Rock Shape and Size

If you are using the Rigid Body analysis method in *RocFall*, which requires Rock Shapes to be defined, the size of the rocks is determined from:

- the rock mass (m)
- rock density (ρ)
- rock shape

as described below. The mass moment of inertia for each shape is also given.

Sphere

For a sphere, the volume is given by:

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

From which we get the sphere radius:

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3}$$

Mass moment of Inertia:

$$I_m = \frac{2}{5}mr^2$$

2D Rock Shapes

For rock shapes other than the sphere (e.g. Circle, Square, Rhombus, Ellipse, Super-Ellipse, Polygon), the shape is generated in 2-dimensions, and then extruded in the out-of-plane dimension, in order to obtain a 3D (extruded/prismatic) rock shape of the specified mass.

The out-of-plane dimension of these rock shapes is calculated such that it is equal (or approximately equal) to the in-plane (2D) dimensions. In order to compute the exact dimensions of a rock shape, an assumption has to be made regarding the relationship between the out-of-plane and in-plane shape dimensions. These assumptions are described below, for the various rock shapes available in *RocFall*.

Circle

A Circular rock is a cylinder in 3-dimensions. For a Circle with:

- rock mass (m)
- rock density (ρ)
- radius (r)
- out-of-plane (depth) dimension (d)

The cylinder volume is given by:

$$\pi r^2 d = \frac{m}{\rho}$$

If we make the **assumption** that d = 2r, then:

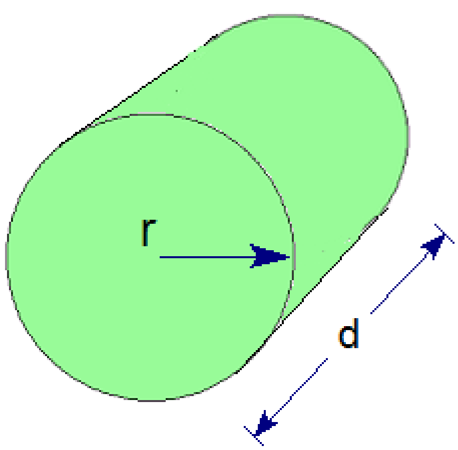
$$2\pi r^3 = \frac{m}{\rho}$$

The circle radius is:

$$r = \left(\frac{m}{2\pi\rho}\right)^{1/3}$$

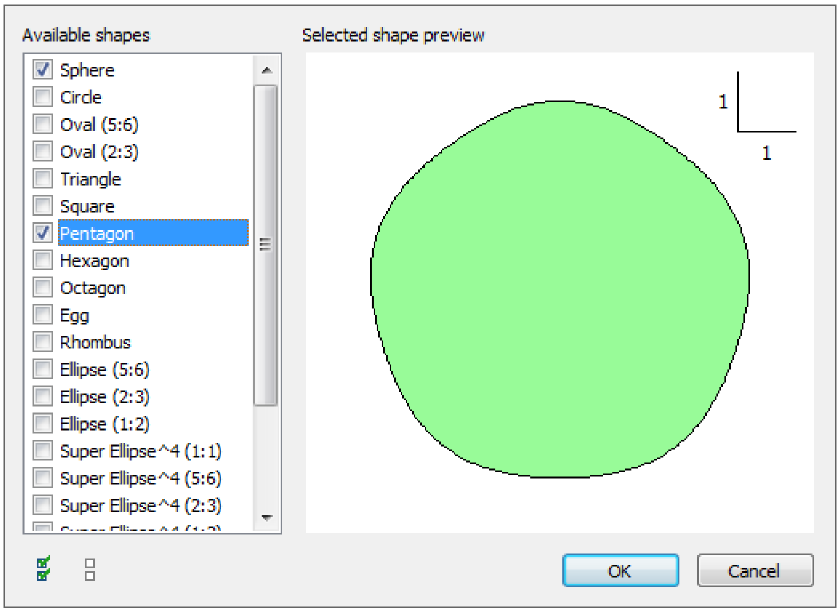
Mass moment of inertia:

$$I_m = \frac{1}{2} m r^2$$



Fourier Shapes

The Fourier shapes in *RocFall* include the following: Oval, Triangle, Square, Pentagon, Hexagon, Octagon, Egg, Rhombus. They are so called because Fourier analysis is used to generate the shapes. As you can see from the shape preview window, all of these shapes have rounded corners, so that the shapes are smooth, without sharp corners.



