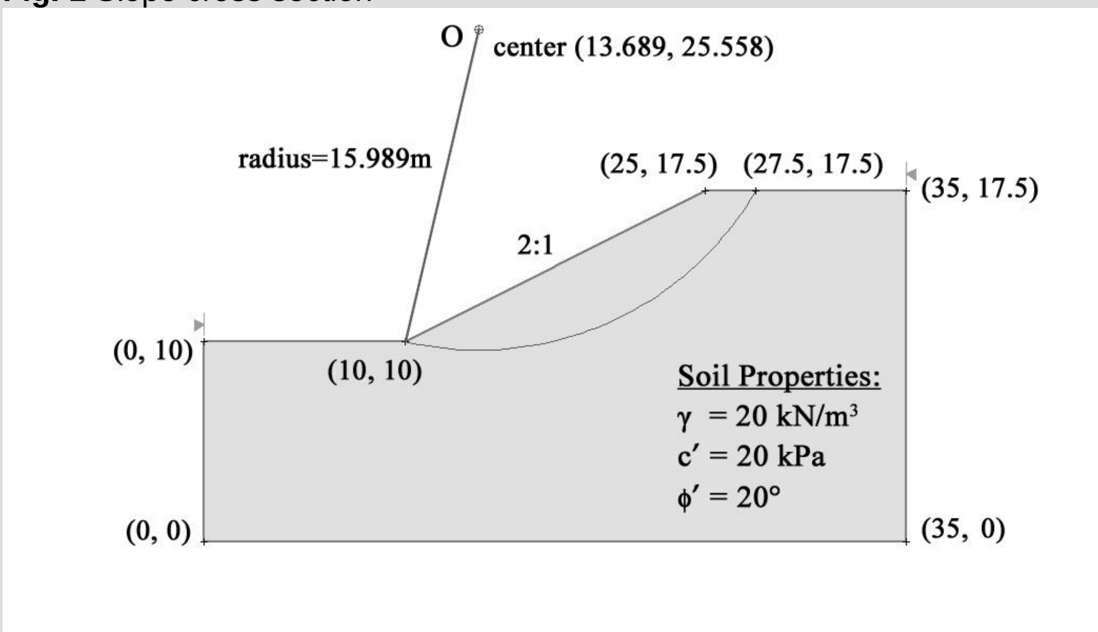
SLIDE HAND CALCULATION #1

Calculate by hand the factor of safety of the circular failure surface shown in Fig. 1 using the following methods:

a) Bishop Simplified (7 slices)
b) Verify parts a) using Slide

Fig. 1 Slope cross section



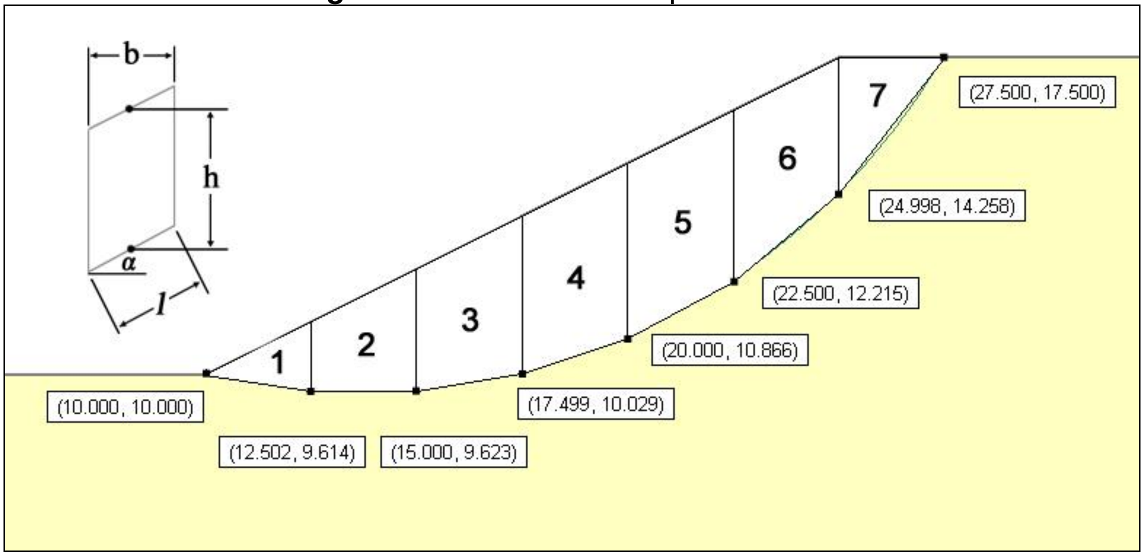
a) Bishop Simplified Solution:

Draw the slope cross section to scale and divide it into 7 vertical slices (see Fig. 2). Using a ruler and protractor, measure the slice data from the cross sectional diagram. Table 1 summarizes the slice data.

