_T}{\left[\frac{V_{NB}}{C_{F2} R_N}\right]^2 + 1} = \frac{0.8}{\left[\frac{-20.6}{(76.2)(0.5)}\right]^2 + 1} = 0.620 \quad (31)$$

Calculate the outgoing velocities:

$$V_{NA} = R_N V_{NB} = 0.5(-20.6) = -10.28 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = \sqrt{\frac{r^2 [I \omega_B^2 + m V_{TB}^2] F_1 F_2}{I + m r^2}} \quad (32)$$

$$= \sqrt{\frac{(0.104)^2 [0.0436(0)^2 + (10)(5.31)^2] (0.9021)(0.620)}{0.0436 + (10)(0.104)^2}} = 3.36 \frac{\text{m}}{\text{s}}$$

$$\omega_A = \frac{V_{TA}}{r} = \frac{3.36}{0.104} = 32.16 \frac{\text{rad}}{\text{s}} \quad (33)$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-10.28) \sin(4.77) + (3.36) \cos(4.77) = 2.49 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (3.36) \sin(4.77) - (-10.28) \cos(4.77) = 10.52 \frac{\text{m}}{\text{s}}$$

Step 1 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(2.49)^2 + (10.52)^2} = 10.81 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (= 10.81 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 2:

The final rock conditions for Step 1 are used as the initial conditions for Step 2. That is:

$$X_{0(step2)} = X_{I(step1)}$$

$$Y_{0(step2)} = Y_{I(step1)}$$

$$V_{X0(step2)} = V_{XA(step1)}$$

$$V_{Y0(step2)} = V_{YA(step1)}$$

$$\omega_{0(step2)} = \omega_{A(step1)}$$

The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the third slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(16.90) \pm \sqrt{(16.90)^2 - 4(-4.90)(-8.683)}}{2(-4.90)} \cong 0.627 \text{ or } 2.824 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(22 - 40)}{(26 - 19)} \cong -2.5714$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 10.52 - (-2.571)2.49 \cong 16.93$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 39.727 - 40 + (-2.5714)(19 - 15.729) \cong -8.683$$

$t = 0.627$  s is rejected because the slope is not defined at this time. The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 2.49(2.824) + 15.729 = 22.764 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(2.824)^2 + 10.52(2.824) + 39.727 = 30.322 \text{ m}$$

$$V_{XB} = V_{X0} = 2.49 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 10.52 + (-9.81)(2.824) = -17.18 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = -68.75^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-17.18) \cos(-68.75) - (2.49) \sin(-68.75) = -3.90 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-17.18) \sin(-68.75) + (2.49) \cos(-68.75) = 16.92 \frac{\text{m}}{\text{s}}$$

The friction function ( $F_1$ ) and scaling function ( $F_2$ ) are calculated:

$$F_1 = R_T + \frac{(1 - R_T)}{\left[\frac{V_{TB} - \omega_B r}{C_{F1}}\right]^2 + 1.2} = 0.8 + \frac{(1 - 0.8)}{\left(\frac{16.92 - (32.16)(0.104)}{6.096}\right)^2 + 1.2} = 0.8325$$

$$F_2 = \frac{R_T}{\left[\frac{V_{NB}}{C_{F2} R_N}\right]^2 + 1} = \frac{0.8}{\left[\frac{-3.90}{(76.2)(0.5)}\right]^2 + 1} = 0.7917$$

Calculate the outgoing velocities:

$$V_{NA} = R_N V_{NB} = 0.5(-3.90) = -1.953 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = \sqrt{\frac{r^2 [I \omega_B^2 + m V_{TB}^2] F_1 F_2}{I + m r^2}} = \sqrt{\frac{(0.104)^2 [0.0436(32.16)^2 + (10)(16.92)^2] (0.8325)(0.7917)}{0.0436 + (10)(0.104)^2}} = 11.70 \frac{\text{m}}{\text{s}}$$

$$\omega_A = \frac{V_{TA}}{r} = \frac{11.70}{0.104} = 112.1 \frac{\text{rad}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-1.953) \sin(-68.75) + (11.70) \cos(-68.75) = 6.06 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (11.70) \sin(-68.75) - (-1.953) \cos(-68.75) = -10.19 \frac{\text{m}}{\text{s}}$$

Step 2 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(6.06)^2 + (-10.19)^2} = 11.86 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (= 11.86 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.



### Step 3:

In a similar fashion to the previous step, the final rock conditions for Step 2 are used as the initial conditions for Step 3.

The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the fourth slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9.185) \pm \sqrt{(-9.185)^2 - 4(-4.90)(7.782)}}{2(-4.90)} \cong -2.506 \text{ or } 0.6332 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(20 - 22)}{(38 - 26)} = -0.1667$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = -10.19 - (-0.1667)6.06 \cong -9.185$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 30.322 - 22 + (-0.1667)(26 - 22.764) \cong 7.782$$

$t = -2.506$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ . The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 6.06(0.6332) + 22.764 = 26.601 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(0.6332)^2 + (-10.19)(0.6332) + 30.322 = 21.900 \text{ m}$$

$$V_{XB} = V_{X0} = 6.06 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = -10.19 + (-9.81)(0.6332) = -16.41 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = -9.4623^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-16.41) \cos(-9.462) - (6.06) \sin(-9.462) = -15.19 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-16.41) \sin(-9.462) + (6.06) \cos(-9.462) = 8.67 \frac{\text{m}}{\text{s}}$$

The friction function ( $F_1$ ) and scaling function ( $F_2$ ) are calculated:

$$F_1 = R_T + \frac{(1 - R_T)}{\left[\frac{V_{TB} - \omega_B r}{C_{F1}}\right]^2 + 1.2} = 0.8 + \frac{(1 - 0.9)}{\left(\frac{8.67 - (112.1)(0.104)}{6.096}\right)^2 + 1.2} = 0.969$$

$$F_2 = \frac{R_T}{\left[\frac{V_{NB}}{C_{F2} R_N}\right]^2 + 1} = \frac{0.9}{\left[\frac{-15.19}{(76.2)(0.6)}\right]^2 + 1} = 0.810$$

Calculate the outgoing velocities:

$$V_{NA} = R_N V_{NB} = 0.6(-15.19) = -9.112 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = \sqrt{\frac{r^2 [I\omega_B^2 + mV_{TB}^2] F_1 F_2}{I + mr^2}} = \sqrt{\frac{(0.104)^2 [0.0436(112.1)^2 + (10)(8.67)^2] (0.969)(0.810)}{0.0436 + (10)(0.104)^2}} = 8.541 \frac{\text{m}}{\text{s}}$$

$$\omega_A = \frac{V_{TA}}{r} = \frac{8.541}{0.104} = 81.83 \frac{\text{rad}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-9.112) \sin(-9.462) + (8.541) \cos(-9.462) = 9.92 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (8.541) \sin(-9.462) - (-9.112) \cos(-9.462) = 7.58 \frac{\text{m}}{\text{s}}$$

Step 3 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(9.92)^2 + (7.58)^2} = 12.49 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  (=12.49 m/s) is greater than the minimum velocity,  $V_{MIN}$  (= 1.0 m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step.