The size of rocks based on Fourier shapes, uses:

- rock mass (m)
- rock density (ρ)
- equivalent radius (R)
- out-of-plane (depth) dimension (d)
- rock shape parameters

The exact dimensions of a Fourier shape rock are calculated as follows. For the Fourier shapes, we first define an equivalent radius R, such that:

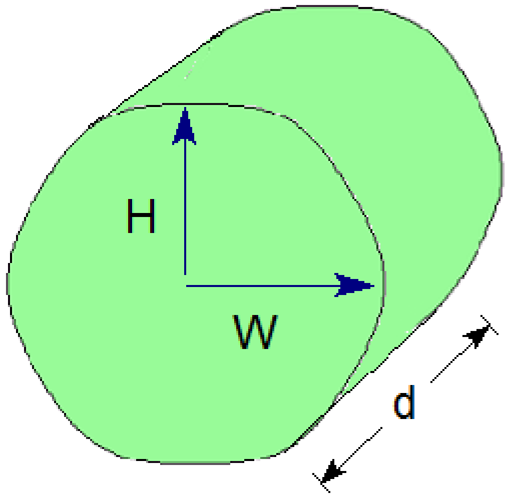
$$A_F = \pi R^2$$

Where Af = area of the Fourier shape (in 2D) and R = equivalent radius (i.e. radius of a circle with equivalent area).

The volume of a Fourier shape rock is then:

$$A_F d = \pi R^2 d = \frac{m}{\rho}$$

Where d = out-of-plane (depth) dimension.



Now we make the same assumption as we did for the circular shape, except we use the equivalent radius R. We assume that:

d = 2R

Where d = out-of-plane (depth) dimension of the Fourier shape and R = equivalent radius. The equivalent radius R can then be expressed as:

$$R = \left(\frac{m}{2\pi\rho f}\right)^{1/3}$$

The variable f in the above equation, is a Fourier shape factor which is constant for each shape, and is given in the table below.

For each Fourier shape, we define an effective aspect ratio S:

$$S = W/H$$

where H and W are shown in the above figure for a Hexagonal shape. The value of S is constant for each shape. Finally, the shape aspect ratio parameters W and H are given by:

$$W = \beta R$$

$$H = W/S$$

The above equations are used to determine the exact final dimensions of each shape, which give the specified rock mass.

Values of S, f, Beta and i are given in the following table.

Shape	S	Factor f	Beta	i
Fat Oval	1.2	1.05	1.1	1.02491294
Oval	1.5	1.02	1.2	1.09862745
Triangle	1.1	1.0018	1.06	1.00898868
Square	1	1.0003125	1.025	1.00156216
Pentagon	1	1.0008	1.02	1.00399776
Hexagon	1	1.0002	1.02	1.00099986
Octagon	1	1.00005	1.01	1.00024999
Egg	1.14	1.0058	1.14	1.02889487
Rhombus	1.15	1.000625	1.15	1.03262345

Mass moment of inertia:

$$I_m = \frac{1}{2} m i R^2$$

Where i is listed in the table for each shape.

Ellipse Shapes

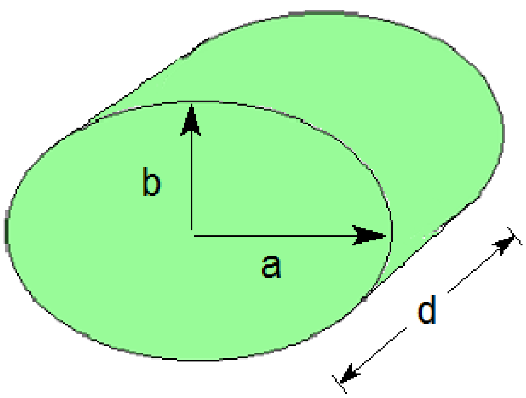
The size of rocks based on Ellipse or Super-Ellipse shapes, uses:

- rock mass (m)
- rock density (ρ)
- out-of-plane (depth) dimension (d)
- ellipse axes (a,b)

The following general equation defines an ellipse in the XY plane:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1$$

Where a and b are the major and minor axes of the ellipse, and n is the order of the ellipse (n=2 gives a standard ellipse, n = 4 or 6 define super-ellipses).



