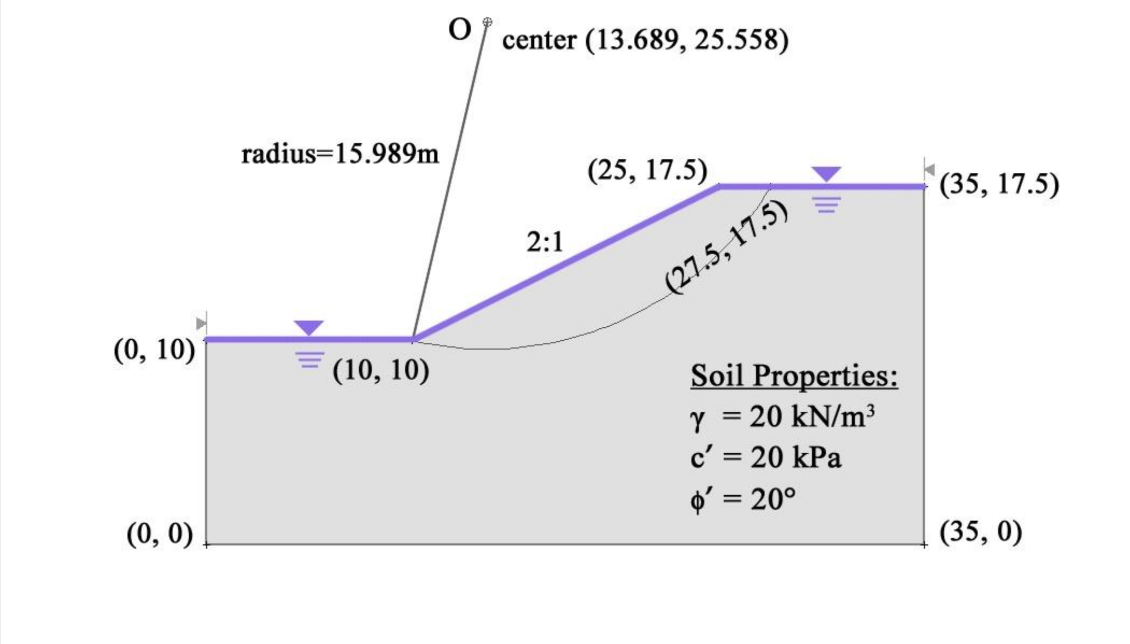
SLIDE HAND CALCULATION #2

Calculate by hand the factor of safety of the circular failure surface shown in Fig. 1 using the following methods:

a) Bishop Simplified (7 slices)
b) Verify parts a) using Slide

Fig. 1 Slope cross section



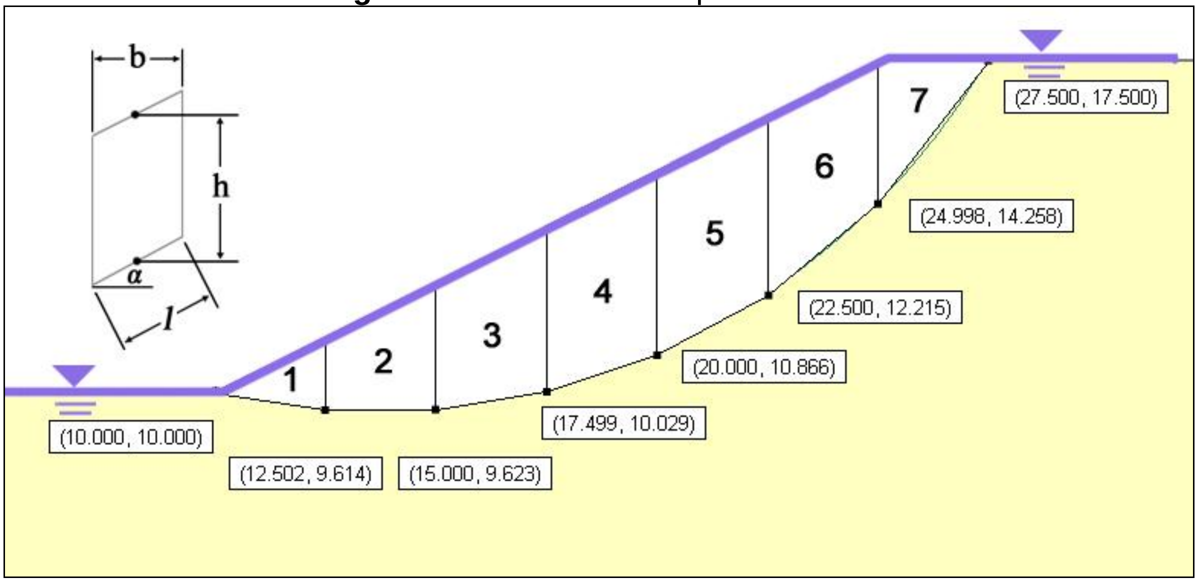
a) Bishop Simplified Solution:

Draw the slope cross section to scale and divide it into 7 vertical slices (see Fig. 2). Using a ruler and protractor, measure the slice data from the cross sectional diagram. Table 1 summarizes the slice data.

Table 1 Slice Data

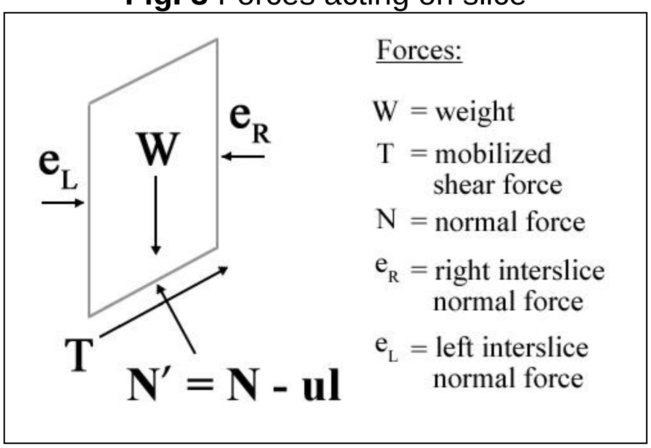
Slice	Base (m)	Height(m)	α (deg)	ϕ (deg)	c (kPa)	γ (kN/m^3)
1	2.5	0.8	-9	20	20	20
2	2.5	2.3	0.5	20	20	20
3	2.5	3.3	9	20	20	20
4	2.5	3.9	18.5	20	20	20
5	2.5	4.1	28	20	20	20
6	2.5	3.6	39	20	20	20
7	2.5	1.65	52	20	20	20

Fig. 2 Cross section of slope – 7 slices



Label the forces acting on each slice (see Fig. 3). Note that interslice shear forces are not depicted because they are assumed to be zero in the Bishop Simplified method.

Fig. 3 Forces acting on slice



The Bishop Simplified method satisfies two equilibrium conditions: overall moment equilibrium about the center O and vertical force equilibrium on each slice.

First consider the overall moment equilibrium about the center O

$$\sum M_O = \sum_{i=1}^{7} W_i r \sin\alpha_i - \sum_{i=1}^{7} T_i r = 0 \tag{1}$$

Dividing both sides of the equation by *r* yields

$$\sum_{i=1}^{7} W_i \sin\alpha_i - \sum_{i=1}^{7} T_i = 0 \tag{2}$$

If we assume that the factor of safety *F* is the same for all 7 slices, the mobilized shear force *T* is given by

$$T_i = \frac{c' l_i + \tan\varphi' N_i}{F} \qquad (3)$$

Summing up equation (3) for all the slices followed by substituting the sum into (2) and rearranging yields

$$F = \frac{\sum_{i=1}^{7}(c' l_i + \tan\varphi' N_i)}{\sum_{i=1}^{7} W_i \sin\alpha_i} \qquad (4)$$

To solve for N_i, consider the equilibrium of forces in the vertical direction on each slice

$$N_i \cos\alpha_i + T_i \sin\alpha_i - W_i = 0 \xrightarrow{\text{from (3)}}$$

$$N_i \cos\alpha_i + \frac{c' l_i + \tan\varphi' N_i}{F}\sin\alpha_i - W_i = 0 \xrightarrow{\text{factoring } N_i \text{ and simplifying}}$$

$$N_i = N_i' - u l_i = \frac{W_i - \frac{c' l_i}{F}\sin\alpha_i}{\cos\alpha_i\left(1 + \frac{\tan\varphi' \tan\alpha_i}{F}\right)} \qquad (5)$$

