Settle3D

Ground Improvement Feature

Verification of Settlement Calculations for Stone Columns

Problem Description

The hypothetical embankment problem from '*Simplified homogenization method in stone column designs'* by **K.S. Ng, and S.A. Tan** (2014) was used to verify the Stone Column calculations for Settle3D's newest Ground Improvement feature. However, some changes were made from the original problem including removal of the 1 m top crust layer and replacement with soft soil. The stone columns, 10 m in length, were used to support a 4 m high embankment fill constructed above a 20 m soft soil layer. To simplify calculations we assumed a constant loading stress (80 kPa) across the entire depth of the model, neglecting the effects of Poisson's Ratio. The embankment had a 1:2 (V:H) slope gradient with a top width of 40 m. Figure 1 shows the geometry used for the model. The stone columns were 1 m in diameter with center-to-center spacing of 2m in a square grid pattern. The material properties are summarized in Table 1.

Table 1: Material Properties

Name	**Depth**	**Unit Weight [kN/m^3]**	**Elastic Modulus [kPa]**
Soft Soil	1-20 m	18	5,000
Embankment fill	4 m high	20	15,000
Stone Column	10 m deep	19	50,000

Stone Column Calculation Methods

a) Area Replacement Ratio

When the columns are installed, the area replacement ratio is defined as the ratio of the cross-sectional area of a column to the tributary area of the column, as shown in Figure 1.

$$a_s = \frac{A_c}{A_e} = C(\frac{d_c}{s})^2 \quad (1)$$

where a_s = area replacement ratio
A_c = cross sectional area of the column
A_e = tributary area of the column
d_c = diameter of the column
s = center-to-center spacing between columns in a square of equilateral triangular pattern
C = constant (π/4 or 0.785 for a square pattern or π/(2$\sqrt{3}$) or 0.907 for an equilateral triangular pattern)

b) Stress Concentration Ratio

Barksdale and Bachus (1983) developed an empirical design chart to determine the stress concentration ratio, which can be approximated as follows for the average ratio (Han, 2010):

$$n = 1 + 0.217\left(\frac{E_c}{E_s} - 1\right) \quad (2)$$

where E_s = elastic modulus of the soil
E_c = elastic modulus of the column

Based on field data, the modulus ratio (E_c/E_s) should be limited to 20.

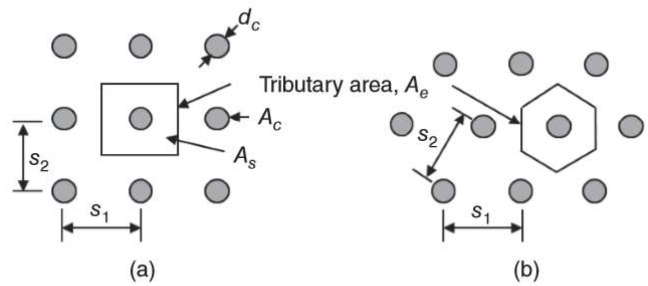


Figure 1: Typical patterns of compaction probe points or columns: (a) rectangular and (b) triangular

c) *Stress Reduction Factor*

Under rigid loading, the stress distribution on the columns and the soil can be simplified as shown in Figure 2.

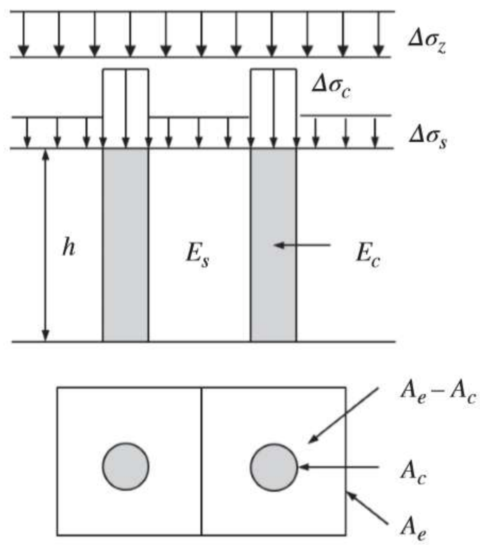


Figure 2: Stress Distribution Model

Based on force equilibrium, the following relationship can be established:

$$F_{total} = F_{on\ soil} + F_{on\ column}$$

$$\Delta\sigma_z A = \Delta\sigma_s(A_e - A_c) + \Delta\sigma_c A_c \tag{3}$$

where A_e = Influence area of one column (also called tributary area)
$\Delta\sigma_z$ = average vertical stress applied on the composite foundation
$\Delta\sigma_s$ = vertical stress on the soil
A_c = thickness of the soil layer
$\Delta\sigma_c$ = vertical stress on the column

Dividing both sides by A_e yields

$$\Delta\sigma_z = \Delta\sigma_s(1 - a_s) + \Delta\sigma_c a_s \tag{4}$$

where a_s is the area replacement ratio, defined as the ratio of the column cross-section area to the influence area.
Considering the stress concentration ratio (i.e., $n_{1D} = \frac{\Delta\sigma_c}{\Delta\sigma_s}$) we can rewrite the equation above as

$$\Delta\sigma_z = [(1 - a_s) + na_s]\Delta\sigma_s = [1 + (n - 1)a_s]\Delta\sigma_s \quad (5)$$

The stress on the soil is

$$\Delta\sigma_s = \mu\Delta\sigma_z \quad (6)$$

$$\mu = \frac{1}{1+(n-1)a_s} \quad (7)$$

Where μ is the stress reduction factor.