Table 1 Slice Data

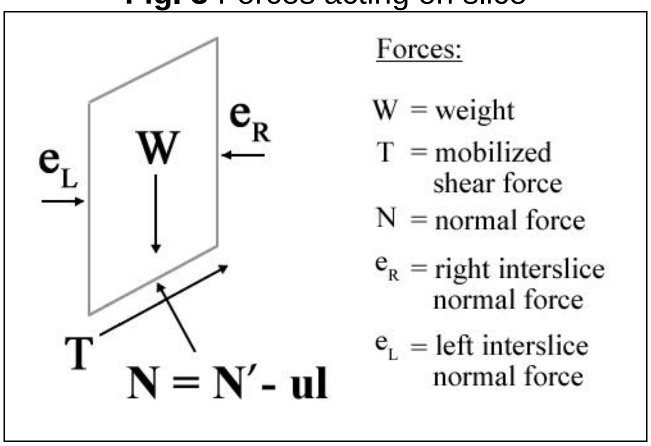
Slice	Base (m)	Height(m)	α (deg)	ϕ (deg)	c (kPa)	γ (kN/m^3)
1	2.5	0.8	-9	20	20	20
2	2.5	2.3	0.5	20	20	20
3	2.5	3.3	9	20	20	20
4	2.5	3.9	18.5	20	20	20
5	2.5	4.1	28	20	20	20
6	2.5	3.6	39	20	20	20
7	2.5	1.65	52	20	20	20

Fig. 2 Cross section of slope – 7 slices



Label the forces acting on each slice (see Fig. 3). Note that interslice shear forces are not depicted because they are assumed to be zero in the Bishop Simplified method.

Fig. 3 Forces acting on slice



The Bishop Simplified method satisfies two equilibrium conditions: overall moment equilibrium about the center O and vertical force equilibrium on each slice.

First consider the overall moment equilibrium about the center *O*

$$\sum M_O = \sum_{i=1}^{7} W_i r \sin\alpha_i - \sum_{i=1}^{7} T_i r = 0 \tag{1}$$

Dividing both sides of the equation by *r* yields

$$\sum_{i=1}^{7} W_i \sin\alpha_i - \sum_{i=1}^{7} T_i = 0 \tag{2}$$

If we assume that the factor of safety *F* is the same for all 7 slices, the mobilized shear force T_i per slice is given by

$$T_i = \frac{c' l_i + \tan\varphi' N_i}{F} \tag{3}$$

Summing up equation (3) for all the slices followed by substituting the sum into (2) and rearranging yields

$$F = \frac{\sum_{i=1}^{7} (c' l_i + \tan\varphi' N_i)}{\sum_{i=1}^{7} W_i \sin\alpha_i} \tag{4}$$

To solve for N_i, consider the equilibrium of forces in the vertical direction on each slice,

$$N_i \cos\alpha_i + T_i \sin\alpha_i - W_i = 0 \xrightarrow{from\ (3)}$$

$$N_i \cos\alpha_i + \frac{c' l_i + \tan\varphi' N_i}{F} \sin\alpha_i - W_i = 0 \xrightarrow{factoring\ N_i\ and\ simplifying}$$

$$N_i = N_i' - u l_i = \frac{W_i - \frac{c' l_i}{F}\sin\alpha_i}{\cos\alpha_i \left(1 + \frac{\tan\varphi' \tan\alpha_i}{F}\right)} \tag{5}$$

Substituting (5) into (4) yields

$$F = \sum_{i=1}^{7} \left(c' l_i + \tan\varphi' \left(\frac{\left(W_i - \frac{c' l_i \sin\alpha_i}{F}\right)}{\cos\alpha_i \left(1 + \frac{\tan\varphi' \tan\alpha_i}{F}\right)} \right) \right) \times \frac{1}{\sum_{i=1}^{7} W_i \sin\alpha_i} \tag{6}$$

Note that according to Fig.2, $l = b \times \sec(\alpha)$

Equation (6) shows that the factor of safety is defined implicitly in an expression containing F. To solve for F from (6), we start with an initial estimated value for F. Iterations of successive approximation are performed until value of F converges to within a given tolerance.