Substituting (5) into (4) yields

$$F = \sum_{i=1}^{7}\left(c' l_i + \tan\varphi'\left(\frac{\left(W_i - \frac{c' l_i \sin\alpha_i}{F}\right)}{\cos\alpha_i\left(1 + \frac{\tan\varphi' \tan\alpha_i}{F}\right)}\right)\right) \times \frac{1}{\sum_{i=1}^{7} W_i \sin\alpha_i} \qquad (6)$$

Note that according to Fig.2, $l = b \times \sec(\alpha)$

Equation (6) shows that the factor of safety is defined implicitly in an expression containing F. To solve for F from (6), we start with an initial estimated value for F. Iterations of successive approximation are performed until value of F converges to within a given tolerance.

For this problem, a tolerance of 0.005 is used.

See Table 2 for Excel spreadsheets of the calculations for each iteration.

Answer: $\text{FOS}_{\text{Bishop Simplified}} = 1.553$

Table 3 Bishop Simplified Method – Calculations

Iteration 1 F1 = 5

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮
Slice	b	h	α	u	W =bhγ	sinα	Wsinα	c'l-tanϕ'ul	⑤-(⑧*⑥)/F	cosα	tanΦ'tanα	⑩*(1+⑪/F)	N=⑨/⑫	N*tanϕ'	⑧+⑭
1	2.5	0.8	-9	7.84	40	-0.156434	-6.25738	43.4005	41.3578673	0.9876883	-0.0576472	0.97630084	42.36181	15.41844	58.818952
2	2.5	2.3	0.5	22.54	115	0.008727	1.00355	29.4914	114.948528	0.9999619	0.00317632	1.00059716	114.8799	41.81287	71.304274
3	2.5	3.3	9	32.34	165	0.156434	25.8117	20.8295	164.348311	0.9876883	0.05764722	0.99907584	164.5003	59.87323	80.702678
4	2.5	3.9	18.5	38.22	195	0.317305	61.8744	16.0522	193.981315	0.9483237	0.12178274	0.97142155	199.6881	72.68052	88.732683
5	2.5	4.1	28	40.18	205	0.469472	96.2417	15.2208	203.570851	0.8829476	0.19352641	0.91712233	221.967	80.78936	96.010189
6	2.5	3.6	39	35.28	180	0.62932	113.278	23.0302	177.101325	0.777146	0.29473728	0.82295674	215.2013	78.32685	101.35705
7	2.5	1.65	52	16.17	82.5	0.788011	65.0109	57.3148	73.4670671	0.6156615	0.46586066	0.67302397	109.1597	39.73087	97.045649
							Σ⑦ = 356.962								Σ⑮ = 593.97147

F2 = Σ⑦/Σ⑮ = 1.66396045 ΔF = 3.33604

Iteration 2 F2 = 1.664

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮
Slice	b	h	α	u	W =bhγ	sinα	Wsinα	c'l-tanϕ'ul	⑤-(⑧*⑥)/F	cosα	tanΦ'tanα	⑩*(1+⑪/F)	N=⑨/⑫	N*tanϕ'	⑧+⑭
1	2.5	0.8	-9	7.84	40	-0.156434	-6.25738	43.4005	44.0802271	0.9876883	-0.0576472	0.95347029	46.23136	16.82684	60.227354
2	2.5	2.3	0.5	22.54	115	0.008727	1.00355	29.4914	114.845334	0.9999619	0.00317632	1.00187074	114.6309	41.72223	71.213632
3	2.5	3.3	9	32.34	165	0.156434	25.8117	20.8295	163.041754	0.9876883	0.05764722	1.02190639	159.5467	58.07024	78.899688
4	2.5	3.9	18.5	38.22	195	0.317305	61.8744	16.0522	191.938974	0.9483237	0.12178274	1.01773002	188.5952	68.64303	84.695192
5	2.5	4.1	28	40.18	205	0.469472	96.2417	15.2208	200.705581	0.8829476	0.19352641	0.98563854	203.63	74.11526	89.336085
6	2.5	3.6	39	35.28	180	0.62932	113.278	23.0302	171.289833	0.777146	0.29473728	0.91480181	187.2426	68.15072	91.180916
7	2.5	1.65	52	16.17	82.5	0.788011	65.0109	57.3148	55.3571285	0.6156615	0.46586066	0.78802883	70.24759	25.56803	82.882815
							Σ⑦ = 356.962								Σ⑮ = 558.43568