This equation shows that the stress reduction factor is less than 1, and an increase of the stress concentration ratio and/or the area replacement ratio reduces this factor, which means less stress is applied on the soil.

Assuming the deformations of both column and soil are one dimensional and equal, we get:

$$\varepsilon_c = \varepsilon_s = \varepsilon_z \quad (8)$$

where ε_c = vertical strain of the column
ε_s = vertical strain of the soil
ε_z = average vertical strain

Dividing Equation 7 by Equation 11 results in the following

$$E_{eq} = E_s(1 - a_s) + E_c a_s \quad (9)$$

where E_{eq} = equivalent modulus of the composite foundation
E_s = soil modulus
E_c = column modulus

The above equation can be expressed as

$$E_{eq} = [1 + (n - 1)a_s]E_s \quad (10)$$
$$E_{eq} = \frac{E_s}{\mu}$$

In conclusion, the in-situ elastic modulus of the soil will be multiplied by the stress reduction factor to account for the stone columns and the new equivalent modulus will be used in all strain calculations.

Table 2: Stone Column Parameters

User Inputs		
Square Pattern	Yes	**-**
Elastic Modulus of Clay (E_s)	5000	**kPa**
Elastic Modulus of the Column (E_c)	50	**MPa**
Diameter of Column (d_c)	1	**MPa**
Center-to-center spacing between columns (s)	2	**MPa**

Using the above values, the following parameters were calculated for each method:

a) Area Replacement Ratio
b) Stress Concentration Ratio
c) Stress Reduction Factor

A spreadsheet was created to compare theoretical results to the Settle3D output values for the following ten cases:

- Immediate Settlement
- Immediate Settlement (w/ STONE COLUMNS)
- Primary Consolidation – Linear
- Primary Consolidation – Linear (w/ STONE COLUMNS)
- Primary Consolidation – Non-linear
- Primary Consolidation – Non-Linear (w/ STONE COLUMNS)
- Primary Consolidation – Janbu [a=1]
- Primary Consolidation – Janbu [a=1] (w/STONE COLUMNS)
- Primary Consolidation – Janbu [a=0]
- Primary Consolidation – Janbu [a=0] (w/ STONE COLUMNS)

Settlement Calculation Methods

The vertical settlement of each sublayer is:

$$\delta = \Delta z = \varepsilon h \tag{11}$$

Where h is the original thickness of the bottom sublayer. The settlement of the i^{th} layer is then the settlement of the sublayer below (i+1) plus the settlement in sublayer i:

$$\delta_i = \delta_{i+1} + \varepsilon_i h_i \tag{12}$$

Figure 1 shows a visual representation of the above computation steps.

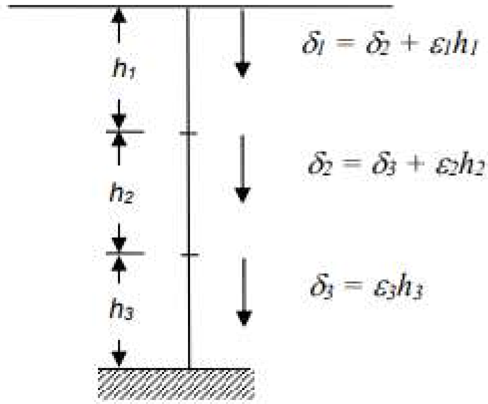


Figure 3: Schematic of the calculation method used to estimate settlement of a soil layer

We have verified the following five methods for calculating settlement after the implementation of Stone Columns by comparing the Settle3D output values with theoretical calculations. These five methods are:

1. **Immediate Settlement**
2. **Primary Consolidation Settlement – Linear Method**
3. **Primary Consolidation Settlement – Non-Linear Method**
4. **Primary Consolidation Settlement – Janbu Method (a=1)**
5. **Primary Consolidation Settlement – Janbu Method (a=0)**
6. **Westergaard Stress Computation Method - Primary Consolidation Settlement – Non-Linear**

1. **Immediate Settlement**

The vertical strain in each sublayer is calculated using:

$$\varepsilon_i = \frac{\Delta\sigma_i}{E} \tag{13}$$

where ε_i = strain in sublayer i
E = constrained modulus of clay
$\Delta\sigma_i$ = change in effective stress in sublayer i

where $\Delta\sigma_i$ is the change in vertical total stress in the ith sublayer. Initial settlement is then calculated from these strains.