Step 4:

The final rock conditions for Step 3 are used as the initial conditions for Step 4. The necessary parameters are determined, and the quadratic equation is solved to find the time to intersection with the sixth slope segment:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(7.584) \pm \sqrt{(7.584)^2 - 4(-4.90)(21.90)}}{2(-4.90)} \cong -1.477 \text{ or } 3.023 \text{ s}$$

Where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{(0 - 0)}{(89 - 46)} = 0$$

$$a = \frac{1}{2}g \cong -4.90$$

$$b = V_{Y0} - qV_{X0} = 7.58 - (0)9.92 \cong 7.584$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) = 21.90 - 0 + 0(46 - 26.58) = 21.90$$

$t = -1.477$  s is rejected because  $t$  must lie in the range  $[0, \infty]$ . The intersection point and pre-impact velocity are determined:

$$X_I = V_{X0}t + X_0 = 9.92(3.023) + 26.601 = 56.598 \text{ m}$$

$$Y_I = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 = \frac{1}{2}(-9.81)(3.023)^2 + 7.58(3.023) + 21.900 = 0.000 \text{ m}$$

$$V_{XB} = V_{X0} = 9.92 \frac{\text{m}}{\text{s}}$$

$$V_{YB} = V_{Y0} + gt = 7.58 + (-9.81)3.023 = -22.07 \frac{\text{m}}{\text{s}}$$

The velocities are transformed into components normal and tangential to the slope segment:

$$\theta = \tan^{-1} q = 0.0^\circ$$

$$V_{NB} = V_{YB} \cos \theta - V_{XB} \sin \theta = (-22.07) \cos(0) - (9.92) \sin(0) = -22.07 \frac{\text{m}}{\text{s}}$$

$$V_{TB} = V_{YB} \sin \theta + V_{XB} \cos \theta = (-22.07) \sin(0) + (9.92) \cos(0) = 9.92 \frac{\text{m}}{\text{s}}$$

The friction function ( $F_1$ ) and scaling function ( $F_2$ ) are calculated:

$$F_1 = R_T + \frac{(1 - R_T)}{\left[\frac{V_{TB} - \omega_B r}{C_{F1}}\right]^2 + 1.2} = 0.8 + \frac{(1 - 0.6)}{\left[\frac{9.92 - (81.83)(0.104)}{6.096}\right]^2 + 1.2} = 0.9197$$

$$F_2 = \frac{R_T}{\left[\frac{V_{NB}}{C_{F2} R_N}\right]^2 + 1} = \frac{0.6}{\left[\frac{-22.07}{(76.2)(0.4)}\right]^2 + 1} = 0.3936$$

Calculate the outgoing velocities:

$$V_{NA} = R_N V_{NB} = 0.5(-22.07) = -8.83 \frac{\text{m}}{\text{s}}$$

$$V_{TA} = \sqrt{\frac{r^2 [I \omega_B^2 + m V_{TB}^2] F_1 F_2}{I + m r^2}} = \sqrt{\frac{(0.104)^2 [(0.0436)(81.83)^2 + (10)(9.92)^2] (0.9197)(0.3936)}{0.0436 + (10)(0.104)^2}} = 5.74 \frac{\text{m}}{\text{s}}$$

$$\omega_A = \frac{V_{TA}}{r} = \frac{5.74}{0.104} = 55.04 \frac{\text{rad}}{\text{s}}$$

The velocities are transformed back into vertical and horizontal components:

$$V_{XA} = V_{NA} \sin \theta + V_{TA} \cos \theta = (-8.83) \sin(0) + (5.74) \cos(0) = 5.74 \frac{\text{m}}{\text{s}}$$

$$V_{YA} = V_{TA} \sin \theta - V_{NA} \cos \theta = (5.74) \sin(0) - (-8.83) \cos(0) = 8.83 \frac{\text{m}}{\text{s}}$$

Step 4 is complete. The velocity of the rock, after impact, is calculated:

$$V_{CHECK} = \sqrt{V_{XA}^2 + V_{YA}^2} = \sqrt{(5.74)^2 + (8.83)^2} = 10.53 \frac{\text{m}}{\text{s}}$$

Since the velocity of the rock,  $V_{CHECK}$  ( $= 10.53$  m/s) is greater than the minimum velocity,  $V_{MIN}$  ( $= 1.0$  m/s), the rock is still considered to be moving. Since the rock is still moving, the simulation must continue for at least one more step. However, the hand calculations will not continue because they are very similar to Step 4, and will not provide much further verification, only repetition.

#### 1.5.4. Results

The same geometry and parameters were input into *RocFall2* and a simulation was performed. The results from *RocFall2* were compared to the manual calculations. The results from the manual calculations were identical to the *RocFall2* results for all practical purposes. The impact locations calculated by hand agreed with the program results up to the third decimal place in all cases (i.e. less

than 0.5 mm difference, everywhere). Therefore, the angular velocity algorithm seems to be working correctly.

### **1.5.5. Input Files**

*RocFall\_LumpMass\_Verification\_#5\_Angular.fal8*

## 2. *RocFall2* Collision Analysis Verification

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This document presents several lump mass rockfall examples, which have been used as collision verification problems for *RocFall2*. The results produced by *RocFall2* agree very well with the results produced by the *Colorado Rockfall Simulation Program (CRSP)* in the following case studies.

## 2.1. RocFall2 Verification Problem #1 – West Rifle Test Site

[RocFall2 Build 8.009]

### 2.1.1. Problem Description

This exercise verifies the average bounce height and translational velocity using the results from the *Colorado Rockfall Simulation Program (CRSP)* [1]. CRSP calibrated its simulation using data collected from a 300 feet high hillside near Rifle, Colorado. The slope material consists of thin desert soil with rocky ledges and sparse vegetation. 100 spherical rocks were used in the CRSP example and 5000 spherical rocks are used in the RocFall2 analysis using the Lump Mass formulation.

### 2.1.2. RocFall2 Analysis

In the RocFall2 **Project Settings** dialog, make sure **Consider angular velocity** checkbox is selected. CRSP always considers angular velocity. When computing angular velocity in RocFall2, the moment of inertia of the rock is always assumed to be that of a sphere. In CRSP, you can choose between a sphere, cylinder, or disc. To compare with RocFall2, you must use the default spherical shape in CRSP. Under **Scaling Functions** pane, make sure the **Scale Rn by Velocity** checkbox is selected and the *K* factor is 30 ft/s for US imperial units or 9.144 m/s for metric units. CRSP always scales  $R_n$  by velocity. Also, make sure that the **Scale Rn by Mass** checkbox is unselected.

Another main difference between RocFall2 and CRSP is the definition of slope roughness. In CRSP, variation in slope geometry is done through a roughness parameter which varies the angle of the slope segment on contact with a rock. In RocFall2, one can statistically vary the angle of the slope segment in the **Material Editor** dialog. Slope roughness ( $S$ ) in CRSP can be translated to maximum slope variation angle ( $\theta_{max}$ ) with the following equation:

$$\theta_{max} = \tan^{-1} \left( \frac{S}{r} \right)$$

Where:

- $S$  is the slope roughness
- $r$  is the rock radius

In RocFall2, you define the standard deviation of the slope roughness. RocFall2 assumes the actual slope angle at any point varies according to a normal distribution. To get similar results, set the standard deviation to be equal to half of  $\theta_{max}$  (e.g., if the maximum slope variation is 10.3 degrees, set the standard deviation of the slope roughness in RocFall2 to 5.15 degrees). The reasoning behind this is simple: In a normal distribution, 95 percent of all values fall within 2 standard deviations of the mean. By setting the standard deviation equal to half the maximum slope variation, you are guaranteed that 95% of all sampled slope angles will fall in the range used by CRSP.

