ROCSCIENCE

RocFall3

Lumped Mass and Rigid Body (Legacy Sphere)

Verification Manual

© 2025 Rocscience Inc.

Table of Contents

RocFall3 Lumped Mass Verifications

This document presents several examples from RocFall2 models and hand calculations, which have been used as verification problems for RocFall3. RocFall3 is a 3D engineering analysis program for assessing rockfall risks in rock slopes, produced by Rocscience Inc. of Toronto, Canada. The purpose of this verification is to confirm that the lumped mass trajectories, impact and sliding algorithm used by the program is working correctly.

The trajectory algorithm calculates the motion of the rocks while they are travelling in the air and finds the closest/next contact point with the slope. The impact algorithm takes the inbound contact geometry and velocities and calculate the outbound velocities. The sliding algorithm computes the rock's motion against friction on the ground.

When comparing with RocFall2, selected RocFall2 models are extruded and replicated in RocFall3. Two scenarios are compared for each of the tutorials. First scenario contains only one rock with no stats variations defined, the second scenario contains the same RocFall2 model (with the same stats variations defined and same number of rock throws) duplicated in RocFall3. Due to the inherent differences in RocFall2 and RocFall3, especially how the random numbers are sampled, you will see that with the second scenarios the results are not exactly identical. Nevertheless, they are statistically identical.

The results produced by RocFall3 agree very well with RocFall2 and hand calculations, which affirms the reliability of RocFall3 results.

1. RocFall2 Tutorial #1

[RocFall2 Build 8.017 & RocFall3 Build 1.002]

1.1. Model

Tutorial1 in RocFall2 contains a simple slope with lumped mass method and rotational velocity considered. The slope vertices don't contain any stats variations.

Table 1-1: Slope Geometry

Vertex	X	X Std. Dev.	Y	Y Std. Dev.
1	0	N/A	0	N/A
2	3.1	N/A	-12.2	N/A
3	6.7	N/A	-12.2	N/A
4	9.8	N/A	-24.4	N/A
5	13.4	N/A	-25	N/A
6	26	N/A	-22.5	N/A

There is one point seeder defined at (0.5,0). We're going to throw down 50 rocks of mass 1000kg and density 2700kg/m^3.

Table 1-1: Seeder Initial Conditions

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Horizontal Velocity (m/s)	1.5	Normal	0.15	0.45	0.45
Vertical Velocity (m/s)	0	None	N/A	N/A	N/A
Rotational Velocity (°/s)	0	None	N/A	N/A	N/A
Initial Rotation (°/s)	0	Uniform	N/A	0	360

There are 3 materials for this slope. The first and 3rd segments have the material “Type One” assigned. The 2nd and 4th segments are assigned the materials “Type Two” and “Type Three” respectively.

Table 1-2: Slope Material Definitions

“Type One” Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Normal Restitution	0.5	Normal	0.03	0.09	0.09
Tangential Restitution	0.9	Normal	0.03	0.09	0.09
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

“Type Two” Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Normal Restitution	0.4	Normal	0.03	0.09	0.09
Tangential Restitution	0.9	Normal	0.03	0.09	0.09
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

“Type Three” Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Normal Restitution	0.3	Normal	0.03	0.09	0.09
Tangential Restitution	0.9	Normal	0.03	0.09	0.09
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

For RocFall3, we just extrude the same geometry 50m into the y-direction and define a line seeder instead of a point seeder.

1.2. Scenario 1: No Stats Variation

For the first no variation scenario, turn all distribution to “None” and set the number of rocks to throw to “1”. In RocFall2 we obtained the following result:

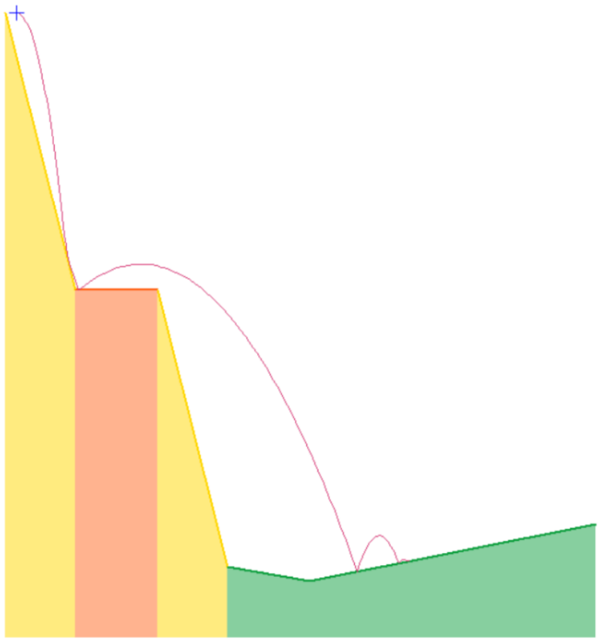

Figure 1-1: RocFall2 Scenario 1 Result

In RocFall3 we obtained the following result:

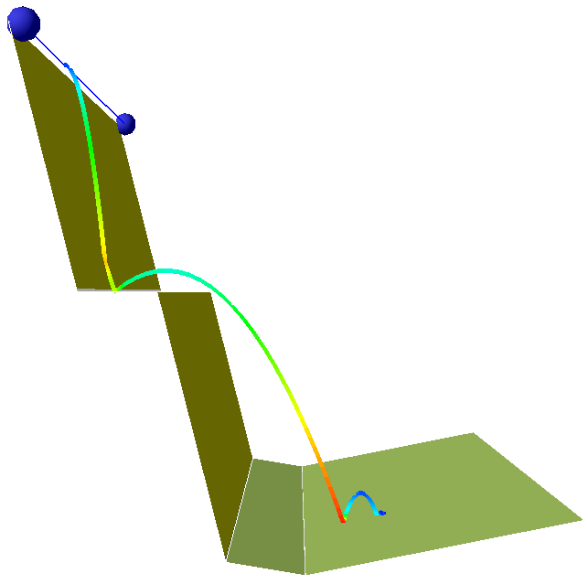

Figure 1-2: RocFall3 Scenario 1 Result

Detailed path results are included in Appendix A. For the most parts, the differences are less than 1%. The results produced by RocFall3 agree very well with RocFall2.

1.2.1. Energy Conservation

To ensure energy is conserved, we plot the Total Energy, Kinetic Energy and Potential Energy along the rock's x location along the slope.

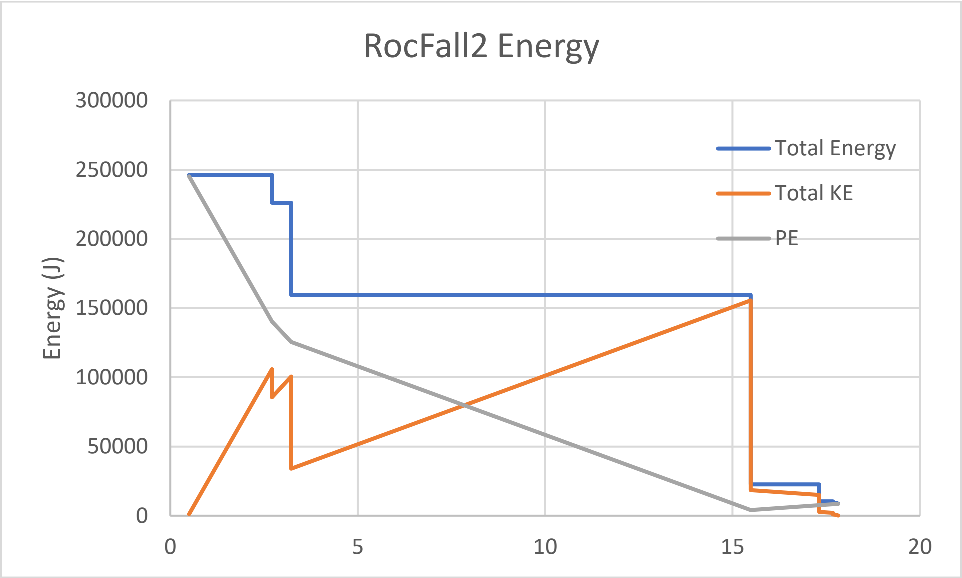

Figure 1-1: RocFall2 Energy Plot

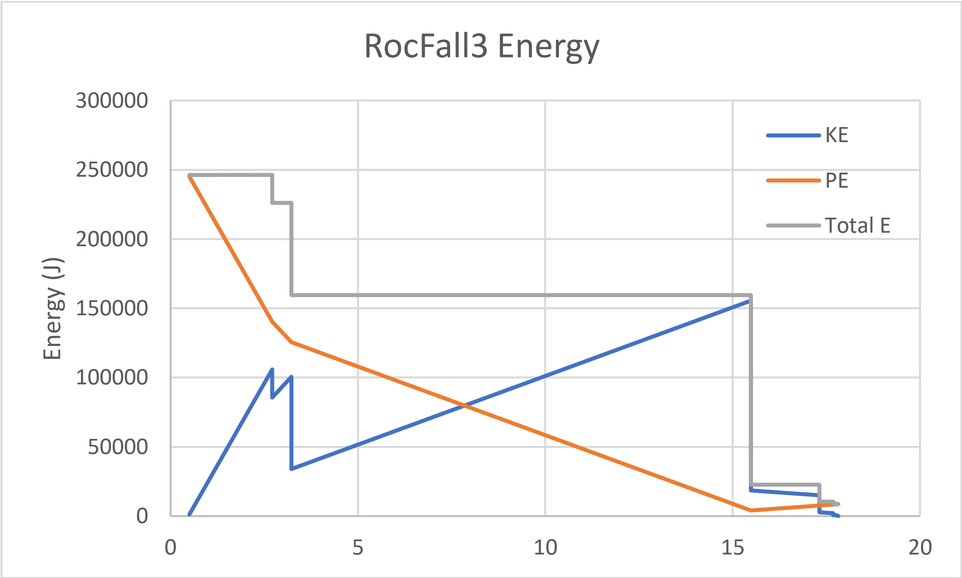

Figure 1-2: RocFall3 Energy Plot

It is observed that no energy is gained, and that energy is conserved.

1.3. Scenario 2: With Stats Variation

For the second scenario with stats variations, the model as described in the “Model” section is used. In RocFall2 we obtained the following result:

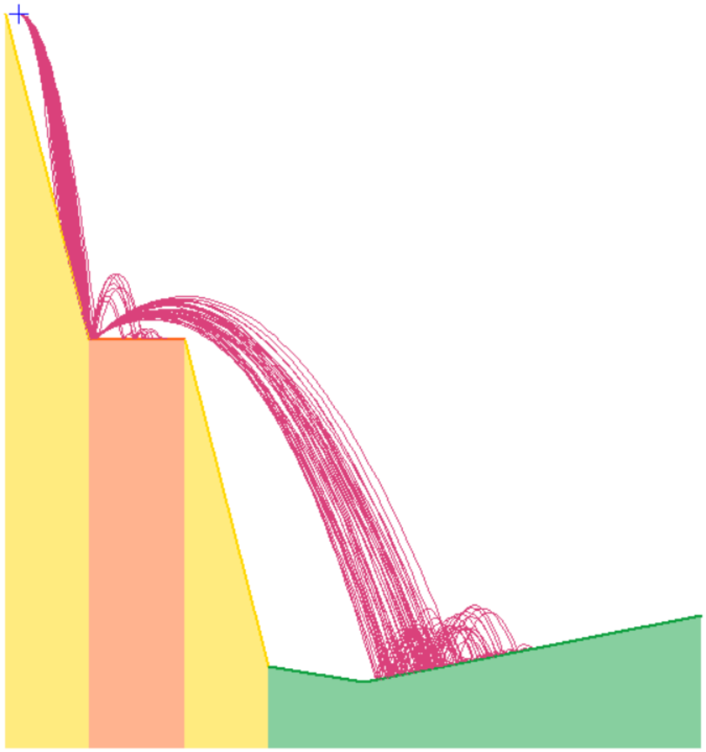

Figure 1-3: RocFall2 Scenario 2 Result

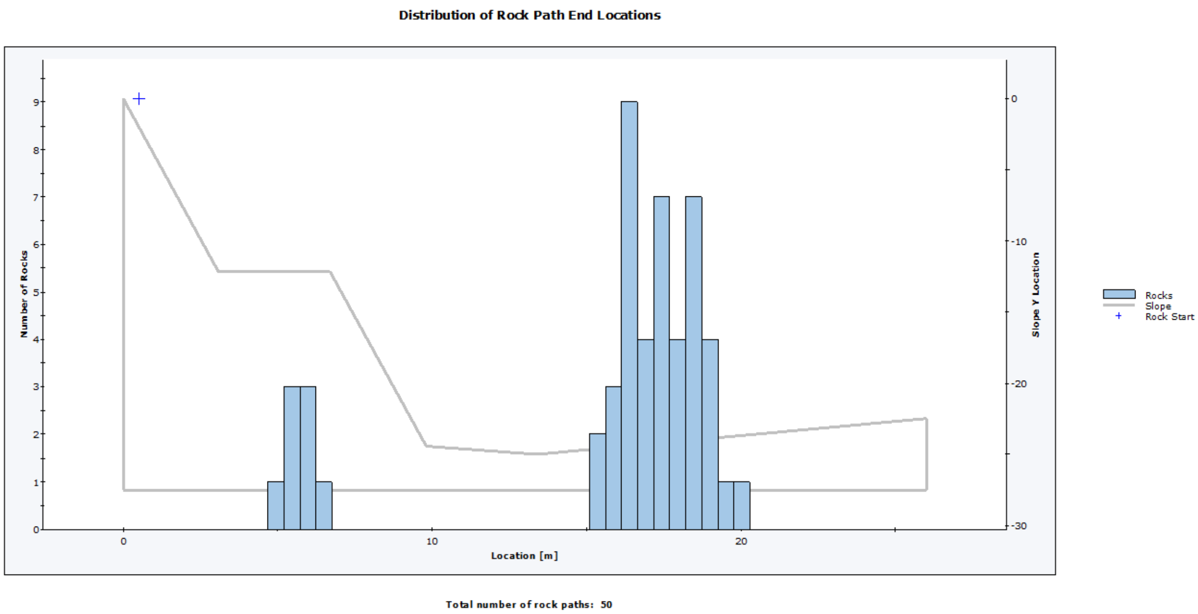

Figure 1-4: RocFall2 Scenario 2 End Points Plot

In RocFall3 we obtained the following result:

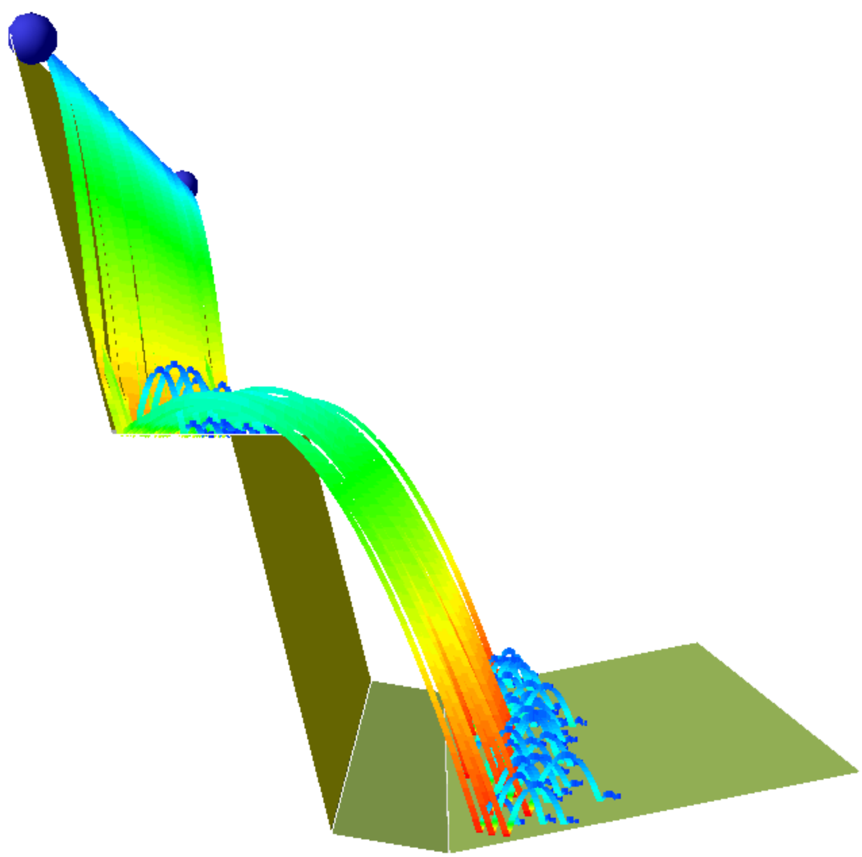

Figure 1-5: RocFall3 Scenario 2 Result

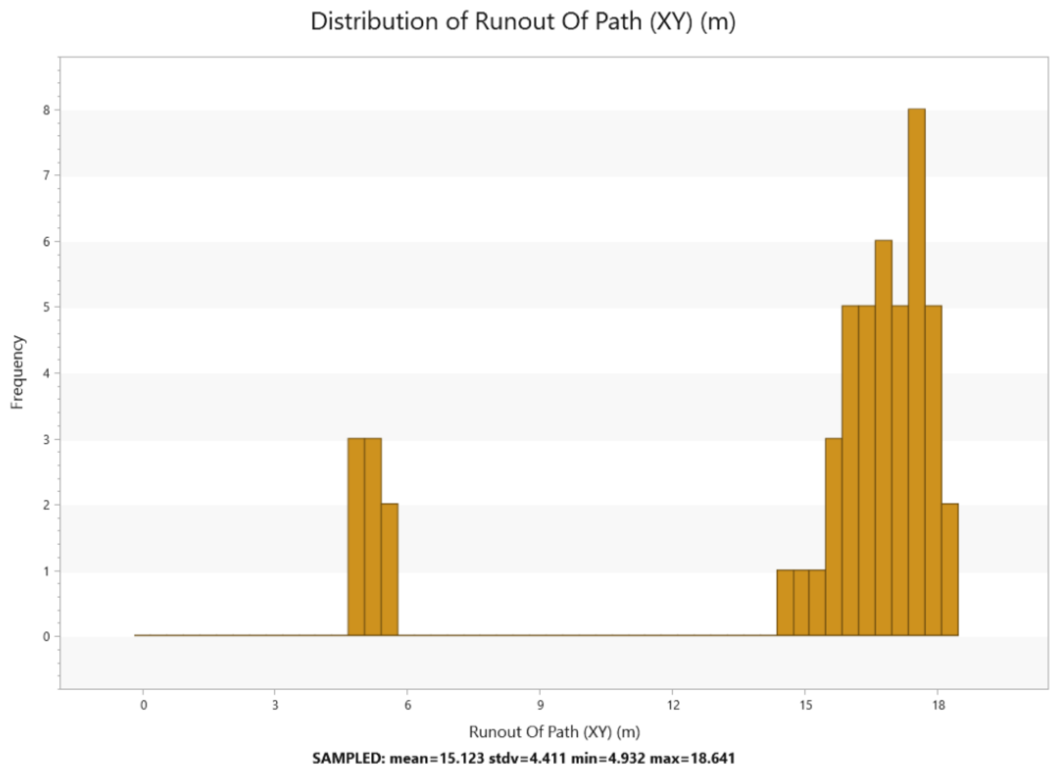

Figure 1-6: RocFall3 Scenario 2 End Points Plot

If we overlay the 2 end points histogram plots:

Figure 1-7: Scenario 2 2D vs 3D End Points Plot

It is observed that RocFall2 and RocFall3's end locations have very similar range and distribution. We run basic statistical analysis on the runout distances and got the following:

Table 1-3: Statistical Analysis on Runout Distances

	RocFall2	**RocFall3**
Mean	15.07462	15.12252
Standard Error	0.632232	0.623768
Median	16.35	16.86221
Standard Deviation	4.470553	4.410708
Sample Variance	19.98585	19.45434
Kurtosis	1.265138	1.434973
Skewness	-1.66522	-1.76917
Range	14.765	13.70954
Minimum	4.575	4.931751
Maximum	19.34	18.64129
Sum	753.731	756.1262
Count	50	50
Confidence Level(95.0%)	1.270517	1.253509

We also analyze and plot the end location in a box and whisker plot:

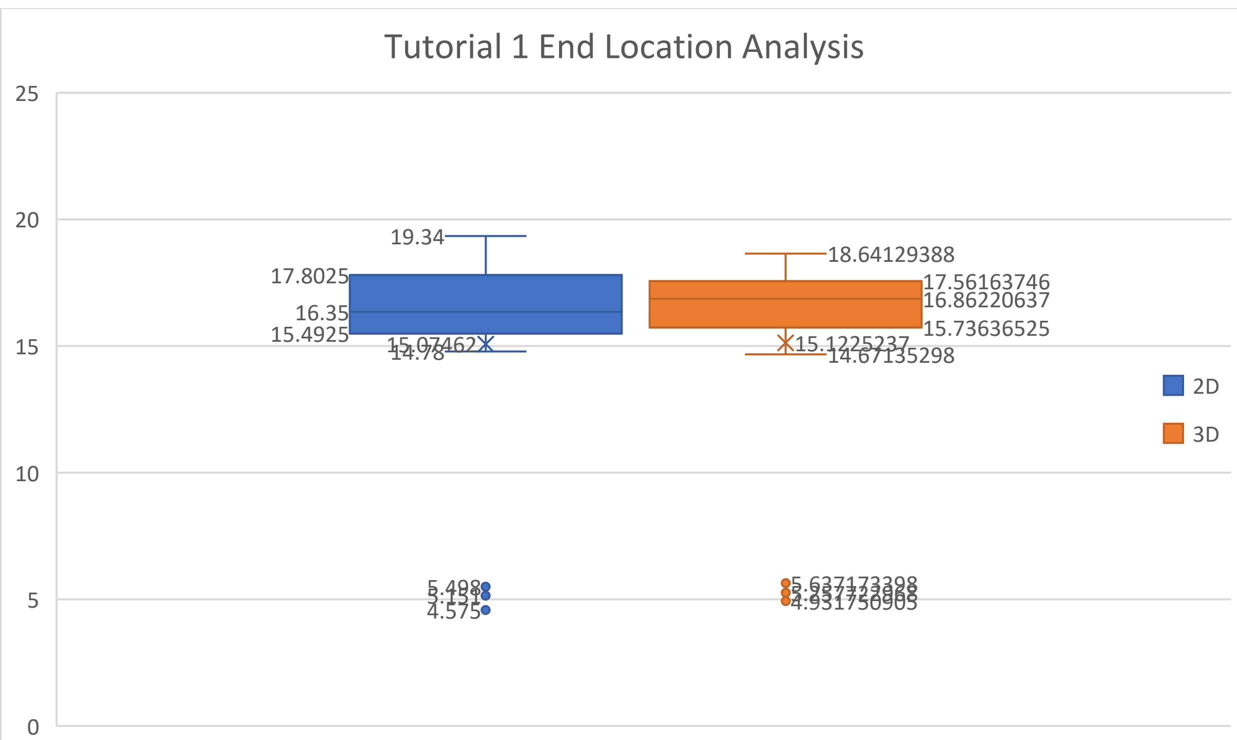

Figure 1-8: Scenario 2 2D vs 3D End Points Analysis Plot

We can conclude that the 2D and 3D end points are statistically identical. If we throw down more rocks than 50, we can expect to obtain even closer results.