F3 = Σ⑦/Σ⑮ = 1.56440996 ΔF = 0.09955

Table 3 Bishop Simplified Method – Calculations (cont'd)

Iteration 3 F3 = 1.5644

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮
Slice	b	h	α	u	W =bhγ	sinα	Wsinα	c'l-tanϕ'ul	⑤-(⑧*⑥)/F	cosα	tanΦ'tanα	⑩*(1+⑪/F)	N=⑨/⑫	N*tanϕ'	⑧+⑭
1	2.5	0.8	-9	7.84	40	-0.156434	-6.25738	43.4005	44.3398704	0.9876883	-0.0576472	0.95129284	46.61012	16.9647	60.365211
2	2.5	2.3	0.5	22.54	115	0.008727	1.00355	29.4914	114.835492	0.9999619	0.00317632	1.00199221	114.6072	41.7136	71.204999
3	2.5	3.3	9	32.34	165	0.156434	25.8117	20.8295	162.917142	0.9876883	0.05764722	1.02408384	159.0857	57.90248	78.731928
4	2.5	3.9	18.5	38.22	195	0.317305	61.8744	16.0522	191.744188	0.9483237	0.12178274	1.02214666	187.5897	68.27707	84.329229
5	2.5	4.1	28	40.18	205	0.469472	96.2417	15.2208	200.432307	0.8829476	0.19352641	0.99217323	202.0134	73.52687	88.747696
6	2.5	3.6	39	35.28	180	0.62932	113.278	23.0302	170.735566	0.777146	0.29473728	0.92356148	184.8665	67.2859	90.316097
7	2.5	1.65	52	16.17	82.5	0.788011	65.0109	57.3148	53.6299046	0.6156615	0.46586066	0.79899734	67.12151	24.43023	81.745012
							Σ⑦ = 356.962								Σ⑮ = 555.44017

F4 = Σ⑦/Σ⑮ = 1.55601829 ΔF = 0.008392

Iteration 4 F4 = 1.556

Calc.	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮
Slice	b	h	α	u	W =bhγ	sinα	Wsinα	c'l-tanϕ'ul	⑤-(⑧*⑥)/F	cosα	tanΦ'tanα	⑩*(1+⑪/F)	N=⑨/⑫	N*tanϕ'	⑧+⑭
1	2.5	0.8	-9	7.84	40	-0.156434	-6.25738	43.4005	44.3632755	0.9876883	-0.0576472	0.95109655	46.64434	16.97715	60.377669
2	2.5	2.3	0.5	22.54	115	0.008727	1.00355	29.4914	114.834605	0.9999619	0.00317632	1.00200316	114.605	41.71282	71.204221
3	2.5	3.3	9	32.34	165	0.156434	25.8117	20.8295	162.905909	0.9876883	0.05764722	1.02428013	159.0443	57.88739	78.71684
4	2.5	3.9	18.5	38.22	195	0.317305	61.8744	16.0522	191.726629	0.9483237	0.12178274	1.02254479	187.4995	68.24423	84.296395
5	2.5	4.1	28	40.18	205	0.469472	96.2417	15.2208	200.407673	0.8829476	0.19352641	0.99276229	201.8687	73.47421	88.695037
6	2.5	3.6	39	35.28	180	0.62932	113.278	23.0302	170.685602	0.777146	0.29473728	0.9243511	184.6545	67.20875	90.238944
7	2.5	1.65	52	16.17	82.5	0.788011	65.0109	57.3148	53.4742071	0.6156615	0.46586066	0.79998608	66.84392	24.3292	81.64398
							Σ⑦ = 356.962								Σ⑮ = 555.17309

F5 = Σ⑦/Σ⑮ = 1.55527007 ΔF = 0.000748

b) Verification Using Slide:

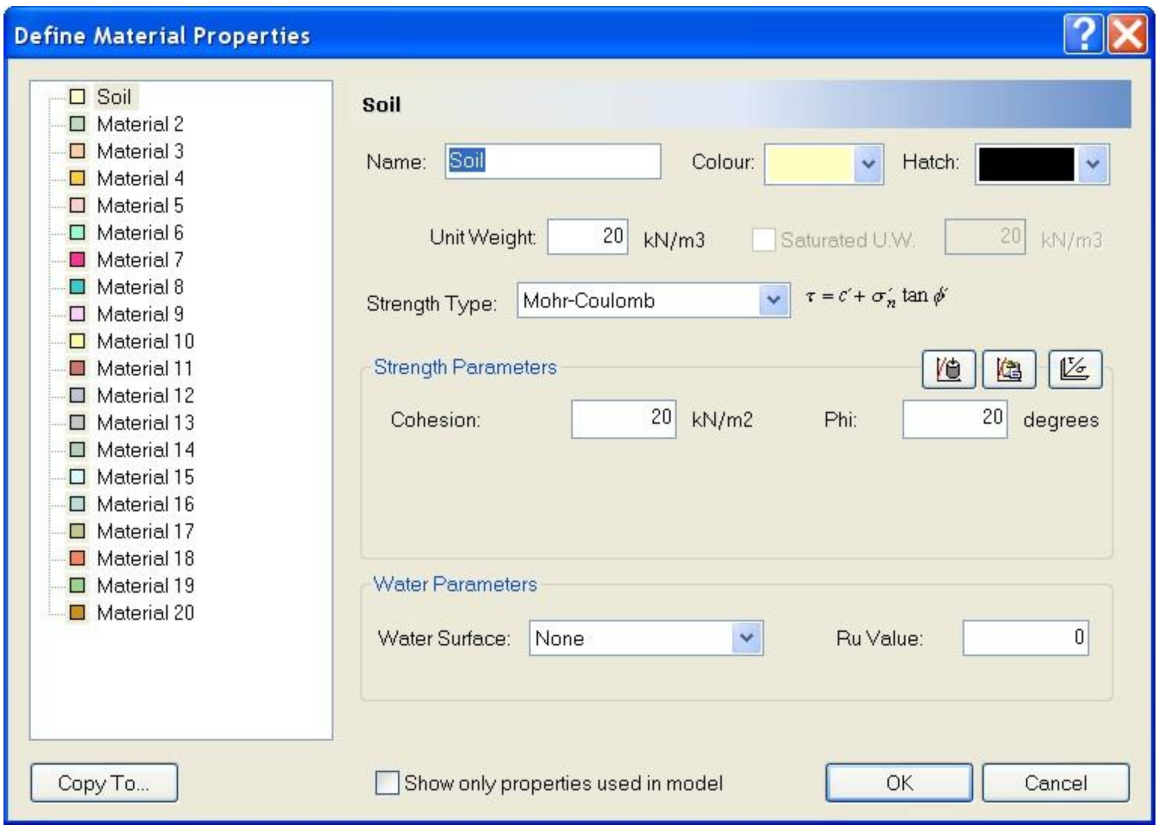
Start a new file and define the slope cross section.