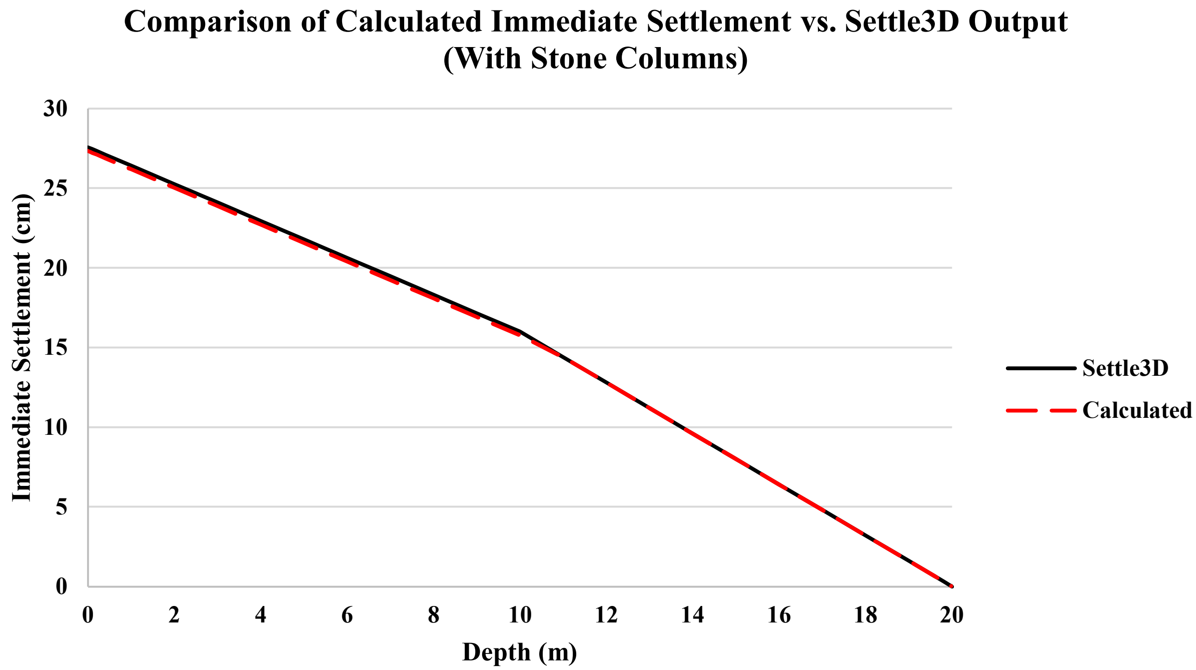
For the Immediate Settlement case, the stress concentration ratio, n, was computed with the elastic moduli of clay and stone column set to 5000 and 50000 kPa, respectively.

Using the values from Table 2, the area replacement ratio and the stress concentration ratio were computed yielding 0.20 and 2.95, respectively.

With the area replacement ratio and stress concentration ratio computed, the last step was computing the stress reduction factor which resulted in a value of 0.723. The elastic modulus of the clay was then divided by the stress reduction factor to yield a new corrected modulus of 6917.35 kPa.

The strains for each sublayer were then computed using Equation (13) using the equivalent modulus. After computing the strains of each sublayer, Equation (12) was used calculate a total settlement of 27.34 cm.

A comparison of the output values from Settle3D and the theoretical calculations for immediate settlement is shown in the following graph.



The implementation of Stone Columns decreased the total immediate settlement by approximately 15%. Table 3 below shows the more detailed calculations of each sublayer and its associated error.

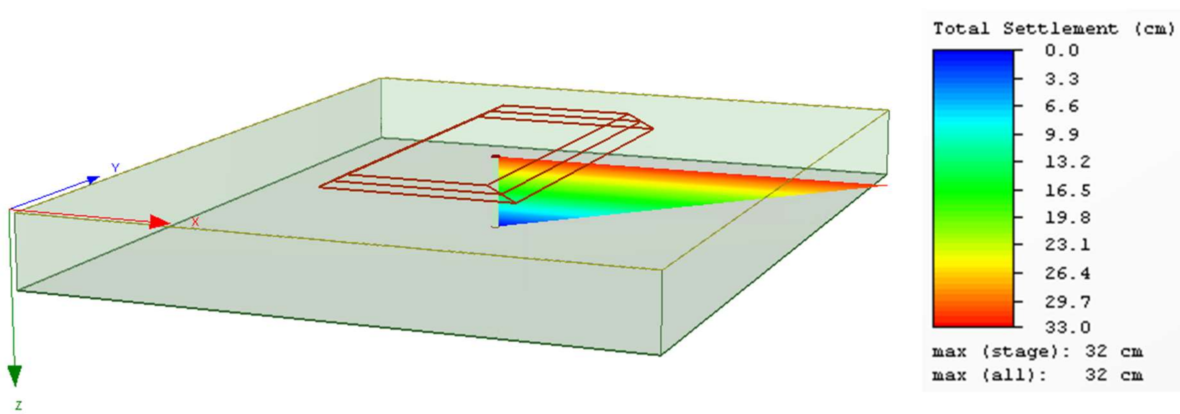


Figure 3: Settle3D Model using Immediate/Linear/Janbu Method Settlement without Stone Columns