For this problem, a tolerance of 0.005 is used.

See Table 2 for Excel spreadsheets of the calculations for each iteration.

Answer: $FOS_{Bishop\ Simplified} = 2.113$

Table 3 Bishop Simplified Method – Calculations

Iteration 1	F1 = 5													
Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭
Slice	b	h	α	W =bhγ	sinα	Wsinα	c'l	④-(⑦*⑤)/F	cosα	tanΦ'tanα	⑨*(1+⑩/F)	N=⑧/⑪	N*tanΦ'	⑦+⑬
1	2.5	0.8	-9	40	-0.156434	-6.25738	50.6233	41.5838444	0.9876883	-0.0576472	0.97630084	42.59327	15.50268	66.125938
2	2.5	2.3	0.5	115	0.008727	1.00355	50.0019	114.912731	0.9999619	0.00317632	1.00059716	114.8442	41.79985	91.801756
3	2.5	3.3	9	165	0.156434	25.8117	50.6233	163.416156	0.9876883	0.05764722	0.99907584	163.5673	59.53364	110.15689
4	2.5	3.9	18.5	195	0.317305	61.8744	52.7246	191.654047	0.9483237	0.12178274	0.97142155	197.2924	71.80855	124.53316
5	2.5	4.1	28	205	0.469472	96.2417	56.6285	199.682906	0.8829476	0.19352641	0.91712233	217.7277	79.24639	135.87489
6	2.5	3.6	39	180	0.62932	113.278	64.338	171.90216	0.777146	0.29473728	0.82295674	208.8836	76.02741	140.36539
7	2.5	1.65	52	82.5	0.788011	65.0109	81.2135	69.7005837	0.6156615	0.46586066	0.67302397	103.5633	37.69396	118.90742
						Σ⑥ = 356.962								Σ⑭ = 787.76545

F2 =	Σ⑥/Σ⑭ =	2.20685775	ΔF =	2.793142

Iteration 2	F2 = 2.2069													
Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭
Slice	b	h	α	W =bhγ	sinα	Wsinα	c'l	④-(⑦*⑤)/F	cosα	tanΦ'tanα	⑨*(1+⑩/F)	N=⑧/⑪	N*tanΦ'	⑦+⑬
1	2.5	0.8	-9	40	-0.156434	-6.25738	50.6233	43.5884606	0.9876883	-0.0576472	0.96188809	45.31552	16.4935	67.116757
2	2.5	2.3	0.5	115	0.008727	1.00355	50.0019	114.802278	0.9999619	0.00317632	1.00140116	114.6416	41.72615	91.728051
3	2.5	3.3	9	165	0.156434	25.8117	50.6233	161.411539	0.9876883	0.05764722	1.01348859	159.2633	57.9671	108.59036
4	2.5	3.9	18.5	195	0.317305	61.8744	52.7246	187.419192	0.9483237	0.12178274	1.00065573	187.2964	68.17031	120.89492
5	2.5	4.1	28	205	0.469472	96.2417	56.6285	192.953246	0.8829476	0.19352641	0.96037609	200.9143	73.12681	129.75531
6	2.5	3.6	39	180	0.62932	113.278	64.338	161.653008	0.777146	0.29473728	0.88093783	183.501	66.78892	131.12689
7	2.5	1.65	52	82.5	0.788011	65.0109	81.2135	53.5008125	0.6156615	0.46586066	0.74562566	71.75291	26.11592	107.32939
						Σ⑥ = 356.962								Σ⑭ = 756.54168

F3 =	Σ⑥/Σ⑭ =	2.11938702	ΔF =	0.087471

Table 3 Bishop Simplified Method – Calculations (cont’d)