### Slope Geometry and Material Properties

The location of the slope vertices and material parameters for all slope segments are presented in the following tables:

Table 2.1-1: Slope Geometry and Materials

	X	Y	Normal Restitution $R_N$	Tangential Restitution $R_T$	Friction Angle $\phi$ (deg)	Surface Roughness Std. Dev. (deg)
Vertex 1	0	320				
Segment 1			0.25	0.82	1	5.15
Vertex 2	8	314				
Segment 2			0.32	0.84	1	7.62
Vertex 3	18	304				
Segment 3			0.32	0.84	1	9.99
Vertex 4	34	290				
Segment 4			0.32	0.84	1	21.14
Vertex 5	66	258				
Segment 5			0.3	0.84	1	9.99
Vertex 6	92	240				
Segment 6			0.3	0.84	1	9.99
Vertex 7	120	214				
Segment 7			0.3	0.83	1	9.99
Vertex 8	199	164				
Segment 8			0.33	0.82	1	12.22
Vertex 9	260	140				
Segment 9			0.33	0.82	1	9.99
Vertex 10	269	133				
Segment 10			0.34	0.84	1	16.24
Vertex 11	305	108				
Segment 11			0.34	0.84	1	14.31
Vertex 12	335	87				
Segment 12			0.34	0.84	1	9.99
Vertex 13	396	51				
Segment 13			0.34	0.85	1	5.15
Vertex 14	410	49				

### Initial Conditions

The rock starts at location  $X_0 = 1.32$  ft,  $Y_0 = 325$  to  $330$  ft, which is added as a **Line Seeder** in *RocFall2*. The rock was given an initial velocity of  $V_{X0} = 1$  ft/s,  $V_{Y0} = -1$  ft/s. The rock has a radius of 2.2 ft, mass of 7358 lb and density of 165 lb/ft<sup>3</sup>.

Enter the seeder and slope geometry values from Table 2.1-1 into *RocFall2*.

The RocFall2 model looks like this:

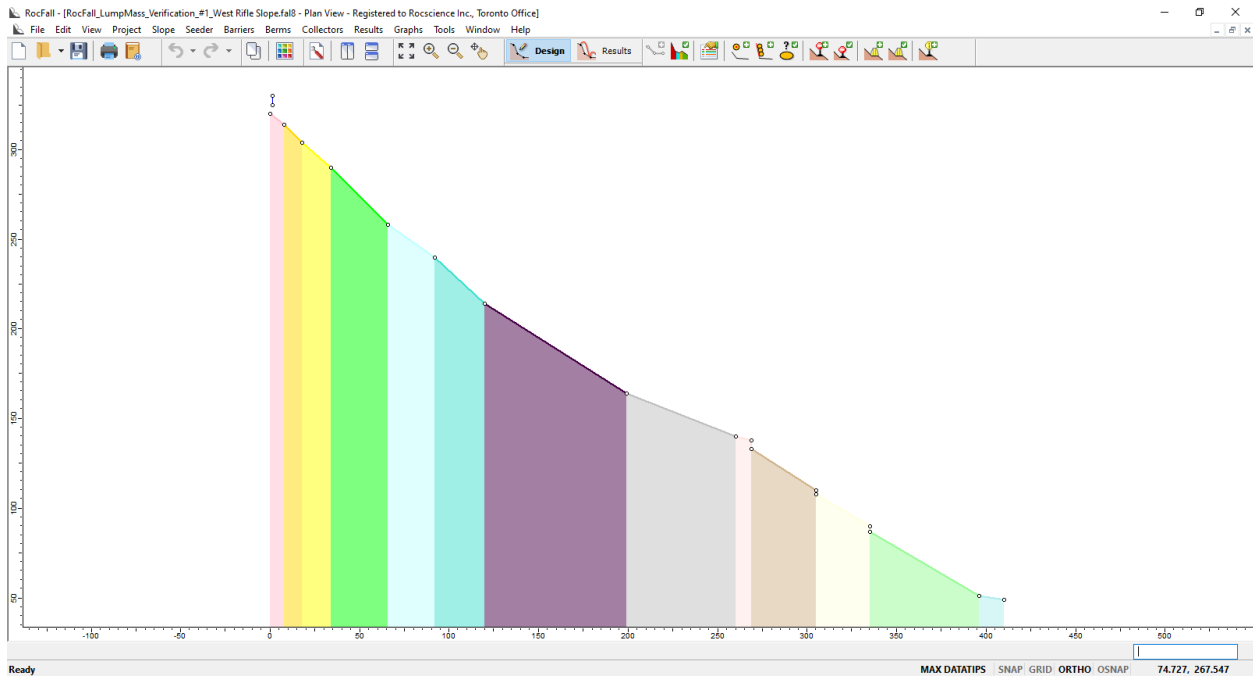


Figure 2.1-1: RocFall2 Model Geometry

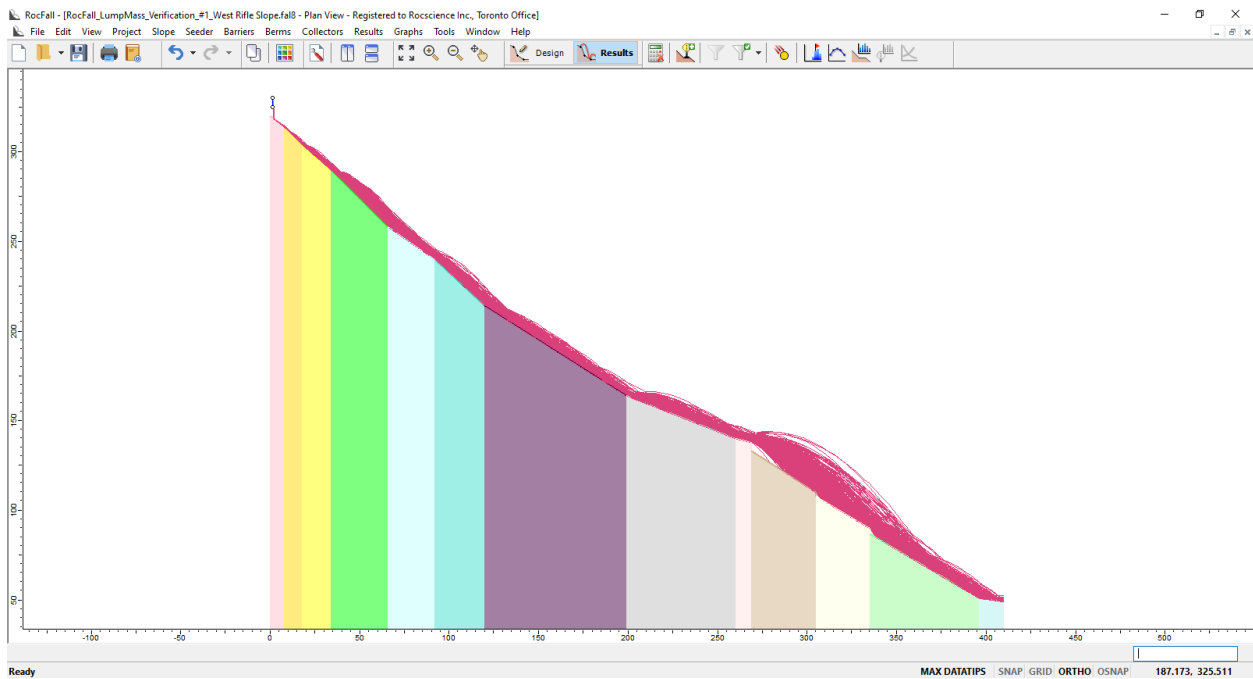


Figure 2.1-2: RocFall2 Rock Trajectory Model Results



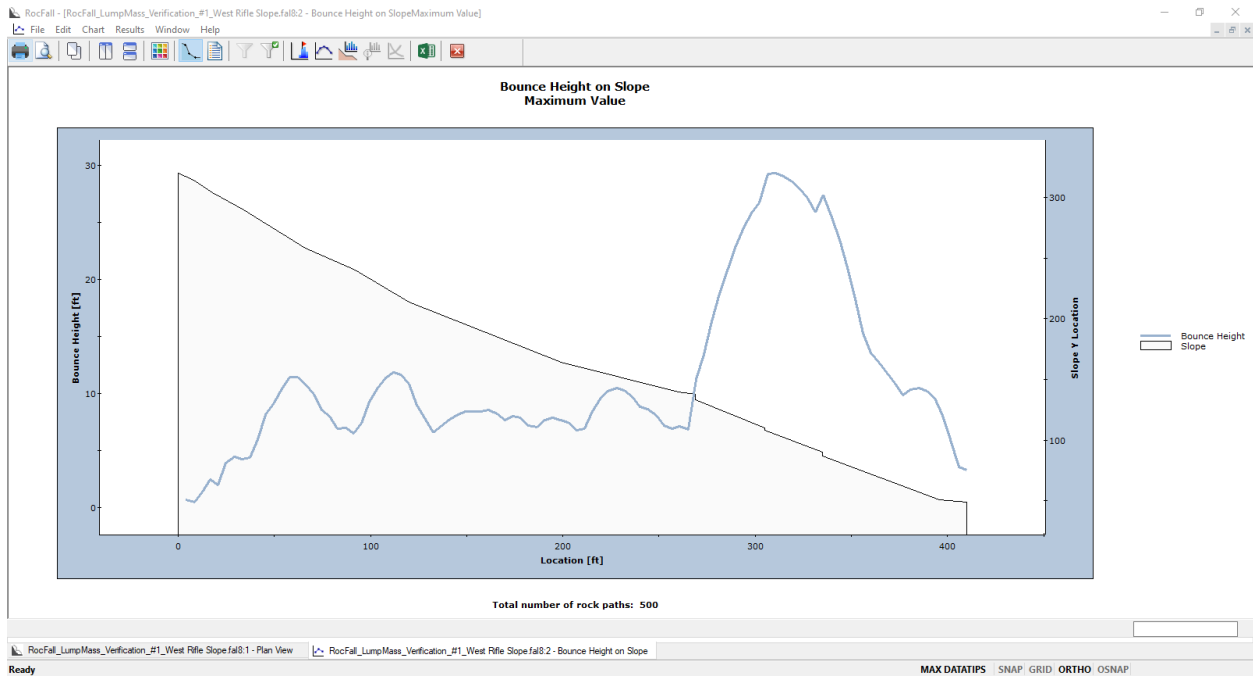


Figure 2.1-3: RocFall2 Maximum Bounce Height Results

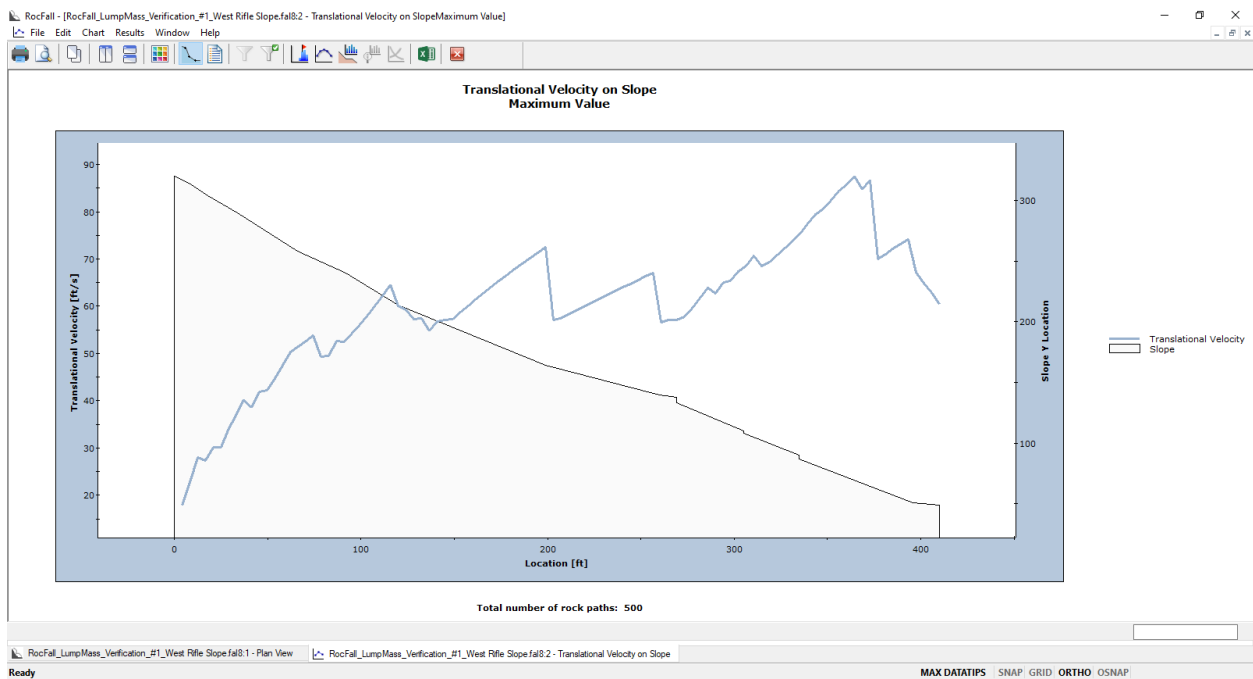


Figure 2.1-4: RocFall2 Maximum Translational Velocity Results

### 2.1.3. Building a Compatible CRSP Model

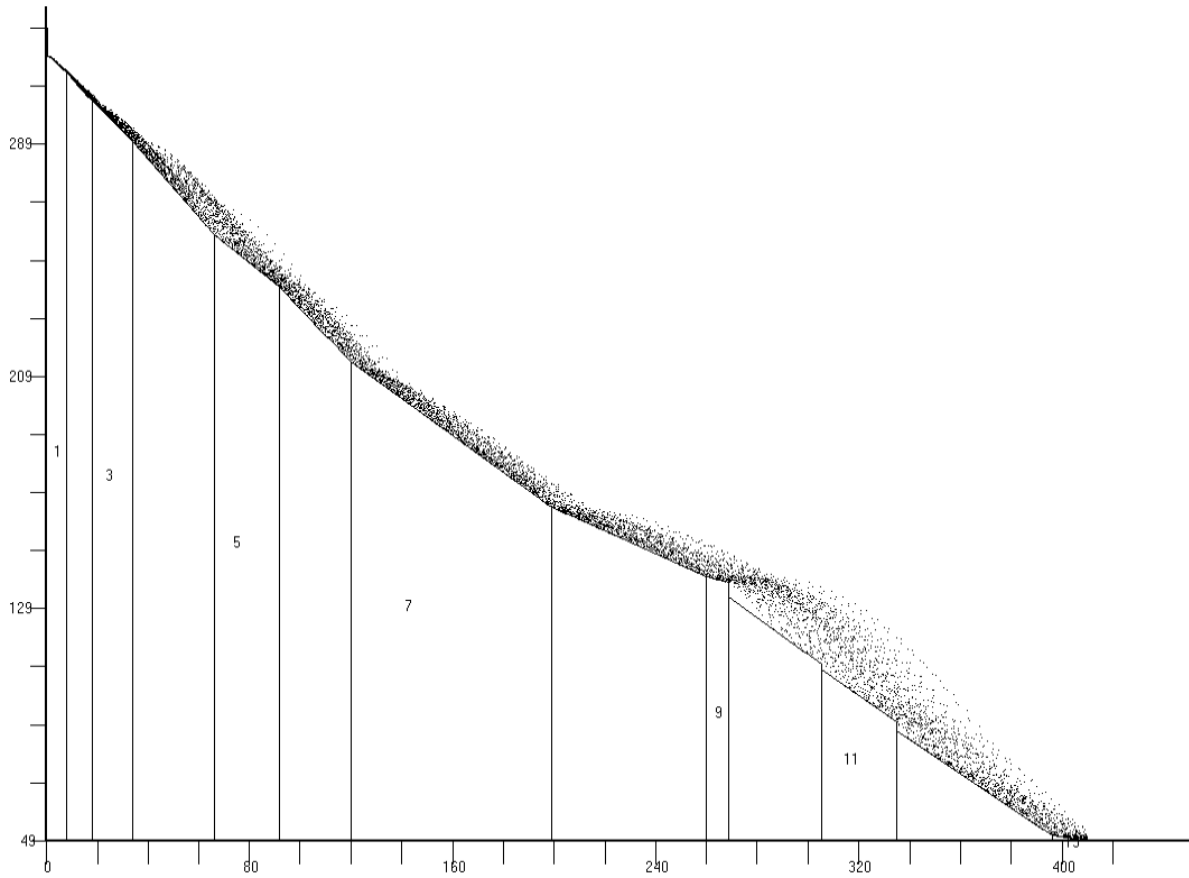


Figure 2.1-5: CRSP Rock Trajectory Model Results

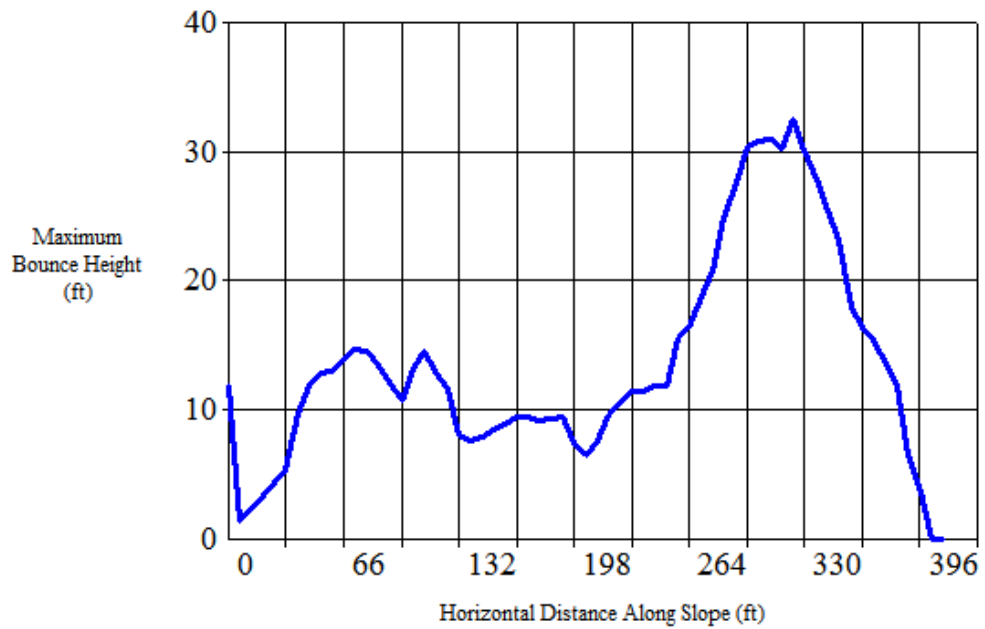


Figure 2.1-6: CRSP Maximum Bounce Height Results

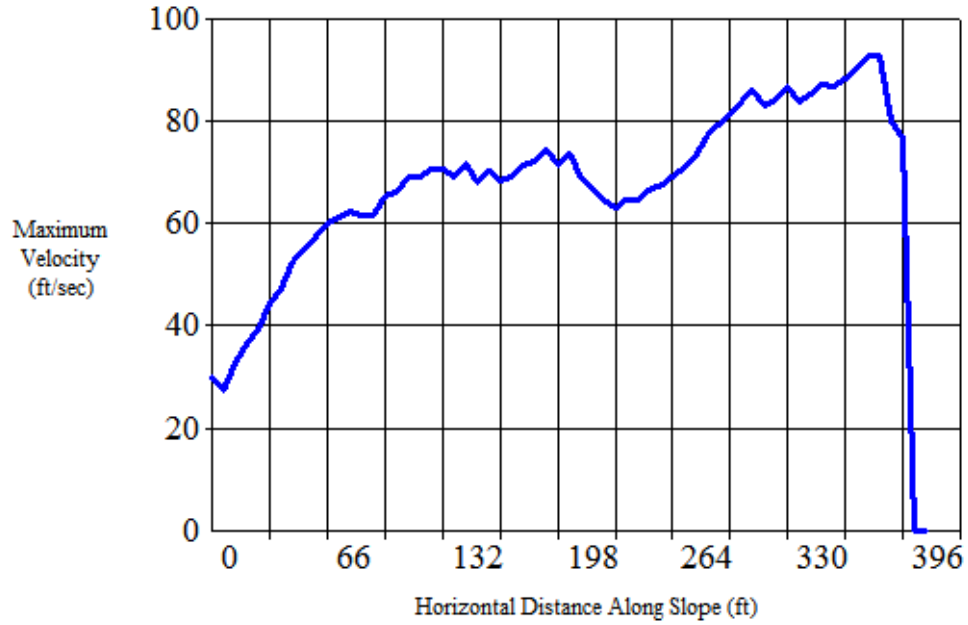


Figure 2.1-7: CRSP Maximum Translational Velocity Results

#### 2.1.4. Results

The maximum bounce height and translational velocity are presented in Figure 2.1-3, Figure 2.1-6, Figure 2.1-4, and Figure 2.1-7. The results obtained from *RocFall2* compare well with the *CRSP* model.

#### 2.1.5. Input Files

*RocFall\_LumpMass\_Verification\_#1\_West Rifle Slope.fal8*

## 2.2. RocFall2 Verification Problem #2 – Glenwood Canyon

[RocFall2 Build 8.009]

### 2.2.1. Problem Description

This exercise verifies the average bounce height and translational velocity using the results from the *Colorado Rockfall Simulation Program (CRSP)* [1]. *CRSP* calibrated its simulation using data collected from quartzite cliffs 750 ft above Interstate 70 (I-70) in