ROCFALL:
A TOOL FOR PROBABILISTIC ANALYSIS, DESIGN OF REMEDIAL MEASURES AND PREDICTION OF ROCKFALLS

by

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Abstract

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Accurate prediction of rockfalls is *practically* impossible. Variability in slope geometry, poorly defined initial conditions, uncertain material properties (especially coefficients of restitution) and an analysis method that is sensitive to minor changes in these parameters are contributing factors that make accurate prediction extremely difficult. Performing probabilistic simulation and statistical analyses has proven to be an effective and acceptable method for overcoming these difficulties and thereby enabling the production of rational engineering designs.

The computer program RocFall is a tool to assist engineers with probabilistic simulation of rockfalls and the design of remedial measures. This thesis details the difficulties with rockfall analyses and explains how RocFall can be used to overcome these difficulties. This thesis presents the equations and the algorithm used by the program to simulate the rockfalls. Essential to the use of a computer program in engineering practice, this thesis also presents a thorough verification of the program's output.

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Δh	= height of the rock above the slope
ϕ	= friction angle of the line segment
μ	= mean of a normal distribution
θ	= slope of the line segment
σ	= standard deviation of a normal distribution
a	= coefficient of the quadratic term in the quadratic equation
b	= coefficient of the linear term in the quadratic equation
c	= constant term in the quadratic equation
g	= acceleration due to gravity, sign is negative in all equations
H_S	= height of the slope
k	= friction & slope angle coefficient in the sliding algorithm
KE_A	= kinetic energy of the rock immediately after impact
KE_B	= kinetic energy of the rock immediately before impact
KE_{PEAK}	= kinetic energy of the rock at the peak of the trajectory
m	= additive constant coefficient for a random variable
n	= multiplicative constant coefficient for a random variable
q	= tangent of the, slope of the line segment in the sliding algorithm
R_N	= coefficient of normal restitution
R_T	= coefficient of tangential restitution
s	= distance the rock slides on the slope in the sliding algorithm
s_D	= distance from the initial position of the rock to the end of the line segment
Si	= first in a pair of successive samples from a uniform distribution
Sj	= second in a pair of successive samples from a uniform distribution
t	= parameter in the parametric form of the parabola equations

List of Symbols

V_A = velocity of the rock immediately after impact
V_B = velocity of the rock immediately before impact
V_{CHECK} = velocity of the rock between steps in the projectile algorithm
V_{EXIT} = velocity of the rock at the end of the line segment in the sliding algorithm
V_{MIN} = minimum velocity still considered to be moving
V_{NA} = velocity of the rock immediately after impact, normal to the line
V_{NB} = velocity of the rock immediately before impact, normal to the line
V_{PEAK} = velocity of the rock at the peak of the parabolic path
V_{TA} = velocity of the rock immediately after impact, tangential to the line
V_{TB} = velocity of the rock immediately before impact, tangential to the line
V_{X0} = initial horizontal velocity of the rock
V_{XA} = horizontal velocity of the rock immediately after impact
V_{XB} = horizontal velocity of the rock immediately before impact
V_{Y0} = initial vertical velocity of the rock
V_{YA} = vertical velocity of the rock immediately after impact
V_{YB} = vertical velocity of the rock immediately before impact
X = a random variable
x = horizontal location of the rock
X_0 = initial horizontal location of the rock
X_1 = horizontal location of the first vertex of the line segment
X_2 = horizontal location of the second vertex of the line segment
X_I = horizontal location of the intersection between the parabola and the line
Y_0 = initial vertical location of the rock
Y_1 = vertical location of the first vertex of the line segment
Y_2 = vertical location of the second vertex of the line segment
Y_I = vertical location of the intersection between the parabola and the line
y = vertical location of the rock
Zj = sample from a normal distribution

1. Introduction

The accurate prediction of rockfalls is *practically* impossible. An engineer attempting to perform a rockfall analysis will encounter a number of difficulties. The slope geometry is highly variable. The location where the rocks begin is often unknown. The slope material can be variable or the relevant material properties not well known. The calculations used to simulate the rockfall events are sensitive to small changes in these parameters. Taken together, these factors all contribute to make accurate prediction of rockfalls extremely difficult. Determination of a single factor of safety, or creation of a design that is certain to prevent *all* rockfalls, are generally not realistic goals. This thesis will begin by detailing the difficulties that are encountered when performing a rockfall analysis and present a solution to those difficulties.

Employing probability and statistics in the analysis of rockfall simulations has proven to be an effective and acceptable method for dealing with these difficulties. The goal of this thesis was to create a tool to assist engineers with the probabilistic analysis of rockfalls, the result of which is the program RocFall[1]. RocFall is a robust, easy-to-use computer program that performs a probabilistic simulation of rockfalls and can be used to design remedial measures and test their effectiveness. An overview of the RocFall program is presented, with a focus on how RocFall helps to deal with the difficulties identified above.

The final section of this thesis presents a verification of the program's results and the computer code that is used to calculate the motion of the rocks. The verification is contained in the first appendix and the computer code is contained in the second appendix.

The majority of the effort necessary to prepare this thesis was devoted to the creation and coding of the RocFall program. A great deal of effort was required to ensure that the program was robust, correct, fully functional and easy-to-use. RocFall meets these objectives.

[1] RocFall is available from Rocscience Inc. Rocscience can be found on the internet at: www.rocscience.com or can be contacted by telephone: 1-416-698-8217, or by fax: 1-416-698-0908

2. Difficulties with Rockfall Analyses

Prediction of rockfalls is a difficult task. Slopes that are at risk of rockfall often have highly variable geometry. The location and mass of the rocks that will, eventually, become the rockfall are uncertain. The materials that make up the slope can vary considerably from one section of the slope to the other and the relevant material properties are usually not well known. The equations used to simulate the rockfalls are sensitive to small changes in these parameters. Each of these difficulties will be discussed in more detail below.

The responsibility for the sensitivity of the simulation to small changes in these parameters does not lie entirely with the computer program or the equations used; the physical process of a rockfall is sensitive to small changes in these parameters as well.

2.1 Slope Geometry

Since the area at risk of rockfalls is often very large (e.g. a long mountain highway), the slope geometry can vary considerably along this distance. Performing a detailed survey and analysis of the entire area is usually not feasible for budgetary reasons. Often the engineer is only able to obtain a survey of a few cross sections, those that appear to be most at risk of rockfalls. Therefore, the geometry used in the simulation is not always exact.

Even when the slope geometry is well known, most rockfall simulations are sensitive to small changes in the slope geometry. For example, if a small block of rock were to slide down a long inclined section of slope (gaining considerable speed along the way), the slope geometry at the end of the inclined section plays a *super*-critical role in determining the trajectory of the rock. If the edge of the slope were to drop-off quickly, the rock would simply fall off the end, landing fairly close to the edge of the slope. Alternatively, if the rock were to encounter a small "ramped" section, the rock could be deflected up and away from the slope. When this occurs, the rock can land quite far from the slope. Trajectories like these are the most important to predict, because they can send rocks far away from the slope and well above the height of any realistic barriers.