The ratio S = a/b defines the ratio of the major and minor axes of the ellipse.

$$S = a/b$$

The area of an ellipse is given by:

$$A = fab$$

Where f is a factor which is a function of n. For a standard ellipse f = π. Values of f, S and i are summarized in the tables below for different values of n and shape.

n	f	i
2	3.14159265 (π)	0.25
4	3.70814935	0.299535
6	3.85524259	0.314980

Ellipse Shape	S
1:1	1
5:6	1.2
2:3	1.5
1:2	2

The volume of an ellipse with out-of-plane dimension d is given by:

$$Ad = fabd = \frac{m}{\rho}$$

Now we make a similar assumption as we did for the other shapes, except we use the minor ellipse axis b. We assume that:

d = 2b

Where d = out-of-plane (depth) dimension of the ellipse. Given that:

b = a/S

We can derive an expression for major axis a:

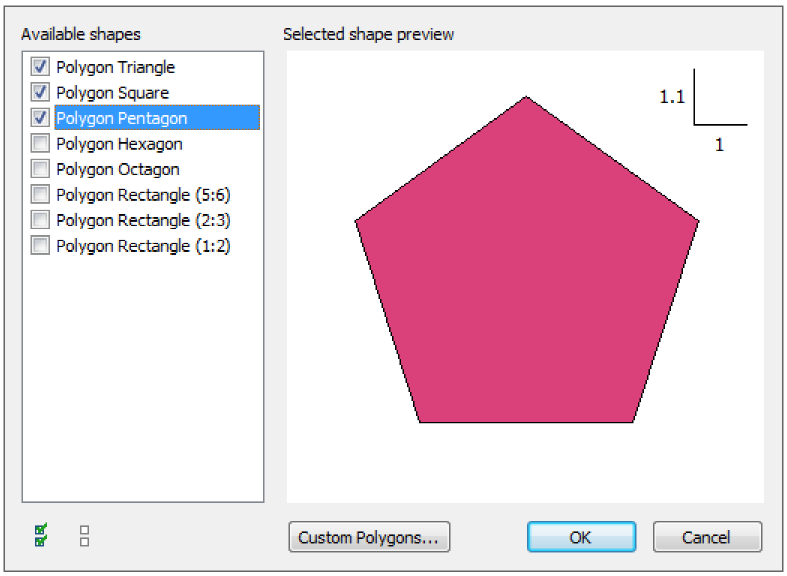
$$a = S\left(\frac{m}{2S\rho f}\right)^{1/3}$$

Mass moment of inertia:

$$I_m = im(a^2 + b^2)$$

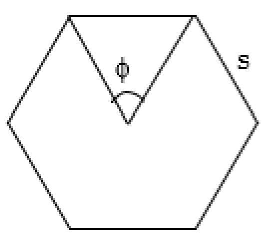
Polygonal Shapes

Regular polygons in *RocFall* include the following: Triangle, Square, Pentagon, Hexagon, Octagon, Rectangle (5:6), Rectangle (2:3) and Rectangle (1:2). As you can see from the shape preview window, all of these shapes have sharp corners. In *RocFall* 6.0, smooth or polygonal shapes are selected in separate dialogs. However, they can still be combined within the same analysis.



Triangle, Square, Pentagon, Hexagon and Octagon consist of n equal sides. Unit size shapes (side length, s = 1.0) are built and scaled later depending on the rock mass (m) and density (ρ) defined in the “Rock Type Library” dialog.

A regular hexagon with 6 sides:



Area of a unit n-sided regular polygon is:

$A = 1/4 * n / \tan(\pi/n)$

Same as Fourier Shapes, we define an equivalent radius R such that:

$A = \pi R^2$

Where A is the area of the polygon calculated earlier and R is the equivalent radius. We then make the same assumption as we did for the circular shape and Fourier shapes that:

$d = 2R$

Where d is the out-of-plane (depth) dimension of the regular polygon. The volume (V) of the polygonal rock is then:

$V = A * d = 1/4 * n * s^3 * \text{sqrt}(n/\pi) * \tan^{-1.5}(\pi/n) = m/\rho$

For a unit polygon s = 1.0. From the above equation, we can calculate the actual s based on the rock mass and density, which is also equivalent to the scaling factor.