Iteration 3 F3 = 2.1194

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭
Slice	b	h	α	W =bhγ	sinα	Wsinα	c'l	④-(⑦*⑤)/F	cosα	tanΦ'tanα	⑨*(1+⑩/F)	N=⑧/⑪	N*tanΦ'	⑦+⑬
1	2.5	0.8	-9	40	-0.156434	-6.25738	50.6233	43.7365625	0.9876883	-0.0576472	0.96082327	45.51988	16.56788	67.191139
2	2.5	2.3	0.5	115	0.008727	1.00355	50.0019	114.794118	0.9999619	0.00317632	1.00146056	114.6267	41.72071	91.72261
3	2.5	3.3	9	165	0.156434	25.8117	50.6233	161.263438	0.9876883	0.05764722	1.01455341	158.9502	57.85313	108.47639
4	2.5	3.9	18.5	195	0.317305	61.8744	52.7246	187.106319	0.9483237	0.12178274	1.00281557	186.581	67.90993	120.63454
5	2.5	4.1	28	205	0.469472	96.2417	56.6285	192.456056	0.8829476	0.19352641	0.96357169	199.732	72.69649	129.32499
6	2.5	3.6	39	180	0.62932	113.278	64.338	160.895796	0.777146	0.29473728	0.8852215	181.7577	66.15438	130.49236
7	2.5	1.65	52	82.5	0.788011	65.0109	81.2135	52.3039664	0.6156615	0.46586066	0.7509895	69.64673	25.34934	106.5628
						Σ⑥ = 356.962								Σ⑭ = 754.40483

F4 = Σ⑥/Σ⑭ = 2.1134008 ΔF = 0.005986

Iteration 4 F4 = 2.1134

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭
Slice	b	h	α	W =bhγ	sinα	Wsinα	c'l	④-(⑦*⑤)/F	cosα	tanΦ'tanα	⑨*(1+⑩/F)	N=⑧/⑪	N*tanΦ'	⑦+⑬
1	2.5	0.8	-9	40	-0.156434	-6.25738	50.6233	43.7471463	0.9876883	-0.0576472	0.96074717	45.5345	16.5732	67.196461
2	2.5	2.3	0.5	115	0.008727	1.00355	50.0019	114.793535	0.9999619	0.00317632	1.00146481	114.6256	41.72032	91.722221
3	2.5	3.3	9	165	0.156434	25.8117	50.6233	161.252854	0.9876883	0.05764722	1.01462951	158.9278	57.845	108.46825
4	2.5	3.9	18.5	195	0.317305	61.8744	52.7246	187.08396	0.9483237	0.12178274	1.00296992	186.53	67.89136	120.61598
5	2.5	4.1	28	205	0.469472	96.2417	56.6285	192.420525	0.8829476	0.19352641	0.96380006	199.6478	72.66584	129.29435
6	2.5	3.6	39	180	0.62932	113.278	64.338	160.841683	0.777146	0.29473728	0.88552762	181.6337	66.10927	130.44725
7	2.5	1.65	52	82.5	0.788011	65.0109	81.2135	52.218436	0.6156615	0.46586066	0.75137282	69.49737	25.29497	106.50844
						Σ⑥ = 356.962								Σ⑭ = 754.25294

F5 = Σ⑥/Σ⑭ = 2.11297531 ΔF = 0.000425

b) Verification Using Slide:

Start a new file and define the slope cross section.

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Select: Boundaries → Add External Boundary
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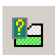
Enter the slope cross section coordinates as given in Fig. 1