2.2 Material Properties

The materials that constitute the slope can vary considerably from the crest of the slope to the toe and from cross-section to cross-section. Even when the material is uniform, the material properties relevant to the rockfall analysis (the coefficients of restitution) may not be well known.

Typical values for the coefficient of normal restitution (R_N) used in rockfall analyses range from 0.3 to 0.5. Typical values used for the coefficient of tangential restitution (R_T) range from 0.8 to 0.95. Vegetated areas and soft soils occupy the lower end of the ranges, and bedrock and asphalt the higher end. Unfortunately, the algorithm used for the rockfall simulation is sensitive to small changes in the coefficients of restitution. For example, a slope segment with $R_N = 0.4$ will exhibit behaviour that is very different from the same slope segment with $R_N = 0.5$.

To make matters worse, the value of the coefficient of restitution can be highly variable within a small section of the slope. For example, an area that contained mainly loose gravel with a few sections of exposed bedrock would exhibit very different behaviour depending on whether the falling rocks were to strike a section of bedrock ($R_N = 0.5$) or a section of gravel ($R_N = 0.35$).

Most engineers are familiar with the concept of "friction angle", and would be able to specify the friction angle of each slope segment with a good degree of certainty. The typical engineer is much less familiar with coefficients of restitution and does not have a great deal of certainty about what values are appropriate in each situation. In fact, most engineers have a difficult time accurately determining coefficients of restitution *a priori*. A popular method for determining coefficients of restitution is to perform a back-analysis after a rockfall has occurred. Usually the field data for this back analysis provides an endpoint for the rock (this will be obvious), a mass for the rock (easy to measure), a starting point (the original location of the rock may appear less weathered), and the location of a few impact points (marks or dents along the slope profile). The empirical values for the coefficients of restitution are determined by adjusting the coefficients of restitution in the computer program, until the program is able to reproduce the same impact locations and rock endpoints that occurred in the field. This technique is fairly crude and the engineer is usually left with a good deal of uncertainty about the proper values for the coefficients of restitution.

2.3 Initial Conditions

Most slopes that pose a risk of rockfall are quite steep and have a large number of loose rocks or debris along the entire slope profile. This implies that a rock can start from almost anywhere along the slope. While small variations in the initial position of the rocks are not as critical in determining the trajectory of the rock, as the material properties or the slope geometry, they are still significant.

Naturally occurring slopes often exhibit a large degree of variation in the mass of the rocks that comprise a rockfall. Virtually all naturally occurring slopes have a number of very small (under 5 kg) rocks scattered around. The size of the largest rocks varies widely, and depends entirely on the local conditions, but it is not uncommon for the mass of the rocks to vary by three orders of magnitude from the smallest rock to the largest. Determining a mass for the rocks is important if barriers are used for remedial measures, since barrier capacities are specified in units of energy.

2.4 Probabilistic Analysis

This combination of uncertainty about input parameters and sensitivity to those parameters requires the use of some additional technique to turn this pseudo-analysis into a valid scientific effort. Performing probabilistic simulation of rockfalls, combined with a proper statistical analysis has proven to be an effective and acceptable method for dealing with these difficulties.

It is impractical to perform a probabilistic analysis by hand (performing enough simulations to create a statistically valid data set would be extremely time consuming). The purpose of this thesis was to produce a computer program that could perform a probabilistic simulation of rockfalls, which could then be used in the context of a proper statistical analysis.

Despite all of the difficulties rockfall prediction is faced with it does have one thing in its favour - the particle analysis that is used to simulate the rockfalls requires very little computer time. A typical computer today (a 200 MHz Pentium) can perform approximately 50 particle simulations per second. This speed of calculation allows the engineer to resort to probability to aid in the prediction of rockfalls. If the coefficients of restitution (or some other parameter) are not well known, but an *expected* range can be determined, then a large number of analyses can be performed, randomly sampling from this range. This will produce a distribution of outcomes, based on the sampled range of input. This distribution can be analysed and a *probable* outcome is obtained.

2.5 Solution to the Difficulties

The variability in the slope geometry, the uncertain material properties, and the unknown initial conditions can all be taken into account using a probabilistic approach. The application of the probabilistic approach to solving the difficulties presented in the previous sections, is detailed below.

Slope Geometry The uncertainty about the slope geometry could be modelled by assigning a random distribution to the location of each of the slope vertices. This can be used to simulate the change of the slope geometry from one section of slope to the other. This can also be used to determine the sensitivity of the current slope profile to changes in the location of vertices, which is helpful when determining where remedial measures would be of most use.

Material Properties The area of gravel and exposed bedrock that was discussed in the preceding section could be modelled with a large standard deviation for the coefficient of normal restitution (R_N). The range for R_N could be large enough so that the low end of the range (the gravel) and the high end of the range (the bedrock) would both be covered by the distribution.

Initial Conditions The uncertainty about the initial location of the rock could be modelled by randomly starting the rocks at various locations along the slope profile. Typically, locations near the crest of the slope are chosen, because the rocks beginning at these locations have the most potential energy and are likely to be the most hazardous. The mass of the rocks could be specified with a standard deviation that includes the largest blocks that are likely to come free and the small debris that will fall from the slope.

2.6 Conclusion

When using the results from a probabilistic analysis, it is usually wise to be conservative with the design. This is a prudent choice because, as with any statistical simulation, the actual outcome is not guaranteed to be in the range predicted by the simulation, and the “worst case” may not have appeared in the simulation. For example, a barrier that was adequate to catch all the rocks run in the simulation, may not be tall enough to catch a rock that is in the extreme “tail” of a distribution.

How conservative the design must be depends entirely on the application. For example, when designing an infrequently-used logging road it may be sufficient to catch ninety-five percent of the rocks predicted by the simulation (in order to keep the road free of debris). Alternatively, when designing the slope beside a busy highway it may be required that the design catch all of the rocks predicted by the simulation, and include an additional measure of conservatism.

3. RocFall: Overview & Details

The primary goal of this thesis was to create a tool to assist engineers with the probabilistic analysis of rockfalls. The result of this work is the program RocFall. RocFall is a robust, easy-to-use program that can be used to simulate almost all rockfall events. RocFall can be also be used to design remedial measures and test their effectiveness. RocFall employs a particle analysis for the calculation of the rock movement, which will be discussed in much more detail in the following chapter. This chapter will outline the major features of RocFall and detail how they are useful to the engineer who is performing a probabilistic rockfall analysis.

