A New Era in Slope Stability Analysis: Shear Strength Reduction Finite Element Technique

This article examines the merits of using finite element analysis versus limit-equilibrium methods for slope design and analysis

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Article prepared for RocNews
Summer 2004

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Finite Element (FE) analysis is now widely accepted in routine excavation and support design and analysis. It may also revolutionize routine slope stability analysis.

Practicing engineers and academics alike are becoming increasingly convinced of the benefits of the Shear Strength Reduction (SSR) technique for determining slope factor of safety. This method is particularly useful when several different modes of failure are possible; it automatically finds the critical mechanism. But has the SSR technology matured to the stage where it can be used for routine analysis? In this article, we will attempt to answer this question, as well as offer insights into today's slope engineering practices.

Limit-Equilibrium Slope Stability Analysis

Limit-equilibrium methods are the most commonly used approaches for analyzing the stability of slopes. The fundamental assumption at their core is that failure occurs through sliding of a block or mass along a slip surface. The popularity of limit-equilibrium methods is primarily due to their relative simplicity, ready ability to evaluate the sensitivity of stability to various input parameters, and the experience geotechnical engineers have acquired over the years in interpreting calculated factor of safety values. Limit-equilibrium methods require minimal input data. The factor of safety values they output help engineers to guard against uncertainties such as ignorance about the reliability of input parameters and loadings, and the possibility that identified failure mechanisms may differ from actual behaviour. As well, recommended factor of safety values for slopes and excavations generally ensure that deformations are within acceptable range.

Despite all the benefits, the limit-equilibrium approach has some important deficiencies.

"Things should be made as simple as possible, but not any simpler."
— Albert Einstein

The technique neglects stress-strain behaviour of soils and rocks. It also makes arbitrary assumptions (mostly regarding inter-slice forces) to ensure static determinacy. It is awkward to use for analyzing stability problems, such as the failure of cantilever and retaining walls, in which failure involves deformed wedges.

The Shear Strength Reduction Technique

Rapid advances in computer technology and sustained development have pushed the finite element method (FEM) and other numerical analysis approaches to the forefront of geotechnical practice. Since it was first applied to geotechnical engineering in 1966, the FEM has grown tremendously in popularity, primarily due to its ability to analyze a very broad range of problems, while yielding realistic results. It can accommodate practically all kinds of geometry, and can model key aspects of material behaviour such as stress paths (construction sequence), and coupled stress-pore pressure variations.

In the mid 1970s, techniques for applying the FEM to slope stability analysis started appearing in geotechnical literature. They were mostly based on an approach that flows naturally from the definition of slope factor of safety, and is now commonly referred to as the Shear Strength Reduction (SSR) technique. By definition, the factor of safety of a slope is the "ratio of actual soil shear strength to the minimum shear strength required to prevent failure," or the factor by which soil shear strength must be reduced to bring a slope to the verge of failure (Duncan, 1996). In the SSR finite element technique elasto-plastic strength is assumed for slope materials. The material shear strengths are progressively reduced until collapse occurs.

Since the finite element method was first applied to geotechnical engineering in 1966, it has grown tremendously in popularity.

For Mohr-Coulomb material shear strength reduced by a factor (of safety) F can be determined from the equation

$$\frac{\tau}{F} = \frac{c'}{F} + \frac{\tan\phi'}{F}$$

This equation can be re-written as

$$\frac{\tau}{F} = c^* + \tan\phi^*$$

In this case,

$$c^* = \frac{c'}{F} \quad \text{and} \quad \phi^* = \arctan\left(\frac{\tan\phi'}{F}\right)$$

are reduced Mohr-Coulomb shear strength parameters, and these values can be input into an FE model and analyzed.

Basic Algorithm

For Mohr-Coulomb materials, the steps for systematically searching for the critical factor of safety value, F, which brings a previously stable slope to the verge of failure, are as follow

Step 1: Develop an FE model of a slope, using the deformation and strength properties established for the slope materials. Compute the model and record the maximum total deformation in the slope.

Step 2: Increase the value of F and calculate factored Mohr-Coulomb material parameters as described above. Enter the new strength properties into the slope model and re-compute. Record the maximum total deformation.

Step 3: Repeat Step 2, using systematic increments of F, until the FE model does not converge to a solution, i.e. continue to reduce material strength until the slope fails. The critical F value just beyond which failure occurs will be the slope factor of safety.

(For a slope that is initially unstable, factor of safety values in steps 2 and 3 must be reduced until the FE model converges to a solution.)

Advantages

The elasto-plastic SSR FE approach offers a number of significant advantages over traditional limit-equilibrium analysis. First, it eliminates the need for a priori assumptions on failure mechanisms (the type, shape, and location of failure surfaces). The SSR technique automatically establishes the critical failure mechanism. It eliminates artificial separation of slope problems into those involving slip surface failures, and those involving failure of deformed wedges.