`Select: Boundaries → Add External Boundary`

Enter the slope cross section coordinates as given in Fig. 1

Define the Material Properties as given in the problem

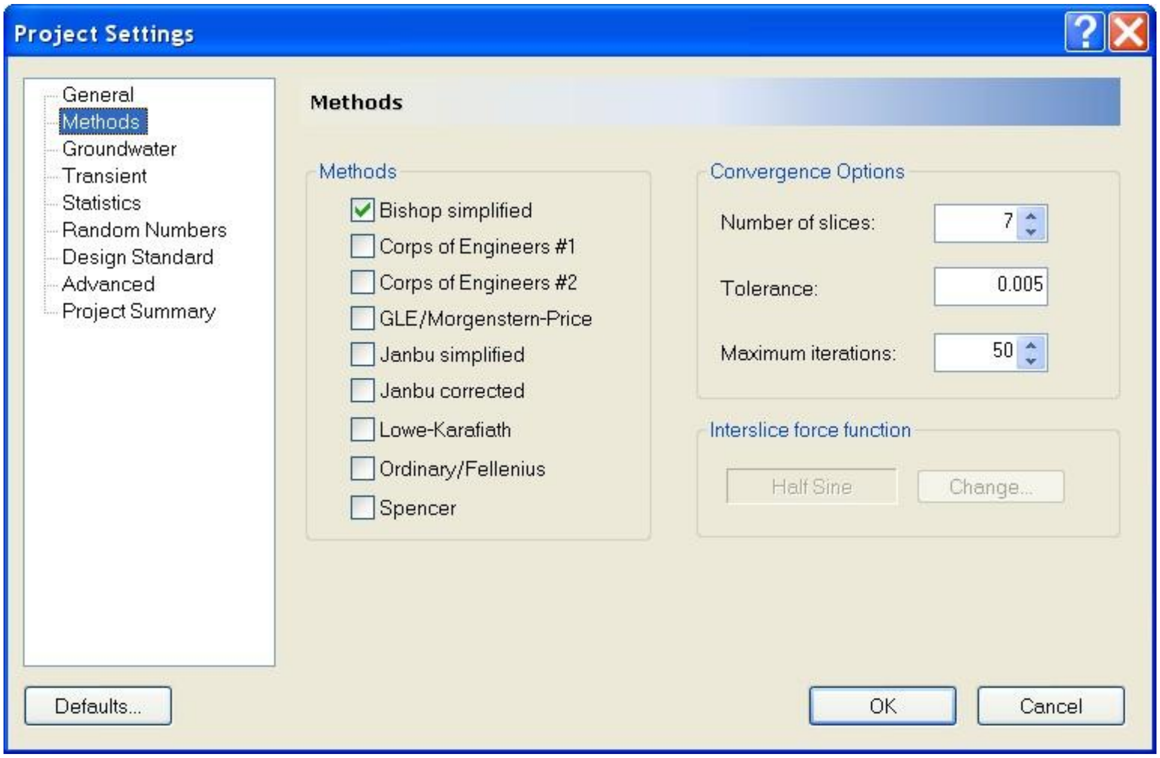
`Select: Properties → Define Materials`



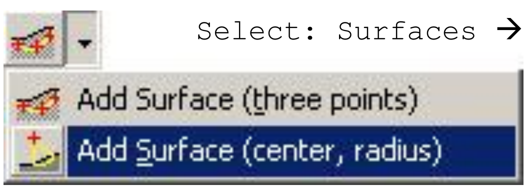
Define the Project Settings as given in the problem

`Select: Analysis → Project Settings`

Under the Methods tab, select Bishops method of analysis and fill in the number of slices and the tolerance.