The simplest simulation that can be performed in RocFall has two essential components: a slope and a rock. More advanced simulations can include barriers and incorporate random variation in the mass, velocity and position of the rock, and random variation in the location and material properties of each segment of the slope.

RocFall produces many forms of output to assist with statistical analyses and to aid in the design of remedial measures. RocFall produces plots displaying the *maximum* velocity, kinetic energy and bounce-height of the rocks, along the length of the entire slope profile (referred to as "envelopes" in the program). These envelopes are useful when deciding where remedial measures should be placed. The program also produces histograms displaying the *distribution* of the velocity, kinetic energy and bounce-height of the rocks at any location along the slope profile (referred to as "data collectors" in the program). The data collectors are useful when designing the remedial measures (e.g. deciding the capacity of a barrier).

3.1 Slope

The slope creation process in RocFall is relatively unrestricted; virtually any slope geometry can be modelled. The slope profile can contain any number of overhanging sections. The slope can be made up of any number of segments and each segment can have different material properties (i.e. R_N, R_T, ϕ). It is important to be able to model overhanging sections because the slopes that are at risk of rockfall are the same slopes that are most likely to contain overhanging sections.

3.2 Random Variables

One of the requirements of a probabilistic analysis tool is to be able to specify some of the input parameters as random distributions when the simulation is performed. In order to provide a thorough probabilistic analysis, almost all of the parameters in RocFall can be defined by either a constant value or by a random variable. The mass of the rock, the initial position of the rock, the velocity of the rock, the location of each of the slope vertices and the coefficients of restitution and friction angle (for each slope segment and for each barrier) can all be defined by random variables. Each distribution is specified separately and each distribution is independent of all the others.

Although it is unlikely that all of these items will be assigned a random distribution at the same time, this feature allows the program user to perform a sensitivity study on *any* of the input parameters. Determining the most sensitive parameter in the simulation is very useful when deciding where remedial measures would be most effective. If the remedial measure can be taken on the most sensitive parameter, this will provide the most economical design.

The only parameter that cannot be assigned a distribution is the location of the barriers. If barriers are going to be employed in a design, the location will be known (and usually specified) by the program user, so varying the location is not necessary.

3.3 Vertex Variation

Assigning a normal distribution to the slope vertices allows the engineer to statistically simulate the effect of the variation in the slope geometry, as it changes from cross-section to cross-section. This can be thought of as simulating the three-dimensional shape of the slope.

This feature is also useful if the exact location of the vertices is not known. This can occur if the slope geometry was scaled-off of a diagram produced for some other reason (e.g. highway construction) and is not exact, or if the model geometry is supposed to represent many similar (but not identical) cross-sections.

Assigning a random distribution to a vertex can also be used to determine the sensitivity of the current slope profile to changes in the location of vertices. This is the most useful application of this feature and can often be helpful when determining where remedial measures would be of most use. For example, if it were found that the location of the rock endpoints is particularly sensitive to the location of one of the benches on a slope, removing the bench (or otherwise changing the geometry) would be the best choice for the remedial measure.

3.4 Initial Conditions for the Rocks

Before a simulation can begin, the initial location, velocity and mass of the rocks must be defined. This section will detail how these initial conditions are specified.

The starting location for the rocks can be specified anywhere on or above the slope surface (i.e. anywhere, except underground). The starting location can be defined by a single point in space (referred to as a “point seeder” in the program). In this case, all of the rocks will begin the simulation at the same location. This would be useful if the engineer was modelling two parallel roadways (one further up the slope than the other), and the majority of rockfalls were caused by debris originating at the side of the higher road. Placing the point seeder at the side of the higher road would model this situation quite well.

The starting location can also be defined by a poly-line (referred to as a “line seeder” in the program). The initial position of each rock is determined by randomly generating a location somewhere along the length of the poly-line. The location generated has equal probability of being generated at any location along the poly-line (i.e. a uniform distribution). This method of “line seeding” is useful when the engineer is uncertain exactly where the rockfall will be initiated, but would like to specify a likely range for the starting points (e.g. along one of the upper segments of the slope.)

3.5 Multiple Seeders

Any number and combination of point seeders and line seeders can be added to the simulation. Regardless of how many seeders are present, only one rock is generated at a time, and the generation is independent of the generation of the other rocks in the simulation.

When there are multiple point seeders present, or a combination of point seeders and line seeders, the rock will start from any one of the seeders with equal probability. When there are multiple line seeders present, the user of the program is allowed to choose between two options. The first option is to have the rock start with equal probability on any of the seeders (this is the same probability behaviour as described above). The second option is to have the rock location generated with a probability proportional to the length of the seeder (relative to the other seeders). For example, if there are three seeders present, with lengths of 1 m , 3 m, and 6 m then the probability that the rock with begin on each of the seeders, respectively, is 0.1, 0.3 and 0.6.

The mass, initial horizontal velocity and initial vertical velocity of the rocks can each be specified by a constant value or sampled from a random distribution. The sampling of the mass and velocity are independent of the sampling technique that is used to generate the location of the rock.

3.6 Data Collectors

In order to assist with the design of remedial measures, RocFall provides an ability to determine the current state of the rocks as they pass certain locations on the slope (referred to as a "data collector" in the program). A data collector is a single line segment that can be placed anywhere along the slope profile. The data collector records the position, velocity and kinetic energy of every rock that passes through the data collector during the simulation. After the simulation has been performed, the data collectors can present a distribution of the velocity, kinetic energy and position of the all the rocks that passed the data collector.

The data collectors are useful for determining the distribution of velocity, kinetic energy and bounce-height at a certain location. This information is useful when designing barriers. Once the location for the barrier has been decided, a data collector can be placed at the location, and the simulation re-run. The data collector will display the distribution of kinetic energy of the rocks that passed through the data collector. This can be used to specify the required capacity of the barrier. The data collector will also display the distribution of bounce-height of the rocks. This can be used to specify the required height of the barrier.

Checking all of the data collectors requires a substantial amount of effort during the course of a simulation. Since the envelopes use (invisible) data collectors to gather their information, it is not uncommon for there to be 500 data collectors present in each simulation. This involves performing 500 parabola-line intersections each time the rock strikes a slope segment or barrier. In the course of a typical simulation, checking the data collectors requires the parabola-line intersection routine to be executed tens of millions of times.

3.7 Envelopes

The program produces three “envelopes”: the kinetic energy envelope, the velocity envelope and the bounce-height envelope. Each envelope is defined by the maximum value (e.g. maximum velocity) at a number of evenly spaced horizontal locations along the slope profile. The kinetic energy envelope measures the highest kinetic energy that any rock attained while passing each horizontal location. The velocity envelope measures the highest velocity that any rock attained while passing each horizontal location. The bounce-height graph measures the maximum height that any rock reached minus the slope height at each horizontal location (i.e. the maximum height *above* the slope). Any horizontal location that is not crossed by a rock is given a value of zero when creating the envelope.

