RocTunnel3

Kinematics Computations

Theory Manual

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Introduction

This theory manual documents the calculations used in the Kinematics Engine in RocTunnel3 to determine the safety factor of a block. This involves the following series of steps:

1. Determine whether the block is geometrically removable
2. Determine all the individual forces acting on the block to calculate the resultant active and passive force vectors for the block
3. Determine the sliding direction of the block
4. Determine the normal forces on each block plane
5. Compute the resisting forces due to joint shear strength
6. Calculate the safety factor

All blocks are analyzed independently by the procedure outlined above.

However, when Successive Failure analysis is turned on, the analysis is computed in iterations, as is detailed in Section 7.

1. Removability

A testing matrix, as explained in Chapter 6 of the text by Goodman and Shi, "Block Theory and Its Application to Rock Engineering" (1985), is used to determine the geometric removability of a block.

Let (T) denote the matrix in question. Each row (T_{ij}) represents a pair of joints i, j. The matrix has n columns, where n is the number of joints. The value of an entry of the matrix equals:

$$1.1 \qquad \left(T_{ij}\right)^k = \text{sign}\left[\left(\widehat{n_i} \times \widehat{n_j}\right) \cdot \widehat{n_k}\right] I(a_k)$$

Where:

$\widehat{n_i}$ is the upward unit normal of joint face i

$I(a_k)$ represents the value corresponding to joint i in the signed block code

For a joint k, its value in the signed block code is defined as follows:

$$1.2 \qquad I(a_k) = \begin{cases} +1, & \textit{if the block lies above the plane defining joint face } k \\ -1, & \textit{if the block lies below the plane defining joint face } k \\ \pm 1, & \textit{if the block lies between } 2 \textit{ planes parallel to joint face } k \end{cases}$$

Then, according to the Rule for Testing Finiteness in the referenced text, if every row of the testing matrix includes both positive and negative terms, the block is not removable; otherwise, the block is geometrically removable.

2. Block Forces

All forces on the block can be classified as either Active or Passive. In general, Active forces represent driving forces in the safety factor calculation, whereas Passive forces represent resisting forces.

The individual force vectors are computed for each quantity (e.g. wedge weight, bolt force, water force, etc.), and then the resultant Active and Passive force vectors are determined by a vector summation of the individual forces.

2.1. Active Force Vector

The resultant Active force vector is comprised of the following components:

$$2.1 \qquad A = W + C + X + E + B_a + W_s$$

Where:

A is the resultant active force vector

W is the block weight vector

C is the load force vector

X is the active pressure force vector

U is the water force vector

E is the seismic force vector

B_a is the active bolt force vector

W_s is the shotcrete weight vector

2.1.1. Block Weight Vector

The block weight is usually the primary driving force in the analysis.

$$2.2 \qquad W = \sum_i (\gamma_{r,i} V_i) \cdot \hat{g}$$

Where:

$\gamma_{r,i}$ is the unit weight of rock of material i

V_i is the volume of material i in the block

$\hat{g}$ is the gravity direction

2.1.2. Load Force Vector

External loads can be defined on a surface, line, or vertex.

$$2.3 \qquad C = \sum_i c_i$$

Where:

c_i is the force vector resulting from applying load i on the block

$$2.4 \qquad c_i = \int_S c_{i,p} \widehat{n_{i,p}} dA$$

Where:

$c_{i,p}$ is the magnitude of applying load i at point p

$\widehat{n_{i,p}}$ is the unit direction vector of load i at point p

S is the surface of the slope face of the block

2.1.3. Active Pressure Force Vector

Support pressure can be applied on any surface of the slope as either active or passive.

$$2.5 \qquad \boldsymbol{X} = \sum_i x_i$$

Where:

x_i is the force vector resulting from the application of support pressure i on the block

$$2.6 \qquad x_i = \int_S x_{ip} \widehat{n_{i,p}} dA$$

Where:

$c_{i,p}$ is the magnitude resulting from applying support pressure i at point p

$\widehat{n_{i,p}}$ is the unit direction vector of support pressure i at point p

S is the surface of the slope face of the block

2.1.4. Water Force Vector

$$2.7 \qquad \boldsymbol{U} = \sum_i u_i \widehat{n_i} + U_p$$

Where:

u_i is the magnitude of the water force on joint i

$\widehat{n_i}$ is the inward (into the block) normal of joint face i

U_p is the ponded water force vector

Define the following variable:

γ_w is the unit weight of water

Water Pressure on Joints

The following water pressure methods are available to the user in RocTunnel3.