Define the Material Properties as given in the problem



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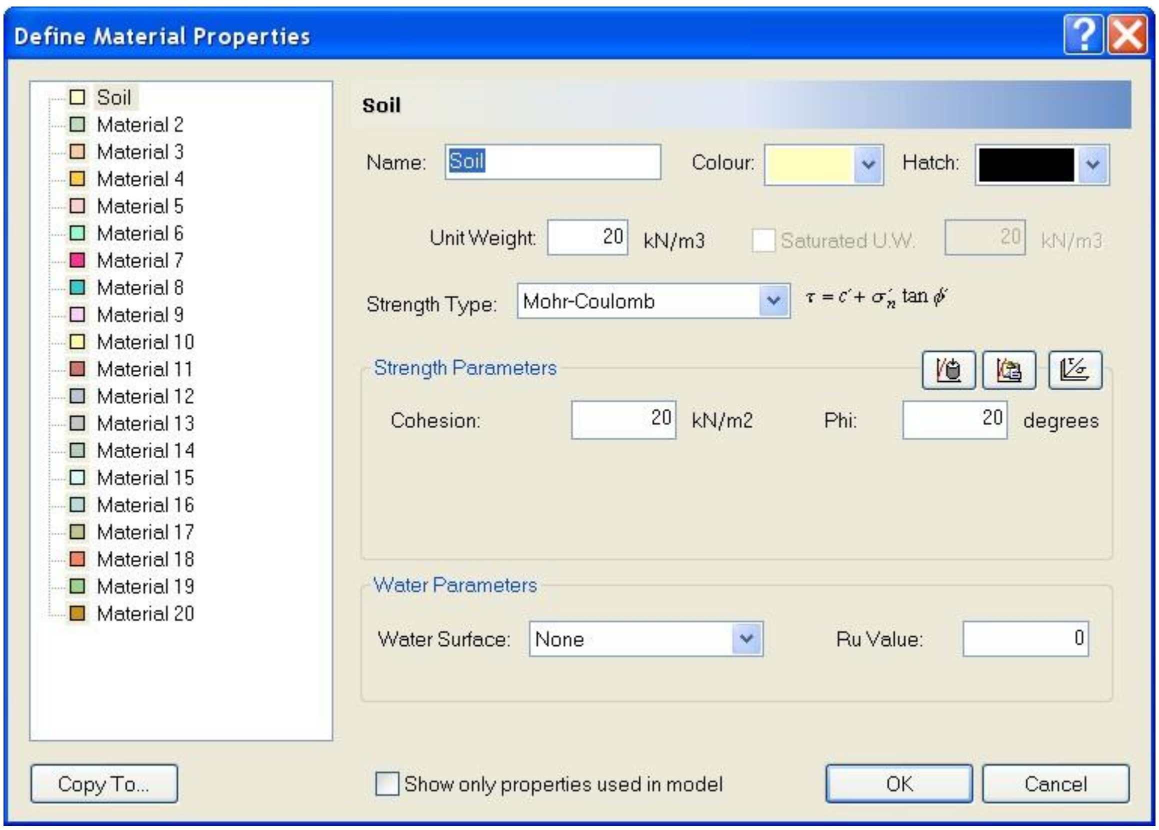
Select: Properties → Define Materials
```



Define the Project Settings as given in the problem

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Select: Analysis → Project Settings
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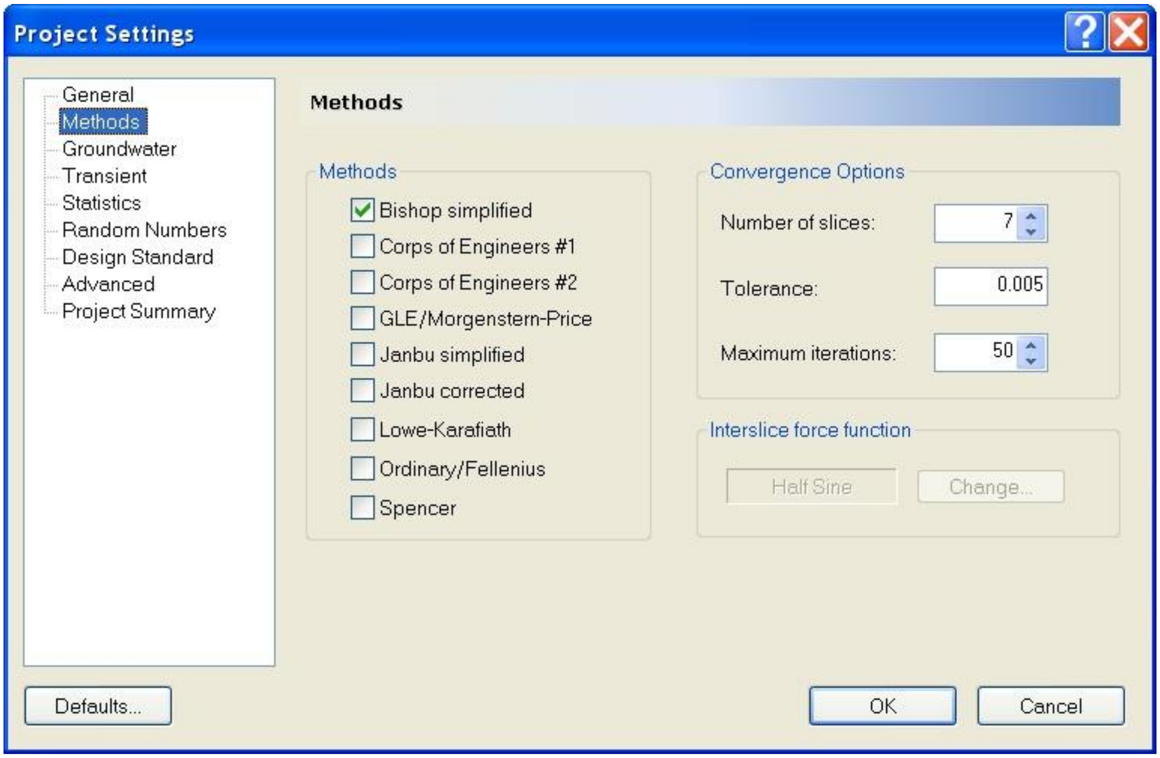
