rocscience

RocTopple

Toppling Analysis and Support Design of Rock Slopes

Theory Manual

Safety Factor Calculations – Block Toppling

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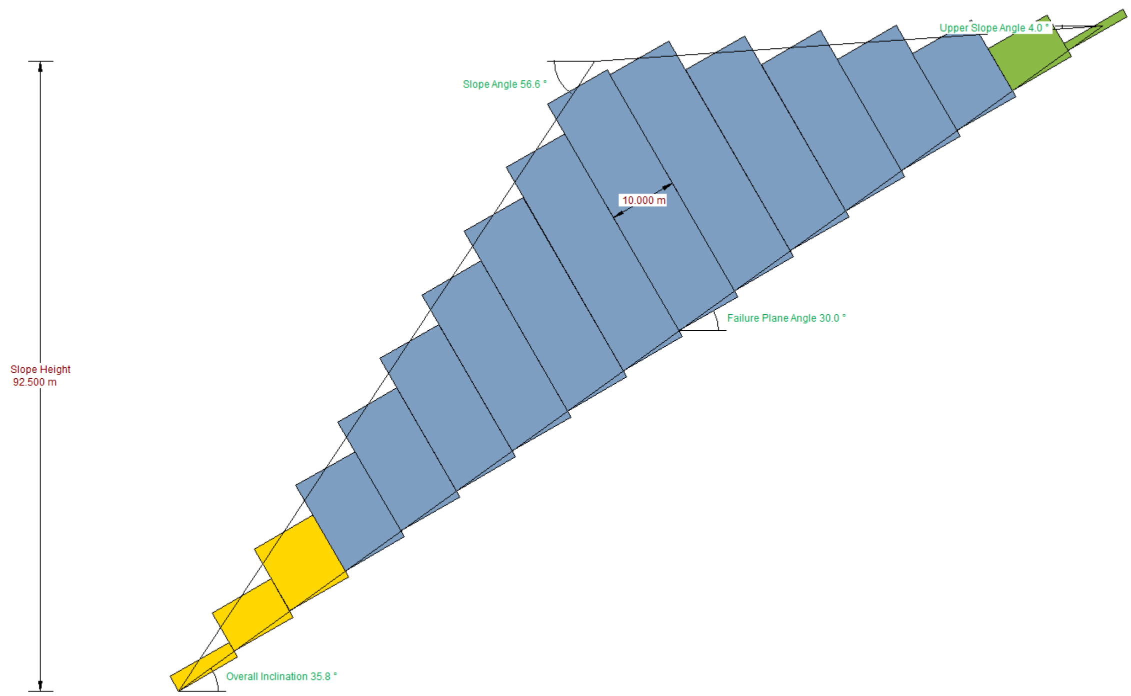
Block Toppling Calculations

1. Introduction

This paper documents the calculations used in *RocTopple* to determine the safety factor of **toppling blocks**.