The envelopes provide an overview of the condition of the rocks as they travel from one section of the slope to the other. The envelopes are very helpful in determining where remedial measures, particularly barriers, would be most effective. If there are few restrictions on the placement of a barrier, a good choice of location would be at a local minimum on the bounce-height envelope. This is a good choice of location because it is a section of the slope where the rocks are not likely to be travelling far above the ground, and the barrier would not have to be very tall to be able to intercept all of the rocks.

Since the envelopes only display the maximum value, at any location, it is typical to use a data collector in combination with the envelope. The data collector can be used to determine the distribution of energies at a specific location once the envelopes have been used to narrow down a location of interest.

The data that is required to create the envelopes is gathered by placing a number of vertical data collectors (invisible to the program user) equally spaced along the length of the slope profile. The number of locations is specified by the program user in the “number of intervals to use when plotting” option in RocFall.

3.8 Location of Rock Endpoints

The location of the rock endpoints is, arguably, the most important single piece of output from the program. This is considered an essential piece of output because it is usually the final location of the rocks that determines whether a design is successful or not. The adequacy of a design can often be summarised in a yes or no question (e.g. did the rocks reach the highway, or not?)

The location of the rock endpoints is presented as a distribution. The distribution can either be displayed graphically in the program or pasted into another program for further statistical analysis.

3.9 Barriers

Barriers are modelled in the program very much like a slope segment that happens to be attached at an odd angle to the remainder of the slope. The barriers and slope segments are both modelled by a straight line and have the same set of material properties (R_N, R_T, ϕ). The rock bounces and slides on the barriers in the same manner as the slope segments.

The barriers must have one end attached to a slope vertex. The other end must be placed such that the barrier does not intersect any of the other barriers or any slope segments (i.e. the barrier must have one end on the ground and not intersect any other items). These restrictions were required because crossing barriers, and barriers suspended in the air, provided too many “special cases” where the rock might become trapped.

The simulation of remedial measures, such as a barrier, is useful to the engineer because it allows them to test their design with more simulation. For example, a barrier proposed as the final design could be placed in the simulation and a sensitivity analysis performed. This study would reveal the required conditions for the design to fail. The likelihood of the conditions that brought about the failure could be evaluated and the adequacy of the design decided.

4. Particle Algorithm

RocFall employs a particle analysis to calculate the movement of the rock. This chapter will detail the particle model as it is used in RocFall: the assumptions that are made, the equations that are used, and the algorithm that was used to implement the model in RocFall. The C++ implementation of the particle algorithm is presented in much more detail in Appendix B.

A simpler version of this particle model was originally presented by Hoek (1987). The algorithm presented here uses the same assumptions about the slope and the rock as Hoek, but places fewer restrictions on the model. This algorithm permits overhanging sections of the slope, uphill sliding, sliding on any slope segment and the inclusion of barriers in the simulation.

The particle model is a fairly crude model of the physical process of a rockfall. The particle model neglects the effects that the size, shape and angular momentum of the particle have on the outcome. However, the particle model does have the advantage of being extremely quick to calculate, which allows sensitivity analyses to be performed. Since the input for most rockfall analyses is so poorly defined, it is equally important to determine the sensitivity of the results as it is to determine the results themselves.

4.1 The Algorithm

There are three distinct sections to the particle analysis: The particle algorithm, the projectile algorithm, and the sliding algorithm. The particle algorithm makes sure all of the simulation parameters are valid, sets up all of the initial conditions in preparation for the projectile and sliding algorithms and then starts the projectile algorithm. The remainder of the simulation (until the rock comes to rest) is spent in either the projectile algorithm or the sliding algorithm. The projectile algorithm is used to calculate the movement of the rock while the rock is travelling through the air, bouncing from one point on the slope to another. The sliding algorithm is used to calculate the movement of the rock while the rock is in contact with the slope. Because the velocity of the rock has to be very low before the rock will leave the projectile algorithm, the majority of the simulation is spent in the projectile algorithm. A better understanding of these three algorithms and the interaction between them can be gained by looking at Figures 4.1, 4.2 and 4.3. Thorough examples of the use of these algorithms can be found in the first two chapters of the verification appendix.

The largest problem that was encountered when implementing the particle analysis was the numerical instabilities of the projectile algorithm. These problems are explained in much more detail in the following chapter.

4.2 Assumptions

Each rock is modelled as a particle. The particle may be thought of as an infinitesimal circle, since the size of the rock does not play a role in the algorithm, but the equations used in the sliding algorithm imply a circular shape. Because each rock is assumed to be infinitely small, there is no interaction between particles, only with the slope segments and barriers. Because there is no interaction between particles, each rock behaves as if it were the only rock present in the simulation.

Although the rocks are not considered to have any size (for the purpose of interaction with other rocks, the slope or barriers) they are considered to have a mass. The mass it is not used in any of the equations used to calculate the motion of the rocks, it is only used to calculate the kinetic energy when creating the graphs and presenting results. The mass is determined at the beginning of the simulation and stays constant throughout the simulation. The rocks cannot break or split into multiple pieces during the simulation. The mass can be specified by a constant value or samples sampled from a random distribution.

The frictional resistance of the air is not taken into account in any of the equations. It is assumed that the rocks are massive enough and travelling at low enough speeds that this can be ignored. Incorporating air resistance would make the analysis much more complicated and would have little effect on the outcome of the simulation.

The slope is modelled as one continuous group of straight line segments, connected end to end. In order to be considered valid, a slope segment cannot cross any other slope segments, and vertices cannot be coincident. Otherwise, the geometry is unrestricted. The barriers and data collectors are modelled as single straight line segments.

4.3 Projectile Algorithm

The projectile algorithm assumes that the rock has some velocity (even a small velocity) that will move it, through the air, from its present location to a new location where the rock will strike another object (which can be further along the same object). The path the rock will take through the air is, because of the force of gravity, a parabola.

The essence of the projectile algorithm is finding the location of intersection between a parabola (the path of the rock) and a line segment (a slope segment or a barrier). Once the intersection point is found, the impact is calculated according to the coefficients of restitution. If, after the impact, the rock is still moving fast enough the process begins again, with the search for next intersection point. In this context "fast enough" is defined as the minimum velocity (V_{MIN}) and is specified by the program user at the beginning of the simulation. The minimum velocity defines the transition point between the projectile state and the state where the rock is moving too slowly to be considered a projectile and should instead be considered rolling, sliding or stopped. The outcome of the simulation and the time it takes to perform each simulation are not particularly sensitive to changes in V_{MIN}.