Uniform Pressure

$$2.8 \qquad u_i = P_i A_i$$

Where:

P_i is the water pressure on joint i

A_i is the are of joint i

Water Surface

$$2.9 \qquad u_i = \gamma_w \int_{S_i} \max\{(z_{p,w} - z_{p,S_i}), 0\} \, dA$$

Where:

$z_{p,w}$ is the elevation of the water surface at point p

$z_{p,S}$ is the elevation of the surface of joint face i at point p

S_i is the surface of joint face i

Water Grid

For a pore pressure grid, the computation of the magnitude of the water force on a joint is as follows:

$$2.10 \qquad u_i = \int_{S_i} \max\{P_p, 0\} \, dA$$

Where:

P_p is the interpolated pore pressure at point p

S_i is the surface of joint face i

For a pressure head grid, the computation of the magnitude of the water force on a joint is as follows:

$$2.11 \qquad u_i = \gamma_w \int_{S_i} \max\{P_{h,p}, 0\} \, dA$$

Where:

$P_{h,p}$ is the interpolated pressure head at point p

S_i is the surface of joint face i

For a total head grid, the computation of the magnitude of the water force on a joint is as follows:

$$2.12 \qquad u_i = \gamma_w \int_{S_i} \max\{(t_{h,p} - z_{p,S_i}), 0\} \, dA$$

Where:

$t_{h,p}$ is the interpolated total head at point p

z_p is the elevation of the surface of joint face i at point p

S_i is the surface of joint face i

RU Coefficient Method

$$2.13 \qquad u_i = \gamma_w \int_{S_i} \left(\sum_j \Delta z_{j,p} \, RU_j \right) dA$$

Where:

$\Delta z_{j,p}$ is the thickness of material j in the slope above point p

RU_j is the Ru coefficient of material j

S_i is the surface of joint face i

Ponded Water Force

In RocTunnel3, ponded water forces are accounted for only when a water table is defined. Additionally, there can be at most one water table defined.

$$2.14 \qquad U_p = \gamma_w \int_S \max\{(z_{p,w} - z_{p,S}), 0\} \widehat{n_p} dA$$

Where:

$z_{p,w}$ is the elevation of the water table at point p

$z_{p,S}$ is the elevation of the surface of the slope face of the block at point p

$\widehat{n_p}$ is the unit normal vector into the block of the surface of the slope face at point p

S is the surface of the slope face of the block

2.1.5. Seismic Force Vector

Seismic force can be defined in the direction of sliding or in a direction defined by the user, either in terms of trend/plunge angles or a vector.

$$2.15 \qquad \boldsymbol{E} = (k\,W) \cdot \hat{e}$$

Where:

k is the seismic coefficient

W is the magnitude of the block weight vector

$\hat{e}$ is the unit vector of the direction of the seismic force

2.1.6. Active Bolt Force Vector

RocTunnel3 contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the block. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

$$2.16 \qquad \boldsymbol{B_a} = \sum_{i=1}^{n} c_i \, \widehat{e_\iota}$$

Where:

c_i is the capacity of bolt i

$\widehat{e_\iota}$ is the unit direction vector of bolt i

2.1.7. Shotcrete Weight Vector

$$2.17 \qquad \boldsymbol{W_s} = A_s \gamma_s t$$

Where:

A_s is the area where the slope face of the block and the shotcrete overlap

γ_s is the unit weight of the shotcrete

t is the shotcrete thickness

2.2. Passive Force Vector

The resultant Passive Force Vector is the sum of the bolt, shotcrete, and pressure (passive) support force vectors.