2. Slope Geometry



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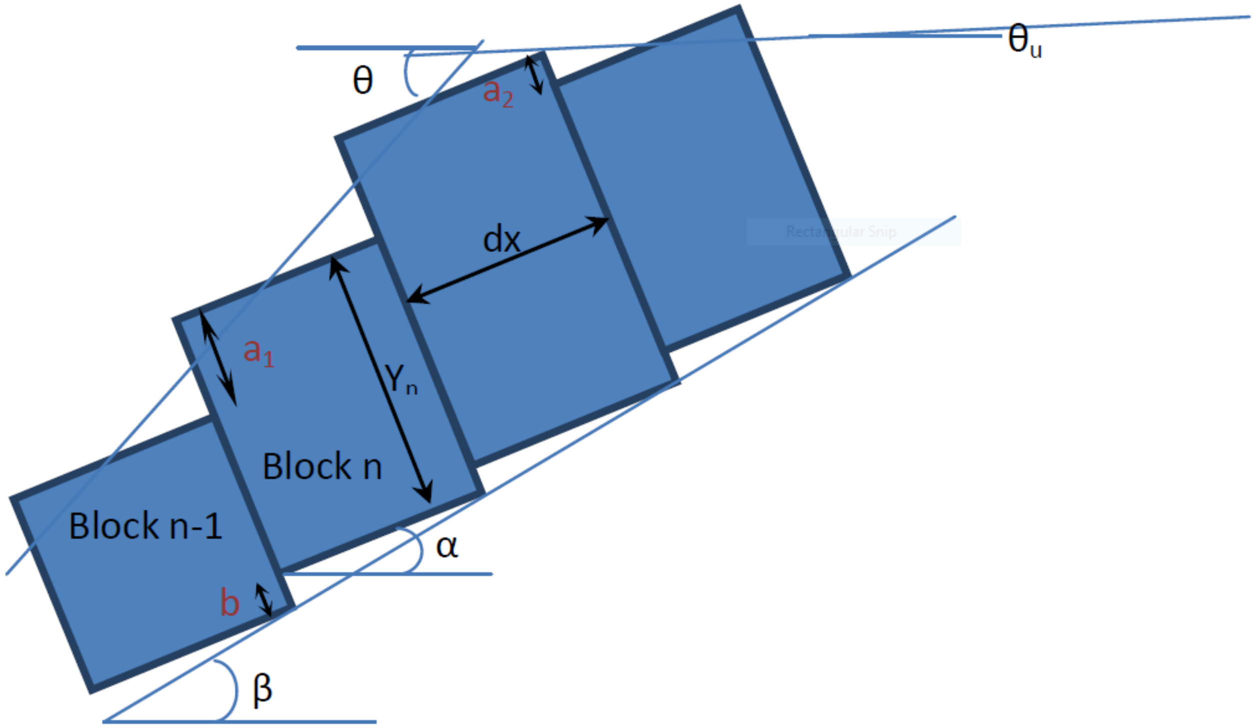
Where:

H	is the slope height
θ	is the slope angle
θ_u	is the upper slope angle
β	is the overall base inclination
α	is the block failure plane angle (assumed to be perpendicular to toppling joint dip)
dx	is the uniform toppling joint spacing

A bench width may also be specified to control the number of blocks above the crest.



$$a_1 = (dx)\tan(\theta - \alpha)$$

$$a_2 = (dx)\tan(\alpha - \theta_u)$$

$$b = (dx)\tan(\beta - \alpha)$$

There are 2 methods for generating block geometry in *RocTopple*.

In the "Equal Area" method, the height of the first block is generated such that its top surface intersects the slope line at the midpoint. The heights of blocks after the first and up until the crest are calculated as follows:

$$Y_n = Y_{n-1} - b + a_1$$

This method ensures that the volume of the wedge is conserved.

The crest block is assumed to be the block with the largest block height and is usually the block at the wedge crest. The midpoint of the top surface of the crest block is assumed to be less than or equal to the slope height. Heights for blocks above the crest block are calculated as follows:

$$Y_n = Y_{n-1} - a_2 - b$$

In the "Goodman and Bray" method, block heights are generated using the same formulae, except for the very first block. The height of the first block is assumed to be

$$Y_1 = a_1 - b$$

3. Limitations in Slope Geometry

The relationships between slope geometry variables are critical in the generation of the blocks. There are four main limitations in the input of geometry.

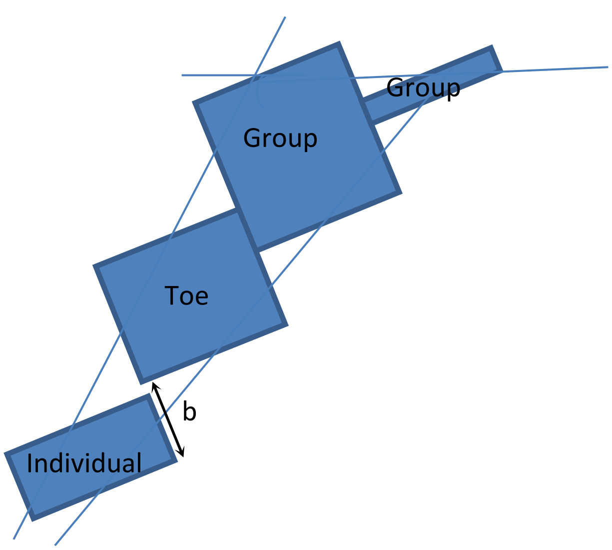
1. The upper slope angle must be less than the overall base inclination; otherwise, the upper slope would never meet with the wedge base, and an infinite number of blocks can be generated.
2. The upper slope angle can be negative, but it cannot exceed the toppling joint dip.
3. The slope angle must be greater than the overall base inclination.
4. The block failure plane angle (perpendicular to the toppling joint dip) must be less than the overall base inclination.