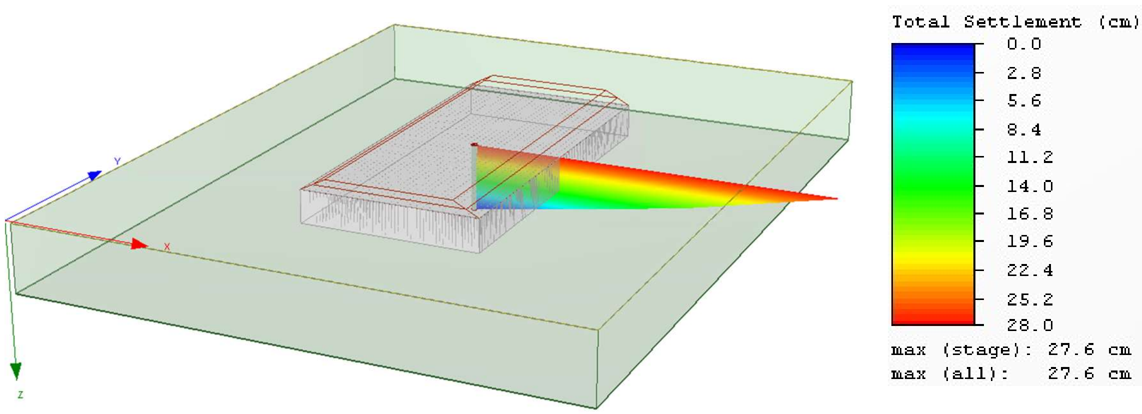


Figure 4: Settle3D Model using Immediate/Linear/Janbu Method Settlement with Stone Columns

2. Primary Consolidation Settlement – Linear

The settlement of a foundation under a large loading area (i.e. the width of the loading area is at least three times the thickness of the soft soil) is

$$S = m_{v,s} \Delta\sigma_z h \tag{14}$$

where $m_{v,s}$ = coefficient of volume compressibility of soil
$\Delta\sigma_z$ = additional vertical stress
h = thickness of soil layer

The coefficient of volume compressibility of soil can be determined by the following relationship:

$$m_{v,s} = \frac{1}{D_s} \tag{15}$$

The change in vertical strain for any given linear elastic element for a change in vertical effective stress is:

$$\Delta\varepsilon = \Delta\sigma m_{v,s} \tag{16}$$

According to Equation (15), the coefficient of volume compressibility of the soil is equal to the inverse of the constrained modulus. Therefore, m_v was set to 1/5000 = 0.0002.

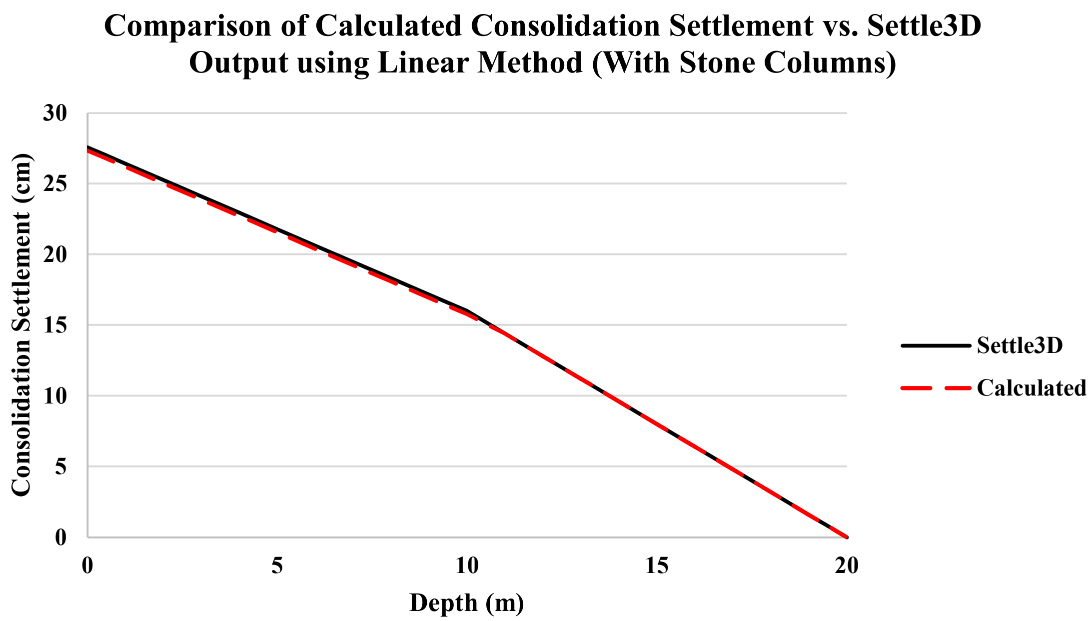
For the Linear method, instead of using the elastic modulus of the clay in Equation (2), the relationship from Equation (15) was used to replace it with the coefficient of volume compressibility as shown below.

$$n = 1 + 0.217\left(\frac{E_c}{(\frac{1}{m_{v,s}})} - 1\right)$$

Because of the inversely proportional relationship of the constrained modulus with the coefficient of volume compressibility, the stress reduction factor of 0.723 was multiplied with m_v to compute an 'equivalent' coefficient of volume compressibility of the composite foundation.

With the new equivalent m_v, the strain for each sublayer was calculated using Equation (16). Following the same steps as immediate settlement, the primary consolidation settlement was calculated to be 27.43 cm.

A comparison of the output values from Settle3D and the theoretical calculations using the linear method is shown in the following graph.



The implementation of Stone Columns with the Linear method decreased the total immediate settlement by approximately 15%.

3. Primary Consolidation Settlement – Non-Linear

Non-linear materials exhibit a changing modulus as opposed to a constant one as seen with immediate settlement and linear materials. The modulus is a function of the effective stress and the over consolidation ratio (OCR). Figure 4 shows the relationship between void ratio and the logarithm of effective stress.