2.18 $$\boldsymbol{P} = \boldsymbol{H} + \boldsymbol{Y} + \boldsymbol{B_p}$$

Where:

$\boldsymbol{P}$ is the resultant passive force vector

$\boldsymbol{H}$ is the shotcrete shear resistance force vector

$\boldsymbol{Y}$ is the passive pressure force vector

$\boldsymbol{B_p}$ is the resultant passive bolt force vector

2.2.1. Shotcrete Shear Resistance Force Vector

The shotcrete support model in RocTunnel3 assumes that the shotcrete fails by punching shear. Basically, the block shears through the shotcrete on the slope face. The shear zones are along the lines of intersection of the joints with the slope face. To account for adhesion failure coupled with bending failure, factor the shear strength accordingly.

2.19 $$\boldsymbol{Y} = \sum_i y_i$$

Where:

y_i is the shear resistance force vector resulting from shotcrete i

2.20 $$y_i = \tau_s t \int_L \widehat{n_{S,p}} \, dl$$

Where:

τ_s is the shotcrete shear strength

t is the shotcrete thickness

$\widehat{n_S}$ is the unit normal direction to the slope face at point p

L is the line where the shotcrete and traces of the joints on the slope overlap

2.2.2. Passive Pressure Force Vector

Support pressure can be applied on any surface of the slope as either active or passive.

2.21 $$\boldsymbol{X} = \sum_i x_i$$

Where:

x_i is the force vector resulting from the application of support pressure i on the block

2.22
$$x_i = \int_S x_{ip} \, \widehat{n_{i,p}} \, dA$$

Where:

$c_{i,p}$ is the magnitude resulting from applying support pressure i at point p

$\widehat{n_{i,p}}$ is the unit direction vector of support pressure i at point p

S is the surface of the slope face of the block

2.2.3. Passive Bolt Force Vector

RocTunnel3 contains a very simple bolt model. The bolt force is taken as the capacity of the bolt with a direction defined by the user (trend/plunge). The force is assumed to go through the centroid of the wedge. The user must determine the capacity based on the tensile strength of the bolt, bond strength, and plate capacity. The efficiency of the bolt depending on its orientation is not accounted for.

2.23
$$\boldsymbol{B_p} = \sum_i c_i \hat{e}_i$$

Where:

c_i is the capacity of bolt i

$\hat{e}_i$ is the unit direction vector of bolt i

3. Sliding Direction

Next, the sliding direction of the block must be determined. The sliding (deformation) direction is computed by considering active forces only. Passive forces DO NOT influence sliding direction.

In RocTunnel3, the modes of failure are grouped into the following categories: falling (or lifting), sliding on a single joint, sliding on two joints, and sliding on more than two joints.

The calculation algorithm for sliding on up to two joints is based on the method presented in chapter 9 of "Block Theory and its application to rock engineering", by Goodman and Shi (1985).

Define the following variables:

A is the active force vector

$\hat{n}_i$ is the normal to joint face i directed into the block

3.1. Possible Sliding Directions

3.1.1. Falling (or Lifting)

$$3.1 \qquad \hat{s}_0 = \hat{a} = \frac{\mathbf{A}}{\|\mathbf{A}\|}$$

Where:

$\widehat{s_0}$ is the falling or lifting direction

$\hat{a}$ is the unit direction of the resultant active force

3.1.2. Sliding on a Single Face i

$$3.2 \qquad \hat{s}_i = \frac{(\hat{n}_i \times \mathbf{A}) \times \hat{n}_i}{\|(\hat{n}_i \times \mathbf{A}) \times \hat{n}_i\|}$$

Where:

$\widehat{s_i}$ is the sliding direction on joint i

3.1.3. Sliding on Two Faces i and j

$$3.3 \qquad \hat{s}_{ij} = \frac{\hat{n}_i \times \hat{n}_j}{\|\hat{n}_i \times \hat{n}_j\|} \operatorname{sign}\left(\left(\hat{n}_i \times \hat{n}_j \right) \cdot \mathbf{A} \right)$$

Where:

$\widehat{s_{ij}}$ is the sliding direction on joints i and j (along the line of intersection)

3.1.4. Sliding on More Than Two Faces $i_1, i_2, \ldots, i_k$

For any pair of non-parallel joints i_l, i_m, the sliding direction is equal to $\widehat{s_{lm}}$.