4. Block Types

RocTopple categorizes blocks into 3 types:

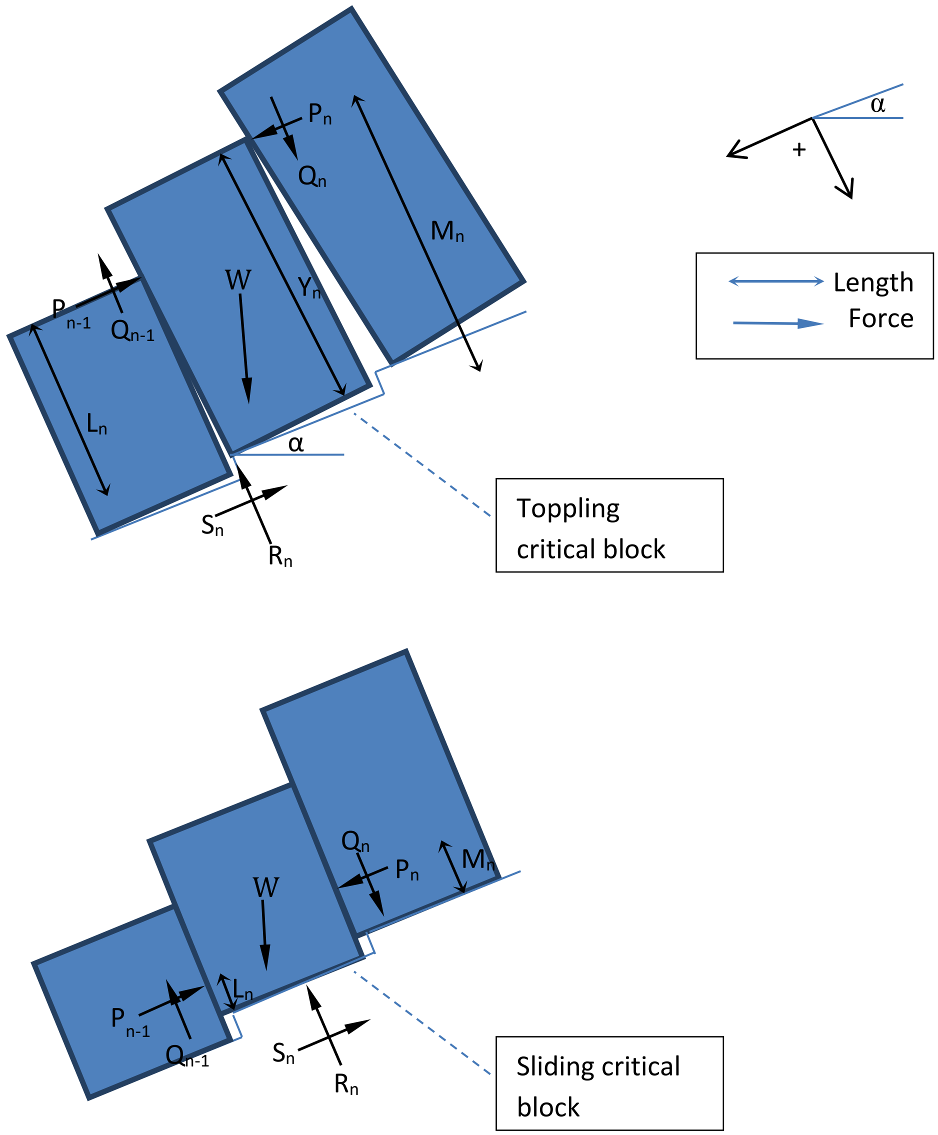
- Individual
- Toe
- Group

In most cases, a block is in contact with the blocks above and below. A block in contact with other blocks is named a "**Group**" block. The bottom-most block of the group is named the "**Toe**" block. There are exceptions in the geometry where a block does not have contact with other blocks. That is, the step height (b) is greater than the block height. This type of block is named the "**Individual**" block. An illustration of the different block types is shown below.



5. Block Equilibrium

Each block in the slope is examined in terms of toppling and sliding failure modes. Shown below are two sets of block force diagrams, one showing a toppling critical block and the other a sliding critical block.



