rocscience

RocFall3

Lumped Mass Basic Physics

Theory Manual

© 2022 Rocscience Inc.

Table of Contents

Introduction

This theory manual documents the basic calculations used in **RocFall3** to simulate the rock paths as rocks travel down the slope. RocFall3 has 2 engine modes: lumped mass and rigid body. The lumped mass mode assumes the rock as a particle (point) and the rigid body mode considers the rock's actual shape and the effects from the shape's size and angular momentum.

Rockfall trajectories consist of 3 main parts: projectile, impacts and sliding/rolling.

1. Projectile

The physics for projectile motion of the rock is the same for both the lumped mass and rigid body modes. While a rock is flying through the air, there are no forces acting on the rock except for gravity. For the rock at any given time $\boldsymbol{t}$, let's define the following variables:

$\vec{\boldsymbol{p}}$ **=** position (x,y,z) vector

$\overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{r}}$= rotation matrix

$\vec{\boldsymbol{v}}$ = velocity (x,y,z) vector

$\vec{\boldsymbol{\omega}}$ = angular velocity (x,y,z) vector

With only gravity as the force, $\vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_{\boldsymbol{g}} = \boldsymbol{M} \cdot \vec{\boldsymbol{g}} = \boldsymbol{M} \cdot (0, 0, -9.81)$.

We can then write:

$$\vec{\boldsymbol{p}}(\boldsymbol{t}) = \vec{\boldsymbol{v}}_{\boldsymbol{o}} \cdot \boldsymbol{t} + \frac{\boldsymbol{1}}{\boldsymbol{2}} \vec{\boldsymbol{g}} \cdot \boldsymbol{t}^{\boldsymbol{2}} \qquad \text{......(1.1)}$$

$$\vec{\boldsymbol{v}}(\boldsymbol{t}) = \vec{\boldsymbol{v}}_{\boldsymbol{o}} + \vec{\boldsymbol{g}} \cdot \boldsymbol{t} \qquad \text{......(1.2)}$$

And with $\vec{\boldsymbol{\omega}}$ and $\boldsymbol{t}$, we can get the rotating axis $\vec{\boldsymbol{u}} = \frac{\vec{\boldsymbol{\omega}} \cdot \boldsymbol{t}}{|\vec{\boldsymbol{\omega}} \cdot \boldsymbol{t}|}$ and the angle to be rotated is $\theta = \|\vec{\boldsymbol{\omega}} \cdot \boldsymbol{t}\|$. We can then construct the rotation matrix [1]:

$$R = \begin{bmatrix} cos\theta + u_x^2(1 - cos\theta) & u_x u_y(1 - cos\theta) - u_z sin\theta & u_x u_z(1 - cos\theta) + u_y sin\theta \\ u_y u_x(1 - cos\theta) + u_z sin\theta & cos\theta + u_y^2(1 - cos\theta) & u_y u_z(1 - cos\theta) - u_x sin\theta \\ u_z u_x(1 - cos\theta) - u_y sin\theta & u_z u_y(1 - cos\theta) + u_x sin\theta & cos\theta + u_z^2(1 - cos\theta) \end{bmatrix}$$

Updated rotation of the rock after time $\boldsymbol{t}$ is then:

$$\overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{r}}(\boldsymbol{t}) = \boldsymbol{R} \cdot \overleftrightarrow{\boldsymbol{R}}_{\boldsymbol{ro}} \cdot \boldsymbol{R}^{\boldsymbol{T}} \qquad \text{......(1.3)}$$

2. Impacts

The lumped mass impacts in **RocFall3** follows the method developed by Pfeiffer and Bowen [2]. The formulae in the paper were written in 2D and they were adapted into 3D to account for the extra dimension. Before the impact calculations, we rotate the rocks to a n/t1/t2 (normal, first tangential and second tangential directions) frame so there are no incoming velocities in the t1 direction. All incoming tangential rotations are along t2. Then we define the following:

$\boldsymbol{r_n}$ and $\boldsymbol{r_t}$ = normal and tangential coefficient of restitutions respectively

$\boldsymbol{v_n, v_{t1}, v_{t2}}$ = incoming translational velocities

$\boldsymbol{\omega_n, \omega_{t1}, \omega_{t2}}$ = incoming angular velocities, where $\boldsymbol{\omega_{t1} = 0}$

$\boldsymbol{v'_n, v'_{t1}, v'_{t2}}$ = outgoing translational velocities

$\boldsymbol{\omega'_n, \omega'_{t1}, \omega'_{t2}}$ = outgoing angular velocities