In Appendix **Error! Reference source not found.** we include the histogram and box and whisker plots w ith 100 rock throws.

2. Impacts Verification against Hand Calculations

[RocFall3 Build 1.002]

2.1. Problem Description

The main purpose of this simulation is to confirm RocFall3's contact finding algorithm and impact calculations. The majority of the simulation time in RocFall3 takes place in the projectile algorithm. Any errors in the projectile algorithm would surely produce incorrect results. Therefore, it is essential that the projectile algorithm work correctly.

This verification example consists of a simple slope with 3 equilateral triangles of sides 20m. The 3 triangles consist of the same material (0.3 normal coefficient of restitution, 0.7 tangential coefficient of restitution and 30 degrees friction angle with no statistical variations). The seeder will start at (0,0,20) m with starting velocities of (3,0,0) m/s with no angular velocities. The rock has an equivalent radius of 0.5 m. With 2700kg/m^3 density, the rock's mass is 1413.7167 kg and its moment of inertia is 141.3717 kg-m^2. The lumped mass method is used, and rotations are considered in impact calculations.

To verify the projectile and impact algorithms, we are comparing the trajectory profile to that performed by hand calculations. Due to the complexity of the calculations, only the first 3 bounces were calculated by hand. Normally for engineering calculations we don't carry that many number of significant digits. We're doing so here to avoid accumulation of rounding errors when comparing with RocFall3 results by the computer.

Table 2-1: Slope Vertices

Vertex	X	Y	Z
A	-10	10	16.32993
B	10	10	16.32993
C	0	-7.32051	16.32993
D	0	4.226497	0

Table 2-2: Slope Geometry

Triangle	Vertex 1	Vertex 2	Vertex 3
1	A	B	D
2	A	C	E
3	B	C	D

Table 2-3: Default Material Properties

Property	Distribution	Mean	Std. Dev.	Rel. Min.	Rel. Max
Normal Restitution	None	0.3	N/A	N/A	N/A
Tangential Restitution	None	0.7	N/A	N/A	N/A
Friction Angle (º)	None	30	N/A	N/A	N/A

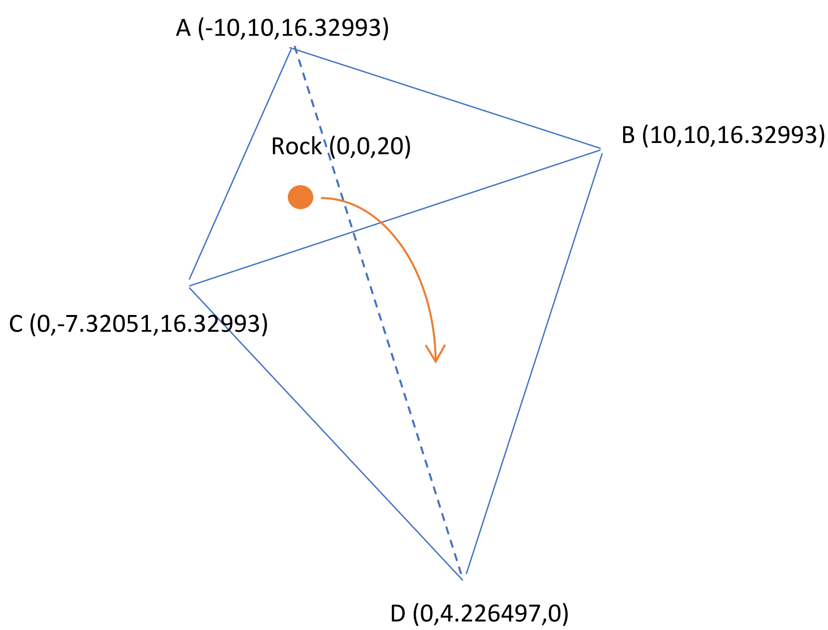

Figure 2-1: Slope Geometry

2.2. RocFall3 Analysis

First, we need to create the model in RocFall3. In a New Project, go to **Geometry** > **Draw Polyline**. In the left pane, make sure **Freehand** is selected in **Plane Orientation** and click on the **Edit Table** button.

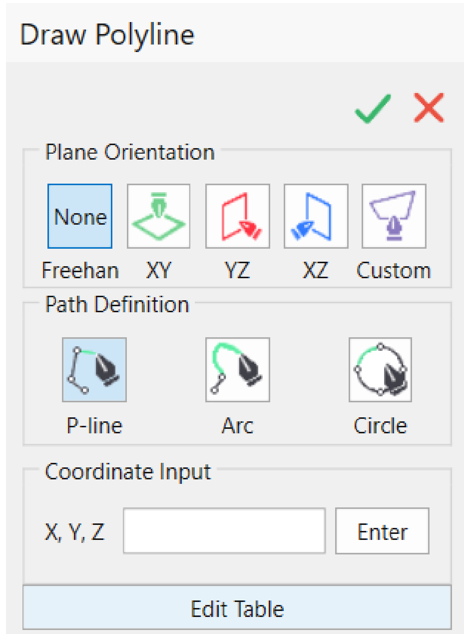

Figure 2-2: Draw Polyline Input Pane

In the **Edit Polyline** dialog, Click on **Insert Row** 4 times till you have 4 rows. Copy and Paste the information for Vertices A, B and D from Table 2-1 into the first 3 rows. Then copy Vertex A into the 4th row so it forms a closed triangle.

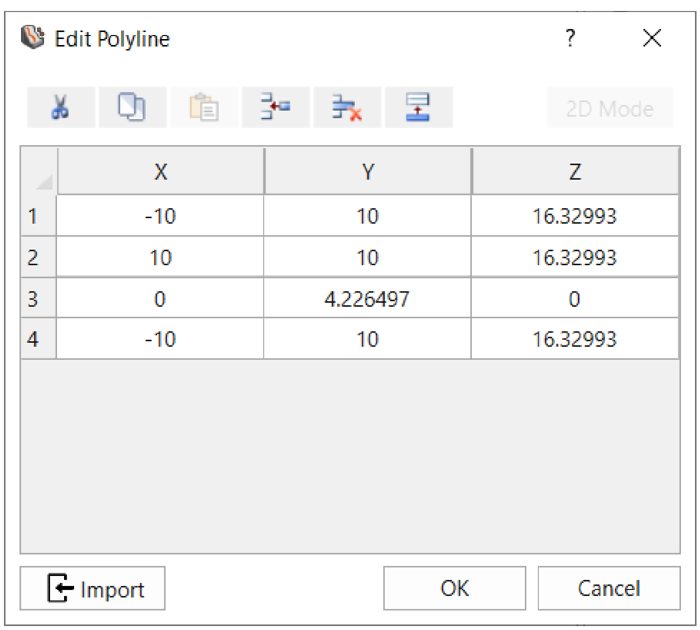

Figure 2-3: Edit Polyline dialog

Click on **OK**, and then the green check mark in the left pane to finish entering the first polyline. Repeat this process 2 more times for vertices A, C, D, A and B, C, D, B.

Now let's turn the 3 polylines into polygons. Select all 3 polylines from the Tree view. Go to **Geometry > Surface Triangulation Tools > Create Triangulation From Closed Polyline**.

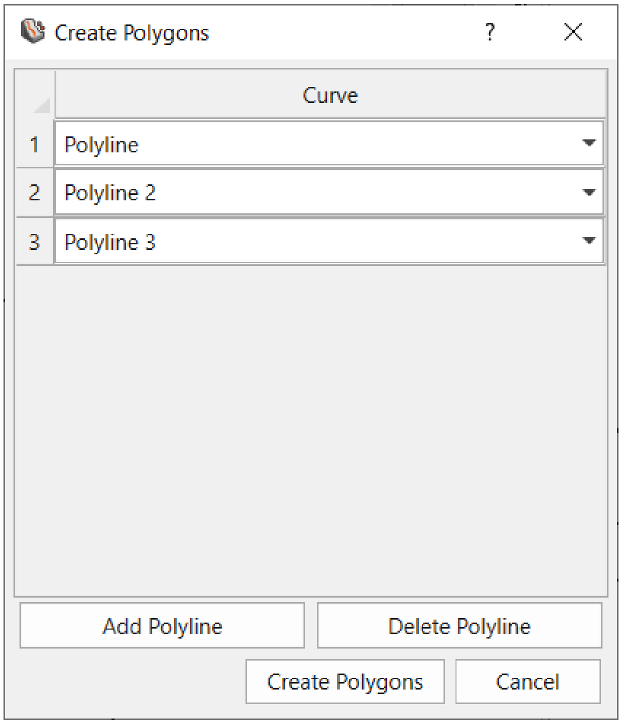

Figure 2-4: Create Polygons dialog

In the **Create Polygons** dialog, make sure all 3 polylines are listed and then click **Create Polygons**. The 3 polylines were turned into 3 polygons.

Now let's merge the 3 polygons into 1 surface. Select all 3 polygons in the Tree view. Go to **Geometry > Surface Triangulation Tools > Merge**. Click **OK** to accept the default tolerance option. The 3 polygons are now 1 polygon.

Go to **Geometry** and set the polygon as the slope surface. Then we define the material as shown in the Table 4-3 under Problem Description. Go to **Materials** > **Add/Edit Material Regions**. Add a new layer with the default material. Select the entire slope then click **on Add Region(s) From Selected**. Click **Save and Close**.

Now let's add a point seeder. Go to **Seeder** > **Add Point Seeder**. In the **Add Point Seeder** dialog, click on **Pick On Viewport** in the **Position** group. Randomly click on the view. Back in the **Add Point Seeder** dialog, manually type in **X = 0**, **Y = 0** and **Z = 20**. Then click **OK**.

In the **Seeder Properties** dialog, set **Translational Velocity = 3 m/s** with no distribution. Define the orientation vector as **<1,0,0>** and click **OK**.

The model is now ready to be run. Save the project and click on **Interpret > Compute**.

2.3. Analytical Solution

The projectile algorithm consists, mainly, of the process of determining the intersection between a parabola (the path the rock follows while it is in the air) and a triangle. The problem can be simplified by first imprinting the trajectory of the rock onto the triangle. Get the equation of the imprinted line and then solve the intersection between the line and the parabola.

For the first segment, the rock's trajectory is (3,0). In this case it's very obvious to see that it's going to intersect Triangle 3 (BCD). Along the X/Z plane the projectile looks like below:

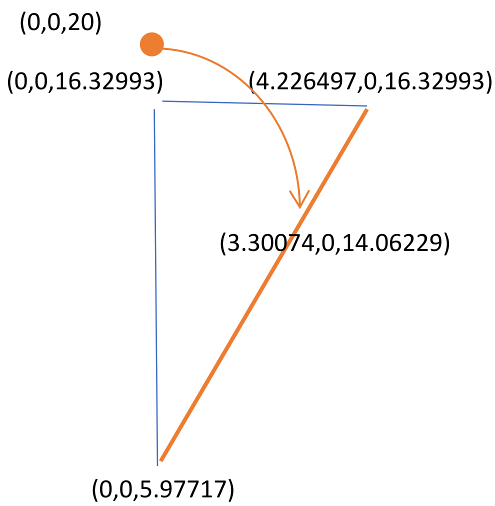

Figure 2-5: First Trajectory Segment

The equation of the imprinted line on the triangle is:

$$z = 5.97717 + x \cdot (16.32993 - 5.97717)/4.22650$$

The rock's trajectory can be described as:

$$\left.\begin{array}{l} x(t) = x_o + Vx \cdot t \\ y(t) = y_o + Vy \cdot t \\ z(t) = z_o + \frac{1}{2} \cdot g \cdot t^2 \\ \text{where } g = -9.81\ m/s^2 \end{array}\right\} \quad \ldots\ldots(2.1)$$

Plug the rock's trajectory back into the imprinted line and solve for t and we get t = 1.100246 sec.

We can update the rock's velocities at t = 1.100246 with the following relations:

$$\left.\begin{array}{l} Vx(t) = Vx_o \\ Vy(t) = Vy_o \\ Vz(t) = Vz_o + g \cdot t \end{array}\right\} \quad \ldots\ldots(2.2)$$

The rock will impact the slope at t = 1.100246 sec at (3.300738,0,14.06229) with velocity of (3,0,-10.7934).

Now we perform impact calculations. To do that, we need to first transform the incoming velocities to a normal and tangents frame. The normal of the triangle can be found with the following:

$$\overrightarrow{n_3} = \frac{\overline{BD}\times\overline{CD}}{|\overline{BD}\times\overline{CD}|} = (-0.8165, 0.471405, 0.333333)$$

The normal component of the velocity vector can be obtained by:

$$\overrightarrow{v_n} = \vec{v}\cdot\vec{n}(\vec{v})$$

The remaining tangential component is just $\overrightarrow{v_t} = \bar{v} - \overrightarrow{v_n}$ and we can make the first tangent direction in line with $\overrightarrow{v_t}$ for simplicity. The velocity vector in normal and tangential frame is then (-6.04729, 9.43017, 0) and there are no angular velocities.

The conversion matrix from x,y,z frame to n/t frame is:

$$M = \begin{bmatrix} -0.8165 & 0.47141 & 0.33333 \\ -0.20547 & 0.3023 & -0.93081 \\ -0.53955 & -0.82849 & -0.14997 \end{bmatrix}$$

The outgoing normal velocity is easy to calculate, it's simply:

$$\boldsymbol{v'_n = r_n \times v_n = -0.3 \times -6.04729 = 1.81419\ m/s}$$

$$SF = \frac{r_t}{\left(\frac{v_n}{f2\cdot r_n}\right)^2+1} = \frac{0.7}{\frac{6.047294534}{76.2\cdot 0.3}^2+1} = 0.65422$$

$$f(F) = r_t + \frac{1-r_t}{\left(\frac{v_{t1-\omega_{t2}\cdot r}}{f1}\right)^2+\left(\frac{v_{t2+\omega_{t1}\cdot r}}{f1}\right)^2+1.2} = 0.7 + \frac{1-0.7}{\frac{9.43017}{6.096}^2+1.2} = 0.78349$$

$$\boldsymbol{v'_{t1}} = \sqrt{\frac{\boldsymbol{r^2\cdot(I\cdot\omega_{t2}{}^2 + M\cdot v_{t1}{}^2)\cdot f(F)\cdot SF}}{\boldsymbol{I + M\cdot r^2}}} = \sqrt{\frac{\boldsymbol{0.5^2\cdot(1413.72\cdot 9.43^2)\cdot 0.654\cdot 0.783}}{\boldsymbol{141.37 + 1413.72\cdot 0.5^2}}}$$
$$= \boldsymbol{5.706\ m/s}$$

$$\boldsymbol{v'_{t2}} = \sqrt{\frac{\boldsymbol{r^2\cdot(I\cdot\omega_{t1}{}^2 + M\cdot v_{t2}{}^2)\cdot f(F)\cdot SF}}{\boldsymbol{I + M\cdot r^2}}} = \boldsymbol{0\ m/s}$$

$$\boldsymbol{\omega'_n = \omega_n = 0}$$

$$\boldsymbol{\omega'_{t1} = \frac{-v_{t2}}{r} = 0}$$

$$\boldsymbol{\omega'_{t2} = \frac{v_{t1}}{r} = \frac{5.706}{0.5} = 11.412\ rad/s}$$

We then need to translate the velocities from the normal and tangential frame to the general x,y,z frame.

$$\overrightarrow{v_{xyz}} = M^{-1}\times\overrightarrow{v_{nt}}$$

After the first impact, the rock is at (3.300738,0,14.06229) with velocity of (-2.65369,2.58014,-4.70648) and angular velocity of (-6.15741, -9.45478, -1.71143). We now need to find where the rock would impact the slope next.

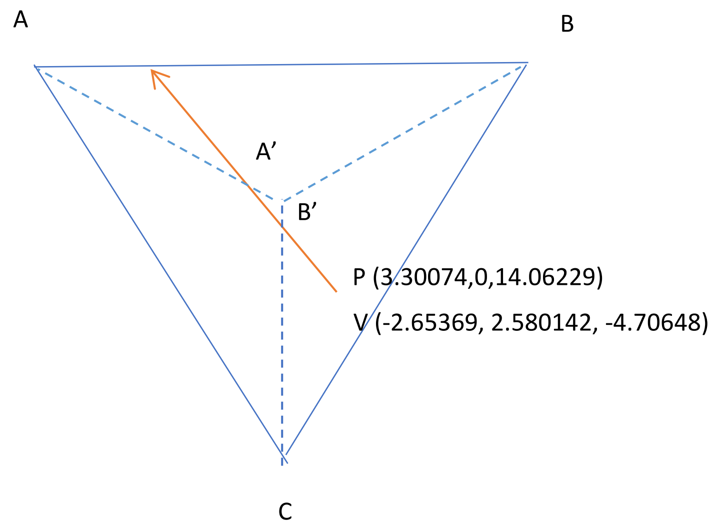

Figure 2-6: Second Trajectory in XY Plane

To find where the rock would hit the slope next, we can first find where points A' and B' are. We found that A' is at (-2.5757, 5.713577, 4.206097) and B' is at (0, 3.20926, 1.43859). Then we calculate where the rock's elevations are when they reach above/below A' and B'. The elevations are -20.4129 and 0.61961. From that we can see that the rock would hit the current triangle (BCD) again before reaching point B'. To find where the exact intersection is, we can find the equation of the imprinted line from the current position P to B'. Plug the rock's trajectory (Equation 2.1) back into the imprinted line and solve for t. We repeat the above procedure till we find 3 bounces on the slope.