3.2. Determine Valid Sliding Direction

The following cases are tested in order. Whichever satisfies the given inequalities is the sliding direction of the block. If none of these tests satisfies the given inequalities, the block is unconditionally stable.

3.2.1. Falling (or Lifting)

For all joints i:

$$3.4 \qquad \boldsymbol{A} \cdot \widehat{n}_i > 0$$

3.2.2. Sliding on More Than Two Joints

For some pair of non-parallel joints i, j, there exist one or more other joints $k_1, \dots, k_m$ such that for l ranging from 1 to m:

$$3.5 \qquad \widehat{s_{ij}} \cdot \widehat{n_{k_l}} = 0$$

$$3.6 \qquad \boldsymbol{A} \cdot \widehat{n_{k_l}} \le 0$$

In this case, the block is sliding on the collection of joints $i, j, k_1, \dots, k_m$.

3.2.3. Sliding on Joint i

$$3.7 \qquad \boldsymbol{A} \cdot \widehat{n}_i \le 0$$

For all joints $j \ne i$:

$$3.8 \qquad \widehat{s}_i \cdot \widehat{n}_j > 0$$

3.2.4. Sliding on the Intersection of Joints i and j

$$3.9 \qquad \widehat{s}_i \cdot \widehat{n}_j \le 0$$

$$3.10 \qquad \widehat{s}_j \cdot \widehat{n}_i \le 0$$

For all joints k that are different from i and j:

$$3.11 \qquad \widehat{s_{ij}} \cdot \widehat{n_k} > 0$$

4. Normal Force

The calculation of normal forces on each joint plane for a block first requires the calculation of the sliding direction. Once the sliding direction is known, the following equations are used to determine the normal forces given a resultant force vector. This force vector is generally the active or the passive resultant force vector.

Define the following variables:

N_i is the normal force on joint i

$\boldsymbol{F}$ is the resultant force vector

$\hat{n}_i$ is the inward (into the block) normal of joint i

4.1. Falling (or Lifting)

For all joints i:

4.1 $\qquad N_i = 0$

4.2. Sliding on Joint i

4.2 $\qquad N_i = -\boldsymbol{F} \cdot \hat{n}_i$

For all joints $j \neq i$:

4.3 $\qquad N_j = 0$

4.3. Sliding on Joints i and j

4.4 $\qquad N_i = -\dfrac{(\boldsymbol{F} \times \hat{n}_j) \cdot (\hat{n}_i \times \hat{n}_j)}{(\hat{n}_i \times \hat{n}_j) \cdot (\hat{n}_i \times \hat{n}_j)}$

4.5 $\qquad N_j = -\dfrac{(\boldsymbol{F} \times \hat{n}_i) \cdot (\hat{n}_j \times \hat{n}_i)}{(\hat{n}_j \times \hat{n}_i) \cdot (\hat{n}_j \times \hat{n}_i)}$

For all joints k different from i and j:

4.6 $\qquad N_k = 0$

4.4. Sliding on Joints $i_1, i_2, \ldots, i_m$

Normal forces on joints $i_1, i_2, \ldots, i_m$ are computed using the methodology from "Stability analysis of rock wedges with multiple sliding surfaces" by Mauldon and Ureta (1996).

For all joints j not in $i_1, i_2, \ldots, i_k$:

4.7 $\qquad N_j = 0$

5. Shear Strength

There are five joint strength models available in RocTunnel3:

1. Mohr-Coulomb
2. Barton-Bandis
3. Hoek-Brown
4. Generalized Hoek-Brown
5. Power Curve

5.1. Compute Normal Stress on Each Joint

First, compute the stress on each joint plane based on the normal forces computed in Section 4.

