ROCSCIENCE

RocFall3 Rock Shape and Size

If you are using the Rigid Body analysis method in **RocFall3**, which requires Rock Shapes to be defined, the size of the rocks is determined from:

- rock mass (m)
- rock density (ρ)
- rock shape

The mass moment of inertia for each shape is also given.

Sphere

For a sphere, the volume is given by:

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

From which we get the sphere radius:

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3}$$

Mass moment of Inertia [1]:

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Polyhedron Shapes

Numerical integration is used for calculating the volume, centre of mass and the mass moment of inertia for any polyhedrons.

The volume of a polyhedron would be the summation of smaller finite volumes with edge lengths dx, dy and dz.

$$V = \sum_1^n dv = \sum_1^n dx\, dy\, dz = \frac{m}{\rho}$$

The coordinates of the centre of mass (COM_x, COM_y, COM_z) from an arbitrary reference point is calculated as:

$$COM_x = \frac{1}{V}\sum_1^n dv\, dy$$

$$COM_y = \frac{1}{V}\sum_1^n dv\, dy$$

$$COM_z = \frac{1}{V}\sum_1^n dv\, dz$$

where dx, dy and dz are distances from the centroid of each finite volume (dv) to the reference point. In RocFall3, the lower left corner of the bounding box of the shape is used as the reference point.

Lastly, the moment of inertia (I) for a polyhedron is calculated:

$$I = \rho v \left\{ (dx^2 + dy^2 + dz^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} dxdx & dxdy & dxdz \\ dydx & dydy & dydz \\ dzdx & dzdy & dzdz \end{bmatrix} \right\} = \rho v \begin{bmatrix} dy^2 + dz^2 & -dxdy & -dxdz \\ -dydx & dx^2 + dz^2 & -dydz \\ -dzdx & -dzdy & dx^2 + dy^2 \end{bmatrix}$$

$$I_{xx} = \sum dv\, (dy^2 + dz^2)$$

$$I_{yy} = \sum dv\, (dx^2 + dz^2)$$

$$I_{zz} = \sum dv\, (dx^2 + dy^2)$$

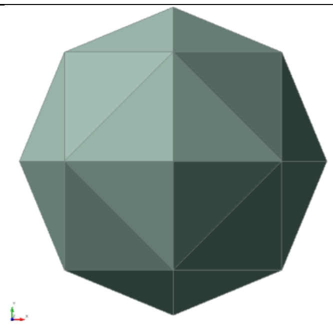
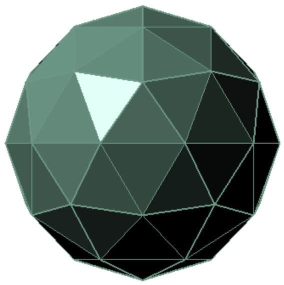
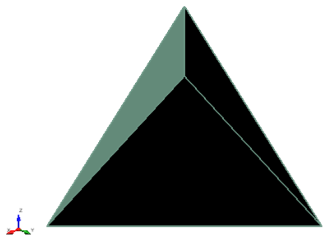
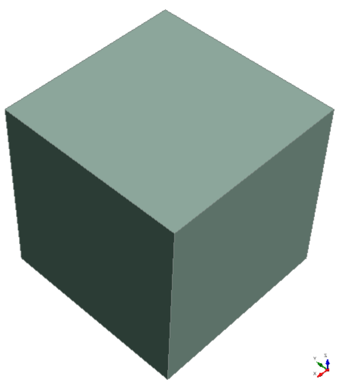
$$I_{xy} = I_{yx} = \sum dv\, (dxdy)$$

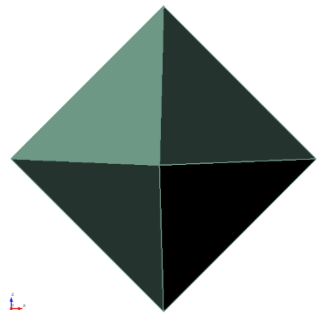
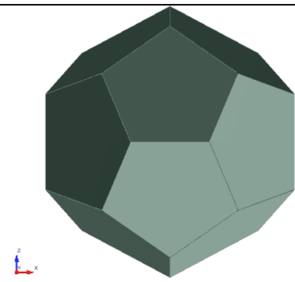
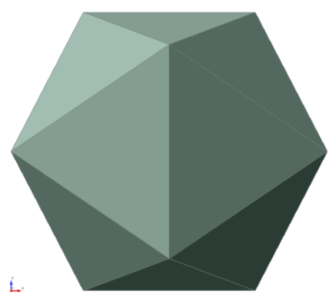
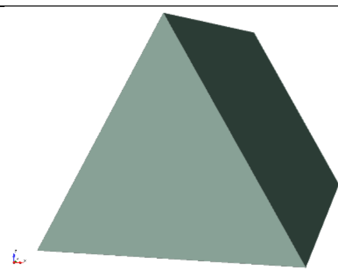
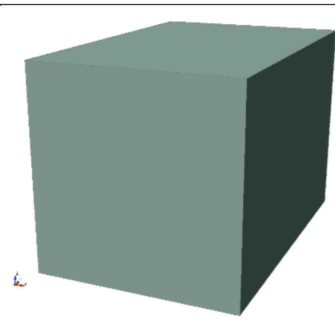
$$I_{xz} = I_{zx} = \sum dv\, (dxdz)$$