2.4. Results

Path details from RocFall3 and hand calculations are summarized in Table 2-4 and Table 2-5 below. The results match quite well with differences less than 0.02%. This affirms the reliability of RocFall3 results.

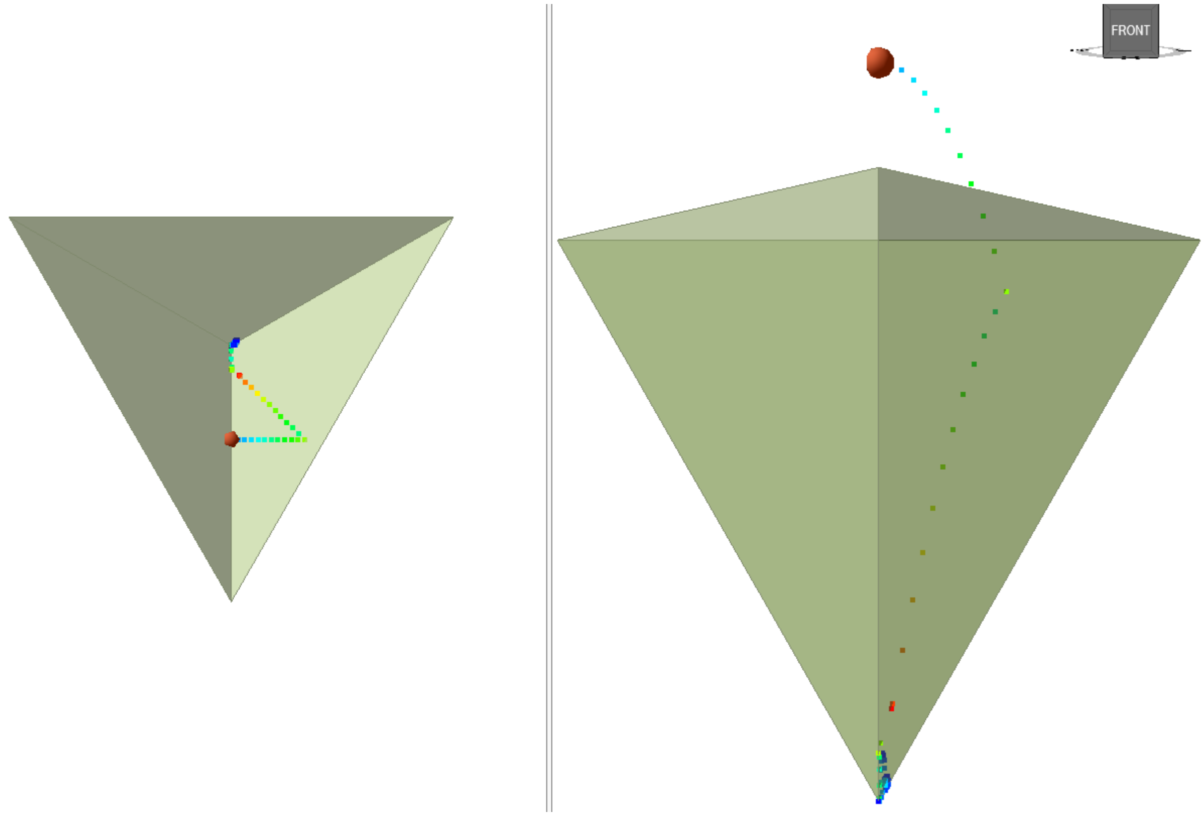

Figure 2-7: RocFall3 Result

2.4.1. Energy Conservation

To ensure energy is conserved, we plot the Total Energy, Kinetic Energy and Potential Energy along the rock's x location along the slope.

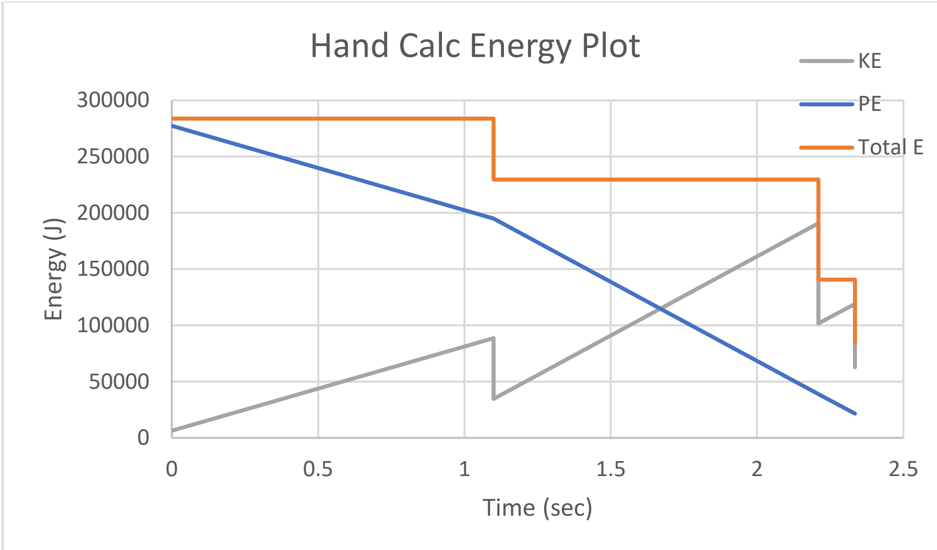

Figure 2-8: Hand Calc Energy Plot

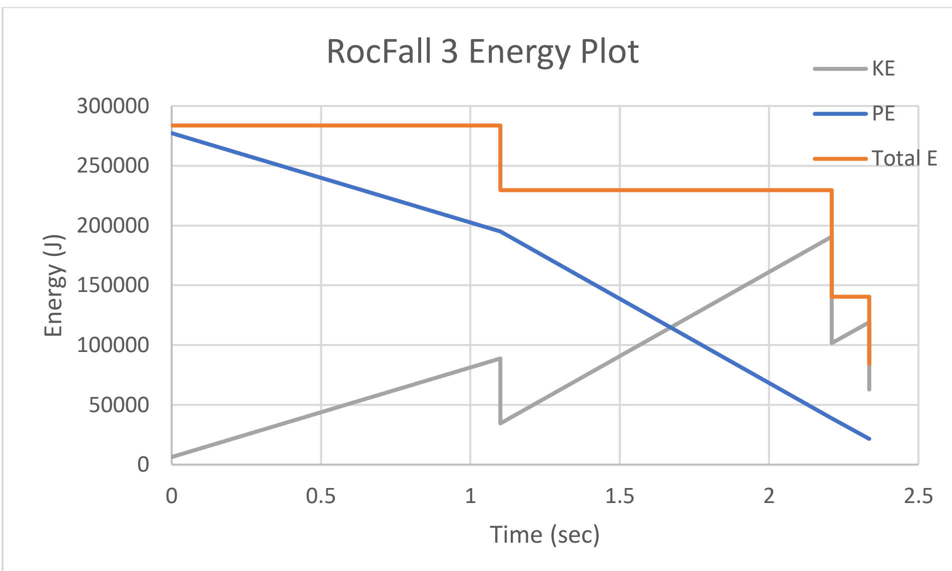

Figure 2-9: RocFall3 Energy Plot

It is observed that no energy is gained, and that energy is conserved.

Table 2-4: Summarized Path Results from RocFall3

ID	Time	Path State	Pos.x	Pos.y	Pos.z	Vel.x	Vel.y	Vel.z	ω.x	ω.y	ω.z	KE	PE	Total E
0	0	Projectile	0	0	20	3	0	0	0	0	0	6362	277276	283638
1	1.1004	slope impact	3.3011	0	14.0631	3	0	-10.7908	0	0	0	88669	194968	283638
2	1.1004	Projectile	3.3011	0.0000	14.0631	-2.6531	2.5798	-4.7053	-6.1561	-9.4525	-1.7115	34531	194968	229500
3	2.2102	slope impact	0.3566	2.8631	2.8017	-2.6531	2.5798	-15.5889	-6.1561	-9.4525	-1.7115	190658	38842	229500
4	2.2102	Projectile	0.3566	2.8631	2.8017	-3.0604	2.4633	-9.3476	-10.4552	-17.3048	-1.1372	101659	38842	140501
5	2.3352	slope impact	-0.0260	3.1711	1.5563	-3.0604	2.4633	-10.5738	-10.4552	-17.3048	-1.1372	118926	21576	140501

Table 2-5: Summarized Path Results by Hand Calculations

ID	Time	Path State	Pos.x	Pos.y	Pos.z	Vel.x	Vel.y	Vel.z	ω.x	ω.y	ω.z	KE	PE	Total E
0	0	Projectile	0	0	20	3	0	0	0	0	0	6362	277371	283733
1	1.10025	slope impact	3.3007	0	14.0623	3	0	-10.7934	0	0	0	88709	195024	283733
2	1.10025	Projectile	3.3007	0	14.0623	-2.6537	2.5801	-4.7065	-6.1574	-9.4548	-1.7114	34547	195024	229571
3	2.20984	slope impact	0.3562	2.8629	2.8010	-2.6537	2.5801	-15.5916	-6.1574	-9.4548	-1.7114	190725	38845	229571
4	2.20984	Projectile	0.3562	2.8629	2.8010	-3.0610	2.4636	-9.3492	-10.4569	-17.3078	-1.1371	101693	38845	140538
5	2.33471	slope impact	-0.0260	3.1705	1.5570	-3.0610	2.4636	-10.5741	-10.4569	-17.3078	-1.1371	118944	21594	140538

3. Sliding Verification against RocFall2 & Hand Calculations

[RocFall3 Build 1.002]

3.1. Problem Description

In Verification Problems #1 and #2, RocFall3 was verified against models where the paths consist predominantly of bounces. The program will now be verified for sliding. In this verification, a simple slope with materials of low normal coefficient of restitution is used for this purpose. Due to the simplified nature of the slope, we will also compare the results against hand calculations with one rock throw.

3.2. Model

The model contains a simple slope with lumped mass method and rotational velocity considered. The slope vertices don't contain any stats variations.

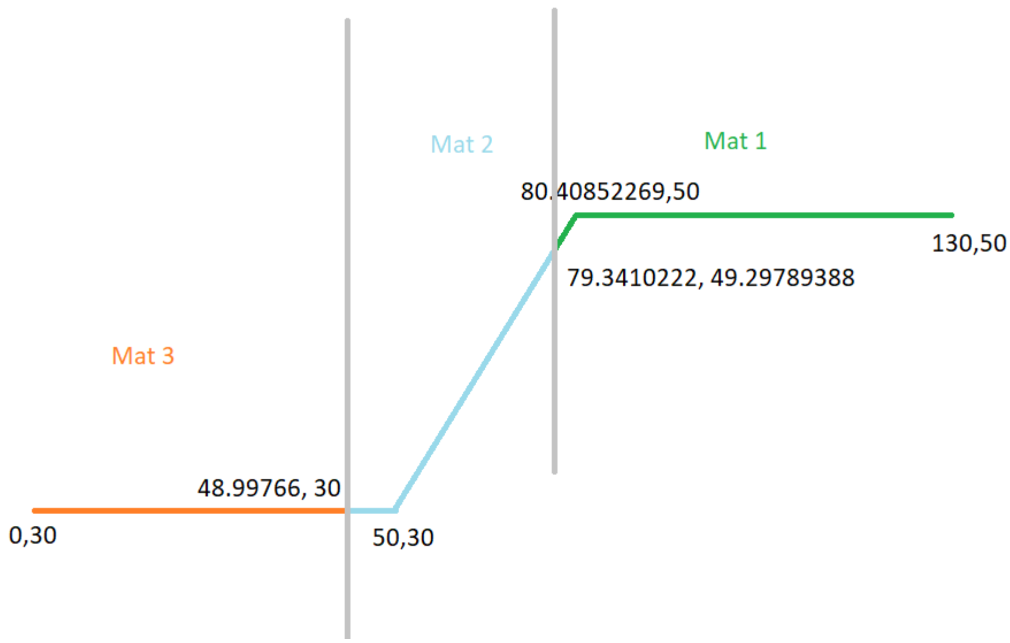

Figure 3-1: Slope Geometry and Material Assignment

Table 3-1: Slope Geometry

Vertex	X	X Std. Dev.	Y	Y Std.Dev.
1	0	N/A	30	N/A
2	48.99766	N/A	30	N/A
3	50	N/A	30	N/A
4	79.34102	N/A	49.29789	N/A
5	80.40852	N/A	50	N/A
6	130	N/A	50	N/A

There is one point seeder defined at (70,45.5). We're going to throw down rock(s) of mass 1000kg and density 2700kg/m^3.

Table 3.2-2: Seeder Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Horizontal Velocity (m/s)	10	Normal	0.5	1.5	1.5
Vertical Velocity (m/s)	0	None	N/A	N/A	N/A
Rotational Velocity (°/s)	0	None	N/A	N/A	N/A
Initial Rotation (°/s)	0	Uniform	N/A	0	360

Table 3-2: Slope Material Definitions

"Type One" Properties

	Mean	Distribution	Std.Dev.	Rel. Min	Rel. Max
Normal Restitution	0.03	Normal	0.01	0.03	0.03
Tangential Restitution	1.0	None	N/A	N/A	N/A
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

"Type Two" Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Normal Restitution	0.04	Normal	0.01	0.03	0.03
Tangential Restitution	1.0	None	N/A	N/A	N/A
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

"Type Three" Properties

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Normal Restitution	0.03	Normal	0.01	0.03	0.03
Tangential Restitution	1.0	None	N/A	N/A	N/A
Friction Angle (°)	20	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

3.3. Scenario 1: No Stats Variation

For this example, we'll remove any stats distributions for the seeder's horizontal velocity and "Type Two" slope material's normal restitution. The horizontal velocity will be 10.1825m/s and the "Type Two" material's normal restitution will be 0.03766. This is to ensure that we remove any effects from the random number generation differences.

3.3.1. RocFall3 Results

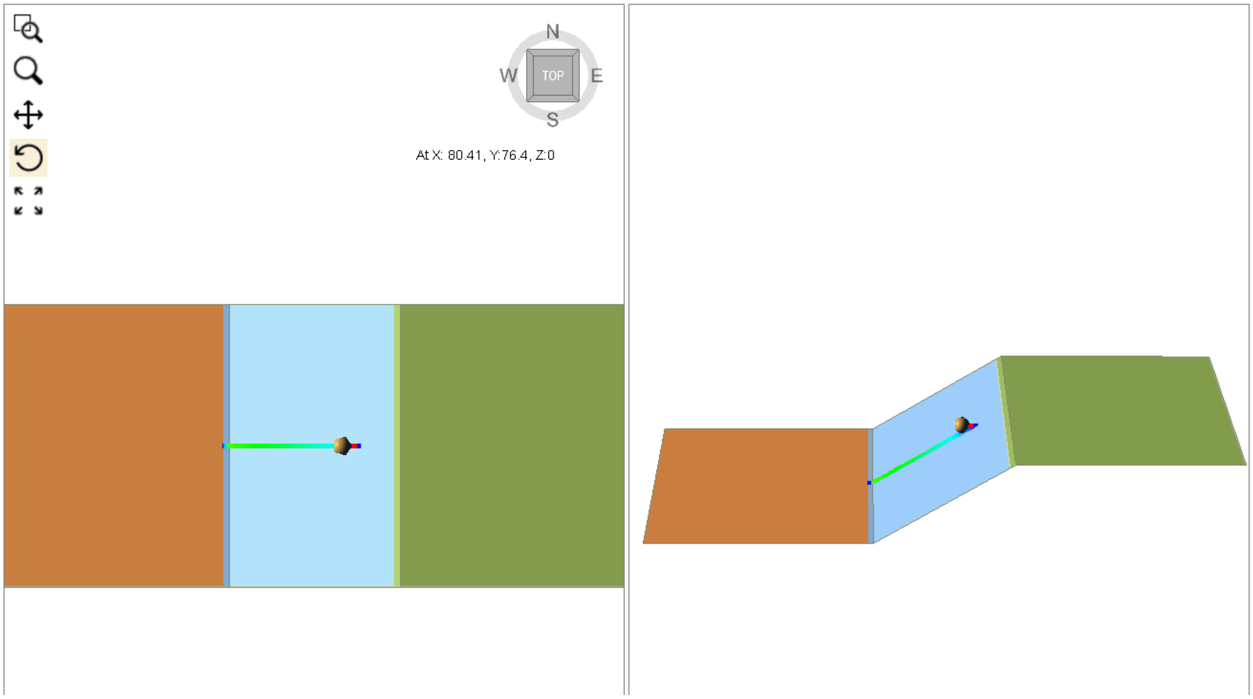

Figure 3-2: RocFall3 Results View

The rock stops at (49.2917, 30) at 10.1223 sec.

3.3.2. RocFall2 Results

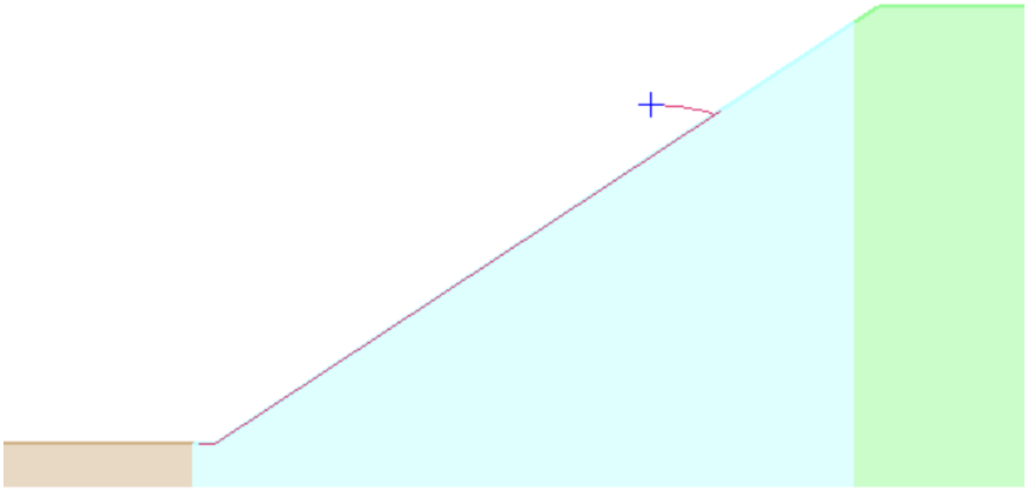

Figure 3-3: RocFall2 Results View

The rock stops at (49.212, 30) at 10.2398 sec.

3.3.3. Hand Calculations

The points of contact and impacts are calculated as per section 2.3. Sliding is calculated as follows:

After 2nd impact, the rock's position is at (73.0533, 45.1624) and velocity is (1.42188, 0.94871). The slope segment normal and tangent are (-0.54951, 0.8355) and (-0.8355,-0.54951) respectively. We calculate the normal and tangential velocity as:

$$v_n = \vec{v} \cdot \vec{n} = 0.0113\, m/s$$

$$v_t = \vec{v} \cdot \vec{t} = -1.7093\, m/s$$

Since the normal velocity is less than 0.1 m/s, the rock goes into sliding and it is sliding back uphill.

The normal force of the rock on the ground is:

$N = M \cdot \vec{g} \cdot \vec{n} = 8193\, N$

$Friction = \tan(\emptyset) \cdot N = \tan(30) \cdot 8193\, N = 4730\, N$ (in the tangential direction)

$Gravitational\ acceleration = \vec{g} \cdot \vec{t} = 5.39\, m/s^2$ (in the tangential direction)

Time to when the rock stops before sliding back down:

$$dt = \frac{v_t}{\vec{g} \cdot \vec{t} + \frac{Friction}{M}} = 0.1689\, sec$$

We update the rock's status at $t = 0.3623 + 0.1689 = 0.5312\, sec$.

$$Position = P(t) = \overrightarrow{Po} + \overrightarrow{v_0} \cdot dt + \frac{1}{2} \cdot \overrightarrow{v_0} \cdot dt^2 = (73.1739, 45.2417)$$

$$v(t) = \overrightarrow{v_0} + \vec{a} \cdot dt = (-0.0062, 0.0094)$$

We repeat this process till the rock ultimately stops at (49.411, 30) at 10.1799 sec on the lower plane.

3.3.4 Summary

Based on the end point locations, the differences with the 3 methods are within 0.5%.