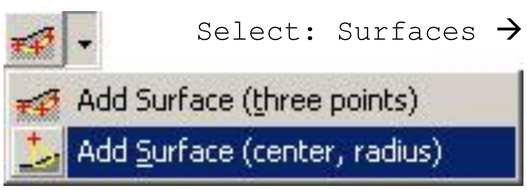
Under the Methods tab, select the Bishops method of analysis and fill in the number of slices and the tolerance.



Add the circular surface defined in the problem



`Select: Surfaces → Add Surface (center, radius)`

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Enter center (x,y) of slip circle [esc=quit]: 13.689 25.558
Enter radius of slip circle [esc=quit]: 15.989
```
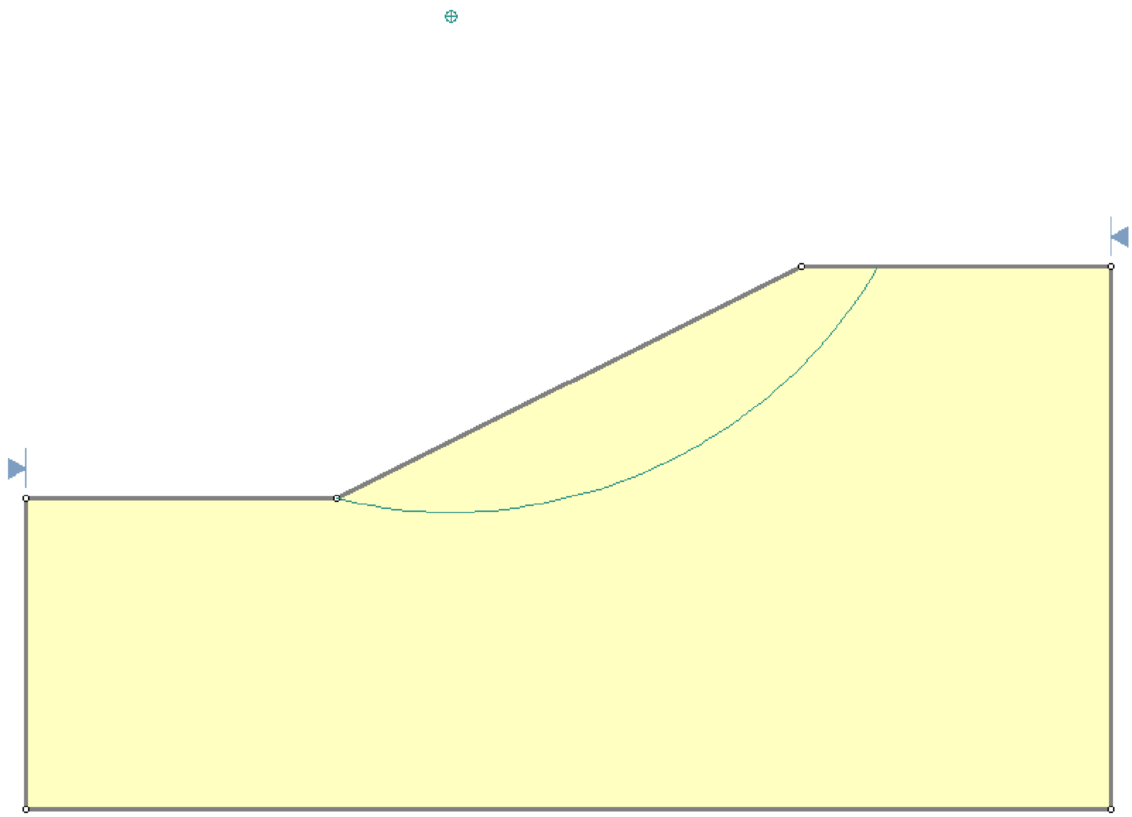