Where:

W	is the block weight
Pn	is the normal force (due to failure of block above)
Qn	is the joint shear force (function of Pn)
P_{n-1}	is the supporting normal force (for block equilibrium)
Q_{n-1}	is the joint shear force (function of P_{n-1})
R_n	is the base normal reaction force
Sn	is the base shear force (function of R_n)

The stability of the entire slope rests on the equilibrium of the toe block. If the P_{n-1} required to stabilize the toe block is 0, then the slope is metastable and in equilibrium. Determining the stability of the toe block entails solving the force equilibrium for each block in the slope starting from the top block, working down to the toe. If any block is unstable, the required P_{n-1} to stabilize the block would be greater than 0 and would add to the driving force for the block below.

A block is considered stable if P_{n-1} for both the toppling and sliding failure modes are less than or equal to 0. A block is considered toppling critical if $P_{n-1,toppling} > 0$ and $P_{n-1,toppling} > P_{n-1,sliding}$. Likewise, a block is considered sliding critical if $P_{n-1,sliding} > 0$ and $P_{n-1,sliding} > P_{n-1,toppling}$. The larger, positive P_{n-1} becomes the driving force for the block below.

The unknown variables for each block are P_{n-1}, Q_{n-1}, $R_n$and S_n. These are solved using the following equations.

For a toppling block, $P_{n-1,toppling}$ is found assuming rotational equilibrium (summing moment about point of rotation, i.e. bottom left corner of block in the force diagram):

$$\sum Moment = 0 = \frac{1}{2}(W\sin(\alpha) \times Y_n - W\cos(\alpha) \times dx) + P_n \times M_n - Q_n \times dx - \boldsymbol{P_{n-1}} \times L_n$$

$$-T \times \frac{2dx}{3} + \sum(external\ force \times moment\ arm)$$

In the above equation,

$$T = tensile\ resistance = tensile\ strength \times dx \times 0.5$$

where the tensile strength assumes a triangular distribution for the toppling mode.

Once $P_{n-1,toppling}$ is determined, solving Q_{n-1}, $R_n$and S_n becomes trivial, where

$$Q_{n-1} = shear\ function\ of\ P_{n-1,toppling}$$

$$\sum F_y = 0 = W\cos(\alpha) + Q_n - Q_{n-1} - \boldsymbol{R_n} + external\ vertical\ forces$$

$$\sum F_x = 0 = W\sin(\alpha) + P_n - P_{n-1} - \boldsymbol{S_n} + external\ horizontal\ forces$$

On the other hand, solving the force equilibrium for a sliding block is less trivial. A block on the verge of sliding must satisfy 4 equations of equilibrium:

$$\sum F_x = 0 = W\sin(\alpha) + P_n - P_{n-1} - S_n + external\ horizontal\ forces$$

$$\sum F_y = 0 = W\cos(\alpha) + Q_n - Q_{n-1} - R_n + external\ vertical\ forces$$

$$Q_{n-1} = shear\ function\ of\ P_{n-1,sliding}$$

$$Constraint: Q_{n-1} \geq 0$$

$$S_n = shear\ function\ of\ R_n$$

$$Constraint: S_n \geq 0$$

Note that if a block is found to be sliding critical, the thrust line ($M_n$for the block below) must be adjusted. *RocTopple* assumes the thrust line is adjusted to a height (relative to block base) equivalent to the step height (b) and 1/3 the joint length. M_n does not exceed the block height.

Also note that the equilibrium equations assume joints are fully mobilized in shear, which is why $Q_{n-1}$is the shear function of P_{n-1} (toppling and sliding block) and S_n is the shear function of R_n (sliding block).

6. Factor of Safety

RocTopple uses shear strength reduction of the joints to determine the factor of safety.

$$Q_{n-1} = \frac{shear\ function\ of\ P_{n-1}}{FS}$$

$$S_n = \frac{shear\ function\ of\ R_n}{FS}$$

Physically, changing the shear strength reduction factor means changing the shear strength of the toppling joints and the block basal joints. In toppling where there is slip in the toppling joints, the shear force in these joints adds to the normal force of the block below and thus the resisting moment. The shear strength of the block base applies to sliding critical blocks.

The factor of safety for "**Group**" blocks is determined by the shear strength reduction factor that brings the toe block to equilibrium (P_{n-1} for toe is 0). *RocTopple* also determines a factor of safety for each "**Individual**" block. The program reports the lowest factor of safety.