Although possibly more demanding in other aspects, the SSR FE technique minimizes the expertise required in finding critical failure mechanisms for certain slope problems. At times this goes unnoticed by slope engineers. A typical example involves the problem of identifying the critical failure mechanism beneath a concrete dam on a foundation that includes a weak layer. The search for the critical limit-equilibrium mechanism, using *Slide*, is described in an AVI movie in

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this issue of RocNews Summer 2004. Regular non-circular limit-equilibrium analysis with the Morgernstern method produced a factor of safety of 3.0. Using advanced features in *Slide* not applied by most modelers – a combination of block search with surface optimization – a more critical slip surface with a 2.36 factor of safety value was located. The images of these results are shown on Figures 1a and 1b.

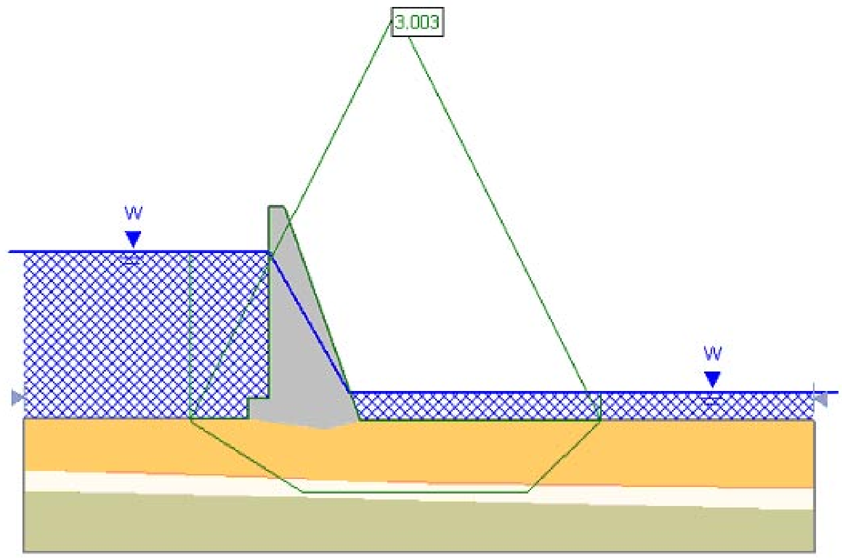

Figure 1a. The critical failure surface and factor of safety from conventional Morgernstern limit-equilibrium analysis of a concrete dam. The thin, beige-coloured material beneath the dam is the weak layer.

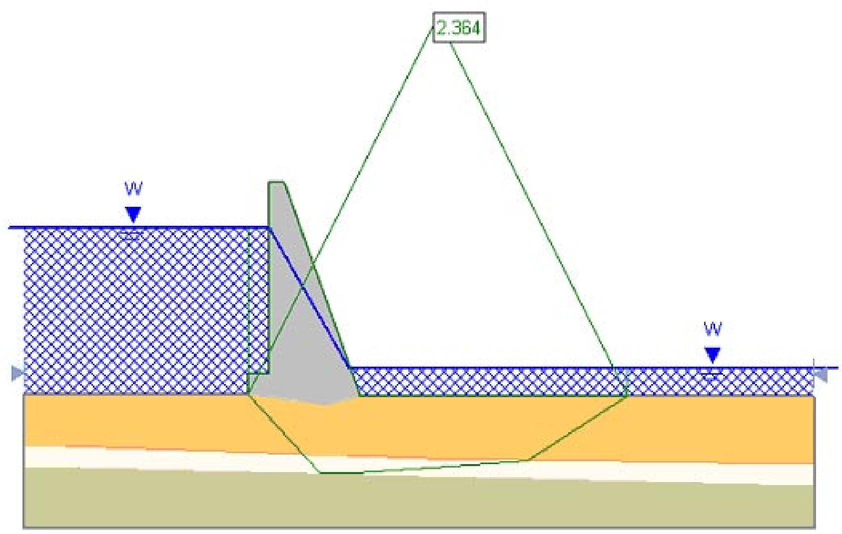

Figure 1b. The critical failure surface and factor of safety obtained using block search, combined with surface optimization. Notice the differences between this surface and that in Figure 1a.

The problem was then modeled in *Phase*[2], the finite element analysis program developed by Rocscience, using the SSR technique. This yielded a factor of safety value of 2.35. Figure 2a shows the contours of total displacement at the modeling stage preceding total failure (non-convergence of the solution), and an exaggerated deformed mesh. Contours of maximum shear strain are shown on Figure 2b. These figures indicate failure in the weak zone and a critical failure surface similar to the non-circular limit-equilibrium surface obtained from the combination of block search and optimization.