Glenwood Canyon, Colorado. The study was for the design of rockfall mitigation after rockfall incidents damaged two retaining walls under construction. The upper third of the slope is granitic bedrock with sparse vegetation and a thin soil cover, while the other two thirds above I-70 is talus cover with scattered shrubs. 100 disk shaped rocks were used in the *CRSP* example and 5000 spherical rocks are used in the *RocFall2* analysis using the Lump Mass formulation.

### 2.2.2. RocFall2 Analysis

In the original *CRSP* example, a disk-shaped rock with radius of 2.2 ft is used. In *RocFall2*, spheres are used. The only difference is the value of the moment of inertia ( $I$ ) when calculating angular velocities.

For a disk:  $I = 0.5mr^2$  ;

For a sphere:  $I = 0.4 \times mr^2$

Where:

$I$  is the moment of inertia  
 $m$  is the mass  
 $r$  is the radius

For a 2.2 ft radius disk,  $I = 9102 \text{ lb-ft}^2$ . We will keep mass the same but enter an equivalent radius, so we have the same moment of inertia. The equivalent radius is  $r_{equivalent} = 2.46 \text{ ft}$ . To have a sphere with a radius  $r = 2.46 \text{ ft}$  and mass  $m = 3761 \text{ lb}$ , we need an adjusted density of  $60.34 \text{ lb/ft}^3$ . These are modifications required to have the same mass and moment of inertia in order to simulate the same behavior as that of a disk as defined in the *CRSP* example.