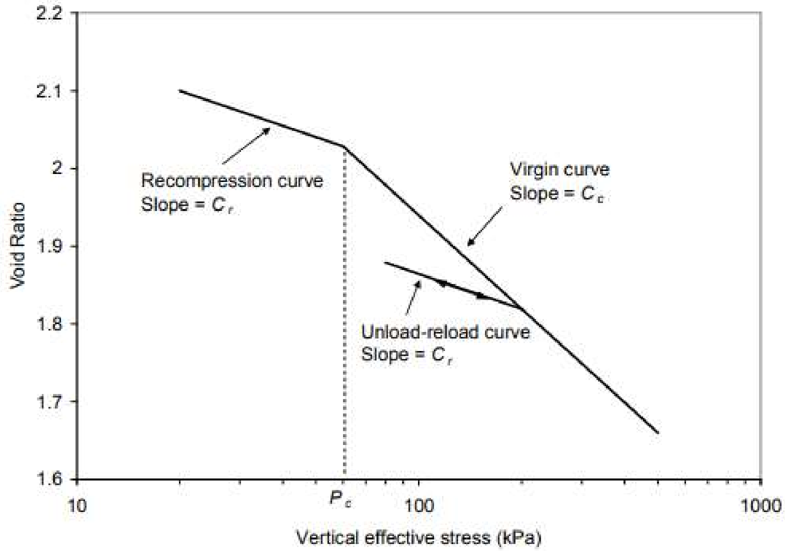


Figure 5: Void Ratio vs. Logarithm of Effective Stress

For the purposes of this report, using non-linear methods we have only considered normally consolidated soils. (OCR =1, $P_c < \sigma'$)

For a stress change in an overconsolidated soil layer, the change in void ratio, Δe, can be calculated from the initial effective stress, σ'_i, and the final effective stress, σ'_f by:

$$\Delta e = -C_r \log\left(\frac{\sigma'_f}{\sigma'_i}\right) \tag{17}$$

Where vertical strain is related to void ratio by:

$$\varepsilon = -\frac{\Delta e}{1+e_0} \tag{18}$$

Where e_0 is the initial void ratio.

Therefore, combining Equations (17) and (18), we get:

$$\Delta\varepsilon = \frac{C_r}{1+e_0} \log\left(\frac{\sigma'_f}{\sigma'_i}\right) \tag{19}$$

As per Jie Han (2015) the non-linear constrained modulus could be estimated using the following relationship:

$$D_s = \frac{2.30(1+e_0)\sigma'_{z0}}{C_c} \tag{20}$$

The following table presents the values that were used for the non-linear material properties in all the calculations.

Table 5: Material Properties of Non-Linear Clay

Non-Linear Material Parameters	
Initial Void Ratio (e_0)	1.1

Virgin Curve Slope (C_c)	0.4
Recompression Curve Slope (Cr)	0.1
OCR	1

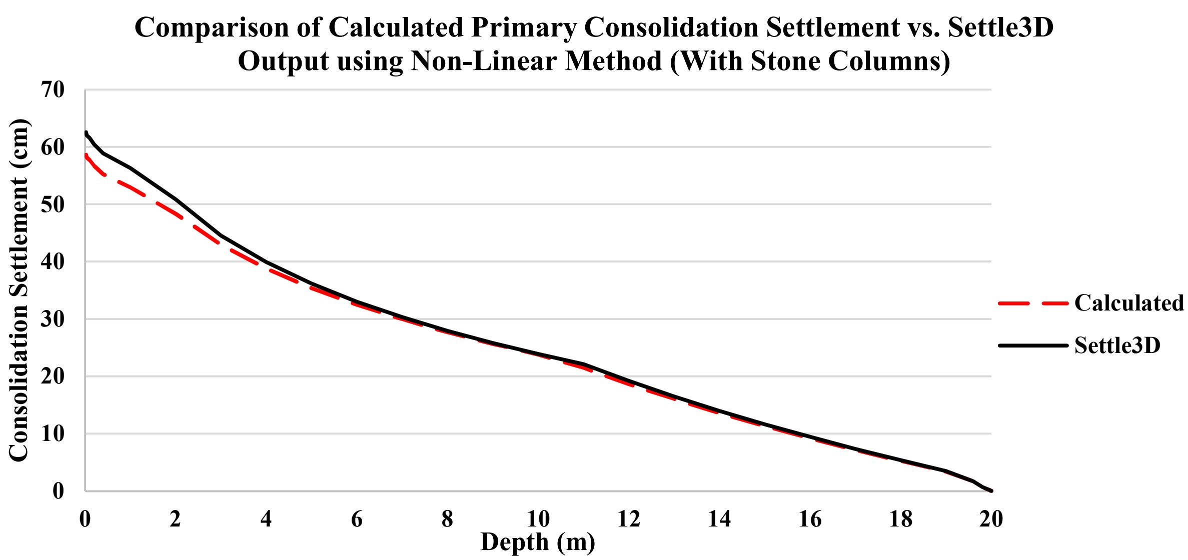
The constrained modulus from Equation (20) was used as the modulus of clay in the stress concentration ratio computation as shown below:

$$n = 1 + 0.217\left(\frac{E_c}{E_s} - 1\right)$$

Using a changing value for the modulus of clay would result in a changing value for the stress concentration ratio as well as the stress reduction factor.