4.4 Equations

Using the parametric form of the equations (for the parabola and the line) is beneficial when overhanging sections of the slope or barriers are permitted. The parametric form of the equations is advantageous because the parabolic path of the rock, may intersect multiple slope segments and barriers, and the order of intersection must be determined. Since the rock only *really* impacts the first intersection point, finding the smallest value of the parameter used in the parabolic equations provides a simple method for determining the correct intersection.

The equations used for the projectile calculations are listed below:

The parametric equation for a line:

$$x = X_1 + (X_2 - X_1)u \tag{4.1}$$

$$y = Y_1 + (Y_2 - Y_1)u \qquad u \in [0,1] \tag{4.2}$$

where:

X_1, Y_1 is the first endpoint of the line

X_2, Y_2 is the second endpoint of the line

The parametric equation for a parabola:

$$x = V_{X0}t + X_0 \tag{4.3}$$

$$y = \frac{1}{2}gt^2 + V_{Y0}t + Y_0 \qquad t \in [0,\infty] \tag{4.4}$$

where:

g is the acceleration due to gravity (sign is negative)

X_0, Y_0 is the initial position of the rock

V_{X0}, V_{Y0} is the initial velocity of the rock

The parametric equations for the velocity of the particle:

$$V_{XB} = V_{X0} \tag{4.5}$$

$$V_{YB} = V_{Y0} + gt \tag{4.6}$$

where:

V_{XB}, V_{YB} is the velocity of the rock at any point along the parabolic path, before impact

Equating the points of the parabola and line equations (i.e. x = x and y = y) and rearranging into the familiar form $ax^2 + bx + c = 0$ gives:

$$\left[\frac{1}{2}g\right]t^2 + \left[V_{Y0} - qV_{X0}\right]t + \left[Y_0 - Y_1 + q(X_1 - X_0)\right] = 0 \tag{4.7}$$

where:

$$q = \frac{(Y_2 - Y_1)}{(X_2 - X_1)} \text{ is the slope of the line segment} \tag{4.8}$$

Equation 4.7 can be solved for t using the quadratic equation:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{4.9}$$

where:

$$a = \frac{1}{2} g \tag{4.10}$$

$$b = V_{Y0} - qV_{X0} \tag{4.11}$$

$$c = Y_0 - Y_1 + q(X_1 - X_0) \tag{4.12}$$

During each pass through the algorithm, the parabola formed by the rock trajectory is checked with every segment of the slope and with every barrier. All of the slope segments and barriers that have a valid intersection with the parabola are inserted into a list. The list is then sorted by the value of the t parameter to determine the correct intersection.

Once the correct intersection is determined, the velocity just prior to impact is calculated according to equations 4.5 and 4.6. These velocities are transformed into components normal and tangential to the slope according to:

$$V_{NB} = (V_{YB})\cos(\theta) - (V_{XB})\sin(\theta) \tag{4.13}$$

$$V_{TB} = (V_{YB})\sin(\theta) + (V_{XB})\cos(\theta) \tag{4.14}$$

where:

V_{NB}, V_{TB} are the velocity components of the rock, before impact, in the normal and tangential directions, respectively

θ is the slope of the line segment

The impact is calculated, using the coefficients of restitution, according to:

$$V_{NA} = R_N V_{NB} \tag{4.15}$$

$$V_{TA} = R_T V_{TB} \tag{4.16}$$

where:

R_N is the coefficient of normal restiution $\in [0,1]$

R_T is the coefficient of tangential restiution $\in [0,1]$

V_{NA} , V_{TA} are the velocity components of the rock, after impact, in the normal and tangential directions, respectively

The post-impact velocities are transformed back into horizontal and vertical components according to:

$$V_{XA} = (V_{NA})\sin(\theta) + (V_{TA})\cos(\theta) \tag{4.17}$$

$$V_{YA} = (V_{TA})\sin(\theta) - (V_{NA})\cos(\theta) \tag{4.18}$$

where:

V_{XA}, V_{YA} are the velocity components of the rock, after impact, in the horizontal and vertical directions, respectively

Once the correct intersection is determined and the velocities have been calculated, all of the data collectors are checked for intersection with the parabola (in a manner similar to checking the slope segments). Any data collector with a parametric value (the value of *t*) less than the value of the actual intersection is informed of the rock's trajectory. The location, velocity and kinetic energy of the rock, at the moment it passes the data collector, are recorded by the data collector.

The velocity of the rock is then calculated and compared to V_{MIN}. If it is greater than V_{MIN} the process starts over again, with the search for the next intersection point. If the velocity is less than V_{MIN} the rock can no longer be considered a particle, and is sent into the sliding algorithm.

4.5 Sliding Algorithm

The sliding algorithm is used to calculate the movement of the rocks after they have exited the projectile algorithm. The rocks can slide on any segment of the slope and on any barrier. For the purpose of the sliding algorithm, the slope segment or barrier that the rock slides on, consists of a single straight-line segment that has properties of slope angle (θ) and friction angle (ϕ). The friction angle can be specified by a constant value or sampled from a random distribution.

The rock can begin sliding at any location along the segment and may have an initial velocity that is directed upslope or downslope. Only the velocity component tangential to the slope is considered in the equations.

Once the sliding is initiated, the algorithm used depends on whether the initial velocity is upslope or downslope. The algorithm used when the initial velocity is downslope will be explained first.

4.6 Sliding Downslope

When the initial velocity of the rock is downslope (or zero) the behaviour of the rock depends on the relative magnitudes of the friction angle (ϕ) and the slope angle (θ).

<u>θ = ϕ</u> If the slope angle is equal to the friction angle, the driving force (gravity) is equal to the resisting force (friction) and the rock will slide off the downslope end of the segment, with a velocity equal to the initial velocity (i.e. $V_{EXIT} = V_0$). There is a special case when $V_0 = 0$; in this case, the rock does not move, and the simulation ends.

<u>θ > ϕ</u> If the slope angle is greater than the friction angle, the driving force is greater than the resisting force and the rock will slide off the downslope endpoint with an increased velocity. The speed with which the rock leaves the slope segment is calculated by:

$$V_{EXIT} = \sqrt{V_0^2 - 2sgk} \qquad (4.19)$$

where:

V_{EXIT}	is the velocity of the rock at the end of the segment
V_0	is the initial velocity of the rock, tangential to the segment
s	is the distance from the initial location to the endpoint of the segment
g	is the acceleration due to gravity (-9.81m/s/s)
k	is $\pm\sin(\theta) - \cos(\theta)\tan(\phi)$

where:

θ is the slope of the segment

ϕ is the friction angle of the segment

± is + if the initial velocity of the rock is downslope or zero

± is – if the initial velocity of the rock is upslope

$\underline{\theta < \phi}$ If the slope angle is less than the friction angle, the resisting force is greater than the driving force and the rock will decrease in speed. The rock may come to a stop on the segment, depending on the length of the segment and the initial velocity of the rock.