Surface roughness is calculated with the following equation using the original radius of 2.2 ft:

$$\theta_{max} = \tan^{-1} \left( \frac{S}{r} \right)$$

Where:

$S$  is the slope roughness  
 $r$  is the rock radius

In the *RocFall2* **Project Settings** dialog, make sure **Consider angular velocity**. Under **Scaling Functions** pane, make sure the **Scale Rn by Velocity** checkbox is selected, the  $K$  factor is set to 30 ft/s for US imperial units or 9.144 m/s for metric units, and **Scale Rn by Mass** checkbox is unselected.

## Slope Geometry and Material Properties

The location of the slope vertices and material parameters for all slope segments are presented in the following tables:

Table 2.2-1: Slope Geometry and Materials

	X	Y	Normal Restitution $R_N$	Tangential Restitution $R_T$	Friction Angle $\phi$ (deg)	Surface Roughness Std. Dev. (deg)
Vertex 1	0	794				
Segment 1			0.35	0.85	1	17.14
Vertex 2	224	620				
Segment 2			0.35	0.85	1	19.64
Vertex 3	248	600				
Segment 3			0.35	0.85	1	24.33
Vertex 4	306	530				
Segment 4			0.32	0.81	1	12.22
Vertex 5	385	480				
Segment 5			0.32	0.81	1	12.22
Vertex 6	500	390				
Segment 6			0.32	0.81	1	14.31
Vertex 7	557	360				
Segment 7			0.31	0.8	1	8.83
Vertex 8	848	157				
Segment 8			0.31	0.8	1	7.63
Vertex 9	925	110				
Segment 9			0.31	0.82	1	12.22
Vertex 10	933	95				
Segment 10			0.32	0.8	1	6.4
Vertex 11	968	78				
Segment 11			0.4	0.9	1	1.3
Vertex 12	1002	60				
Segment 12			0.32	0.8	1	12.22
Vertex 13	1069	25				
Segment 13			0.32	0.82	1	2.6
Vertex 14	1075	27				
Segment 14			0.4	0.9	1	1.3
Vertex 15	1104	27				
Segment 15			0.32	0.82	1	12.22
Vertex 16	1153	4				

### Initial Conditions

The rock starts at location  $X_0 = 0$  ft,  $Y_0 = 800$  to  $810$  ft, which is added as a **Line Seeder** in *RocFall2*. The rock was given an initial velocity of  $V_{X0} = 1$  ft/s,  $V_{Y0} = -1$  ft/s. The rock has a radius of 2.46 ft (equivalent), mass of 3761 lb and density of 60.34 lb/ft<sup>3</sup>.

Enter the seeder and slope geometry values from Table 2.2-1 into *RocFall2*.

The *RocFall2* model looks like this:

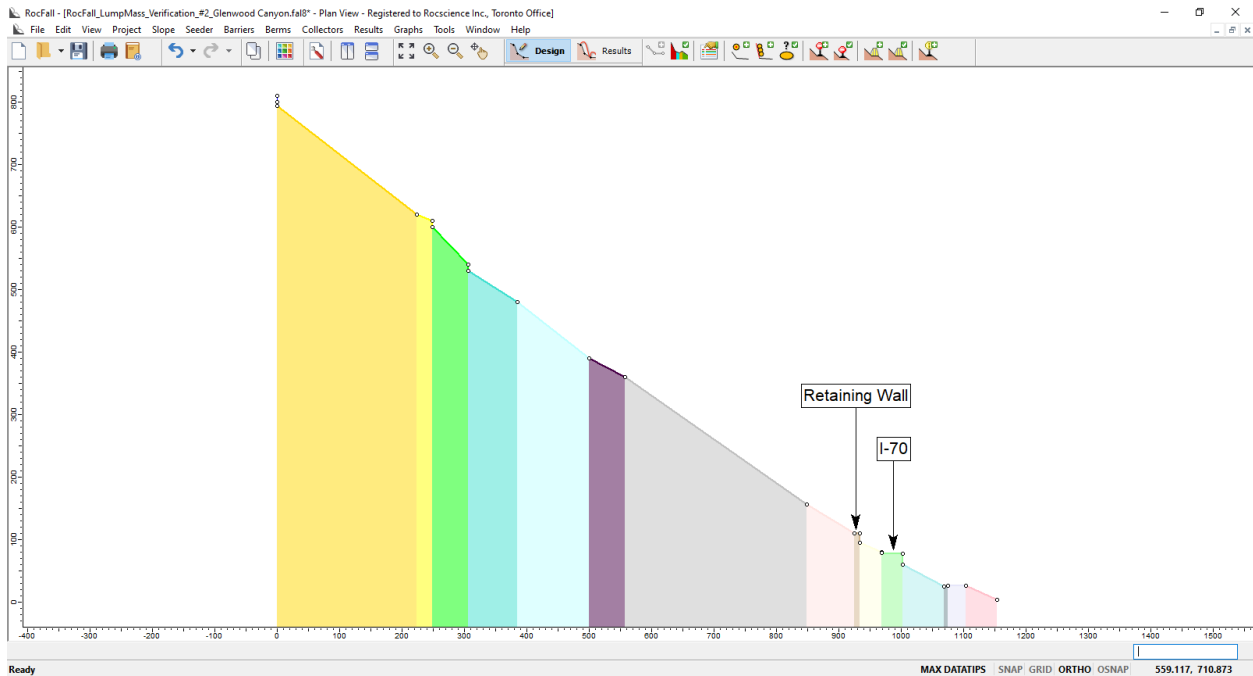


Figure 2.2-1: RocFall2 Model Geometry

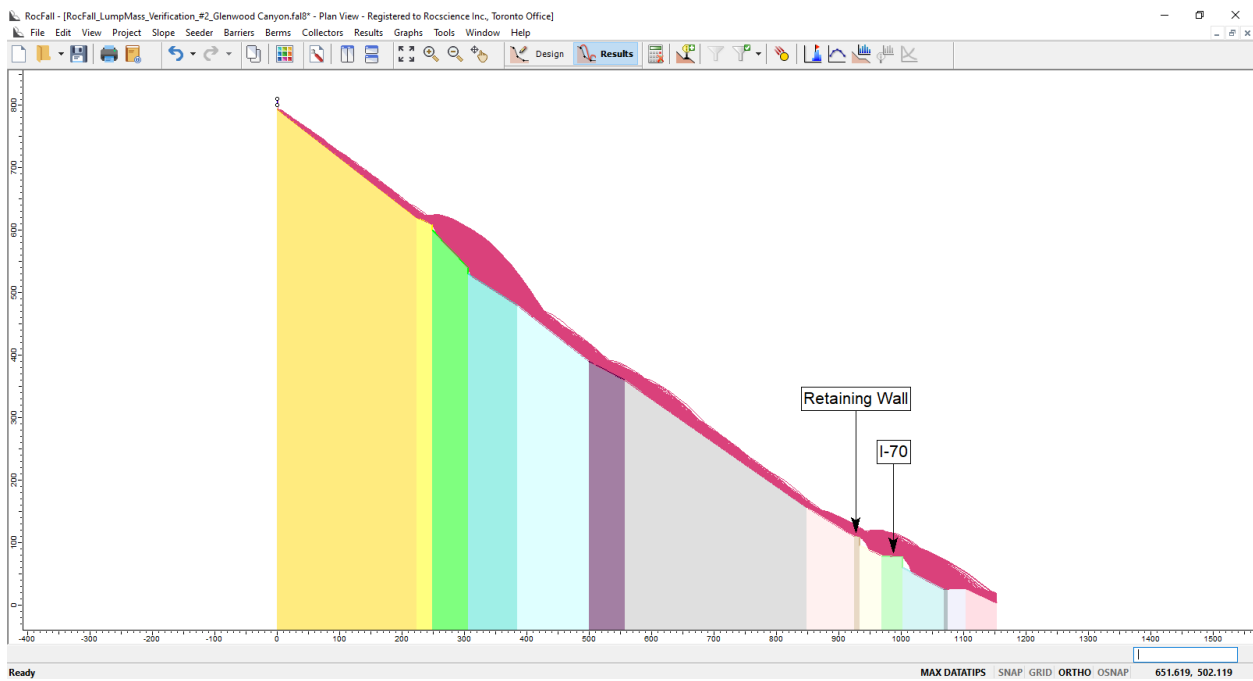


Figure 2.2-2: RocFall2 Rock Trajectory Model Results

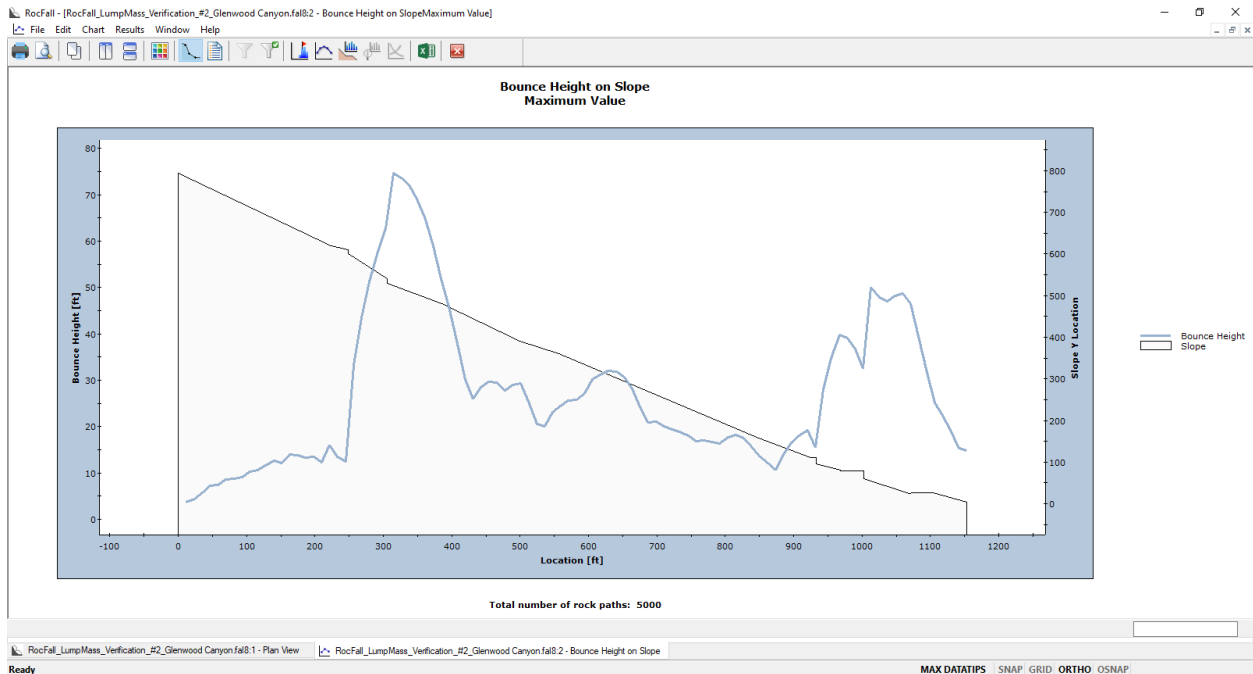


Figure 2.2-3: RocFall2 Maximum Bounce Height Results

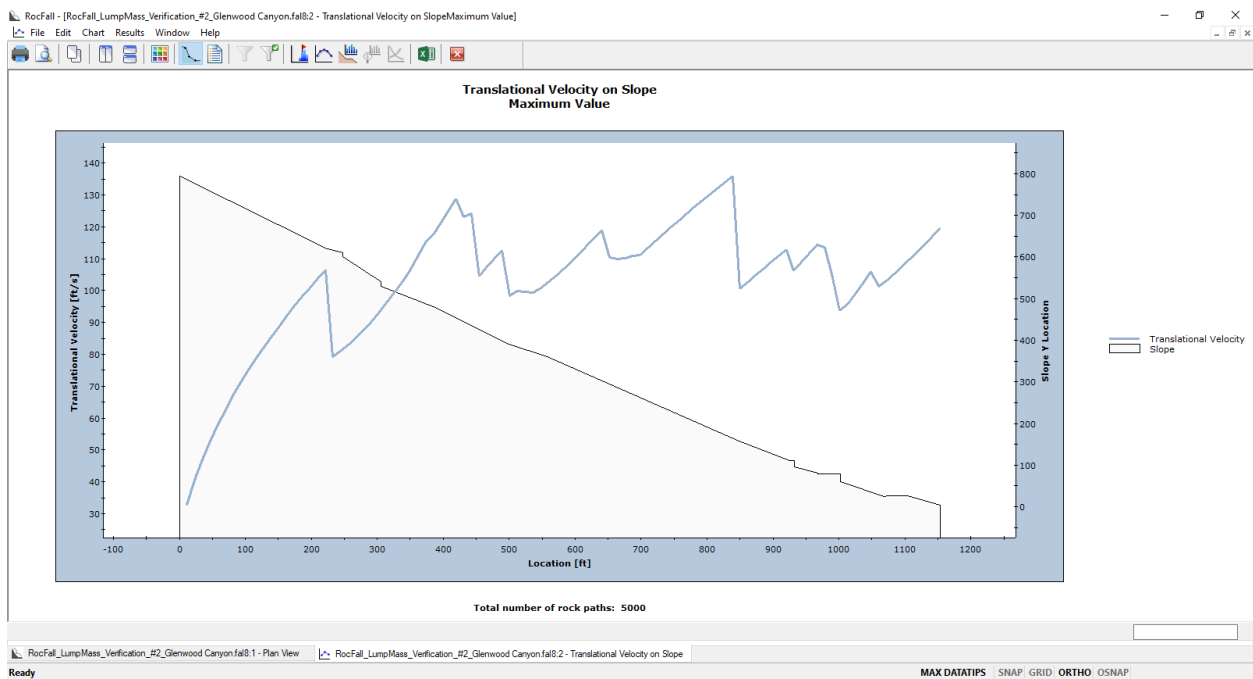


Figure 2.2-4: RocFall2 Maximum Translational Velocity Results

### 2.2.3. Building a Compatible CRSP Model

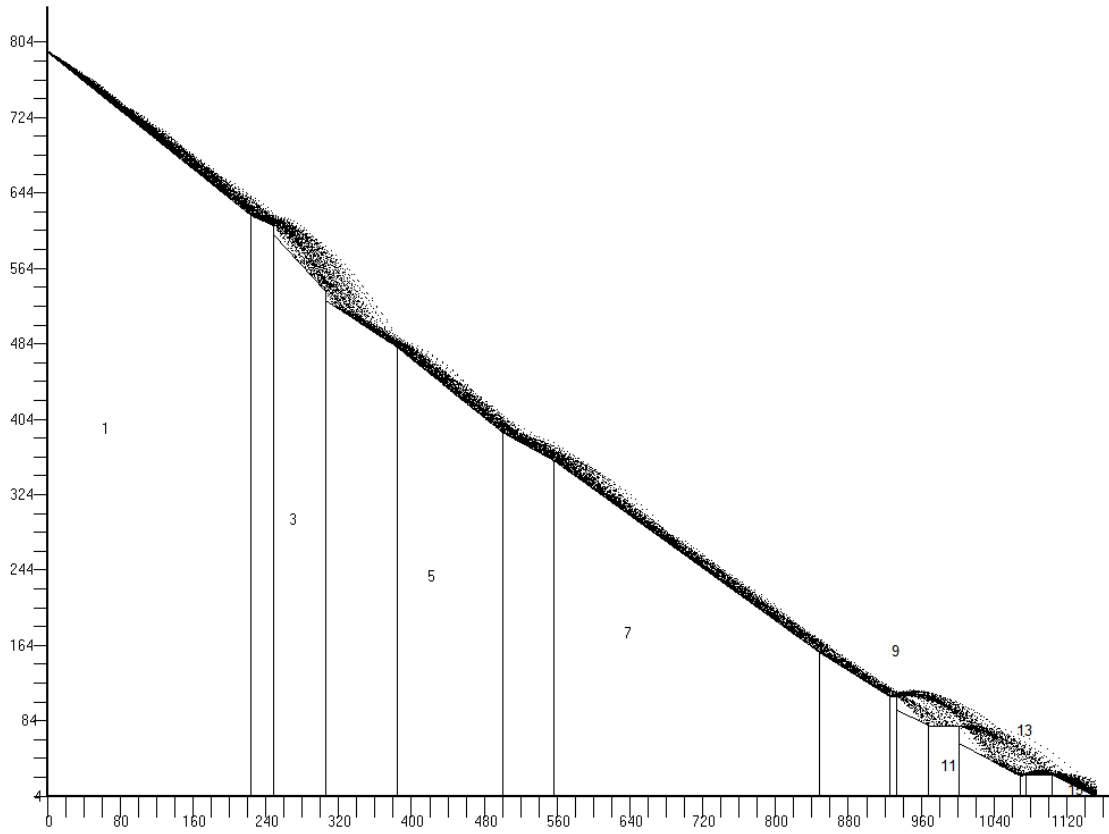


Figure 2.2-5: CRSP Rock Trajectory Model Results

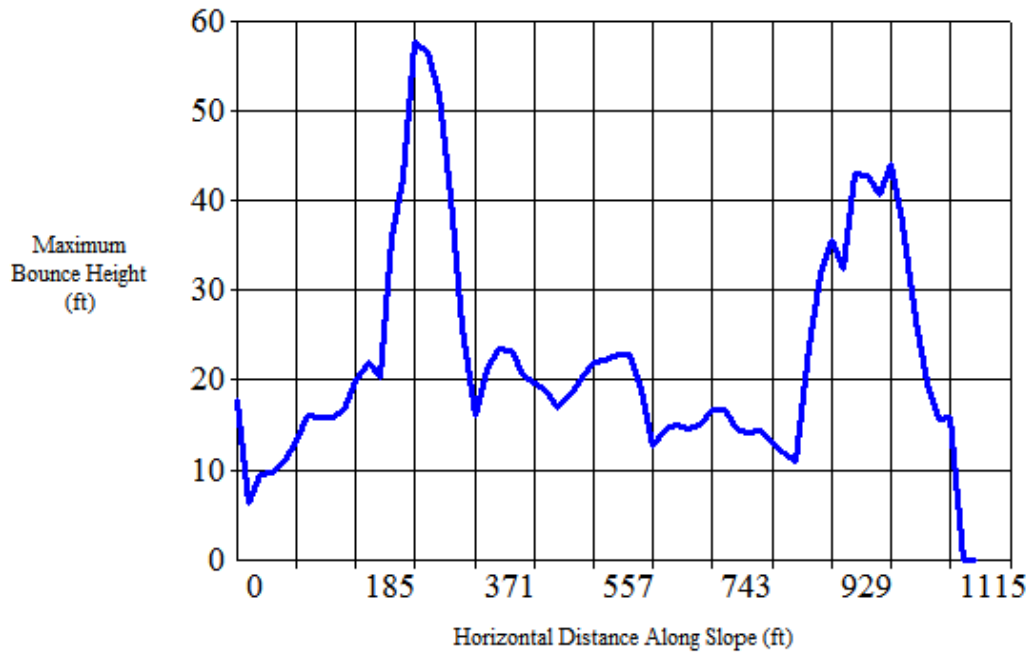


Figure 2.2-6: CRSP Maximum Bounce Height Results