However, recall that the modulus ratio should be limited to 20. Therefore, for the first 13 sublayers, the stress concentration ratio was limited to 5.12.

A comparison of the Settle3D program and the theoretical calculations for primary consolidation settlement using the non-linear method is shown in the following graph.



Our calculations showed a decrease in settlement of about 33 cm from 95.3 cm to 62.6 cm. By implementing the Stone Columns across the total embankment area, there was a 33% decrease in consolidation settlement.

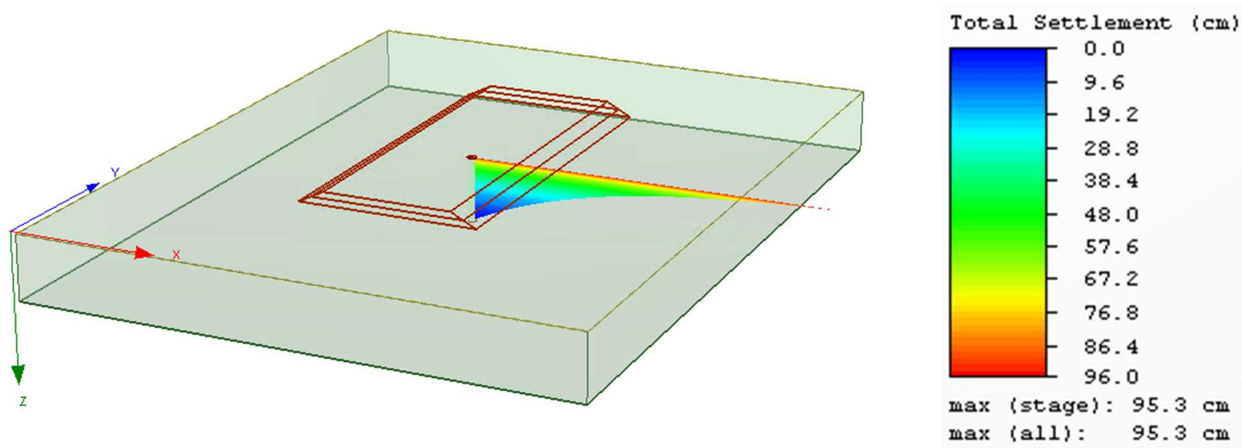


Figure 6: Settle3D Model using Non-Linear/Janbu Method Settlement without Stone Columns

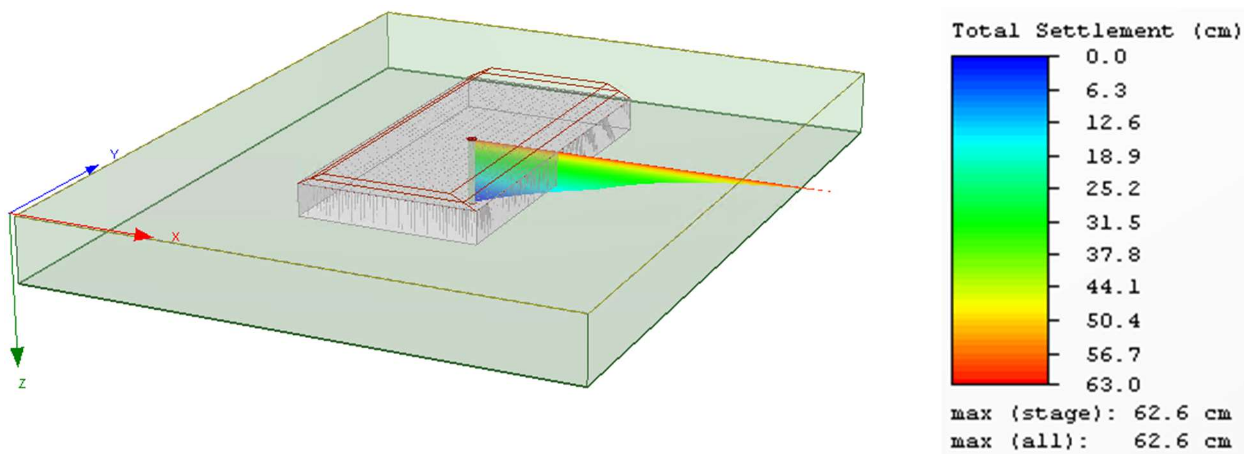


Figure 7: Settle3D Model using Non-Linear/Janbu Method Settlement with Stone Columns

4. Primary Consolidation Settlement – Janbu Method (a=1)

The Janbu method (1963, 1965) can be linear or non-linear depending on the stress exponent, a.