Assuming that the segment is infinitely long, a stopping distance is calculated. The distance is found by setting the exit velocity (V_{EXIT}) to zero in equation 4.19 and rearranging:

$$s = \frac{V_0^2}{2gk} \tag{4.20}$$

The distance from the initial location of the rock to the end of the segment is calculated. If the stopping distance is greater than the distance to the end of the segment, then the rock will slide off of the end of the segment. In this case, the exit velocity is calculated using equation 4.19. If the stopping distance is less than the distance to the end of the segment then the rock will stop on the segment and the simulation is stopped. The location where the rock stops is a distance of *s* downslope from the initial location.

4.7 Sliding Upslope

When sliding uphill both the frictional force and the gravitational force act to decrease the velocity of the particle. Assuming that the segment is infinitely long, the particle will eventually come to rest. The stopping distance is calculated using equation 4.20 and the distance from the initial location of the rock to the upslope end of the segment is calculated. If the stopping distance is greater than the distance to the end of the segment, the rock will slide off of the end of the segment. In this case, the exit velocity is calculated using equation 4.19. If the stopping distance is less than the distance to the end of the segment the rock comes to rest and the simulation is stopped.

If the rock slides up and stops it is then inserted into the downslope sliding algorithm. If the segment is steep enough to permit sliding (i.e. $\theta > \phi$) then the rock will slide off the bottom end of the segment. If the segment is not steep enough, then the location where the rock stopped moving (after sliding uphill) is taken as the final location and the simulation is stopped.

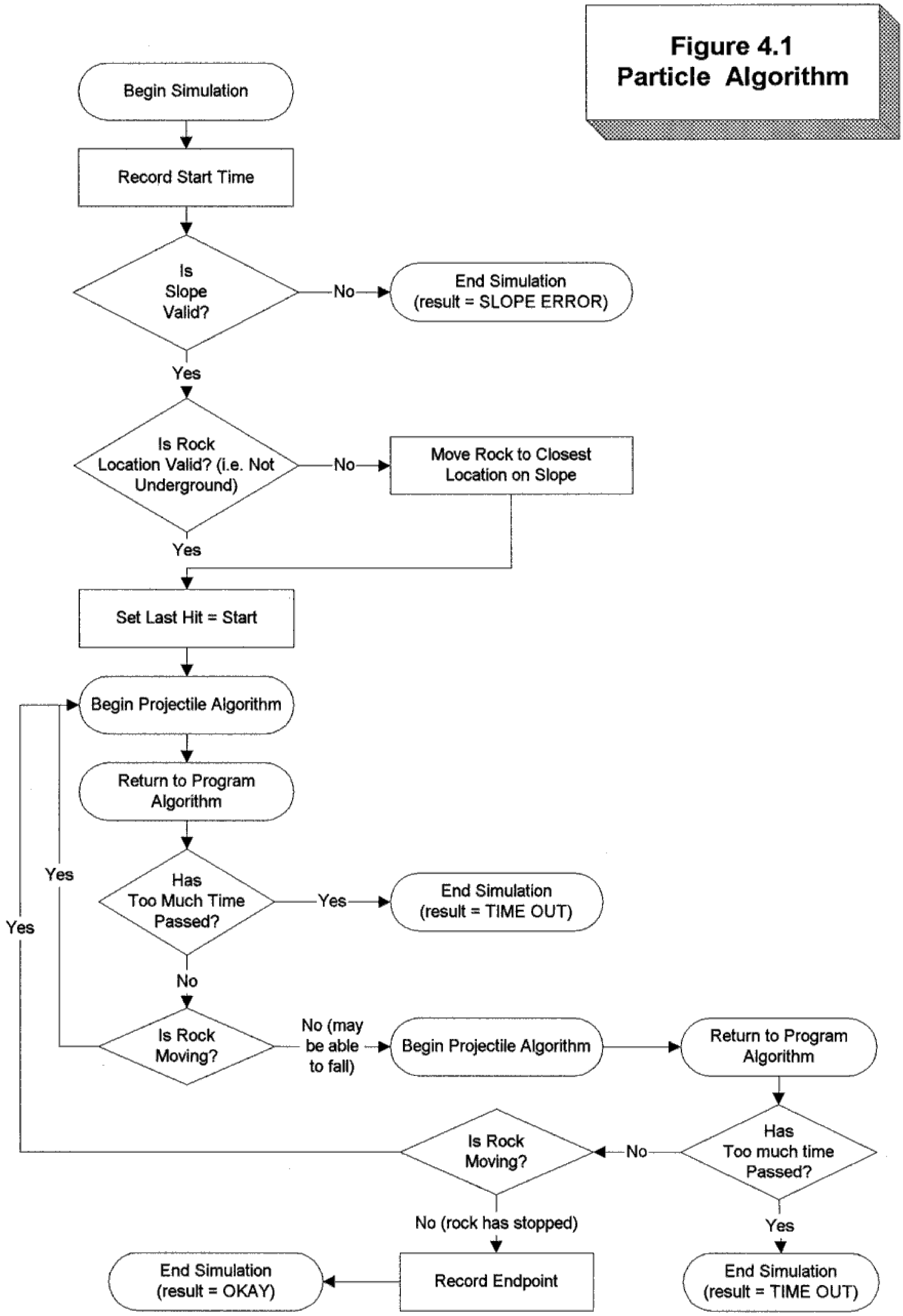


Figure 4.1
Particle Algorithm
Begin Simulation
Record Start Time
Is Slope Valid?
No
End Simulation (result = SLOPE ERROR)
Yes
Is Rock Location Valid? (i.e. Not Underground)
No
Move Rock to Closest Location on Slope
Yes
Set Last Hit = Start
Begin Projectile Algorithm
Return to Program Algorithm
Has Too Much Time Passed?
Yes
End Simulation (result = TIME OUT)
No
Is Rock Moving?
Yes
Yes
No (may be able to fall)
Begin Projectile Algorithm
Return to Program Algorithm
Has Too much time Passed?
No
Is Rock Moving?
No (rock has stopped)
Yes
Record Endpoint
End Simulation (result = OKAY)
End Simulation (result = TIME OUT)