The factor of safety value is limited to 0.1 at the lower bound, and 25 at the upper bound.

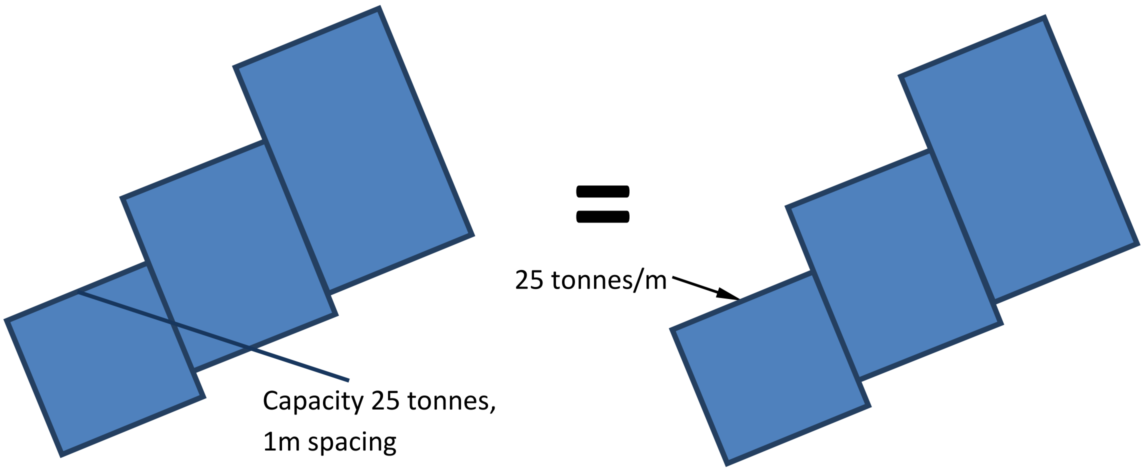
7. Applied Forces

7.1. Line Loads

A line load is treated as an additional force applied on the block. The user is allowed to apply any number of line loads along the surface of the slope.

7.2. Bolts

RocTopple treats bolts as end-anchored bolts, where a force equivalent to the tensile capacity is applied at the two bolt ends. The bolt should pass through the base of a block and be anchored into the bedrock. This is an “active” bolt implementation, where the bolt would influence the mode of failure of the blocks.

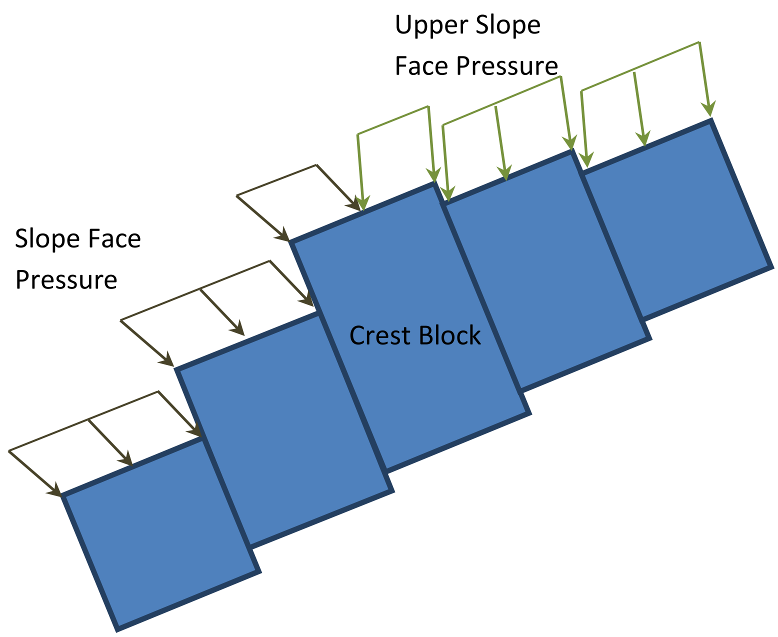


The user can input the bolt tensile capacity as well as the bolt spacing. Since the analysis is per unit depth, the tensile capacity is divided by the spacing to get a capacity per unit depth.

$$bolt\ unit\ capacity = \frac{input\ capacity}{spacing}$$

7.3. Distributed Loads

Distributed loads can be applied to both the slope face and upper slope face. The loading applies to only the top surface of blocks and can be applied at any angle.