Add the circular surface defined in the problem



`Select: Surfaces → Add Surface (center, radius)`

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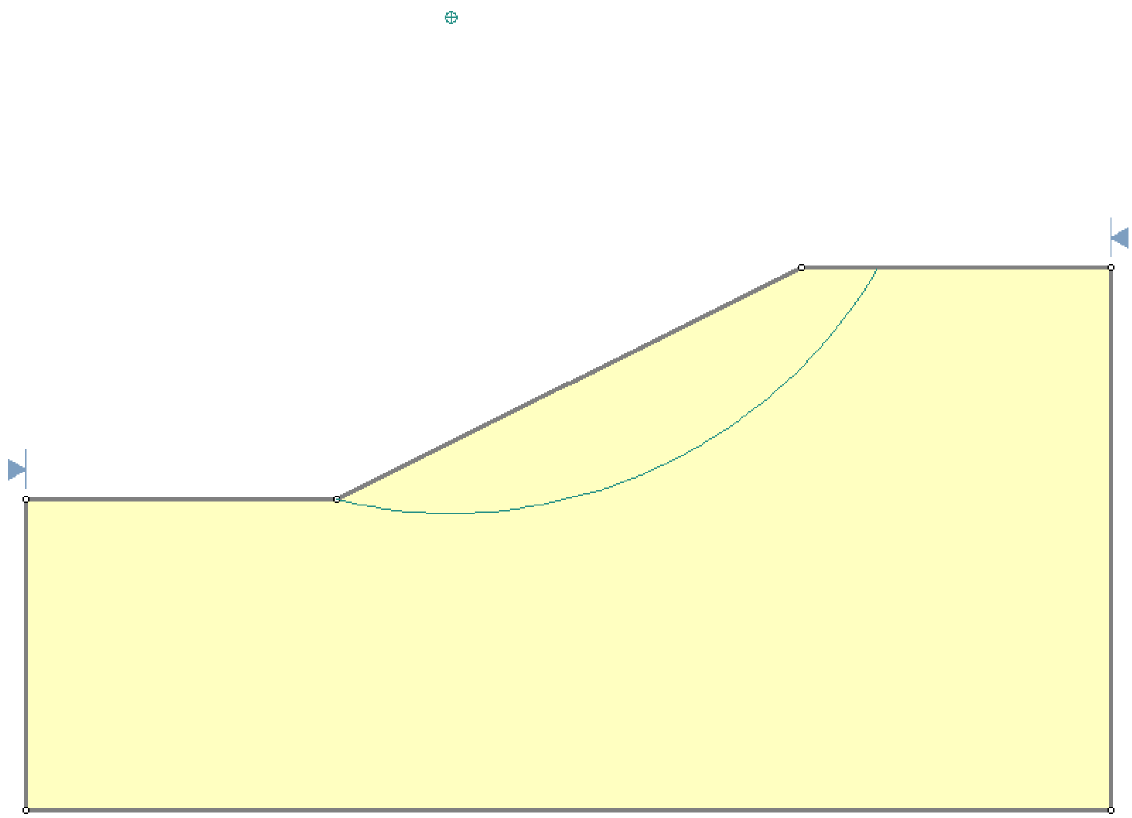
Enter center (x,y) of slip circle [esc=quit]: 13.689 25.558
Enter radius of slip circle [esc=quit]: 15.989
```

Your screen should now look as follows:

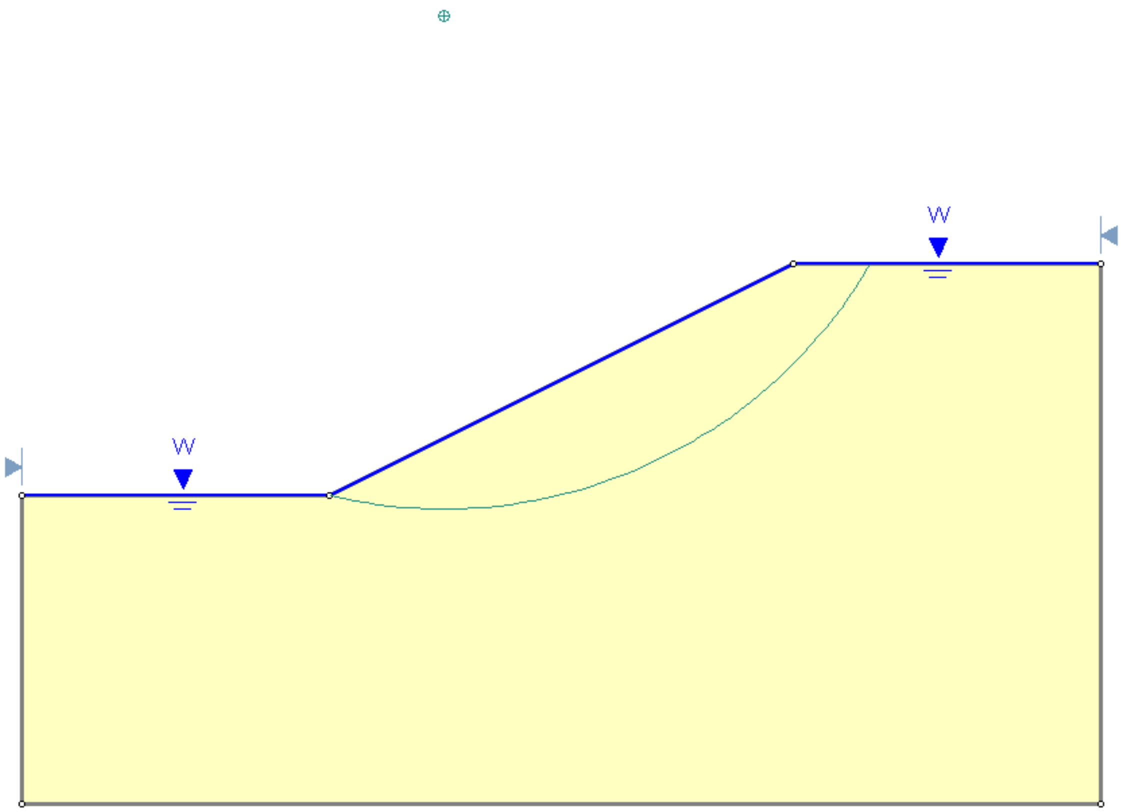


Add the water table

`Select: Boundaries → Add Water Table`

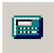
Enter the water table coordinates as given in Fig. 1 or graphically add the water table along the slope surface.