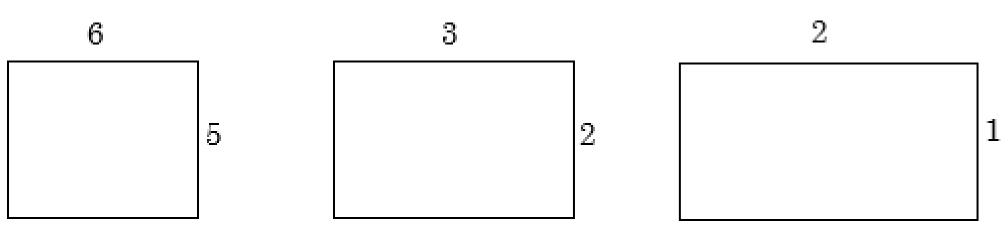
Mass moment of Inertia (I) of the regular polygon is:

$I = m * s^2 /24 * (1 + 3 / \tan^2(\pi/n))$

With known preset values of n, we can develop the following table:

	n	Area (A)	Depth (D)	Volume (V)	MOI (I)
triangle	3	0.433013 s^2	0.742515 s	0.321519 s^3	0.083333 m* s^2
square	4	1.0 s^2	1.128379 s	1.128379 s^3	0.166667 m* s^2
pentagon	5	1.720477 s^2	1.480061 s	2.546411 s^3	0.27847 m* s^2
Hexagon	6	2.598076 s^2	1.818783 s	4.725338 s^3	0.416667 m* s^2
Octagon	8	4.828427 s^2	2.479465 s	11.97191 s^3	0.77022 m* s^2

RocFall 6.0 has **Rectangles** with 3 predefined height to width ratios: 5:6, 2:3 and 1:2.



Unit shapes are built (with shorter side length = 1) and scaled later depending on the rock mass (m) and density (ρ) defined in the “Rock Type Library” dialog. A depth length equal to the shorter edge (height) is used for rectangles. Let h be shorter edge length and w be the longer edge (width) length, areas (A), volume (V) and moment of inertia (I) for rectangles are:

$A = w * h$

$V = w^2 * h = m/\rho$

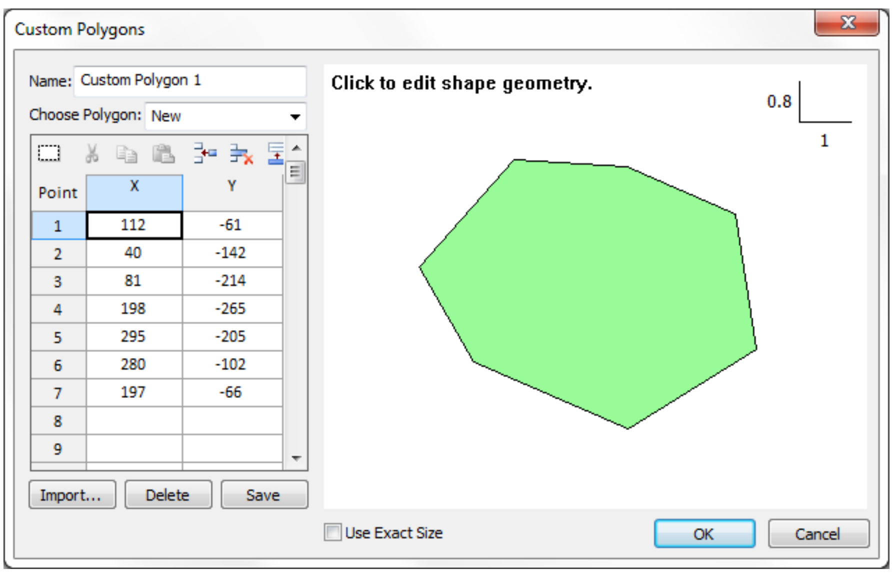
$I = m/12 * (w^2 + h^2)$

With known preset values of ratios, we can develop the following table:

	area (A)	volume (V)	MOI (I)
rectangle (5:6)	1.2 s^2	1.2 s^3	0.203333 m* s^2
rectangle (2:3)	1.5 s^2	1.5 s^3	0.270833 m* s^2
rectangle (1:2)	2.0 s^2	2.0 s^3	0.416667 m* s^2

Where s is the scaling factor calculated based on the input rock mass and density.