5.1 $$\sigma_{n_i} = \frac{N_i}{a_i}$$

Where:

σ_{n_i} is the normal stress on joint i

N_i is the normal force on joint i

a_i is the area of joint i

5.2. Compute Shear Strength of Each Joint

Use the strength criteria defined for the joint and the normal stress to compute the shear strength.

Let τ_i be the shear strength of joint i.

5.2.1. Mohr-Coulomb Strength Criterion

5.2 $$\tau_i = c_i + \sigma_{n_i} \tan \phi_i$$

Where:

c_i is the cohesion of joint i

ϕ_i is the friction angle of joint i

5.2.2. Barton-Bandis Strength Criterion

5.3 $$\tau_i = \sigma_{n_i} \tan \left[JRC_i \log_{10} \left(\frac{JCS_i}{\sigma_{n_i}} \right) + \phi_{r_i} \right]$$

Where:

JRC_i is the joint roughness coefficient of joint i

JCS_i is the joint compressive strength of joint i

ϕ_{r_i} is the residual friction angle of joint i

5.2.3. Hoek-Brown Strength Criterion

Define the following in terms of the intermediate value σ_3:

5.4 $$\sigma_1 = \sigma_3 + \sigma_{ci}\sqrt{m_b \frac{\sigma_3}{\sigma_{ci}} + s}$$

5.5 $$d = \frac{am_b}{\sqrt{m_b \frac{\sigma_3}{\sigma_{ci}} + s}}\, am_b + 1$$

Using a numerical method, find the value of σ_3 that satisfies the following equation:

5.6 $$\sigma_{n_i} = \frac{\sigma_1 + \sigma_3}{2} - \frac{(\sigma_1 - \sigma_3)\,(d-1)}{2(d+1)}$$

Where:

σ_{ci} is the compressive strength

m_b is the mb Parameter

s is the s Parameter

Then,

5.7 $$\tau_i = \frac{(\sigma_1 - \sigma_3)\sqrt{d}}{d+1}$$

5.2.4. Generalized Hoek-Brown Strength Criterion

Define the following in terms of the intermediate value σ_3:

5.8 $$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a$$

5.9 $$d = am_b\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^{a-1} + 1$$

Using a numerical method, find the value of σ_3 that satisfies the following equation:

5.10 $$\sigma_{n_i} = \frac{\sigma_1 + \sigma_3}{2} - \frac{(\sigma_1 - \sigma_3)\,(d-1)}{2(d+1)}$$

Where:

σ_{ci} is the UCS of intact rock

m_b is the mb Parameter

s is the s Parameter

a is the a Parameter

Then,

5.11 $$\tau_i = \frac{(\sigma_1 - \sigma_3)\sqrt{d}}{d+1}$$

When defined by GSI, mi, and D, instead of mb, s, and a, the above method is used by first computing the following values:

$$5.12 \qquad m_b = m_i e^{\frac{GSI-100}{28-14D}}$$

$$5.13 \qquad s = e^{\frac{GSI-100}{9-3D}}$$

$$5.14 \qquad 0.5 + \frac{1}{6}\left(e^{\frac{-GSI}{15}} - e^{\frac{-20}{3}}\right)$$

Where:

GSI, m_i, D are the strength parameters of joint i

5.2.5. Power Curve Strength Criterion

$$5.15 \qquad \tau_i = c_i + a_i\left(\sigma_{n_i} + d_i\right)^{b_i}$$

Where:

a_i, b_i, c_i, d_i are the strength parameters of joint i

5.3. Compute Resisting Force due to Shear Strength

Force acts in a direction opposite to the direction of sliding (deformation).

$$5.16 \qquad J_i = \tau_i a_i \cos\theta_i$$

Where:

J_i is the magnitude of the resisting force due to the shear strenght of joint i

θ_i is the angle between the sliding direction and joint i

6. Factor of Safety

RocTunnel3 computes 3 separate factors of safety:

1. Falling (or lifting) factor of safety
2. Unsupported factor of safety
3. Supported factor of safety

The reported factor of safety is the maximum of the above three factors of safety. The logic of this is simple; support is assumed to never decrease the factor of safety from the unsupported value. The factor of safety can never be less than if the wedge was lifting with only support to stabilize it.