The 1D modulus, M, which is also the constrained modulus, D_s, is given by

$$M = D_s = m\sigma'_r \left(\frac{\sigma'}{\sigma'_r}\right)^{1-a} \tag{21}$$

When a=1, the Janbu method follows the same computations as the linear method since

$$m = \frac{1}{m_v \sigma'_r} \tag{22}$$

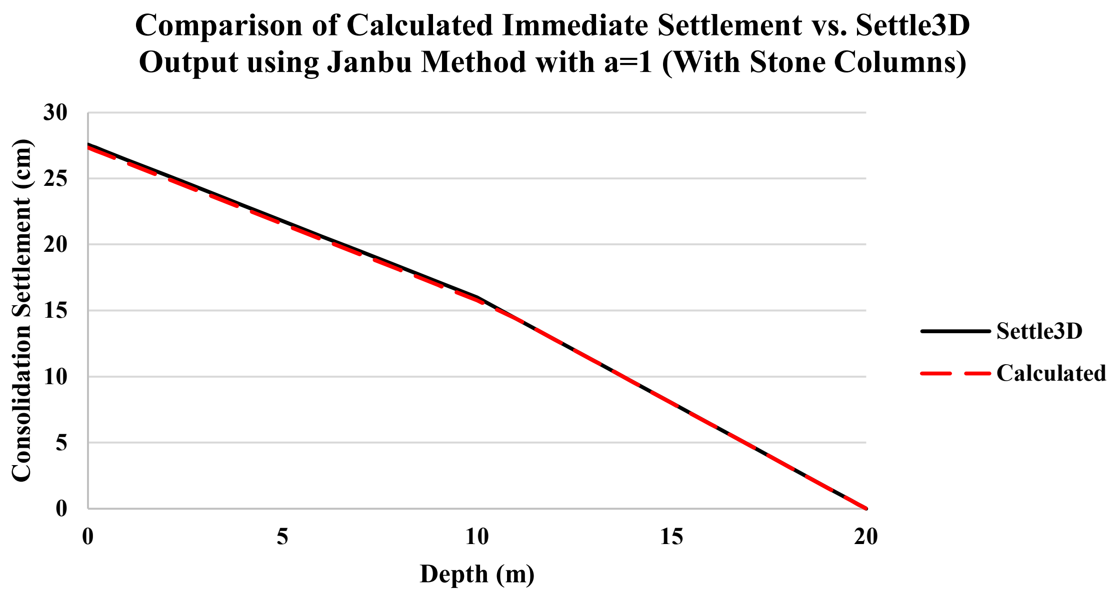
Substituting a = 1 into Equation (21) and using the relationship in Equation (22) this yields

$$D_s = m\sigma'_r = \frac{1}{m_v \sigma'_r}(\sigma'_r) = \frac{1}{m_v}$$

Therefore, the stress concentration ratio becomes

$$n = 1 + 0.217\left(\frac{E_c}{(\frac{1}{m_{v,s}})} - 1\right) \tag{23}$$

We can see that this is in fact the same relationship we encountered in the linear method. A comparison of the Settle3D output values and the theoretical calculations for primary consolidation settlement using the Janbu (a=1) method is shown in the following graph.



Our calculations estimated a decrease of about 2.2 cm in total settlement using the Janbu method with a=1. With the Stone Columns across the total embankment area, there was a 7% decrease in consolidation settlement. As expected, this matches the results from the Linear method presented earlier.

5. Primary Consolidation Settlement – Janbu Method (a=0)

When a=0, the Janbu method follows the same computations as the non-linear method with

$$m = ln(10)\frac{1+e_0}{C_c} \tag{24}$$

and,

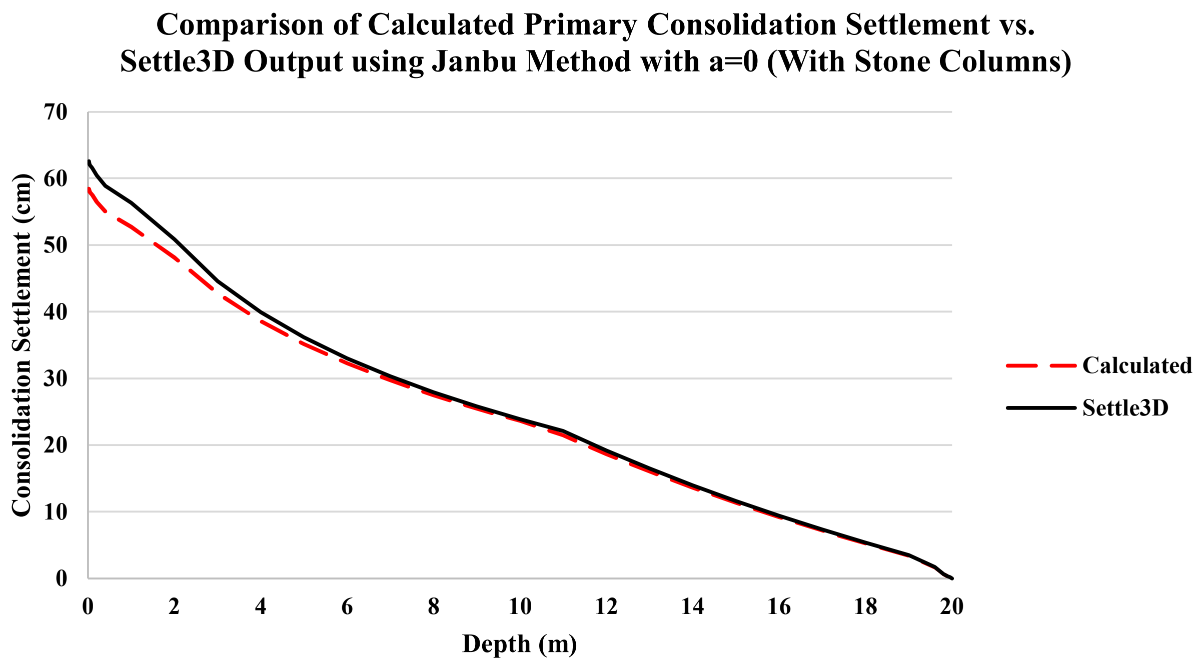
$$\Delta\varepsilon = \frac{1}{m} ln(\frac{\sigma'_f}{\sigma'_i}) \tag{25}$$