Your screen should now look as follows:



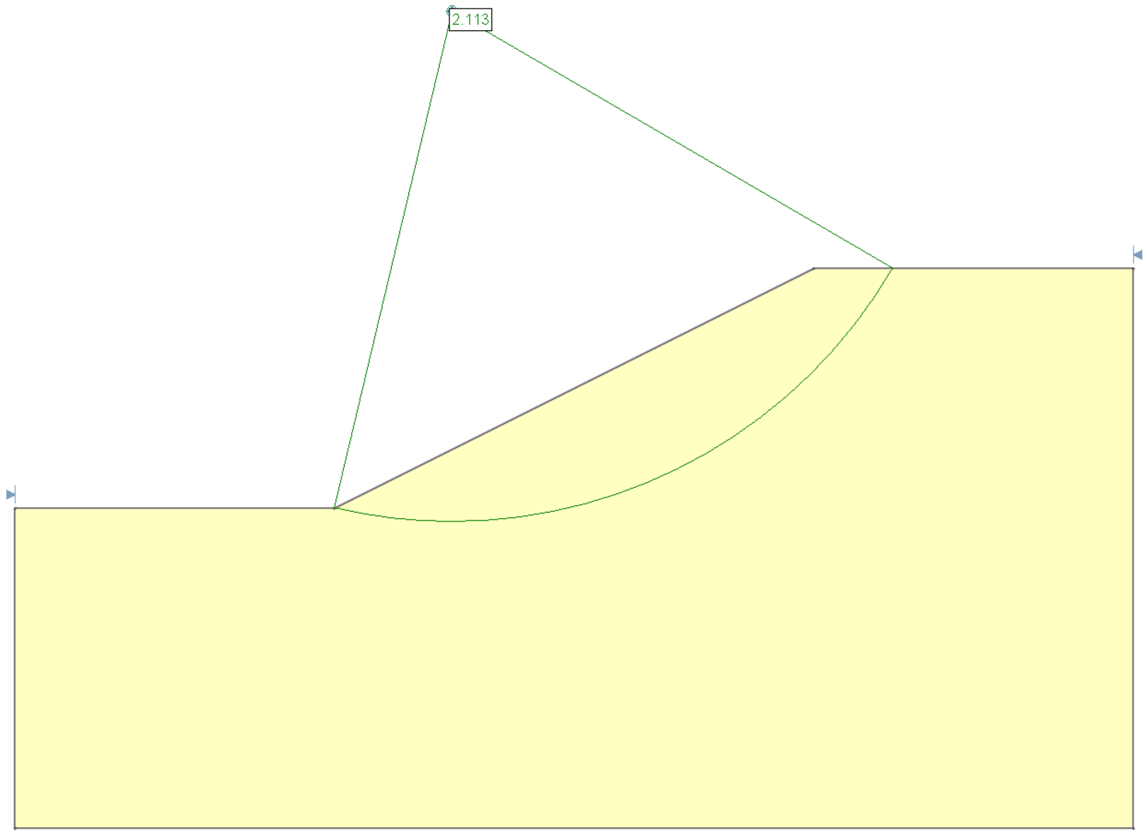
Save the file before analyzing the model.

Select: File → Save

Select: Analysis → Compute

Select: Analysis → Interpret

Your screen should now look as follows:



Answer: Slide calculated $FOS_{Bishops}$ = 2.113

Answer: Hand calculated $FOS_{Bishops}$ = 2.113

REFERENCES

1. Sharma, S., (1996), Chap 6: Slope Stability and Stabilization Methods, Abramson, L.W., Lee, T.S., Sharma, S., and Boyce, G.M. New York: Wiley, pp 408-424.