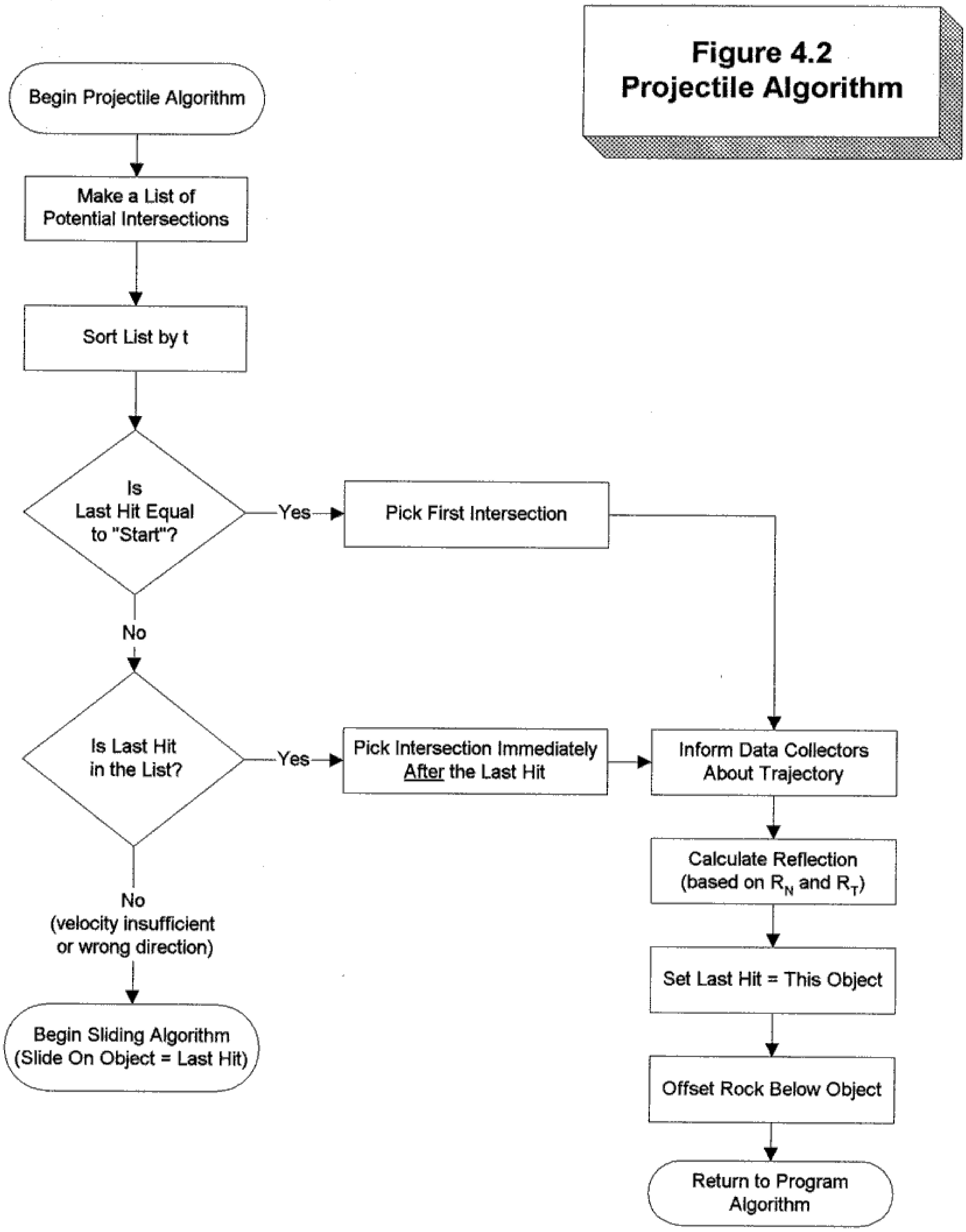
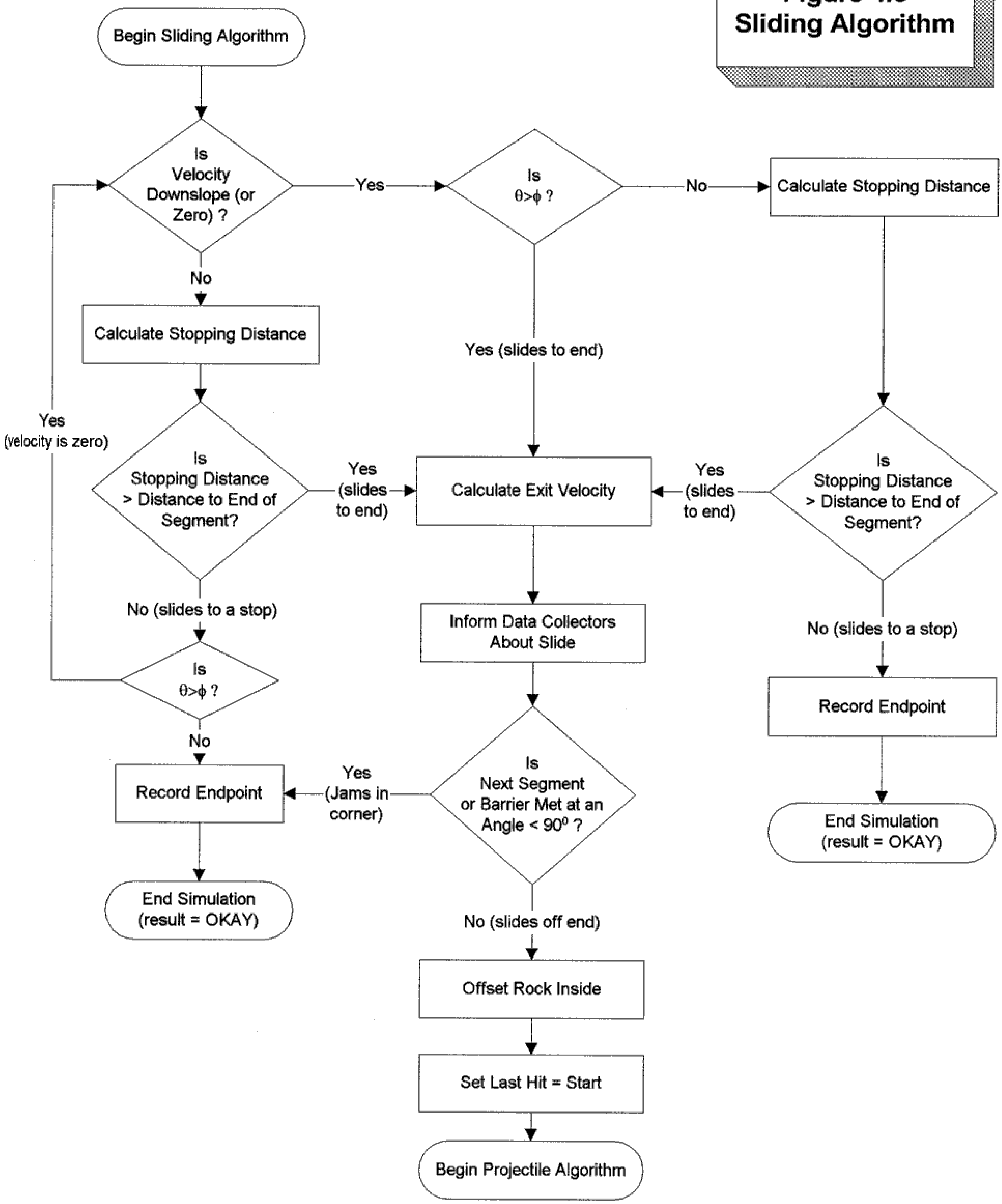