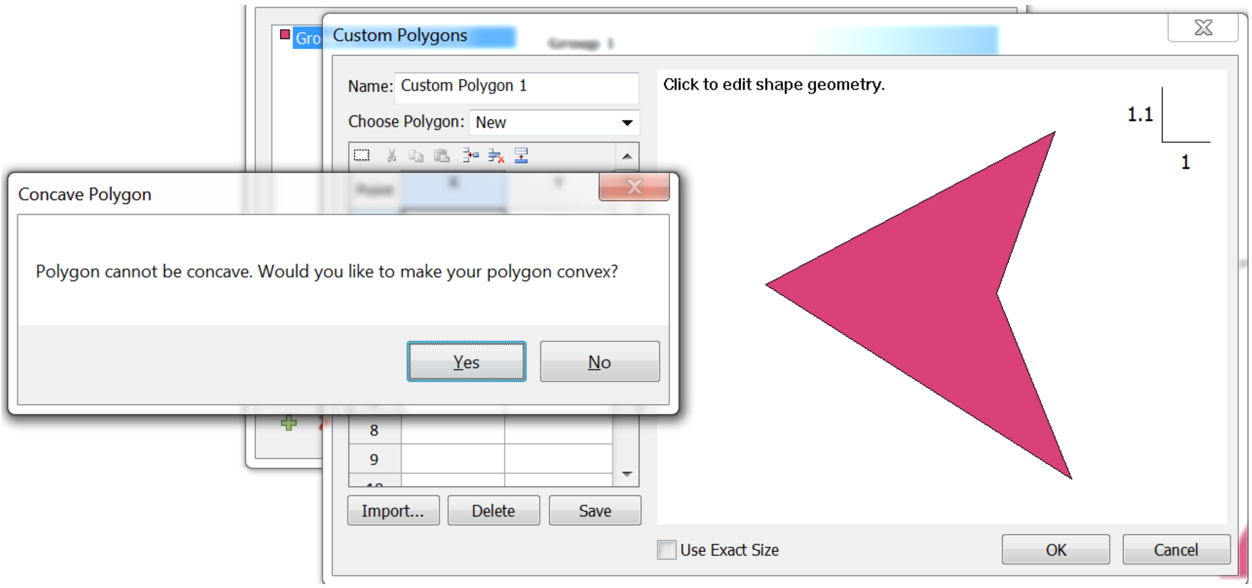
RocFall 6.0 allows users to create custom polygons. Rock shapes can be drawn graphically, entered in a coordinate table, or imported from a DXF or text file. Import from DXF file allows actual rock shapes obtained from laser scanning to be imported as a 2D profile. Custom rock shapes can be automatically scaled or defined as an exact size and mass by selecting the **Use Exact Size** checkbox.



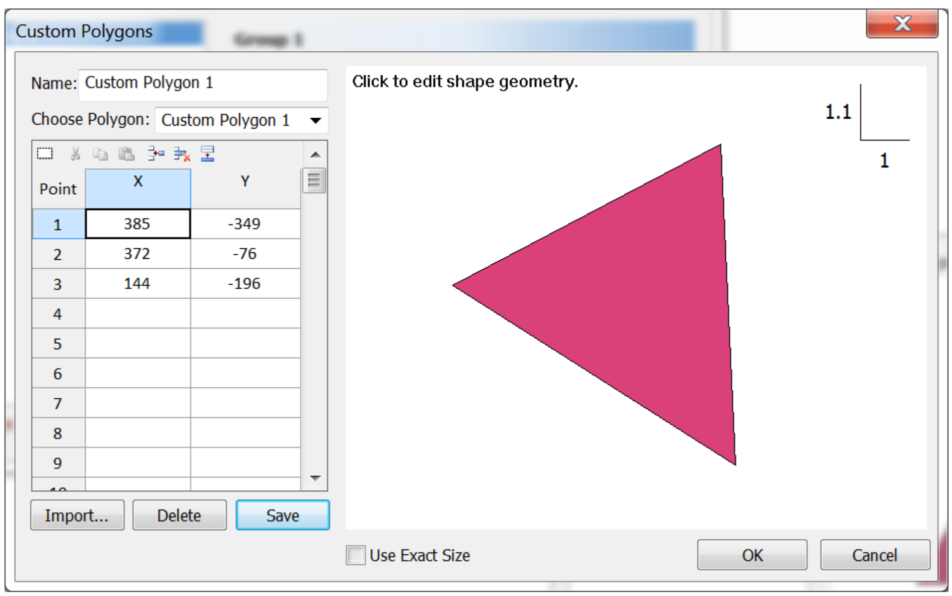
There are a couple restrictions on the types of custom polygons:

1. The polygon must have an area greater than 0.
2. The polygon must be convex.
3. The polygon must have 3 or more vertices.

The program automatically checks the input shape upon clicking "Save" in the "Custom Polygons" dialog. Warning messages are given if any of the above 3 criteria is triggered. Here is an example of what happens when a concave shape is entered.