Your screen should now look as follows:



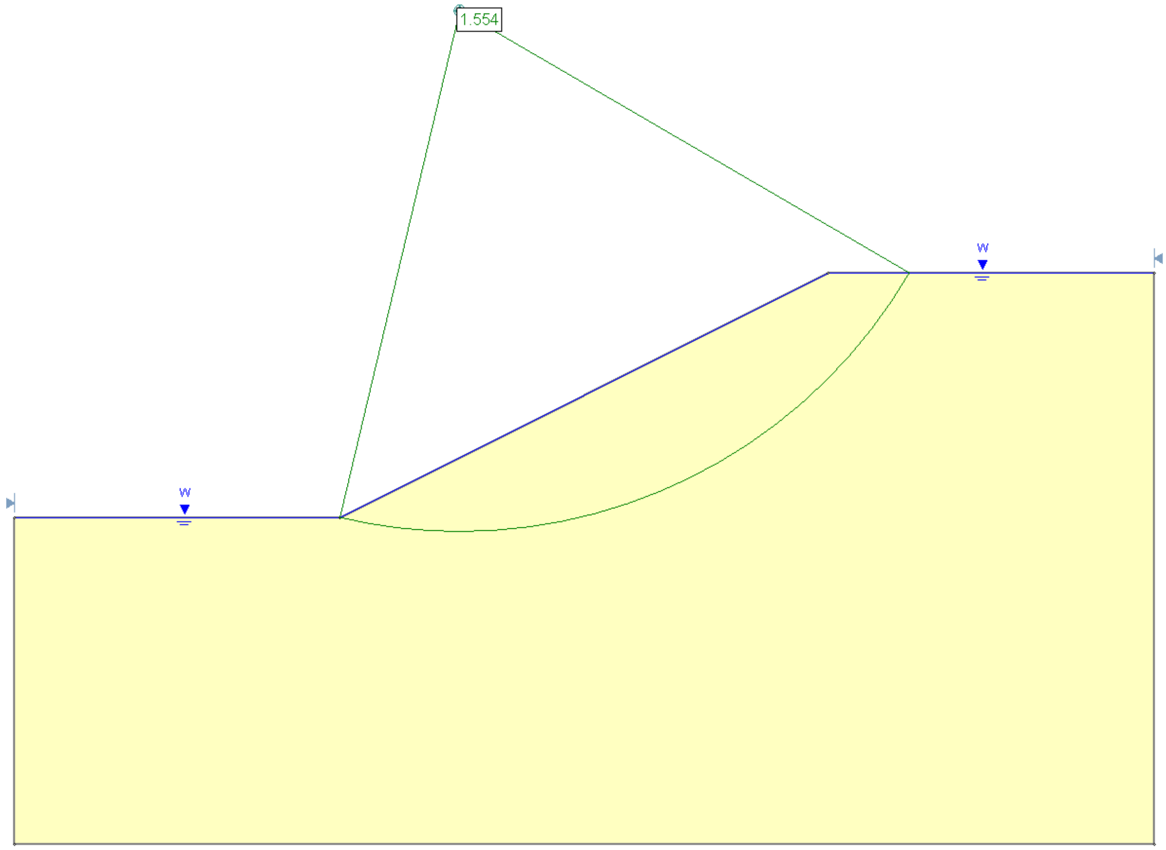
Save the file before analyzing the model.

 Select: File → Save

 Select: Analysis → Compute

 Select: Analysis → Interpret

Your screen should now look as follows:



Answer: Slide calculated $FOS_{Bishops}$ = 1.554

Answer: Hand calculated $FOS_{Bishops}$ = 1.555*

**Minor discrepancy from Slide calculated results due to the estimation of slice height in the hand calculations*

REFERENCES

1. Sharma, S., (1996), Chap 6: Slope Stability and Stabilization Methods, Abramson, L.W., Lee, T.S., Sharma, S., and Boyce, G.M. New York: Wiley, pp 408-424.