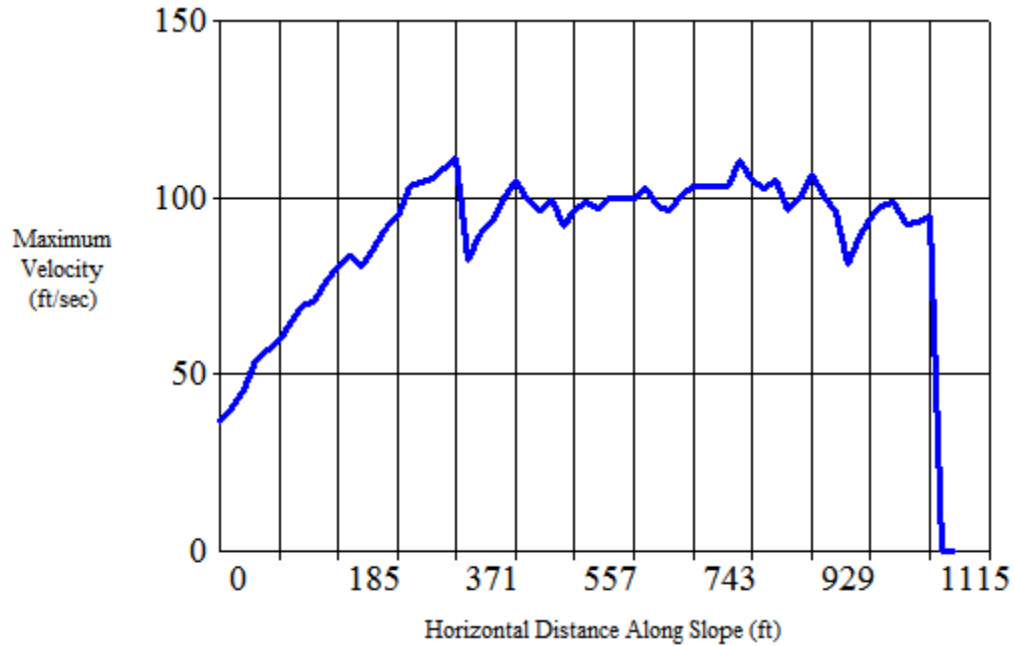


Figure 2.2-7: CRSP Maximum Translational Velocity Results

## 2.2.4. Results

The maximum bounce height and translational velocity are presented in Figure 2.2-3, Figure 2.2-4, Figure 2.2-6, and Figure 2.2-7. The results obtained from *RocFall2* compare well with the *CRSP* model.

## 2.2.5. Input Files

*RocFall\_LumpMass\_Verification\_#2\_Glenwood Canyon.fal8*

## 3. References

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1. Pfeiffer, T.J. (1989). Rockfall Hazard Analysis Using Computer Simulation of Rockfalls. (Doctoral Dissertation). Colorado School of Mines, Golden, Colorado.