Table 3-3: End Locations Compare

	End Locations	Differences
RocFall3	49.2244	-0.12%
RocFall2	49.212	-0.14%
Hand Calculations	49.41078	0.26%

Detailed path results are included in Appendix **Error! Reference source not found.**. For the most parts, t he differences are less than 1%. The results produced by RocFall3 agree very well with RocFall2 and hand calculations.

3.3.5 Energy Conservation

To ensure energy is conserved, we plot the Total Energy, Kinetic Energy and Potential Energy along the rock's x location along the slope.

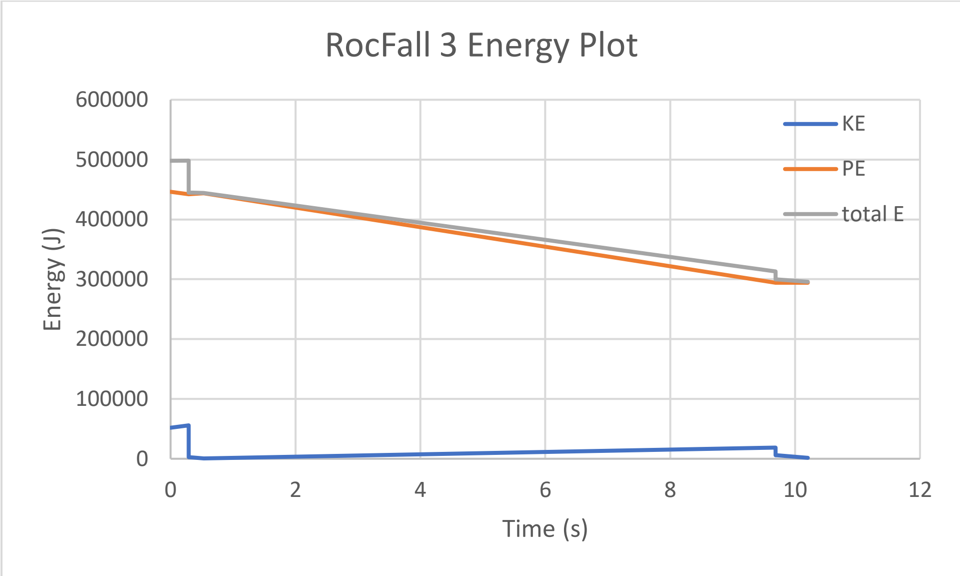

Figure 3-4: RocFall3 Energy Plot

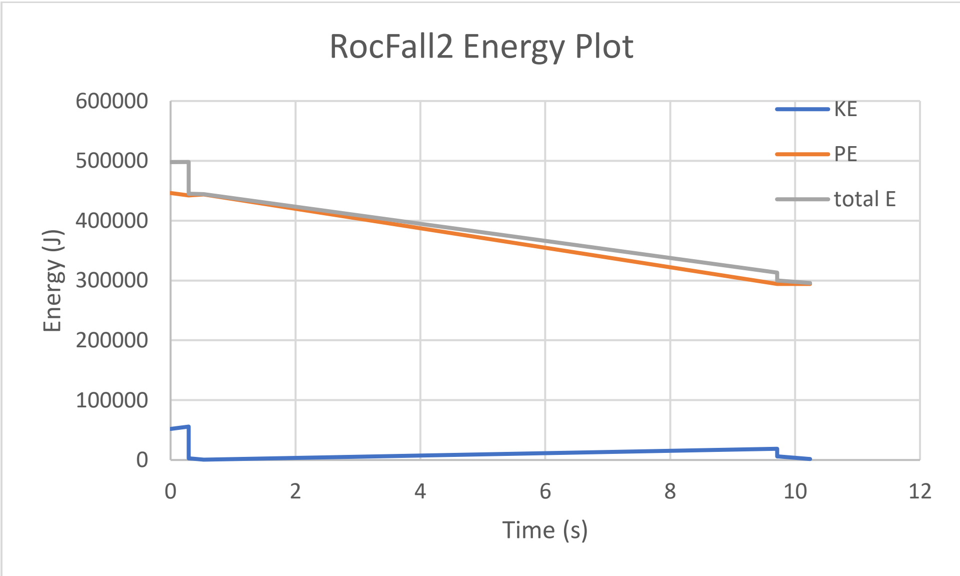

Figure 3-5: RocFall2 Energy Plot

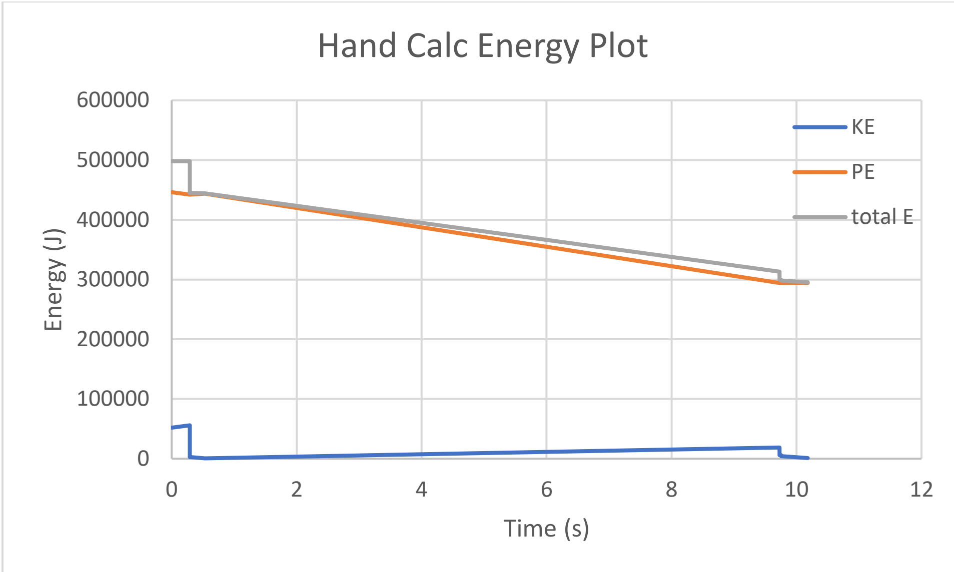

Figure 3-6: Hand Calc Energy Plot

It is observed that no energy is gained, and that energy is conserved.

3.4. Scenario 2: With Stats Variation

For the second scenario with stats variations, the model as described in the “Model” section is used with 100 rock paths. In RocFall2 we obtained the following result:

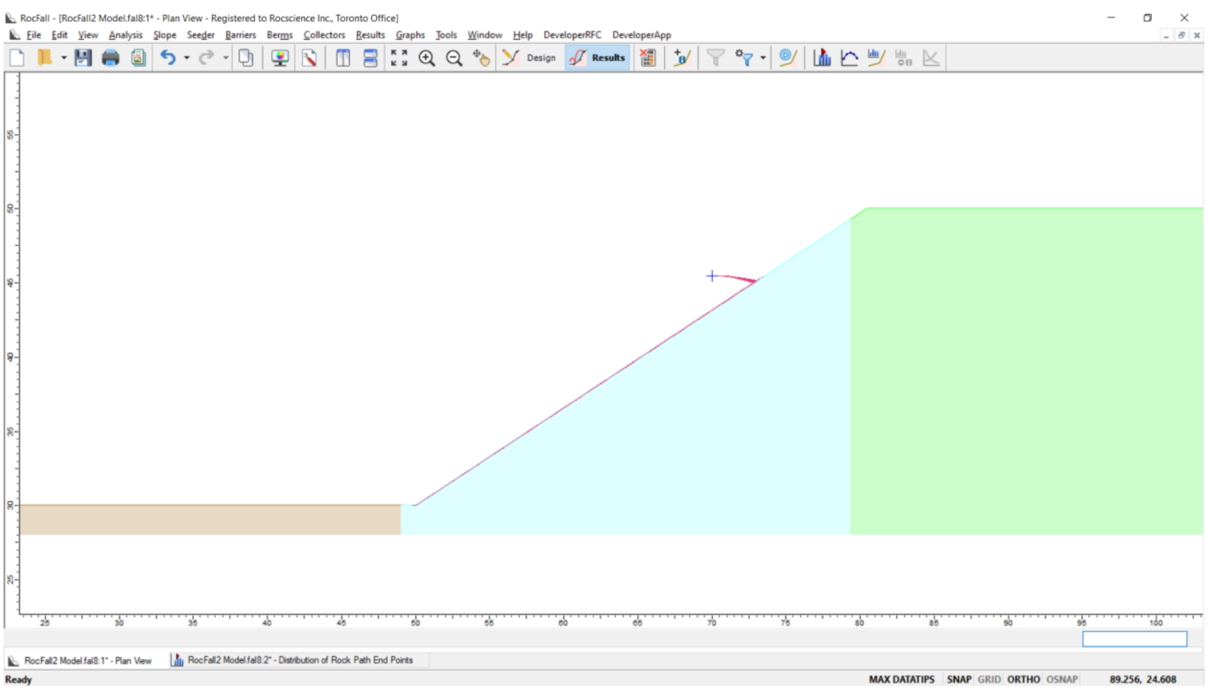

Figure 3-7: RocFall2 Scenario 2 Result

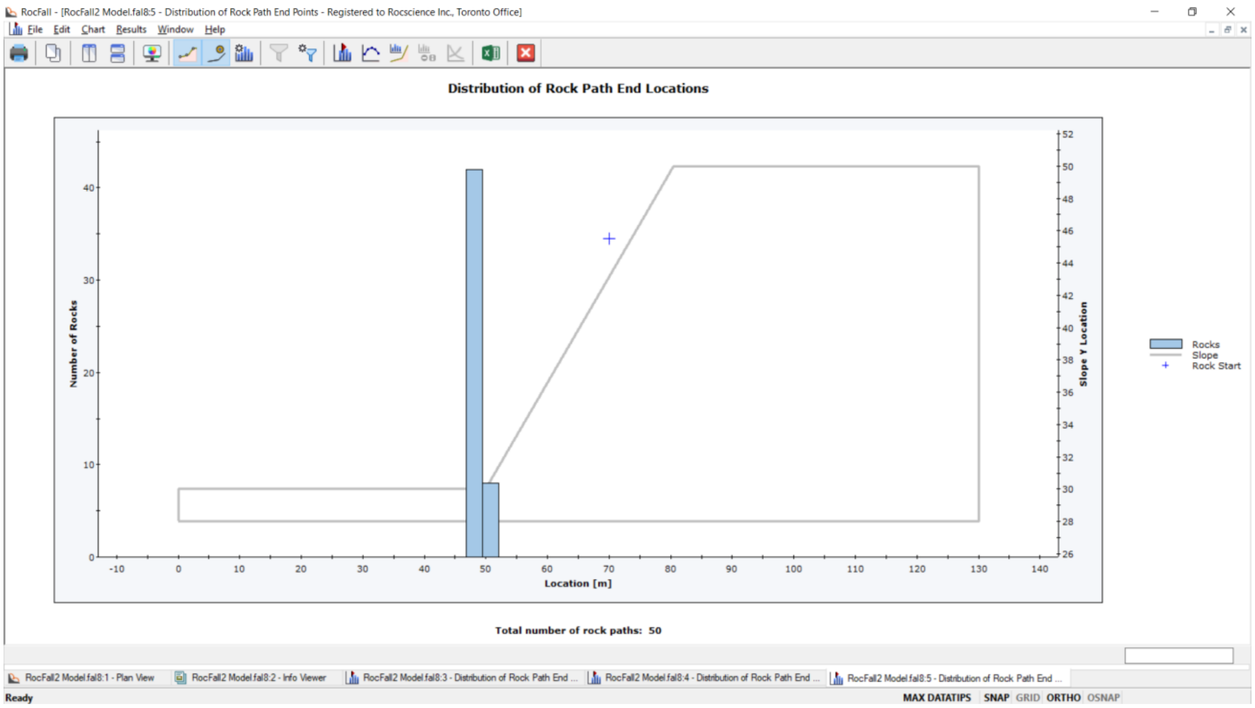

Figure 3-8: RocFall2 Scenario 2 End Points Plot

In RocFall3 we obtained the following result:

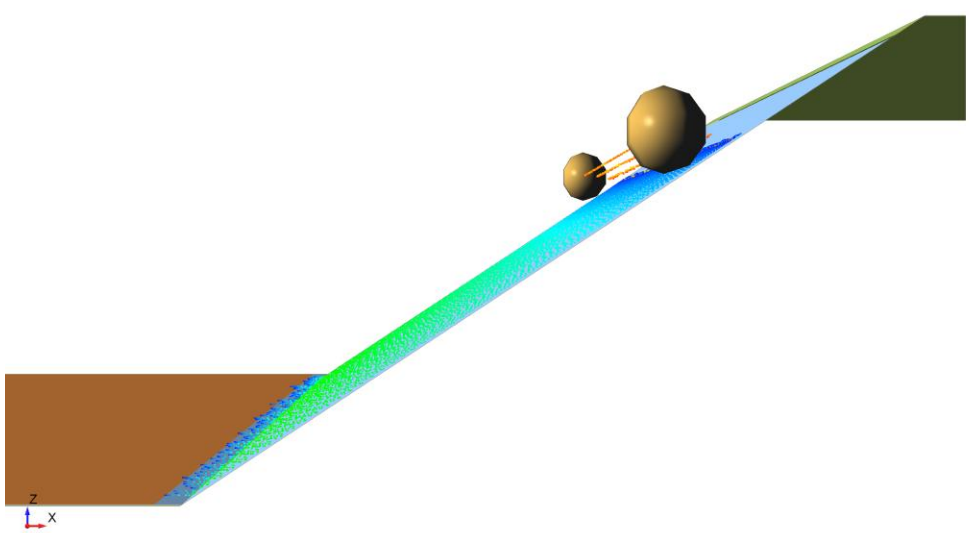

Figure 3-9: RocFall3 Scenario 2 Result

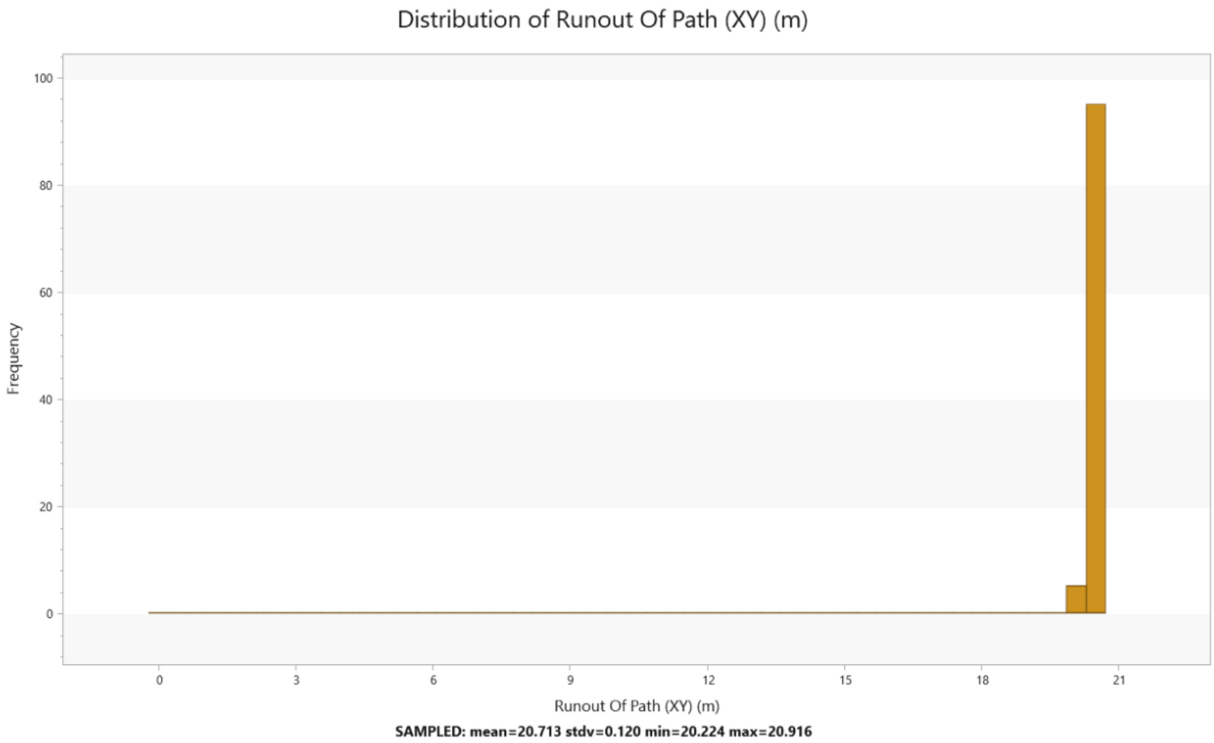

Figure 3-10: RocFall3 Scenario 2 Runout Distance Plot

3.4.1 Results

If we overlay the 2 end points histogram plots:

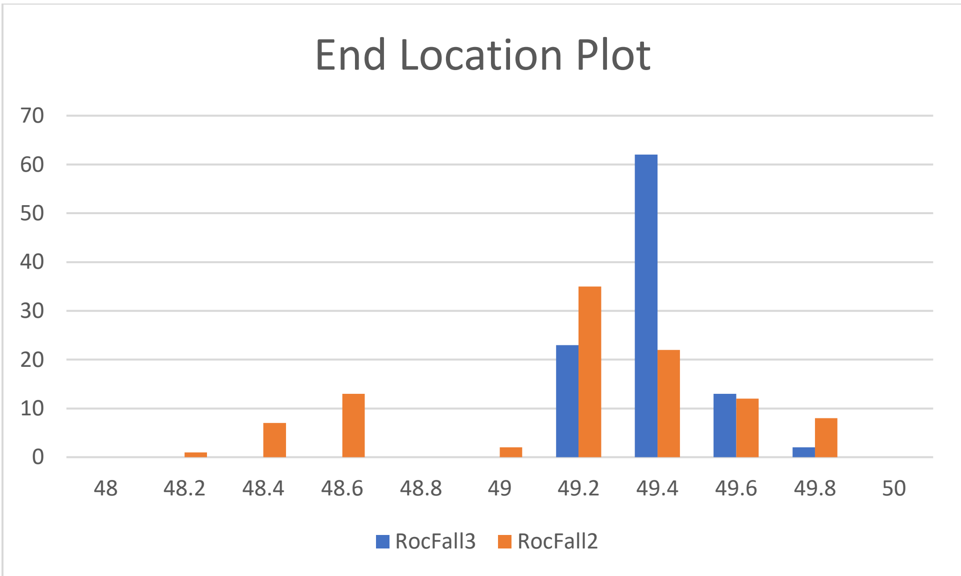

Figure 3-11: Scenario 2 2D vs 3D End Points Plot

It is observed that RocFall2 and RocFall3's end locations have very similar range and distribution. We analyze and plot the end location in a box and whisker plot:

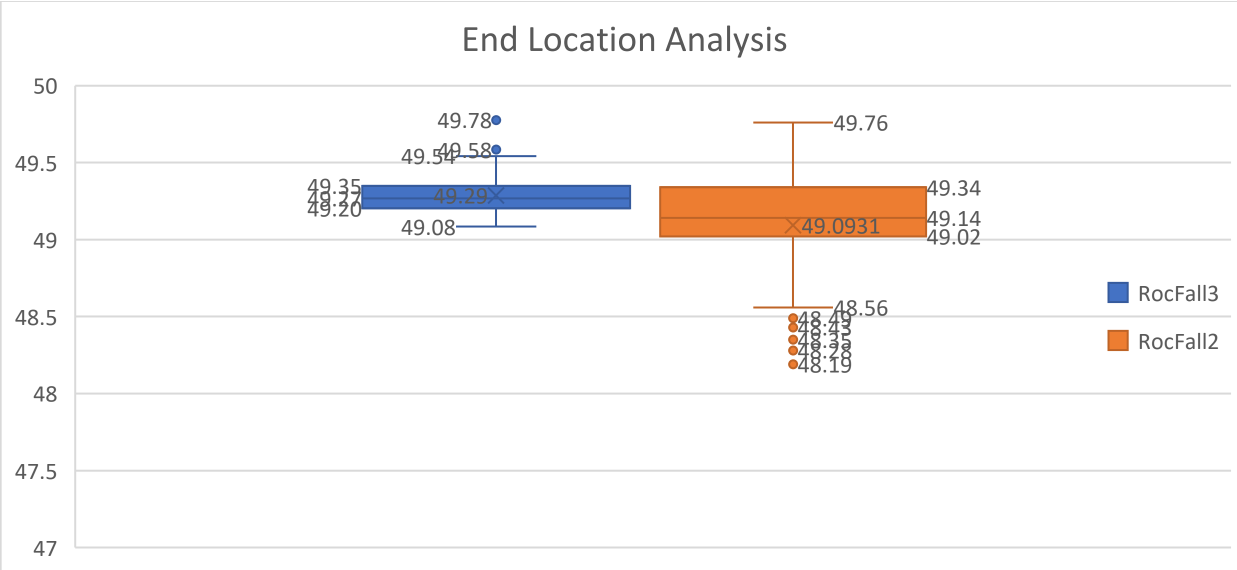

Figure 3-12: Scenario 2 2D vs 3D End Points Analysis Plot

We run basic statistical analysis on the runout distances and got the following:

Table 3-4: Statistical Analysis on Runout Distances

	3D	2D
Mean	49.287279	49.0931
Standard Error	0.011957118	0.03981845
Median	49.26705	49.14
Standard Deviation	0.119571179	0.398184504
Sample Variance	0.014297267	0.158550899
Kurtosis	2.182031306	-0.328533736
Skewness	1.144794449	-0.681774132
Range	0.6919	1.57
Minimum	49.0842	48.19
Maximum	49.7761	49.76
Sum	4928.7279	4909.31
Count	100	100
Confidence Level(95.0%)	0.023725516	0.079008444

We can conclude that the 2D and 3D end points are statistically very similar. If we throw down more rocks than 100, we can expect to obtain even closer results.