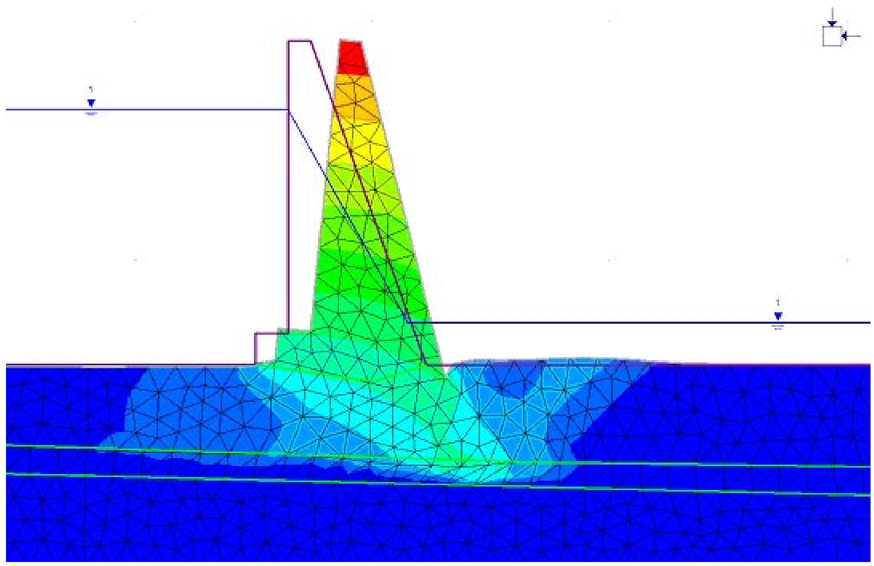

Figure 2a. Contours of total displacement calculated for the FE model of the dam, and the (exaggerated) deformed mesh. These contours indicate a critical failure mechanism similar to that obtained from advanced limit-equilibrium analysis.

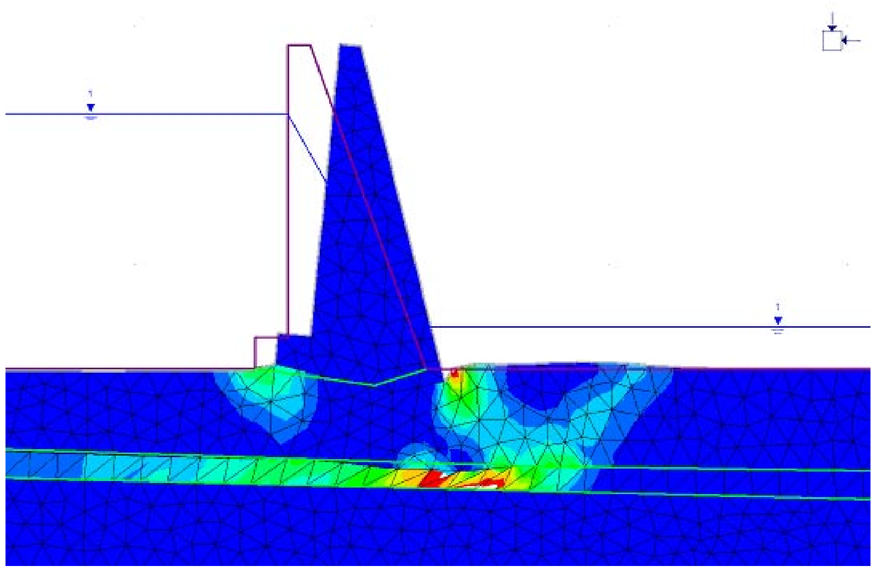

Figure 2b. Contours of maximum shear strain calculated for the FE model of the dam. Like the previous image, these contours indicate the critical failure mechanism as shearing, passing through the weak zone.

Another benefit of the SSR approach is its elimination of arbitrary assumptions regarding the inclinations and locations of inter-slice forces. As well, the method can automatically monitor the development of failure zones, from localized areas all the way to total slope failure. This is particularly important in the analysis of high slopes such as those found in large open-pit mines, and the impact of slope excavation on nearby structures. Given the correct deformation properties of materials, the SSR method can predict expected deformations at the stress levels found in slopes.

Due to the FEM's ability to compute deformations and quantities such as bending moments, it can to be used to design the support elements in a slope. For example, in a slope stabilized with piles, the FEM is capable of predicting axial loads, bending moments and deformations of the piles, making it possible for engineers to select appropriate dimensions and materials that ensure adequate performance.