Therefore, by substituting a = 0 into Equation (21) and using the relationship from Equation (24) we get

$$D_s = m\sigma'_r\left(\frac{\sigma'}{\sigma'_r}\right) = m(\sigma') = \ln(10)\frac{1+e_0}{C_c}(\sigma') \tag{26}$$

Substituting (26) into the stress concentration ratio yields

$$n = 1 + 0.217\left(\frac{E_c}{\ln(10)\frac{1+e_0}{C_c}(\sigma')} - 1\right) \tag{27}$$



Our calculations predicted a total decrease of about 29 cm in settlement using the Janbu method with a=0. By implementing the Stone Columns across the total embankment area, there was a 34% decrease in consolidation settlement. As expected, our results matched the results from using the Non-Linear method.

6. Westergaard Stress Computation Method - Primary Consolidation Settlement – Non-Linear

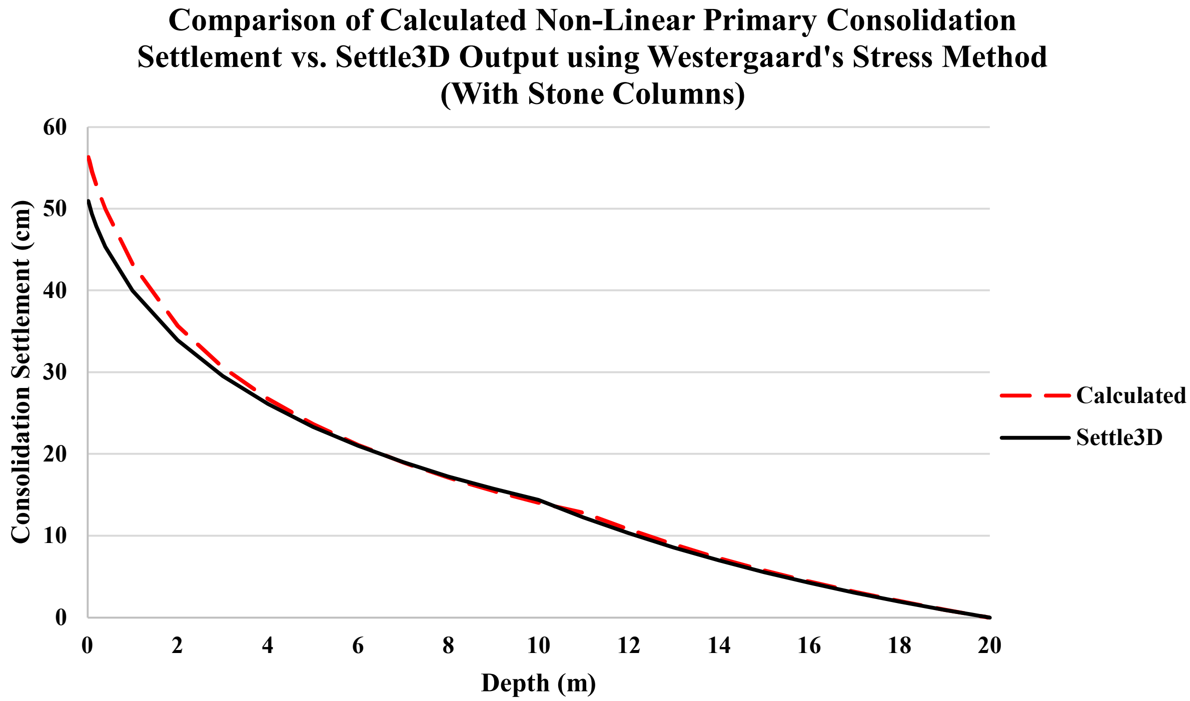
Recall that for the verification cases shown above we assumed a constant effective stress at every depth of the model. For completeness, we have considered one case where the Westergaard stress computation method was used with the Stone Column ground improvement feature. Since the Westergaard stress computation involves a 3D stress distribution, the constrained modulus cannot be used unless it is transformed into an equivalent modulus which considers the 3D effects of stress.

As per Jie Han (2015), the 3D elastic modulus of the soil can be computed from the constrained modulus and Poisson's ratio using Equation (26) below

$$E_s = \frac{(1+\nu_s)(1-2\nu_s)}{(1-\nu_s)} D_s \tag{27}$$

where ν_s = Poisson's Ratio
E_s = Elastic Modulus of the soil
D_s = Constrained Modulus of the soil

We followed the same steps as the Non-Linear method except for an additional step; we transformed the constrained modulus to an equivalent elastic modulus using Equation (27). Then using this new modulus we calculated the stress concentration ratio from Equation (2). The results of our calculations are presented below in comparison to the Settle3D outputs.



References:

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Han, J. (2015). *Principles and Practice of Ground Improvement*. John Wiley & Sons, Inc.

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Settle3D Theory Manual.