4. RocFall3 Barrier Verification Problem

[RocFall3 Build 1.002]

4.1. Problem Description

The main purpose of this verification is to confirm RocFall3's barrier contact finding algorithm and data post processing. We will be comparing the results between RocFall3 and RocFall2.

This verification example consists of a simple slope based on the slope in Verification 1 with only one material. A barrier with infinite capacity of height 3.5m is added at x = 15m. The rock has a 2700kg/m^3 density, and its mass is 1000 kg. The lumped mass method is used, and rotations are considered in impact calculations.

4.2. Model

See Section **Error! Reference source not found.** for slope vertices and seeder location and initial c ondition. Set the number of rocks thrown to 100.

Table 4-1: Slope Material Definitions

"Type One" Properties

	Mean	Distribution	Std.Dev.	Rel. Min	Rel. Max
Normal Restitution	0.45	Normal	0.03	0.09	0.09
Tangential Restitution	0.9	Normal	0.03	0.09	0.09
Friction Angle (°)	30	None	N/A	N/A	N/A
Slope Roughness (°)	N/A	None	N/A	N/A	N/A

4.3. Results

In RocFall2 we obtained the following result:

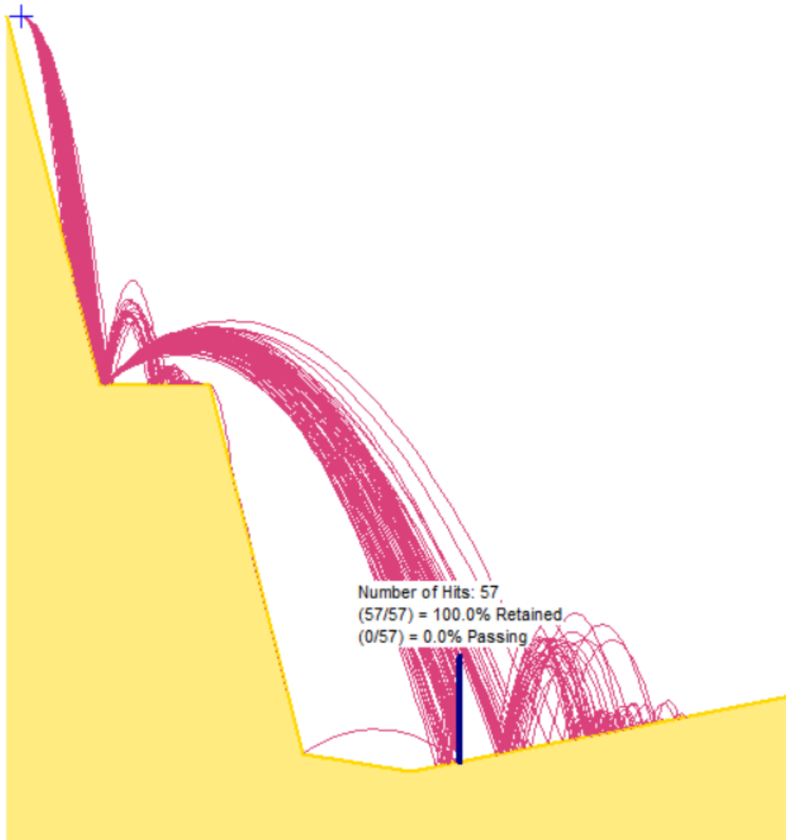

Figure 4-1: RocFall2 Results

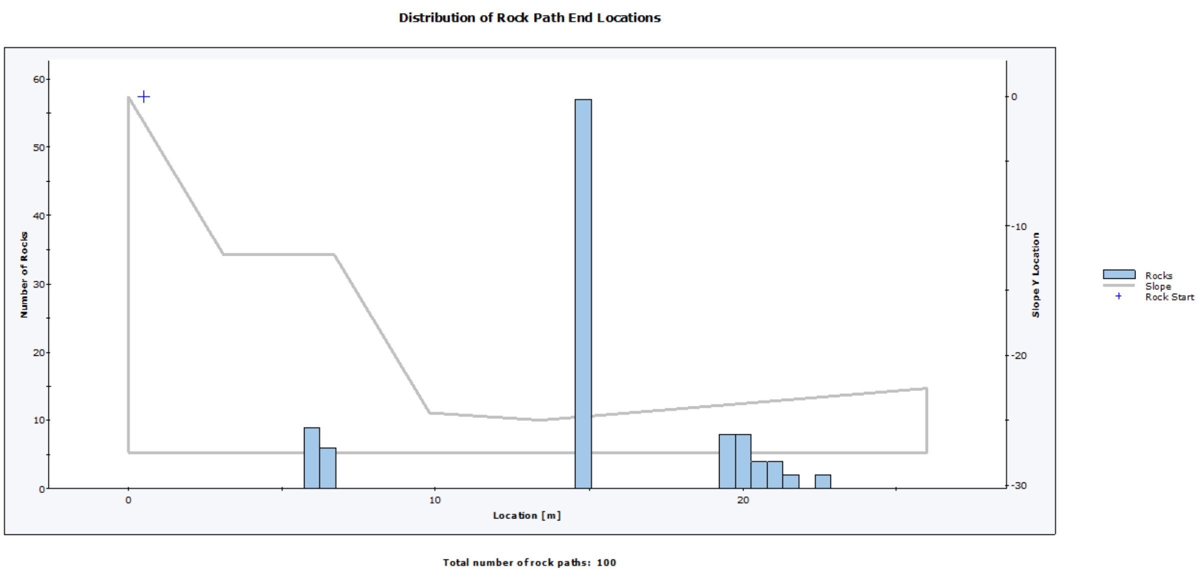

Figure 4-2: RocFall2 End Locations Plot

In RocFall3 we obtained the following result:

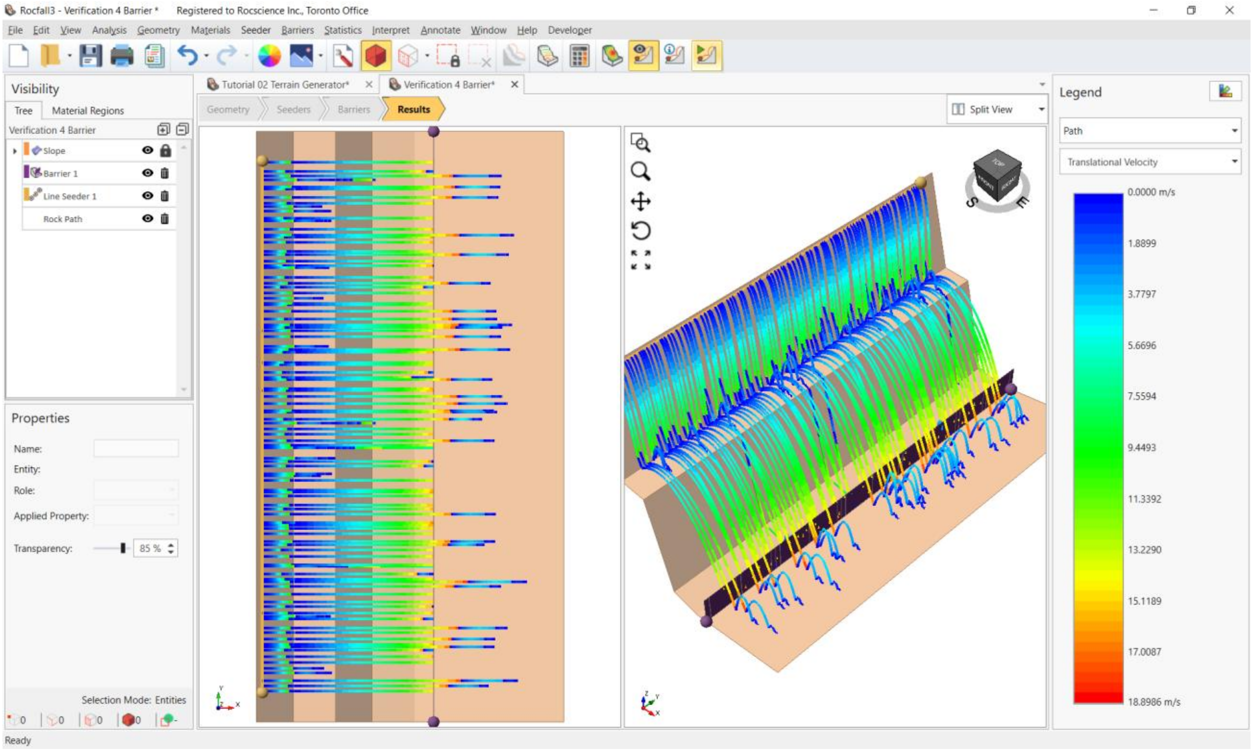

Figure 4-3: RocFall3 Results

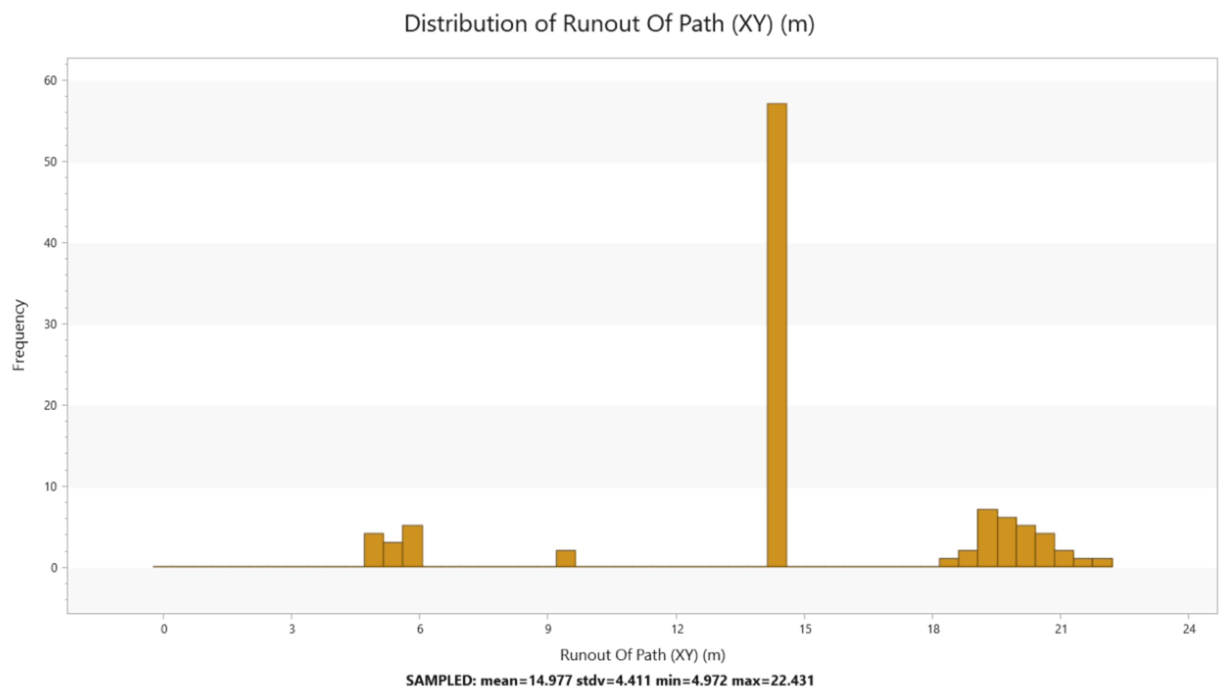

Figure 4-4: RocFall3 Runout Distances Plot

In RocFall2 57 rocks hit the barrier and in RocFall3 57 rocks hit the barrier.

We plot the impact translational kinetic energy on the barrier:

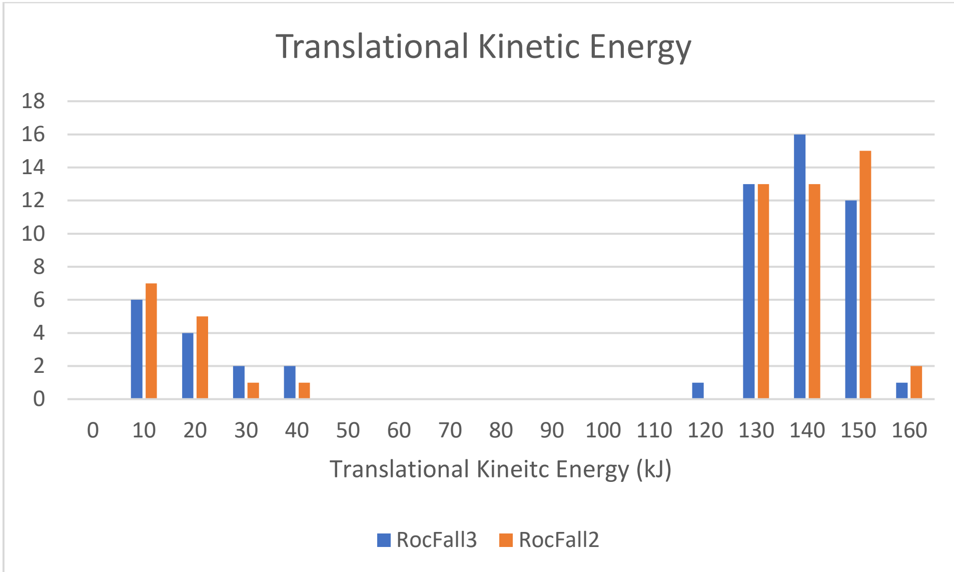

Figure 4-5: Impact Translational Kinetic Energy

It is observed that RocFall2 and RocFall3's impact translational kinetic energy have very similar range and distribution. We run basic statistical analysis on the data and got the following:

Table 4-2: Statistical Analysis on impact translational kinetic energy

	RocFall3	RocFall2
Mean	105.0356	105.2945
Standard Error	7.099613	7.283423
Median	130.2452	130.877
Standard Deviation	53.6009	54.98864
Sample Variance	2873.057	3023.751
Kurtosis	-0.45999	-0.53168
Skewness	-1.17502	-1.16084
Range	150.4422	151.2034
Minimum	0.102618	1.09256
Maximum	150.5448	152.296
Sum	5987.03	6001.784
Count	57	57
Confidence Level(95.0%)	14.22223	14.59045

We then plot the impact heights on the barrier:

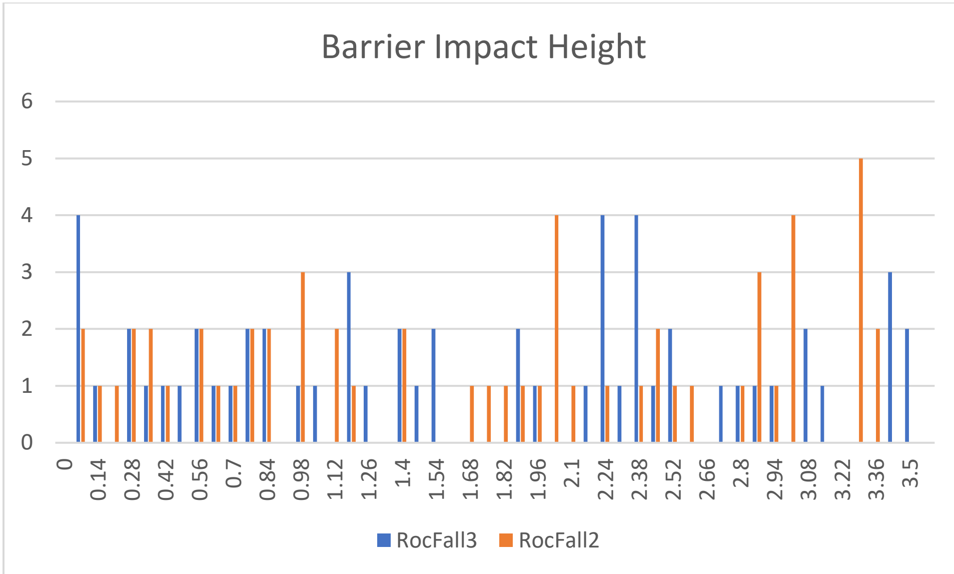

Figure 4-6: Impact Height

We run basic statistical analysis on the data and got the following:

Table 4-3: Statistical Analysis on impact heights

	RocFall3	RocFall2
Mean	1.651113	1.730675
Standard Error	0.14207	0.143783
Median	1.50594	1.84787
Standard Deviation	1.072605	1.085537
Sample Variance	1.150481	1.17839
Kurtosis	-1.23407	-1.40411
Skewness	0.075373	-0.01891
Range	3.476	3.340352
Minimum	3.97E-05	0.017478
Maximum	3.47604	3.35783
Sum	94.11346	98.64848
Count	57	57
Confidence Level(95.0%)	0.2846	0.288032

We can conclude that the 2D and 3D end points are statistically identical. If we throw down more rocks than 50, we can expect to obtain even closer results.

5. Engine Stability Verification

[RocFall3 Build 1.000]

5.1. Problem Description

The main purpose of this verification is to confirm RocFall3's engine's robustness when we rotate the same model. We will be rotating the same model every 30 degrees around a full circle and compare the run out distances from each.

5.2. Model

We will be using the model from Verification #**Error! Reference source not found.** for this purpose. The m odels are rotated around (0,25,0) along the z-axis at every 30 degrees at: 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, and 330 degrees.