In the block equilibrium calculation, the distributed loading on each block is resolved into a single line load and then treated as a force acting at the midpoint of the block's top surface.

7.4. Seismic Loads

RocTopple offers pseudo-static seismic analysis in the form of seismic coefficients. These dimensionless coefficients represent the earthquake acceleration as a fraction of the acceleration due to gravity.

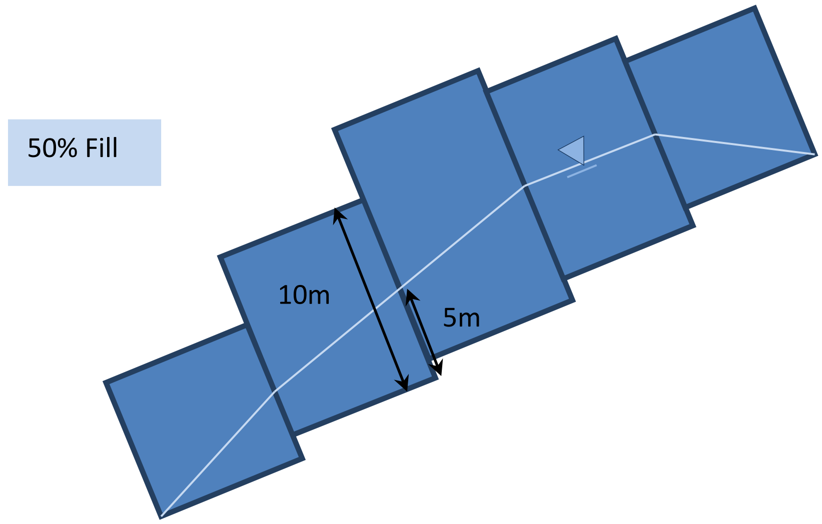
$$seismic\ force\ on\ block = seismic\ coefficient\ \times block\ weight$$

The user can define a horizontal coefficient and a vertical coefficient to represent a horizontal seismic force and a vertical seismic force acting on all blocks. Like the block weight, the seismic forces are applied at the centre of gravity of each block.

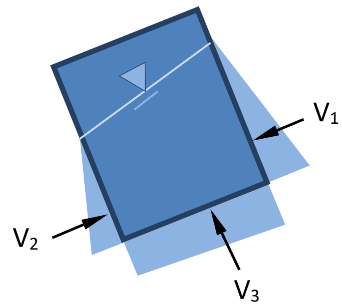
7.5. Water Pressure

There are two ways of specifying water pressure in *RocTopple*.

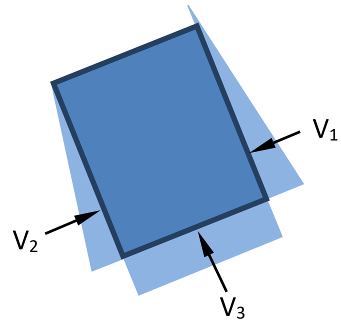
The first is by designating a **percent fill** in joints. This means a certain percent of the joint length is filled with water.



Each block would then be subjected to an additional set of forces due to the water pressure.



The second way for specifying water pressure is by drawing a **phreatic surface**. Calculations for each block are identical to the percent joint fill option. Where a block is fully submerged (block is underneath the phreatic surface), the ponded water is not taken into account. Instead, the block is assumed to experience water pressure along the full length of joints (equivalent to 100% fill).



8. Upslope Toppling

When external forces act on the slope, blocks may be stable for downslope toppling but unstable for upslope toppling. *RocTopple* provides warning for when the slope is unstable in that opposite direction. Upslope toppling is accounted for by checking the rotational equilibrium of group blocks. If the programme finds the very top group block rotationally unstable ($P_{n,toppling} > 0$), then the slope is considered to be unstable for upslope toppling. The programme only checks for upslope toppling when these group blocks extend all the way up to the very top. The rotational equilibrium is calculated in reference to the upper base block corner. Although the step length could be equal to 0, sliding upslope is not considered. Individual blocks are not accounted for because the programme assumes the steps prevent them from rotating upslope.

9. Eurocode

RocTopple upholds the single-source assumption when a design standard is applied. It assumes an action is favorable if it overall produces a resisting moment, and unfavorable if it overall produces a driving moment.