If "Consider Rotations" is not selected in Project Settings, outgoing angular velocities remain unchanged and outgoing translational velocities are calculated as follows:

$$\left.\begin{aligned} \boldsymbol{v'_n} &= \boldsymbol{r_n} \times \boldsymbol{v_n} \\ \boldsymbol{v'_{t1}} &= \boldsymbol{r_t} \times \boldsymbol{v_{t1}} \\ \boldsymbol{v'_{t2}} &= \boldsymbol{r_t} \times \boldsymbol{v_{t2}} \end{aligned}\right\} \qquad \text{......(2.1a)}$$

If to "consider rotations", outgoing translational and angular velocities are calculated as follows:

$$\left.\begin{aligned} \boldsymbol{v'_n} &= \boldsymbol{r_n} \times \boldsymbol{v_n} \\ \boldsymbol{v'_{t1}} &= \sqrt{\frac{\boldsymbol{r^2 \cdot (I \cdot \omega_{t2}{}^2 + M \cdot v_{t1}{}^2) \cdot f(F) \cdot SF}}{\boldsymbol{I + M \cdot r^2}}} \\ \boldsymbol{v'_{t2}} &= \sqrt{\frac{\boldsymbol{r^2 \cdot (I \cdot \omega_{t1}{}^2 + M \cdot v_{t2}{}^2) \cdot f(F) \cdot SF}}{\boldsymbol{I + M \cdot r^2}}} \\ \boldsymbol{\omega'_n} &= \boldsymbol{\omega_n} \\ \boldsymbol{\omega'_{t1}} &= \boldsymbol{-v_{t2}/r} \\ \boldsymbol{\omega'_{t2}} &= \boldsymbol{v_{t1}/r} \end{aligned}\right\} \qquad \text{......(2.1b)}$$

where:

r = equivalent radius = $\sqrt[3]{\frac{3V}{4\pi}}$

I = equivalent moment of inertia = $\frac{2}{5}Mr^2$

f(F) = Friction function

$$= r_t + \frac{1-r_t}{\left(\frac{v_{t1}-\omega_{t2}\cdot r}{f1}\right)^2+\left(\frac{v_{t2}+\omega_{t1}\cdot r}{f1}\right)^2+1.2} \quad \text{……(2.2)}$$

SF = Scaling Factor

$$= \frac{r_t}{\left(\frac{v_n}{f2\cdot r_n}\right)^2+1}$$

f1 = constant. 6.096 with metric units; 20 with imperial units.

f2 = constant. 76.2 with metric units; 250 with imperial units.

3. Sliding/Rolling

Pseudo-sliding in used in the lumped mass method, meaning that velocity is reduced, but angular velocity is unchanged since it's just a particle. Frictional force is simply the coefficient of friction multiplied by the normal force.

$$Friction = \mu \cdot N \quad \text{……(3.1)}$$

The coefficient of friction φ is simply:

$$\mu = \tan(\varphi) \quad \text{……(3.2)}$$

The frictional force is in the tangential direction only, opposite to the rock's direction of travel. It is simply:

$$\vec{F}_{FR} = -\mu \cdot N \cdot \hat{v} \quad \text{……(3.3)}$$

While the rock is sliding on the ground, we have the gravity and the friction. In t1/t2/n frame, we can write:

$$\overrightarrow{F_g} = (F_{g1}, F_{g2}, 0) \quad \text{……(3.4)}$$

$$\vec{F} = \vec{F}_{FR} + \overrightarrow{F_g} \quad \text{……(3.5)}$$

$$\vec{a} = \frac{\vec{F}}{M} \quad \text{……(3.6)}$$

The rock's equation of motion can then be written as:

$$\boldsymbol{\vec{p}(t) = \vec{v}_o \cdot t + \frac{1}{2}\vec{a} \cdot t^2} \quad \text{……(3.7)}$$

$$\boldsymbol{\vec{v}(t) = \vec{v}_o + \vec{a} \cdot t} \quad \text{……(3.8)}$$

4. References

[1] Taylor, Camillo J.; Kriegman, David J. (1994). *"Minimization on the Lie Group SO(3) and Related Manifolds"*. Technical Report No. 9405. Yale University.

[2] Pfeiffer, T.J. Bowen, T.D. (1989). “Computer Simulation of Rockfalls.” *Bulletin of the Association of Engineering Geologists Vol. XXVI, No. 1, 1989 p 135-146.*