5.3. Results

The results from the 30-degree model are presented below:

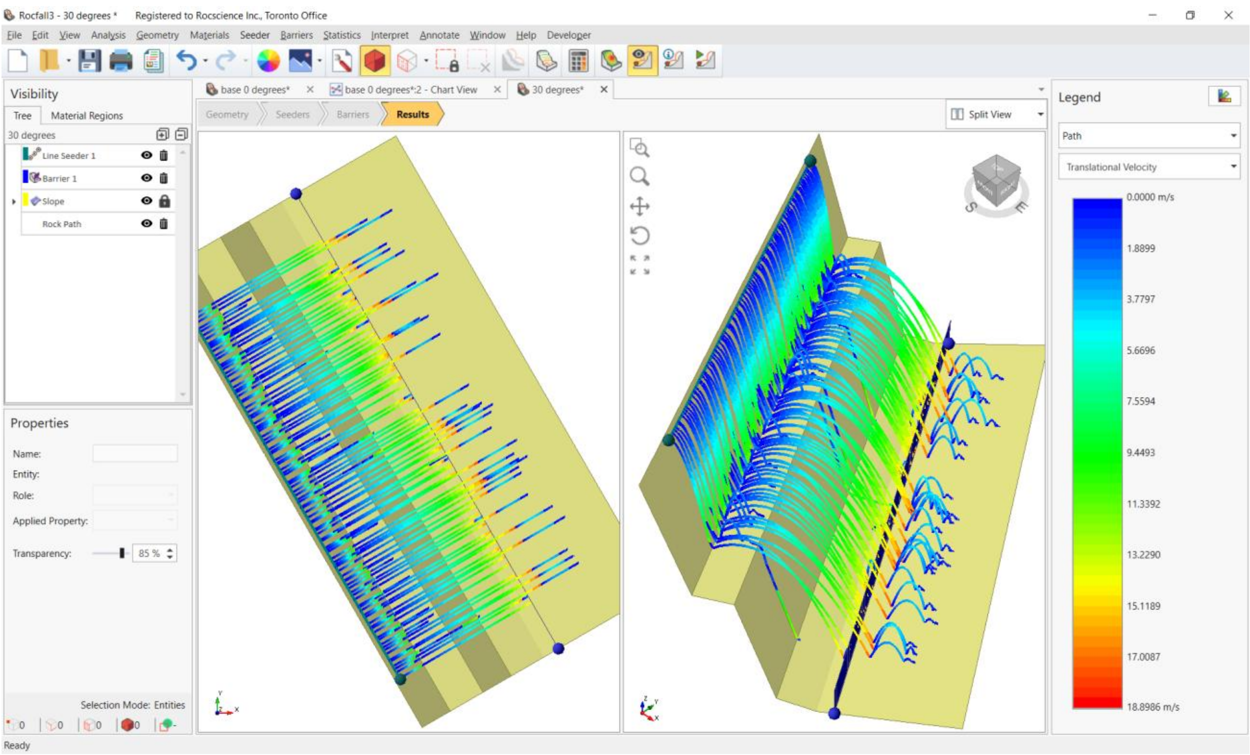

Figure 5-1: 30-degree model result

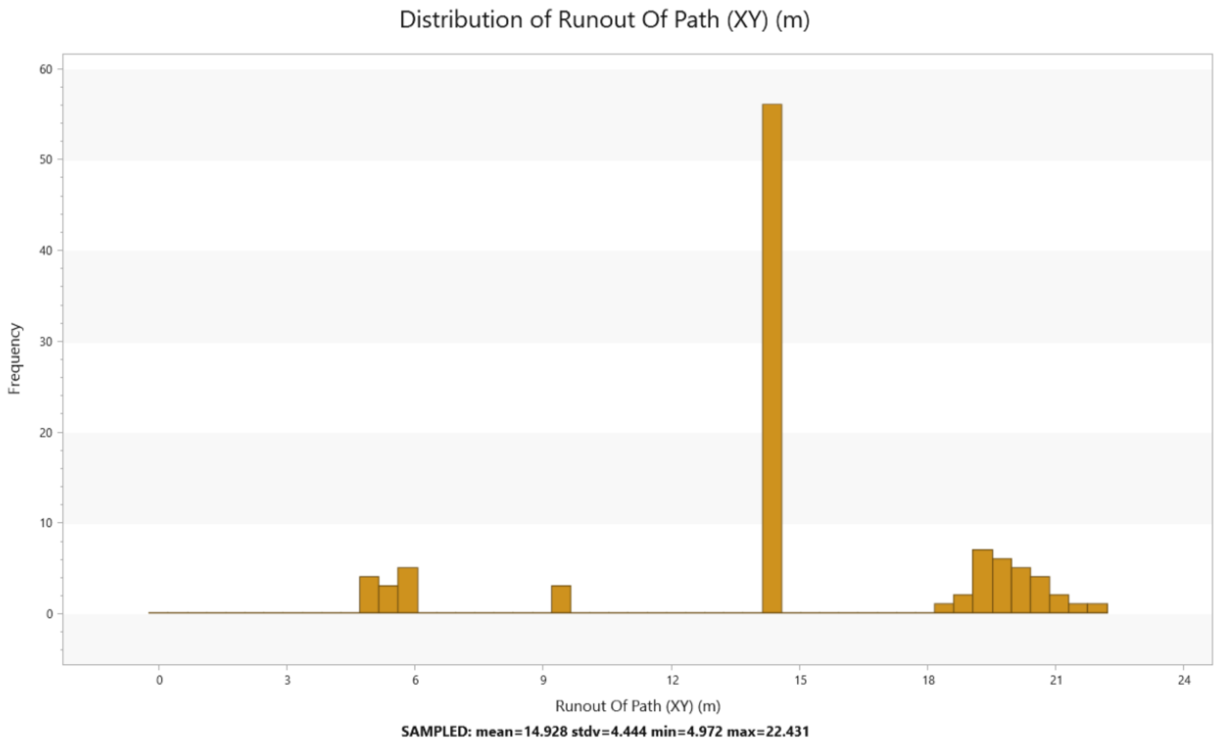

Figure 5-2: 30-degree model runout distance plot

We summarize the statistical distribution (at the bottom of the plots) of the runout distances of all the models in the Table below:

Table 5-1: Statistical Analysis on the Runout Distances

Model Rotation	mean	stdv	min	max
0	15.075	4.614	5.082	21.508
30	15.075	4.614	5.082	21.508
60	15.075	4.614	5.082	21.508
90	15.075	4.614	5.082	21.508
120	15.075	4.614	5.082	21.507
150	15.075	4.614	5.082	21.508
180	15.075	4.614	5.082	21.508
210	15.075	4.614	5.082	21.508
240	15.075	4.614	5.082	21.508
270	15.075	4.614	5.082	21.508
300	15.075	4.614	5.082	21.508
330	15.075	4.614	5.082	21.508

It can be seen that the results are almost identical, with slight difference in one model likely result of precision rounding. The robustness of the RocFall3 engine is verified.

RocFall3 Rigid Body Verifications

This document presents a couple examples from **RocFall2** models and hand calculations, which have been used as verification problems for **RocFall3**. The purpose of this verification is to confirm that the rigid body trajectories of legacy spheres (used in v.1.005 and earlier) and impact calculations used by the program is working correctly.

The trajectory algorithm calculates the motion of the rocks while they are travelling in the air and finds the closest/next contact point with the slope. The impact algorithm takes the inbound contact geometry and velocities and calculate the outbound velocities. The majority of the simulation time in RocFall3 takes place in the projectile algorithm. Any errors in the projectile algorithm would surely produce incorrect results. Therefore, it is essential that the projectile algorithm work correctly.

When comparing with RocFall2, selected RocFall2 models are extruded and replicated in RocFall3. Two scenarios are compared for each of the tutorials. First scenario contains only one rock with no stats variations defined, the second scenario contains the same RocFall2 model (with the same stats variations defined and same number of rock throws) duplicated in RocFall3. Due to the inherent differences in RocFall2 and RocFall3, especially how the impacts are calculated, you will see that with the results are not exactly identical. Nevertheless, the differences are small and/or the results are statistically identical.

The results produced by RocFall3 agree very well with RocFall2 and hand calculations, which affirms the reliability of RocFall3 results.

6. Rigid Body Verification Against RocFall2

[RocFall3 Build 1.005 or earlier]

6.1. Problem Description

The main purpose of this verification is to confirm RocFall3's impact calculations. We will be comparing the results between RocFall3 and RocFall2. Please note that this document only applies to RocFall3 v.1.005 or earlier.

This verification example consists of a simple slope based on the slope in Verification 1 with only one material. The Rigid Body Method is used without using Tangential CRSP damping.

6.2. Model

The slope vertices don't contain any stats variations.

Table 6-1: Slope Geometry

Vertex	X	X Std. Dev.	Y	Y Std. Dev.
1	0	N/A	0	N/A
2	3.1	N/A	-12.2	N/A
3	6.7	N/A	-12.2	N/A
4	9.8	N/A	-24.4	N/A
5	13.4	N/A	-25	N/A
6	26	N/A	-22.5	N/A

There is one point seeder defined at (0.5,0). We're going to throw down 50 rocks of mass 800kg and density 2700kg/m^3.

Table 6-2: Seeder Initial Conditions

	Mean	Distribution	Std. Dev.	Rel. Min	Rel. Max
Horizontal Velocity (m/s)	1.5	Normal	0.15	0.45	0.45
Vertical Velocity (m/s)	0	None	N/A	N/A	N/A
Rotational Velocity (°/s)	0	None	N/A	N/A	N/A

Initial Rotation (°/s)	0	Uniform	N/A	0	360

We will be using the same material for the entire slope. The materials will have the following properties:

Table 6-3: Slope Material Definitions for RocFall2

	Mean	**Distribution**	**Std. Dev.**	**Rel. Min**	**Rel. Max**
Normal Restitution	0.55	None	N/A	N/A	N/A
Dynamic Friction	0.47	None	N/A	N/A	N/A
Rolling Friction	0.1	None	N/A	N/A	N/A

Keep Advanced Parameters, Slope Roughness and Forest/Vegetation Damping disabled.

Table 6-4: Slope Material Definitions for RocFall3

	Mean	**Distribution**	**Std. Dev.**	**Rel. Min**	**Rel. Max**
Normal Restitution	0.55	None	N/A	N/A	N/A
Tangential Restitution	0.68	None	N/A	N/A	N/A
Dynamic Friction	0.47	None	N/A	N/A	N/A
Rolling Friction	0.1	None	N/A	N/A	N/A

6.3. Scenario 1: No Stats Variation

For the first no variation scenario, turn all distribution to “None” and set the number of rocks to throw to “1”. In RocFall2 we obtained the following result:

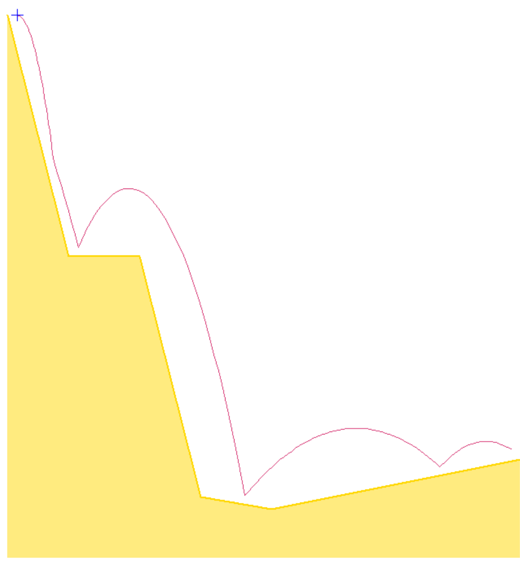

Figure 6-1: RocFall2 Scenario 1 Result

In RocFall3 we obtained the following result:

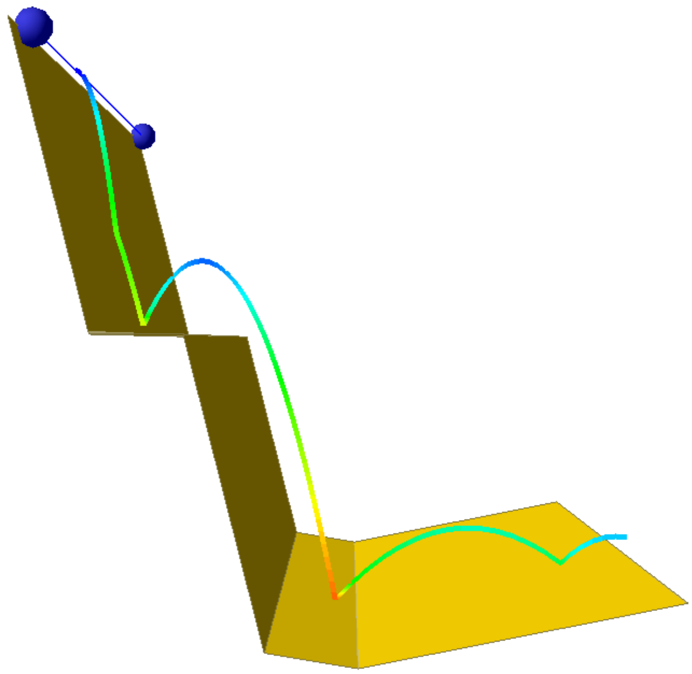

Figure 6-2: RocFall3 Scenario 1 Result

Detailed path results are included in Appendix C. For the first several bounces, the differences are less than 0.1%. It is only further down the slope, the minute differences accumulate and affect enough of that of the contact points. We plot the centre of mass locations of the rocks when they impact the slope:

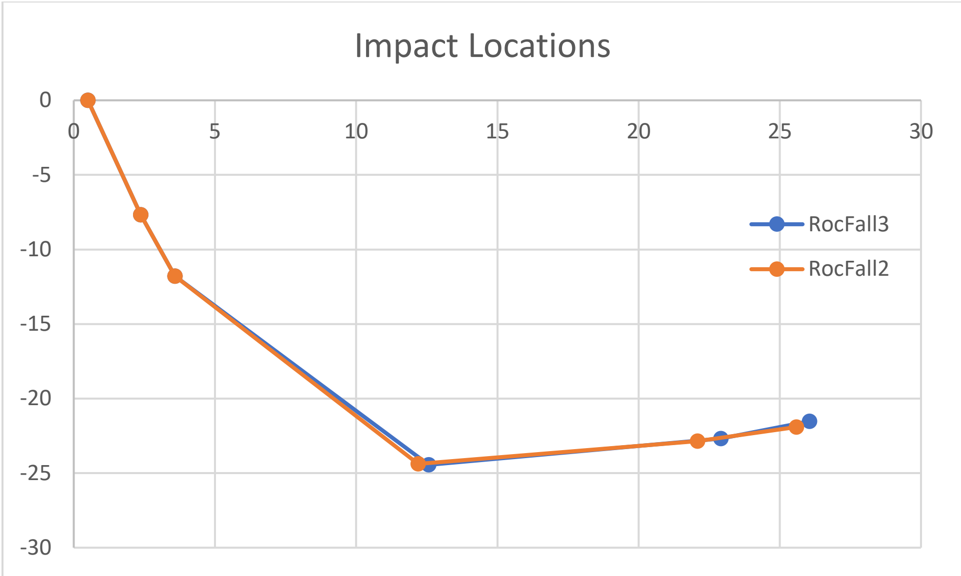

Figure 6-3: Impact Locations Along Slope

We can also plot the energy loss per impact:

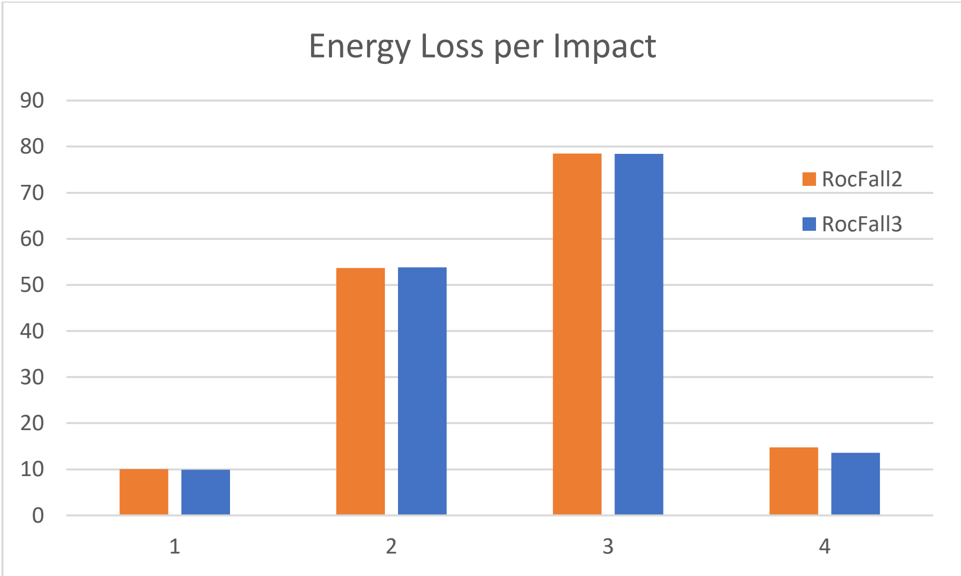

Figure 6-4: Energy Loss per Impact

We conclude the results produced by RocFall3 agree very well with RocFall2.

6.3.1. Energy Conservation

To ensure energy is conserved, we plot the Total Energy, Kinetic Energy and Potential Energy along the rock's x location along the slope.

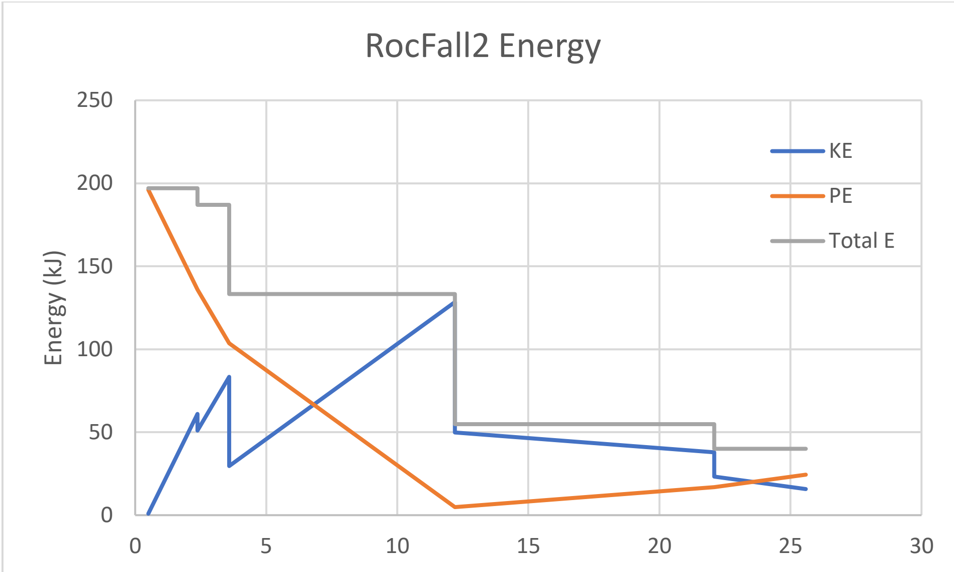

Figure 6-5: RocFall2 Energy Plot

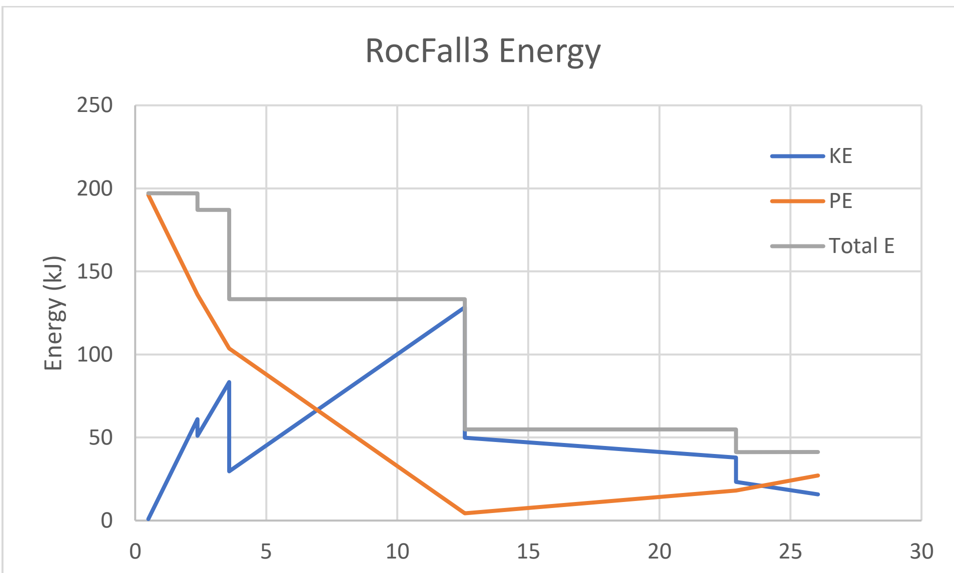

Figure 6-6: RocFall3 Energy Plot

It is observed that no energy is gained, and that energy is conserved.