The limit equilibrium safety factor calculations only consider force equilibrium in the direction of sliding. Moment equilibrium is not considered.

Factor of Safety:

6.1 $$FS = \max(FS_f, FS_u, FS_s)$$

Where:

FS_f is the falling (or lifting) factor of safety

FS_u is the unsupported factor of safety

FS_s is the supported factor of safety

6.1. Factor of Safety Definition

6.2 $$factor\ of\ safety = \frac{resisting\ forces\ (e.g.\ shear\ strength,\ support)}{driving\ forces\ (e.g.\ weight,\ seismic,\ water)}$$

6.2. Falling (or Lifting) Factor of Safety

The falling (or lifting) factor of safety assumes that only passive support acts to resist movement. Basically, the block is assumed to be falling (or lifting) off the failure planes, so no influence of the joint planes (shear strength, failure direction) is incorporated. Driving forces are due to the active forces on the block. The falling (or lifting) direction is calculated from the direction of the active force vector.

6.3 $$FS_f = \frac{-\mathbf{P} \cdot \hat{s}_0}{\mathbf{A} \cdot \hat{s}_0}$$

Where:

$\mathbf{P}$ is the resultant passive force vector (Section 2.2)

$\mathbf{A}$ is the resultant active force vector (Section 2.1)

$\hat{s}_0$ is the falling (or lifting) direction (Section 3.1.1)

6.3. Unsupported Factor of Safety

The unsupported factor of safety assumes that shear strength acts to resist movement. No passive support force is used.

Driving forces are due to the active forces on the block as defined in Section 2.1.

The sliding direction is calculated from the equations in Section 3. The shear strength is calculated based on the normal forces from the active force vector only. Normal forces from the passive force vector are not included.

$$6.4 \qquad FS_u = \frac{\sum_{i=1}^{2} J_i^u}{\mathbf{A} \cdot \hat{s}}$$

Where:

J_i^u is the magnitude of the resisting force due to the unsupported shear strength of joint i (Section 5.3)

$\mathbf{A}$ is the resultant active force vector (Section 2.1)

$\hat{s}$ is the sliding direction (Section 3)

6.4. Supported Factor of Safety

The supported factor of safety assumes that passive support forces and shear strength act to resist movement.

Driving forces are due to the active forces on the block as defined in Section 2.1. The sliding direction is calculated from the equations in Section 3. The shear strength is calculated based on the normal force calculated from the active force vector plus the passive force vector.

$$6.5 \qquad FS_s = \frac{-\mathbf{P} \cdot \hat{s} + \sum_{i=1}^{2} J_i^s}{\mathbf{A} \cdot \hat{s}}$$

Where:

FS_s is the supported factor of safety

J_i^s is the magnitude of the resisting force due to the supported shear strength joint i (Section 5.3)

$\mathbf{P}$ is the resultant passive force vector (Section 2.2)

$\mathbf{A}$ is the resultant active force vector (Section 2.1)

$\hat{s}$ is the sliding direction (Section 3)

7. Successive Failure Analysis

When Successive Failure analysis is turned on, the analysis is computed in iterations.

In the first stage, the daylighting blocks are analyzed independently to compute a factor of safety for each one, using the procedure defined in Sections 1 - 6. Any block whose factor of safety is less than the design factor of safety is identified as a key block, and it is treated as a block that failed.

The blocks that are adjacent to blocks that failed in the first iteration are analyzed in the next one. In this analysis, the portions of the joint surfaces that are adjacent to a block that has failed in any prior iteration are treated as free faces. Once the factors of safety are computed for all blocks analyzed in this stage, their neighbours are used for the analysis in the subsequent stage.

This procedure is repeated until an iteration is reached where no blocks fail.

8. References

Goodman, R., and Shi, G. (1985). *Block Theory and Its Application to Rock Engineering.* Prentice-Hall.

Mauldon, M., and Ureta, J. (1996). *Stability analysis of rock wedges with multiple sliding surfaces.* Geotechnical and Geological Engineering 14, pp. 41-66.