Figure 4.2
Projectile Algorithm
Begin Projectile Algorithm
Make a List of Potential Intersections
Sort List by t
Is Last Hit Equal to "Start"?
Yes
Pick First Intersection
No
Is Last Hit in the List?
Yes
Pick Intersection Immediately After the Last Hit
Inform Data Collectors About Trajectory
Calculate Reflection (based on R_N and R_T)
Set Last Hit = This Object
Offset Rock Below Object
Return to Program Algorithm
No (velocity insufficient or wrong direction)
Begin Sliding Algorithm (Slide On Object = Last Hit)

Figure 4.3
Sliding Algorithm



5. Numerical Instabilities

A lot of time and effort was required to solve the numerical instability problems in RocFall. This chapter outlines some of the difficulties that were encountered when the particle algorithm was implemented and the solution to those difficulties.

It is the author's experience that writing an engineering program that works ninety five percent of the time (a few situations not being handled properly) requires less than half of the time and effort required to write a program that handles *all* of the situations properly. It is important for a program that performs probabilistic simulations to work correctly in all cases. Since each simulation has a slightly different outcome, and hundreds of simulations are performed, a problem that occurs only once in a thousand simulations, will be noticed very quickly.

5.1 Impossible situations

It is trivial to construct a case that is analytically valid (if not realistic) that could cause an unsuspecting rockfall algorithm to continue indefinitely. For example, dropping a rock with zero initial velocity onto a horizontal surface with $R_N = 1$ would cause the rock to bounce up and down – forever. This would cause the program to enter an endless calculation loop, and stop responding to the user (otherwise known as a "crash"). While this example is contrived, it is possible for the user of the program to inadvertently specify a similar situation.

Since the goal was to construct a robust program, this possibility of this situation, and similar situations, was handled. Each rock is given a certain amount of real time (not to be confused with simulation time, which will usually be much longer) to finish all of its calculations. Since the amount of real time permitted for the calculations (approximately 10 s) is many times larger than the typical solution time (approximately 0.02 s) whenever a simulation "time's out" the reason is almost always that the conditions specified are "impossible" and convergence (the rock stopping) will never occur.

5.2 Numerical instabilities

One of the equations used to find the parabola-line intersection (equation 4.8) is unstable in two situations (the instability is caused by the (X_2 - X_1) term in the denominator). The first instability is encountered when the length of the line segment approaches zero. This problem was solved by the forcing the program user to enter geometry such that each of the slope segments is at least 1 mm in length.

The second instability is encountered when the line segment is vertical. Noting that the parameter u can be removed from equation 4.1 when the line is vertical solved this problem. When the parameter u is removed and the horizontal location is equated (i.e. x = x) with equation 4.3 (the parabola equation), these equations can be solved for t.

$$t = \frac{(X_1 - X_0)}{V_{X0}} \tag{4.21}$$

The solution is now stable, for all cases except when the line is vertical and the rock has zero initial horizontal velocity (V_{X0}). When this occurs, there is no intersection between the parabola and the line, and the instability is avoided.

The non-parametric versions of the parabola-line intersection were unstable for horizontal velocities close to zero, which is a much more common occurrence than vertical slope segments. This is another reason why the parametric versions of the equations are superior to the slope, y-intercept form of the equations.

5.3 Machine Error

When equation 4.9 (the quadratic equation) is solved in the projectile algorithm the two roots represent the starting point and ending point of the trajectory. A difficulty presents itself because “machine error” will cause the root at the beginning of the trajectory to have a parameter that is not *exactly* zero. When the bounce-height becomes extremely small, it can become difficult to determine which root represents the beginning and which represents the end. This difficulty was solved by offsetting the rock *slightly* below the segment, before each trajectory, and taking the smallest positive root as the beginning point.

6. Recommendations

RocFall is an ongoing project. While the user-interface of the program can always be improved, these recommendations focus on the calculation aspect of the program. The recommendations are listed in order of desirability:

1. Gathering of empirical data for the coefficients of restitution

The outcome of a rockfall simulation is very dependent on the values used for the coefficients of restitution. Unfortunately, appropriate values are not always well known and a lot of time is usually spent determining what values should be used. Gathering a large sample of previously used values in one location, similar to the work of Tomory (1997), would be extremely useful. Gathering this data would require a substantial amount of time and effort.

2. Mass-based "plasticity" function for the coefficient of restitution

This would involve varying the coefficient of restitution based on the mass of the rock (i.e. more massive rocks would use lower coefficients of restitution when the impact was calculated). This would not require much work to implement.

3. "Breakable" barriers

This would require very little effort to add to the current program (in fact, most of the code has already been written). The reason it was not added is that the outgoing velocity (after the rock has passed through the barrier) is difficult to define. It was thought that the velocity vector of the rock would be changed (tending towards the normal of the barrier, perhaps). Further research into this behaviour should be performed before this feature is added.

4. Three-dimensional particle analysis

This would require a large amount of effort, but is within the realm of possibility. The factor limiting the usefulness of this type of model would likely be the lack of input data (i.e. this would require a large number of cross sections, which might not be available).

5. Consider angular velocity and/or shape effects in the particle model

Although desirable, this would require re-writing and re-verifying the entire calculation section of the program. This would require a substantial amount of work.

7. Conclusion

7.1 Final Product

The final product of this work is the program RocFall. RocFall is a robust, easy-to-use program that is capable of simulating rockfalls on a wide variety of slope geometries, and has been used by dozens of engineers over the course of the last six months. The implementation of the particle analysis is extremely robust and has proven to be stable for all of the realistic and pathological cases that have been attempted.

The goal of this thesis was to create a tool to assist engineers with the probabilistic analysis of rockfalls and the design of remedial measures. RocFall meets these objectives.

7.2 Verification

Essential to the use of a computer program in engineering practice, this thesis presents a thorough verification of the program's output. This verification can be found in appendix A. The verification was placed in an appendix because it is lengthy and it was meant to be readable separate from the body of the thesis. This was done so that an engineer that is using the program can refer to the verification, without having to read the entire thesis.

7.3 The Program

RocFall was written with Microsoft Visual C++ 5.0 using the Microsoft Foundation Classes (MFC) class library. The complete RocFall program consists of slightly more than 26,000 lines of code contained in approximately 400 files. It would be impractical, and of little value, to print out and include all of these files, as this would expand the thesis by more than 800 pages. Instead, only those files that relate to the particle analysis have been included in this thesis. A thorough explanation of the calculation engine used by the particle analysis, and a guide to its use, is presented in appendix B.

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