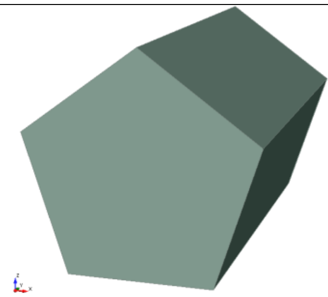
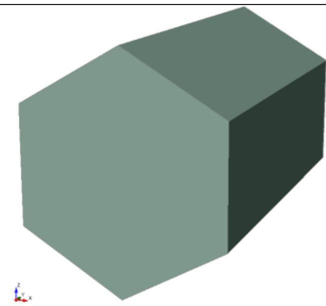
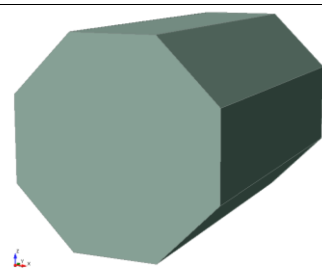
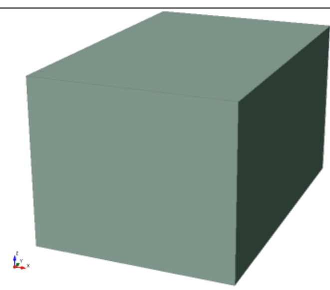
$$I_{yz} = I_{zy} = \sum dv\, (dzdy)$$

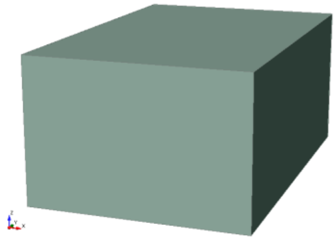
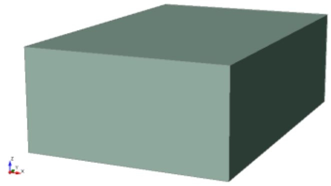
List of Rigid Body Shapes

The following table lists the 3D shapes available in RocFall3 rigid body analysis. For volume calculations, 's' is the edge length, and "depth" is the longitudinal dimension of an extruded polygon.

Name of Shape	3D View of Shape	Notes
Icosphere (32 faces)		32 faces of equal triangles
Icosphere (79 faces)		79 faces of equal triangles
Tetrahedron		4 faces of equal triangles Volume = $\frac{s^3}{6\sqrt{2}}$
Cube		4 faces of equal squares **Volume** = s^3

Octahedron		8 faces of equal triangles Volume = $\frac{\sqrt{2}}{3}s^3$
Dodecahedron		12 faces of equal pentagons Volume = $\frac{1}{4}(15+7\sqrt{5})s^3$ [3]
Icosahedron		20 faces of equal triangles Volume = $\frac{5}{12}(3+\sqrt{5})s^3$ [4]
Extruded Polygon Triangle		Depth = 0.742515 s Volume = 0.321519 s^3
Extruded Polygon Square		Depth = 1.128379 s Volume = 1.128379 s^3

Extruded Polygon Pentagon		Depth = 1.480061 s Volume = 2.546411 s^3
Extruded Polygon Hexagon		Depth = 1.818783 s Volume = 4.725338 s^3
Extruded Polygon Octagon		Depth = 2.479465 s Volume = 11.97191 s^3
Extruded Polygon Rectangle (5:6)		S1 = s S2 = 1.2s Depth = s Volume = 1.2 s^3 $I_{xx} = \frac{1}{12}m(s1^2 + d^2) =$ 0.166667 m s^3 $I_{yy} = \frac{1}{12}m(s1^2 + s2^2) =$ 0.203333 m s^3 $I_{zz} = \frac{1}{12}m(s2^2 + d^2) =$ 0.203333 m s^3

Extruded Polygon Rectangle (2:3)		S1 = s S2 = 1.5s Depth = s Volume = 1.5 s^3 $I_{xx} = 0.166667\ m\ s^3$ $I_{yy} = 0.270833\ m\ s^3$ $I_{zz} = 0.270833\ m\ s^3$
Extruded Polygon Rectangle (1:2)		S1 = s S2 = 2s Depth = s Volume = 2 s^3 $I_{xx} = 0.166667\ m\ s^3$ $I_{yy} = 0.416667\ m\ s^3$ $I_{zz} = 0.416667\ m\ s^3$

Reference

1. ‘List of moments of inertia’ (2021). *Wikipedia.* Available at https://en.wikipedia.org/wiki/List_of_moments_of_inertia (Accessed: 24 February 2021).

2. Tonon, F. (2005). Explicit Exact Formulas for the 3-D Tetrahedron Inertia Tensor in Terms of its Vertex Coordinates. Journal of Mathematics and Statistics, 1(1), 8-11. https://doi.org/10.3844/jmssp.2005.8.11

3. ‘Regular dodecahedron’ (2022). *Wikipedia*. Available at https://en.wikipedia.org/wiki/Regular_dodecahedron

4. ‘Regular icosahedron’ (2022). *Wikipedia*. Available at https://en.wikipedia.org/wiki/Regular_icosahedron