The SSR FE technique can model construction procedures and sequences (i.e. loading paths), a particularly important aspect for some embankments and excavations. Drawing on the robustness of the FEM, the technique performs very well under a wide range of conditions. Lastly, it can be more readily applied to three-dimensional slope modeling than limit-equilibrium methods.

Will Use of the SSR Technique Become Widespread?

In the past, a number of factors limited application of the SSR technique for routine slope stability analysis. Compute times for analyses were long, computing power adequate for such analysis was expensive, and model preparation and results interpretation required extensive effort and time due to clumsy, difficult-to-use interfaces. The above-listed factors combined to push SSR costs very high.

Most practicing engineers had another complaint against the technique: they felt it was data hungry, requiring material input parameters, which were not collected in routine site investigations, at least not with reasonable accuracy. They also felt that the reliability of SSR results was unproven.

The FEM is able to compute deformations and quantities such as bending moments, allowing the method to be used to design the support elements in a slope.

Cheap, and yet very powerful, computers, and tremendous improvements in program interfaces and user-friendliness have radically changed the situation. Today the typical desktop computer can perform two-dimensional FE analysis within minutes. New commercially available computer programs, such as Rocscience's *Phase²*, significantly reduce the amount of time required to build models, to interpret results, and to produce output for reports.

Obtaining input material parameters for finite element analysis is possible through testing, similar to those required for traditional settlement or stability analysis. Triaxial and direct shear tests can be combined with consolidation tests to obtain required input data. For elasto-plastic analysis involving conventional Mohr-Coulomb strength, with the assumption of associative flow rule, the FEM requires the same parameters as limit-equilibrium analysis (except for the modulus of deformation and Poisson's ratio).

The question of the reliability of the SSR technique has been comprehensively answered. Several studies have shown that the technique produces results that are close to those obtained from method-of-slices and other limit-equilibrium approaches. The references provided at the end of this article will provide details for the interested reader.

The advances described above should change perceptions of SSR slope analysis. Most of the issues that hindered application yesterday have been eliminated, or are rapidly being addressed. A few of the outstanding issues with the SSR technique remain and companies like Rocscience are finding ways of addressing them.

Outstanding Issues with the SSR Technique

To date, all published discussions on the application of the FEM to slope stability analysis have assumed Mohr-Coulomb strength. This may be primarily due to the ease with which reduced Mohr-Coulomb strength parameters can be calculated for application in the SSR technique. As shown earlier in the article, reduced cohesion (c^*) and friction angle (φ^*) values can be explicitly determined from original parameters c and φ. With nonlinear criteria such as the Generalized Hoek-Brown and Power Curve strength models, it is impossible to obtain such closed-formed relationships.

To get around the difficulty described above, Rocscience engineers have helped develop an approach that uses a Mohr-Coulomb approximation of a nonlinear strength criterion. The details of this approach are described in a recent paper, (Hammah et al, 2004). We are currently developing a more generalized and more accurate method for performing SSR analysis for nonlinear material strength.

Although the SSR technique is highly intuitive and relatively straightforward to conduct using any existing finite element program, the manual effort involved in calculating reduced material properties for multiple factor of safety values can get tedious, therefore hampering routine use. The SSR technique has been built into a few geotechnical finite element programs, but aspects of the problem have not been sufficiently automated to levels found in limit-equilibrium software packages. Rocscience is working on releasing, in the near future, a version of *Phase²* that more completely automates SSR analysis.

Conclusion

By examining the merits of finite element analysis and limit-equilibrium methods for slope design and analysis, it is our opinion that, for the foreseeable future, these two approaches will coexist. Together, they supply engineers with a more complete toolkit for tackling slope stabillity problems. The experience, amassed by the geotechnical engineering profession over the decades, with limit equilibrium methods is invaluable and cannot be readily displaced. On the other hand, several types of geotechnical problems are not readily analyzed with limit equilibrium methods, but can be handled by the SSR FE approach. Today's technological advances, combined with the reduced costs of computing power, enable the SSR technique to solve these problems easily. In addition, the SSR technique can be used to resolve ambiguities in limit-equilibrium slope stability analysis.

Recent improvements to the SSR technique and its applications do indeed have the potential to enhance the quality of slope designs, and expand our understanding of slope behaviour and interactions between the various factors that influence stability. While we may not be ready to abandon tried and true approaches all together, we may be on the threshhold of a new era in slope engineering.

We invite readers to comment on this article or express their thoughts and observations on the issue of limit-equilibrium versus finite element analysis.

Contact us at: rocnews@rocscience.com.

Useful References on the SSR Technique

The references below supplied most of the technical background and descriptions of the SSR technique described in this article. Although the list is by no means exhaustive, it will provide the interested reader an excellent starting point for learning more about the SSR technique.

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