6.4. Scenario 2: With Stats Variation

For the second scenario with stats variations, the model as described in the “Model” section is used. In RocFall2 we obtained the following result:

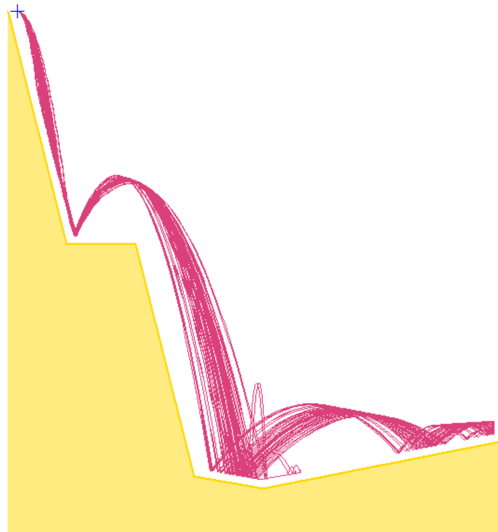

Figure 6-7: RocFall2 Scenario 2 Result

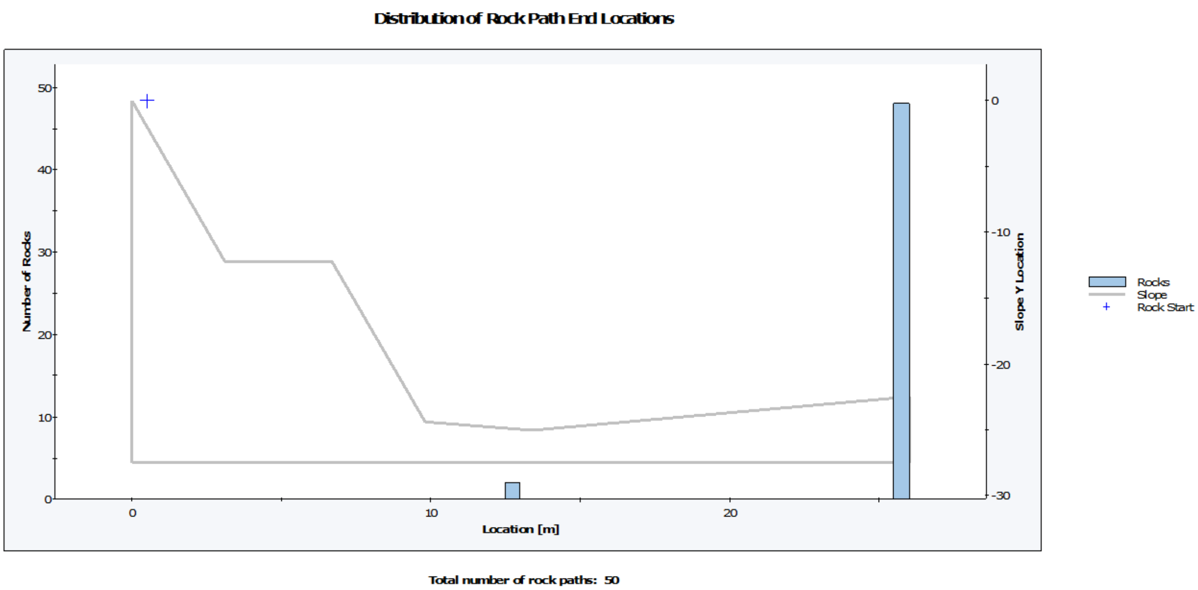

Figure 6-8: RocFall2 Scenario 2 End Points Plot

In RocFall3 we obtained the following result:

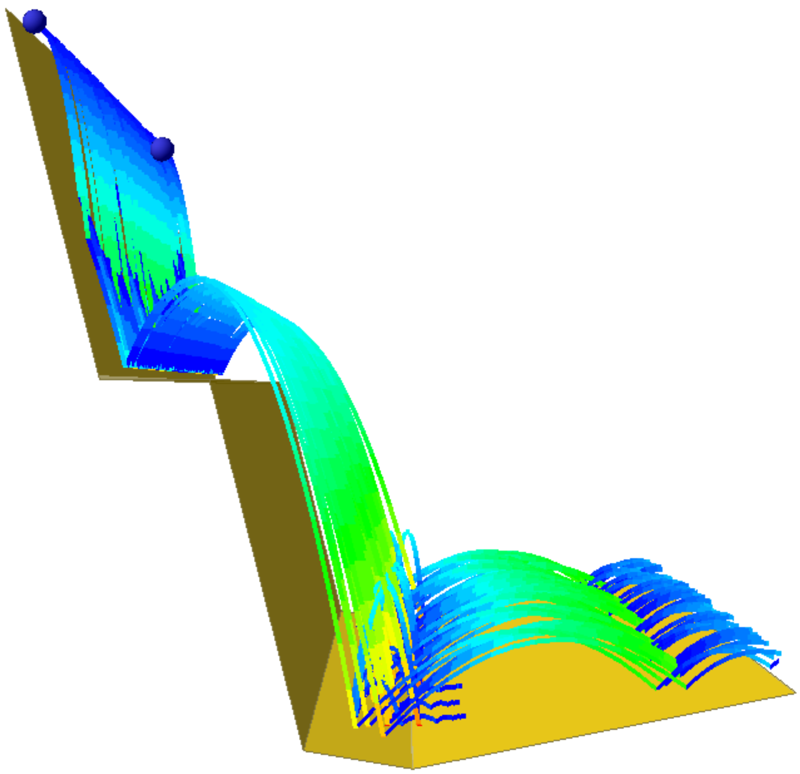

Figure 6-9: RocFall3 Scenario 2 Result

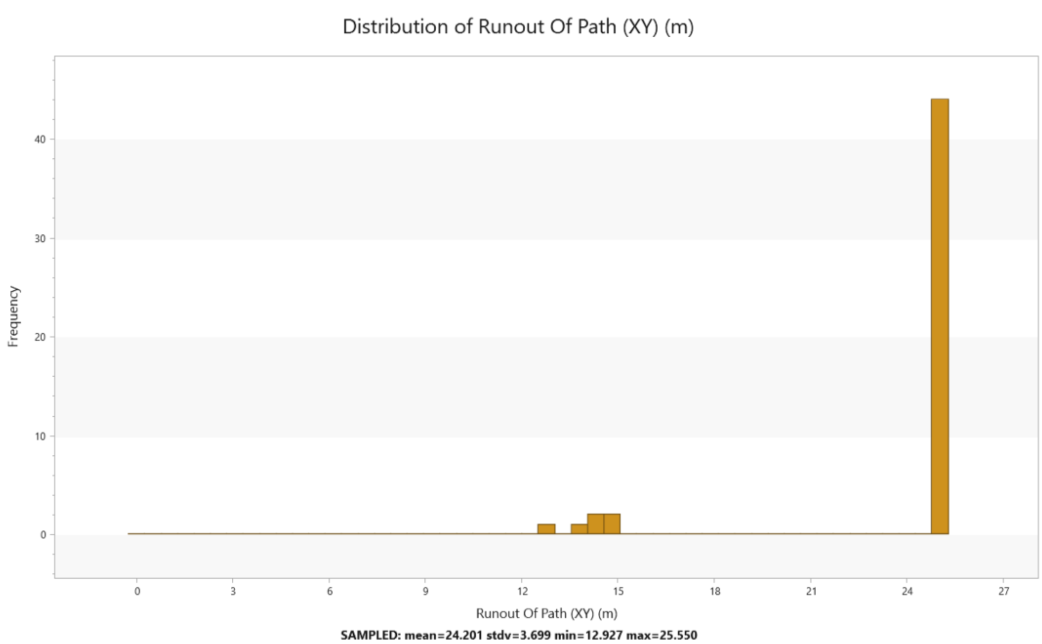

Figure 6-10: RocFall3 Scenario 2 End Points Plot

If we overlay the 2 end points histogram plots:

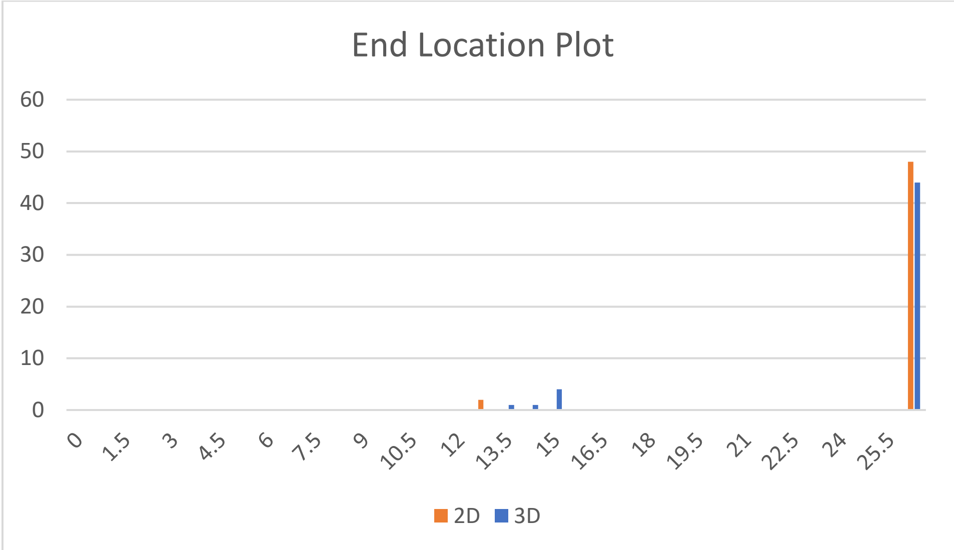

Figure 6-11: End Location Plot

It is observed that RocFall2 and RocFall3's end locations have very similar range and distribution. We analyze and plot the end location in a box and whisker plot:

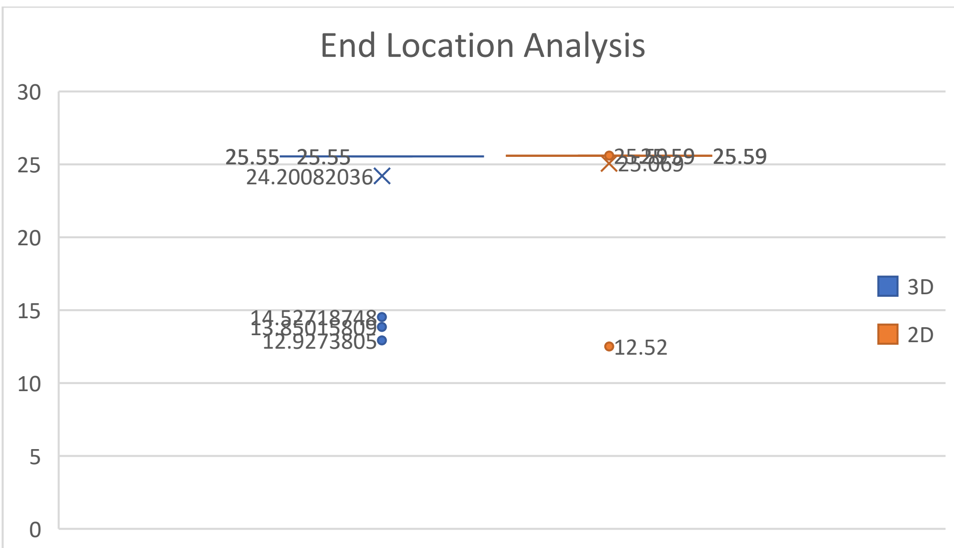

Figure 6-12: End Location Analysis

We can conclude that the 2D and 3D end points are statistically very similar. If we throw down more rocks than 50, we can expect to obtain even closer results.

7. Impacts Verification against Hand Calculations

[RocFall3 Build 1.000]

7.1. Problem Description

The main purpose of this simulation is to confirm RocFall3's contact finding algorithm and impact calculations. We're using the same slope and seeder starting condition as in Verification 0. The only differences are the use of the Rigid Body Method and that the dynamic friction coefficient is tan(30°) = 0.57735. Same as in Verification 0, we're only comparing the first 3 bounces.

7.2. RocFall3 Analysis

First, we need to create the model in RocFall3. In a New Project, follow the steps in Section 2.2 to create the geometry and seeder. Or you can just open the Verification 0 file.

Under Project Settings, change the analysis type to Rigid Body.

The model is now ready to be run. Save the project and click on **Analysis > Compute**.

7.3. Analytical Solution

Finding the contact points with a sphere is slightly more complicated than a point. Instead of a quadratic equation, we're solving a quartic equation. The rock's centre of mass location and velocity are as described in Equations 2.1 and 2.2.

Due to the simplicity of our problem, we can simplify the problem by offsetting the slope by a distance that's equal to the rock's radius along the triangle normal and find the parabola intersection of the rock's trajectory with the offset plane. The rest is the same as described in section 2.3.

We find that the rock will impact the slope at t = 1.015628 sec at (3.0469, 0, 14.9405) with velocity of (3, 0, -9.9599) and all zero angular velocity. The contact point is at (3.4551, -0.2357, 14.7738).

Now we perform impact calculations. To do that, we need to first transform the incoming velocities to a normal and tangents frame. The normal of the triangle can be found with the following:

$$\overrightarrow{n_3} = \frac{\overline{BD} \times \overline{CD}}{|\overline{BD} \times \overline{CD}|} = (-0.8165, 0.471405, 0.333333)$$

The normal component of the velocity vector can be obtained by:

$$\overrightarrow{v_n} = \vec{v} \cdot \vec{n}(\vec{v})$$

The remaining tangential component is just $\overrightarrow{v_t} = \bar{v} - \overrightarrow{v_n}$ and we can make the first tangent direction in line with $\overrightarrow{v_t}$ for simplicity. The centre of mass (COM) velocity vector in normal and tangential frame is then (-5.76946, 8.65552, 0) and there are no angular velocities.

The conversion matrix from x,y,z frame to n,t1,t2 frame is:

$$C = \begin{bmatrix} -0.816496581 & 0.471404521 & 0.333333333 \\ -0.197654221 & 0.314231598 & -0.928542574 \\ -0.542463033 & -0.824036577 & -0.163393933 \end{bmatrix}$$

We then need to construct the rotational matrix $\vec{r}$ from the centre of mass to the contact point.

$$\vec{r} = COM - Contact\ Pt = (-0.40825, 0.235702, 0.16667)$$

Transform $\vec{r}$ to n/t1/t2 frame with $C \cdot \vec{r} = (0.5,0,0)$, which we know is correct since all impacts with spheres are concentric. The rotation matrix:

$$r = \begin{bmatrix} 0 & -r_2 & r_1 \\ r_2 & 0 & -r_3 \\ -r_1 & r_3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -0.5 \\ 0 & 0.5 & 0 \end{bmatrix}$$

We can then get the incoming contact point velocities with: $\vec{\gamma} = \vec{V} + \vec{r} \cdot \vec{\omega}$ = (-5.76946, 8.65552, 0).

The restitution matrix is:

$$\varepsilon = \begin{bmatrix} r_N & 0 & 0 \\ 0 & r_T & 0 \\ 0 & 0 & r_T \end{bmatrix} = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}$$

To get the outgoing contact point velocities: $\vec{\gamma}' = \varepsilon\, \vec{\gamma}$ =(1.73084, 6.05867, 0).

The change in contact point velocities are: $d\vec{\gamma} = \vec{\gamma}' - \vec{\gamma}$ = (7.5003, -2.5966, 0)

The mass inertia matrix for the sphere is:

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{11} & I_{21} & I_{31} \\ 0 & 0 & 0 & I_{12} & I_{22} & I_{32} \\ 0 & 0 & 0 & I_{13} & I_{23} & I_{33} \end{bmatrix} = \begin{bmatrix} 1413.72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1413.72 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1413.72 & 0 & 0 & 0 \\ 0 & 0 & 0 & 141.37 & 0 & 0 \\ 0 & 0 & 0 & 0 & 141.37 & 0 \\ 0 & 0 & 0 & 0 & 0 & 141.37 \end{bmatrix}$$

The w matrix is:

$$w = \begin{bmatrix} 1 & 0 & 0 & 0 & -r_2 & r_1 \\ 0 & 1 & 0 & r_2 & 0 & -r_3 \\ 0 & 0 & 1 & -r_1 & r_3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -0.5 \\ 0 & 0 & 1 & 0 & 0.5 & 0 \end{bmatrix}$$

$$\overleftrightarrow{G} = w \cdot M^{-1} \cdot w^T = \begin{bmatrix} 0.0007074 & 0 & 0 \\ 0 & 0.002476 & 0 \\ 0 & 0 & 0.002476 \end{bmatrix}$$

$$\overleftrightarrow{G}^{-1} = \begin{bmatrix} 1413.72 & 0 & 0 \\ 0 & 403.92 & 0 \\ 0 & 0 & 403.92 \end{bmatrix}$$

The impulses for the impact can be calculated as:

$$d\vec{P} = \overleftrightarrow{G}^{-1} \cdot d\vec{\gamma} = \begin{bmatrix} 1413.72 & 0 & 0 \\ 0 & 403.92 & 0 \\ 0 & 0 & 403.92 \end{bmatrix} \begin{bmatrix} 7.5003 \\ -2.5966 \\ 0 \end{bmatrix} = \begin{bmatrix} 10603.297 \\ -1048.805 \\ 0 \end{bmatrix}$$

The change in COM velocities can be calculated as:

$$d\vec{V} = M^{-1}d\vec{P} = \begin{bmatrix} 0.0007074 & 0 & 0 \\ 0 & 0.0007074 & 0 \\ 0 & 0 & 0.0007074 \end{bmatrix} \begin{bmatrix} 10603.297 \\ -1048.805 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.5 \\ -0.7419 \\ 0 \end{bmatrix}$$

Outgoing velocities are then: $\vec{V'} = \vec{V} + d\vec{V} = \begin{bmatrix} 1.73084 \\ 7.91336 \\ 0 \end{bmatrix}$

And the change in angular velocities can be calculated as:

$$d\vec{\omega} = \overleftrightarrow{I}^{-1} \cdot \overleftrightarrow{r}^{T} \cdot d\vec{P} = \begin{bmatrix} 0.002476 & 0 & 0 \\ 0 & 0.002476 & 0 \\ 0 & 0 & 0.002476 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix} \begin{bmatrix} 10603.297 \\ -1048.805 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3.7094 \end{bmatrix}$$

Since incoming velocities were all zero, the change in angular velocity vector is also the outgoing angular velocity vector.

As a last step, we need to transform the outgoing velocity and angular velocity vectors from the n/t1/t2 frame back to the global x,y,z frame.

$$\overrightarrow{V_{xyz}'} = C^{-1} \cdot \overrightarrow{V'} = \begin{bmatrix} -2.97733 \\ 3.30255 \\ -6.77095 \end{bmatrix}$$

$$\overrightarrow{\omega_{xyz}'} = C^{-1} \cdot \overrightarrow{\omega'} = \begin{bmatrix} -2.0122 \\ -3.0567 \\ -0.6061 \end{bmatrix}$$

With the outgoing velocities, we can then find the next contact point and perform impact calculations. We repeat the above steps still we find 3 bounces on the slope.

7.4. Results

Path details from RocFall3 and hand calculations are summarized in Table 7-1 and Table 7-2 below. The results match quite well with differences less than 1%, that can mainly be attributed to rounding errors. This affirms the reliability of RocFall3 results.

Table 7-1: Summarized Path Results from RocFall3

ID	Time	Path State	Pos.x	Pos.y	Pos.z	Vel.x	Vel.y	Vel.z	ω.x	ω.y	ω.z	KE	PE	Total E
0	0	Projectile	0	0	20	3	0	0	0	0	0	6362	277277	283638
1	1.01573	slope impact	3.0472	0	14.9412	3	0	-9.96089	0	0	0	76496	207142	283638
2	1.01573	Projectile	3.0472	0	14.9412	-2.9777	3.3027	-6.77171	-2.0124	-3.0570	-0.6061	47364	207142	254507
3	2.04827	slope impact	-0.0274	3.4102	2.72145	-2.9777	3.3027	-16.8975	-2.0124	-3.0570	-0.6061	216777	37730	254506
4	2.04827	Projectile	-0.0274	3.4102	2.72145	3.6974	6.7518	-12.7491	-6.04178	2.7536	1.0464	159973	37730	197703
5	2.05917	slope impact	0.0129	3.4838	2.58196	3.6974	6.7518	-12.856	-6.04178	2.7536	1.0464	161908	35796	197704

Table 7-2: Summarized Path Results by Hand Calculations

ID	Time	Path State	Pos.x	Pos.y	Pos.z	Vel.x	Vel.y	Vel.z	ω.x	ω.y	ω.z	KE	PE	Total E
0	0	Projectile	0	0	20	3	0	0	0	0	0	6362	277276	283638
1	1.01563	slope impact	3.0469	0	14.9405	3	0	-9.9599	0	0	0	76482	207132	283614
2	1.01563	Projectile	3.0469	0	14.9405	-2.9773	3.3026	-6.7709	-2.0122	-3.0567	-0.6060	47355	207132	254487
3	2.03982	slope impact	-0.0025	3.3824	2.6957	-2.9773	3.3026	-16.8148	-2.0122	-3.0567	-0.6060	214804	37373	252176
4	2.03982	Projectile	-0.0025	3.3824	2.6957	3.6702	6.7358	-12.6847	-6.0248	2.7250	1.0462	158496	37373	195868
5	2.04049	slope impact	0	3.3869	2.6873	3.6701	6.7358	-12.6913	-6.0248	2.7250	1.0462	158614	37256	195870

7.4.1. Energy Conservation

To ensure energy is conserved, we plot the Total Energy, Kinetic Energy and Potential Energy along the rock's x location along the slope.