Upon clicking "Yes", the closest approximated convex shape is generated automatically.



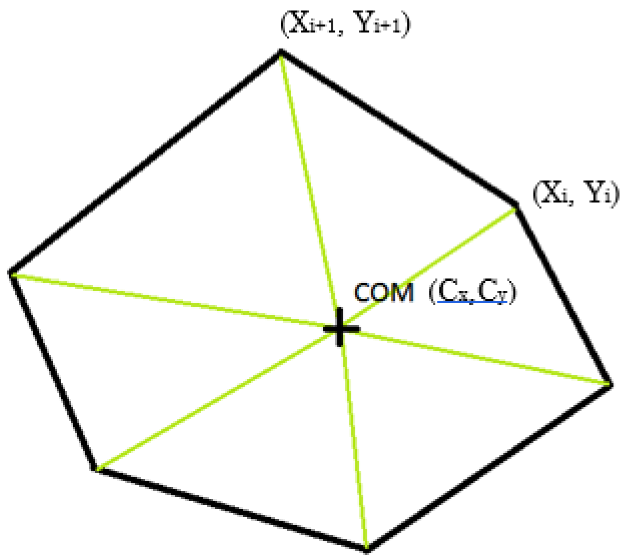
Area (A) of any polygons can be calculated as:

$$A = \frac{1}{2}\sum_{i=0}^{N-1}(X_i \times Y_{i+1} - X_{i+1} \times Y_i)$$

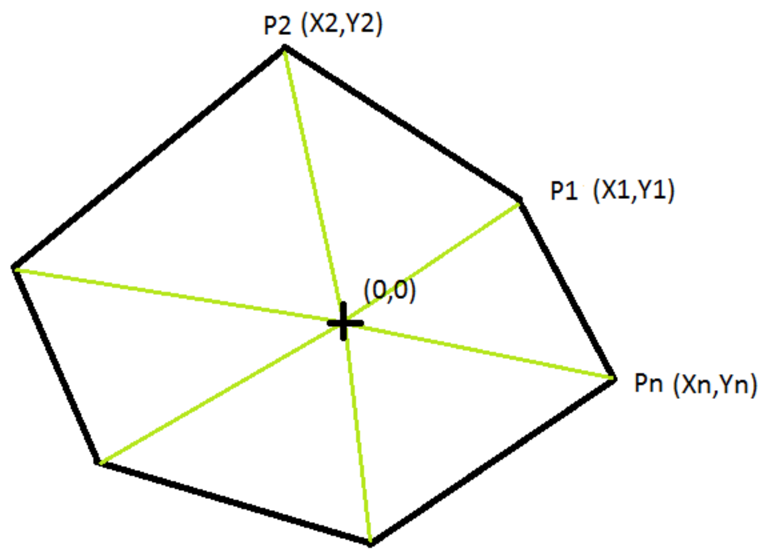
Then we can find the center of mass (COM):

$$C_x = \frac{1}{6A}\sum_{i=1}^{N}(X_i + X_{i+1})(X_i \times Y_{i+1} - X_{i+1} \times Y_i)$$

$C_y = \frac{1}{6A}\sum_{i=1}^{N}(Y_i + Y_{i+1})(X_i \times Y_{i+1} - X_{i+1} \times Y_i)$



The Polygon is then moved so the COM is at (0,0).



If “Use Exact Size” box is checked, a unit depth of 1.0 is used. If not, the depth (d) of the custom polygon is calculated similar to regular polygons.

$A = \pi R^2$

$d = 2R$

The volume (V) of the polygonal rock is then:

$V = m/\rho = A * d * s^3$ if “Use Exact Size” box is not checked. All 3 dimensions are scaled.

$V = m/\rho = A * d * s$ if “Use Exact Size” box is checked. Only the depth is scaled.

Where s is the scaling factor.

Lastly, the moment of inertia (I) for a custom polygon is calculated (in vector formula):

$$I = \frac{m}{6}\left(\frac{\sum_{n=1}^{N} \|\mathbf{P}_{n+1} \times \mathbf{P}_n\| \left((\mathbf{P}_n \cdot \mathbf{P}_n) + (\mathbf{P}_n \cdot \mathbf{P}_{n+1}) + (\mathbf{P}_{n+1} \cdot \mathbf{P}_{n+1})\right)}{\sum_{n=1}^{N} \|\mathbf{P}_{n+1} \times \mathbf{P}_n\|}\right)$$