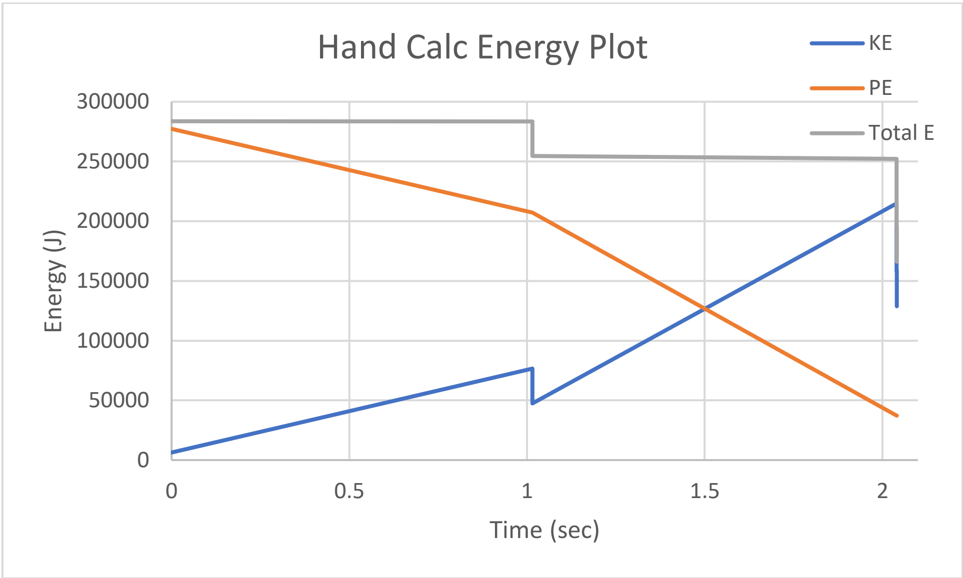

Figure 7-1: Hand Calc Energy Plot

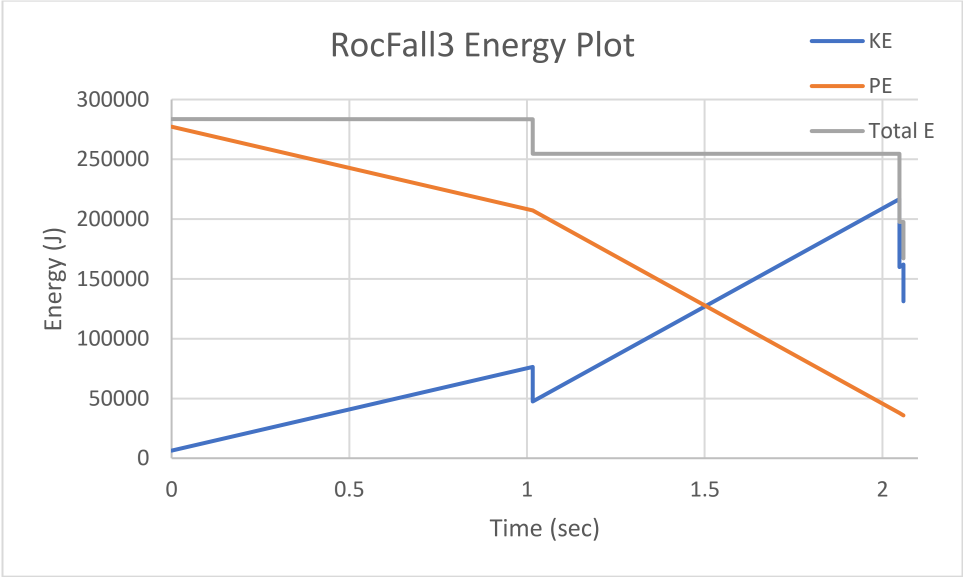

Figure 7-2: RocFall3 Energy Plot

It is observed that no energy is gained, and that energy is conserved.

Appendix

A. RocFall2 Tutorial 1 Path Details

Table A-1: Detailed RocFall2 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	0.50012	3.08E-05	1.5	0	0	0	1.4759	Air	1125	245167	246292
1	2.714	-10.681	1.5	-14.474	0	1.4759	0	Impact	105874	140421	246295
2	2.7139	-10.681	3.7376	-10.424	-24.744	1.4759	0.13692	Air	85619	140421	226040
3	3.2256	-12.2	3.7376	-11.767	-24.744	1.61282	0	Impact	100520	125525	226045
4	3.2256	-12.2	5.7308	4.7068	-12.864	1.61282	2.14014	Air	34067	125525	159592
5	15.49	-24.585	5.7308	-16.281	-12.864	3.75296	0	Impact	155525	4070	159595
6	15.49	-24.585	1.7137	5.5654	-6.2044	3.75296	1.06566	Air	18483	4070	22553
7	17.317	-24.223	1.7137	-4.8851	-6.2044	4.81862	0	Impact	14929	7620	22548
8	17.317	-24.223	1.1373	1.7932	-3.2873	4.81862	0.31961	Air	2683	7620	10303
9	17.68	-24.151	1.1373	-1.3411	-3.2873	5.13823	0	Impact	1975	8326	10301
10	17.68	-24.151	0.89051	0.64671	-2.2432	5.13823	0.09559	Air	805	8326	9131
11	17.765	-24.134	0.89051	-0.2907	-2.2432	5.23381	0	Impact	638	8493	9131
12	17.765	-24.134	0.77856	0.29468	-1.8429	5.23381	0.02765	Air	481	8493	8974
13	17.787	-24.13	0.77856	0.02348	-1.8429	5.26147	0	Impact	438	8532	8970
14	17.787	-24.13	0.72412	0.14367	-1.6548	5.26147	0.09893	Sliding	381	8532	8913
15	17.823	-24.123	0	0	-1.6548	5.3604	0	Sliding	109	8600	8709

Table A-2: Detailed RocFall3 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	0.5	0	1.5	0	0	0	1.47584	Air	1125	245166	246291
1	2.71376	-10.68	1.5	-14.4731	0	1.47584	0	Impact	105860	140431	246292
2	2.71376	-10.68	3.73746	-10.4238	24.7433	1.47584	0.13700	Air	85614	140431	226045
3	3.22578	-12.2	3.73746	-11.7673	24.7433	1.61284	0	Impact	100521	125525	226046
4	3.22578	-12.2	5.73052	4.70691	12.8631	1.61284	2.14017	Air	34065	125525	159590
5	15.4901	-24.5853	5.73052	-16.281	12.8631	3.75301	0	Impact	155523	4067	159590
6	15.4901	-24.5853	1.71352	5.56539	6.20399	3.75301	1.06569	Air	18483	4067	22549
7	17.3161	-24.223	1.71352	-4.88542	6.20399	4.81870	0	Impact	14930	7620	22549
8	17.3161	-24.223	1.13709	1.79324	3.28696	4.81870	0.31971	Air	2683	7620	10303
9	17.6797	-24.1509	1.13709	-1.34201	3.28696	5.13840	0	Impact	1976	8327	10303
10	17.6797	-24.1509	0.890219	0.646918	2.24264	5.13840	0.09591	Air	805	8327	9132
11	17.7651	-24.1339	0.890219	-0.29366	2.24264	5.23431	0	Impact	639	8494	9133
12	17.7651	-24.1339	0.777776	0.295407	1.84151	5.23431	0.02877	Air	481	8494	8974
13	17.7874	-24.1295	0.777776	0.013235	1.84151	5.26309	0	Impact	437	8537	8974
14	17.7874	-24.1295	0.713098	0.183813	1.65036	5.26309	0.09	Sliding	379	8537	8916
15	17.8194	-24.1231	0	0	1.65036	5.35309		Stopped	108	8599	8708

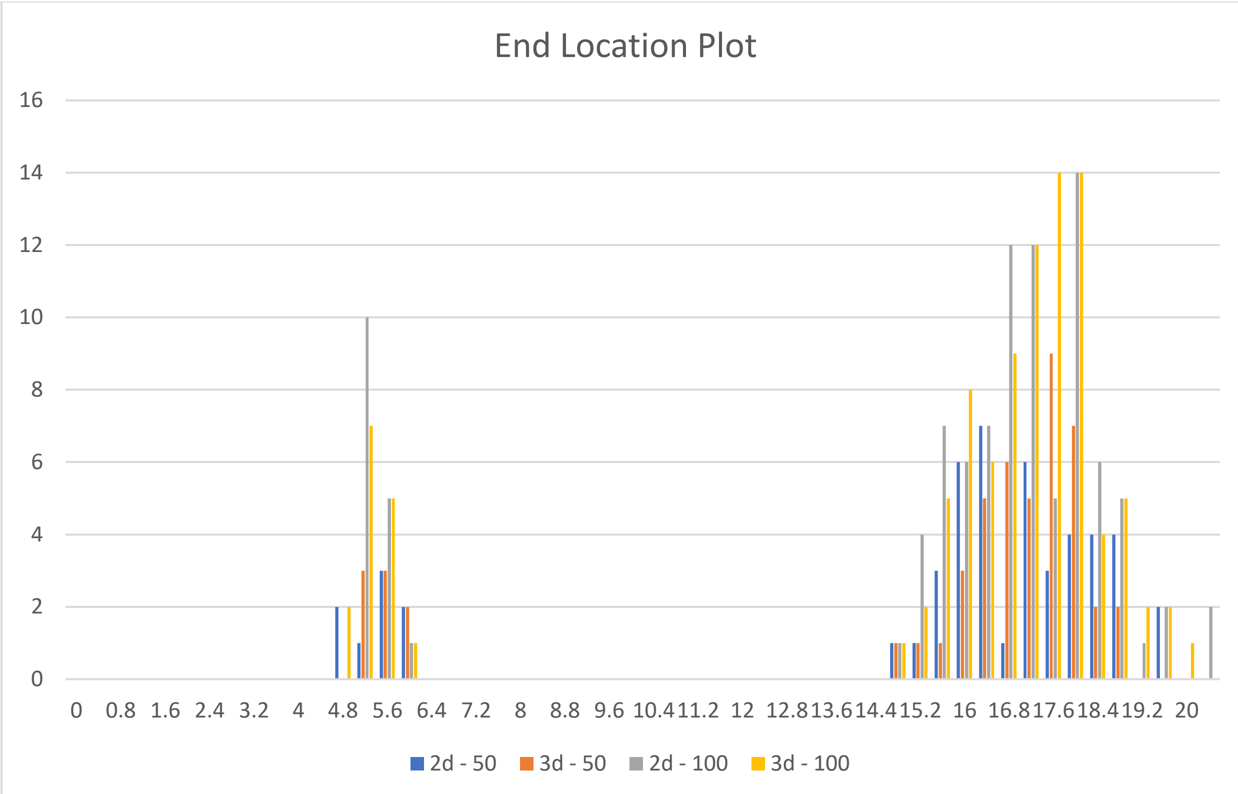

Figure A-1: Scenario 2 2D vs 3D End Points Plot with 100 rocks

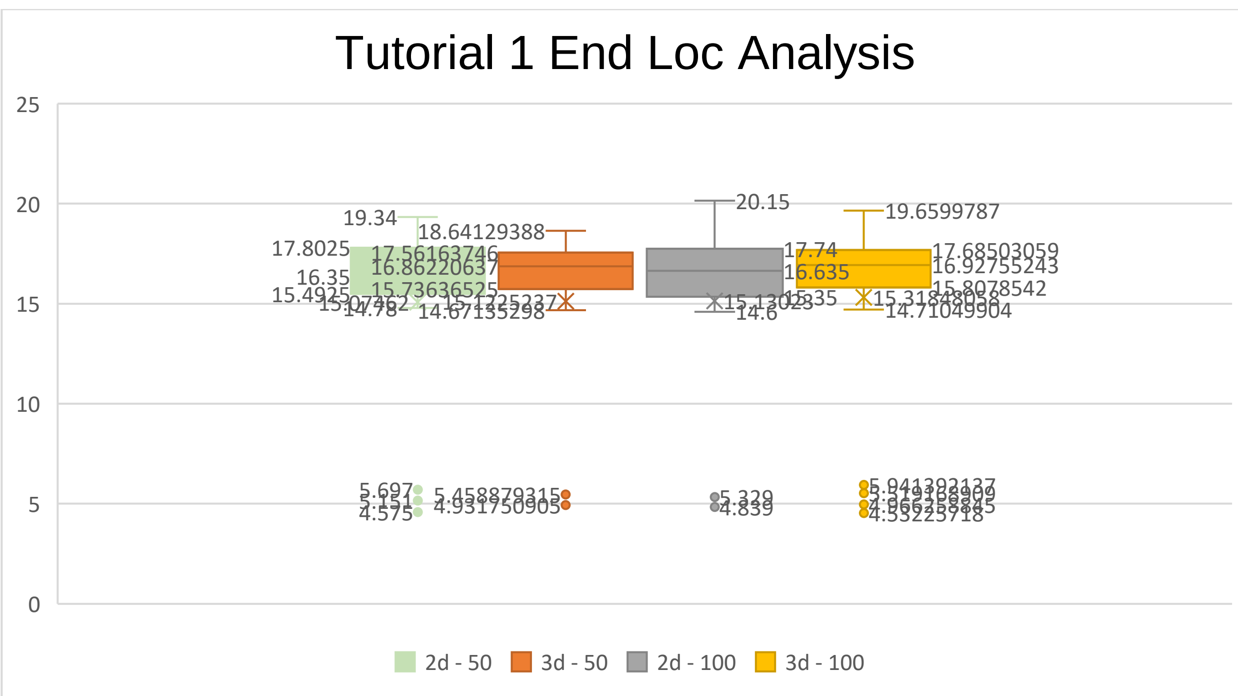

Figure A-2: Scenario 2 2D vs 3D End Points Analysis Plot with 100 rocks

B. Sliding Verification 1 Path Details

Table B-1: Detailed RocFall2 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	70	45.5	10.182	0	0	0	0.28917	Air	51837	446203	498039
1	72.944	45.09	10.182	-2.8358	0	0.28917	0	Impact	55857	442182	498039
2	72.944	45.09	1.4986	1.3446	-4.469	0.28917	0.07282	Air	2820	442182	445001
3	73.054	45.162	1.4986	0.63054	-4.469	0.36199	0	Impact	2114	442888	445002
4	73.054	45.162	1.4293	0.94004	-3.84	0.36199	0.16906	Sliding	2049	442888	444937
5	73.174	45.241	0	0	-3.84	0.53105	9.18066	Sliding	585	443663	444248
6	50	30	-5.0485	-3.3203	-3.84	9.7117	3E-05	Air	18841	294200	313041
7	50	30	-5.0485	-3.3206	-3.84	9.71173	0	Impact	18842	294200	313042
8	50	30	-2.8533	0.12506	6.4047	9.71173	0.02451	Air	5707	294200	299906
9	49.93	30	-2.8533	-0.11527	6.4047	9.73624	0	Impact	5706	294200	299905
10	49.93	30	-2.851	0.00048	6.3996	9.73624	0.50355	Sliding	5690	294200	299889
11	49.2123	30	0	0	6.3996	10.2398	-	Stop	1626	294200	295825

Table B-2: Detailed RocFall3 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	70	45.5	10.1825	0	0	0	-	Air	51842	446203	498044
1	72.9436	45.0902	10.1825	-2.83494	0	0.28908	0.28908	Impact	55860	442184	498044
2	72.9436	45.0902	1.49872	1.3447	4.46932	0.28908	0	Air	2820	442184	445004
3	73.0533	45.1624	1.49872	0.626746	4.46932	0.36229	0.07321	Impact	2112	442892	445004
4	73.0533	45.1624	1.42185	0.948681	3.83667	0.36229	0	Sliding	2045	442892	444937
5	73.1668	45.2371	0	0	3.83667	0.53120	0.16891	Sliding	584	443624	444209
6	50	30	-5.046	-3.31881	3.83667	9.63225	9.10105	Impact	18823	294200	313022
7	50	30	-2.86102	0	-9.55969	9.63225	0	Air	7720	294200	301920
8	49.2917	30	0	0	-9.55969	10.1223	0.49	Impact	3628	294200	297827

Table B-3: Detailed Hand Calculation Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	70	45.5	10.1825	0	0	0	-	Air	51842	446203	498044
1	72.9436	45.0902	10.1825	-2.83493	0	0.28908	0.2891	Impact	55860	442184	498044
2	72.9436	45.0902	1.49877	1.34474	4.46945	0.28908	0	Air	2820	442184	445004
3	73.0533	45.1624	1.49877	0.62676	4.46945	0.36230	0.0732	Impact	2112	442892	445004
4	73.0533	45.1624	1.42189	0.94871	3.83679	0.36230	0	Sliding	2045	442892	444937
5	73.1739	45.2417	-0.00621	0.00944	3.83679	0.53121	0.1689	Sliding	584	443670	444254
6	50.0000	30	-5.06500	-3.31779	3.83679	9.72736	9.1961	Impact	18915	294200	313115
7	50.0000	30	-2.86351	0.12495	-6.42761	9.72736	0	Air	5748	294200	299947
8	49.9270	30	-2.86351	0.12495	-6.42761	9.75284	0.0255	Impact	5748	294200	299947
9	49.9270	30	-2.41782	-0.00471	-5.42718	9.75284	0	Sliding	4092	294200	298292
10	49.4107	30	0	-0.00471	-5.42718	10.1799	0.427	Stop	1169	784532	785701

C. Rigid Body Impacts Path Details

Table C-1: Detailed RocFall2 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	0.5	0	1.5	0	0	0	1.2510	Air	0.9	196.110	197.010
1	2.3764	-7.6732	1.5	-12.2677	0	1.2510	0	Impact	61.099	135.918	197.017
2	2.3765	-7.6732	3.57343	-10.5627	-6.90244	1.2510	0.3363	Air	51.040	135.918	186.958
3	3.5799	-11.7864	3.57343	-13.8653	-6.90244	1.5872	0	Impact	83.310	103.653	186.963
4	3.5799	-11.7864	3.36806	7.6259	-8.14393	1.5872	2.5638	Air	29.614	103.653	133.267
5	12.1986	-24.3805	3.36806	-17.469	-8.14393	4.1510	0	Impact	128.419	4.860	133.279
6	12.1986	-24.3805	6.82136	8.16232	-13.0249	4.1510	1.3870	Air	49.904	4.860	54.764
7	22.0895	-22.8543	6.82136	-6.05718	-13.0249	5.5380	0	Impact	37.931	16.832	54.762
8	22.0895	-22.8542	4.59348	4.98725	-13.2415	5.5380	0.6073	Air	23.187	16.833	40.020
9	25.5864	-21.8993	4.59348	-2.47838	-13.2415	6.1453		Stopped	15.695	24.323	40.018

Table C-2: Detailed RocFall3 Path Output. Single rock no variation.

ID	X	Y	VX	VY	w (rad/s)	T	dT	Type	KE	PE	Total E
0	0.5	0	1.5	0	0	0	1.2510	Air	0.9	196.11	197.01
1	2.3764	-7.67306	1.5	-12.2676	0	1.2510	0	Impact	61.098	135.919	197.017
2	2.3764	-7.67306	3.57856	-10.583	6.77544	1.2510	0.3363	Air	51.179	135.919	187.098
3	3.5798	-11.7864	3.57856	-13.8808	6.77544	1.5872	0	Impact	83.449	103.653	187.102
4	3.5798	-11.7864	3.50757	7.63444	7.20458	1.5872	2.5638	Air	29.656	103.653	133.308
5	12.5725	-24.4428	3.50757	-17.5077	7.20458	4.1510	0	Impact	128.950	4.371	133.320
6	12.5725	-24.4428	7.45832	8.06463	9.06077	4.1510	1.3870	Air	50.513	4.371	54.884
7	22.9172	-22.69	7.45832	-5.53721	9.06077	5.5380	0	Impact	36.762	18.121	54.882
8	22.9172	-22.69	5.15866	4.88291	10.4374	5.5380	0.6073	Air	23.163	18.121	41.284
9	26.05	-21.5331	5.15866	-1.0726	10.4374	6.1453	0	Stopped	14.086	27.196	41.282

References