Stochastic Response Surface Verification Examples

The Stochastic Response Surface method uses a small number of strategically selected computations to create a response surface of factor of safety (FS) values for various combinations of input parameters. It then *predicts* the factor of safety values for any combination of samples and provides an estimated probability of failure. Since an Overall Slope probabilistic analysis can take hours, this method is advantageous in significantly cutting down computation time.

Although many verification examples have shown it to agree well with Latin-Hypercube results, it cannot always guarantee a result that is identical. However, it will be able to give you a ballpark PF value. It is always recommended to run at least one Latin-Hypercube Overall Slope analysis.

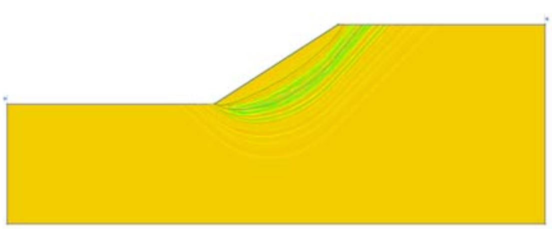
This document focuses on Overall Slope analyses through complex examples. The output data can be found in the accompanying spreadsheet.

Example 1 Worst Case

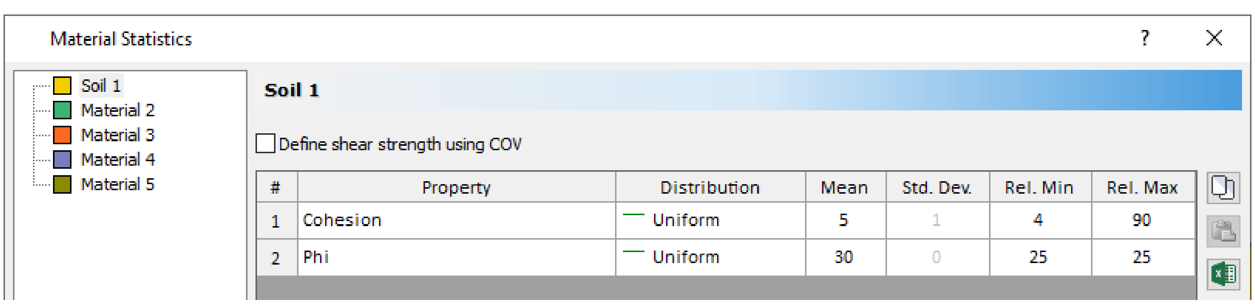
The first example uses a simple slope with two random variables. The random variables have a wide range of variability and uniform distributions, indicating all samples are equally likely.

An Overall Slope analysis with 100 samples was computed with both Latin-Hypercube and Response Surface. Response Surface required 20 computations.

This example was constructed to show the worst case, where response surface has to capture cohesion ranging from 1 kPa to 95 kPa and friction angle ranging from 5 to 55 degrees, all with equal probability, in 20 samples. Furthermore, the use of 100 samples is likely not sufficient.



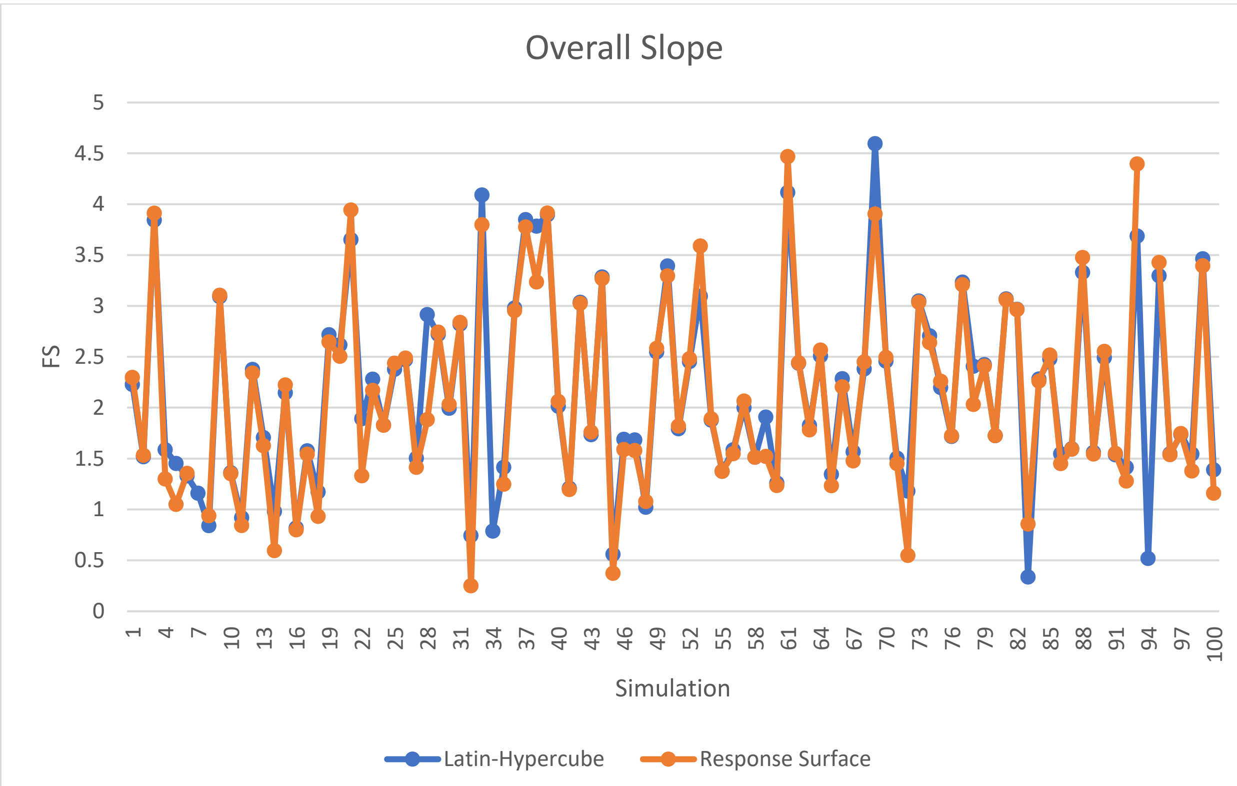
Surface Type	Non-Circular
Search Method	Cuckoo Search
Optimization	Surface Altering
Method	Bishop simplified
Deterministic FS (Bishop)	1.144



The results are found below:

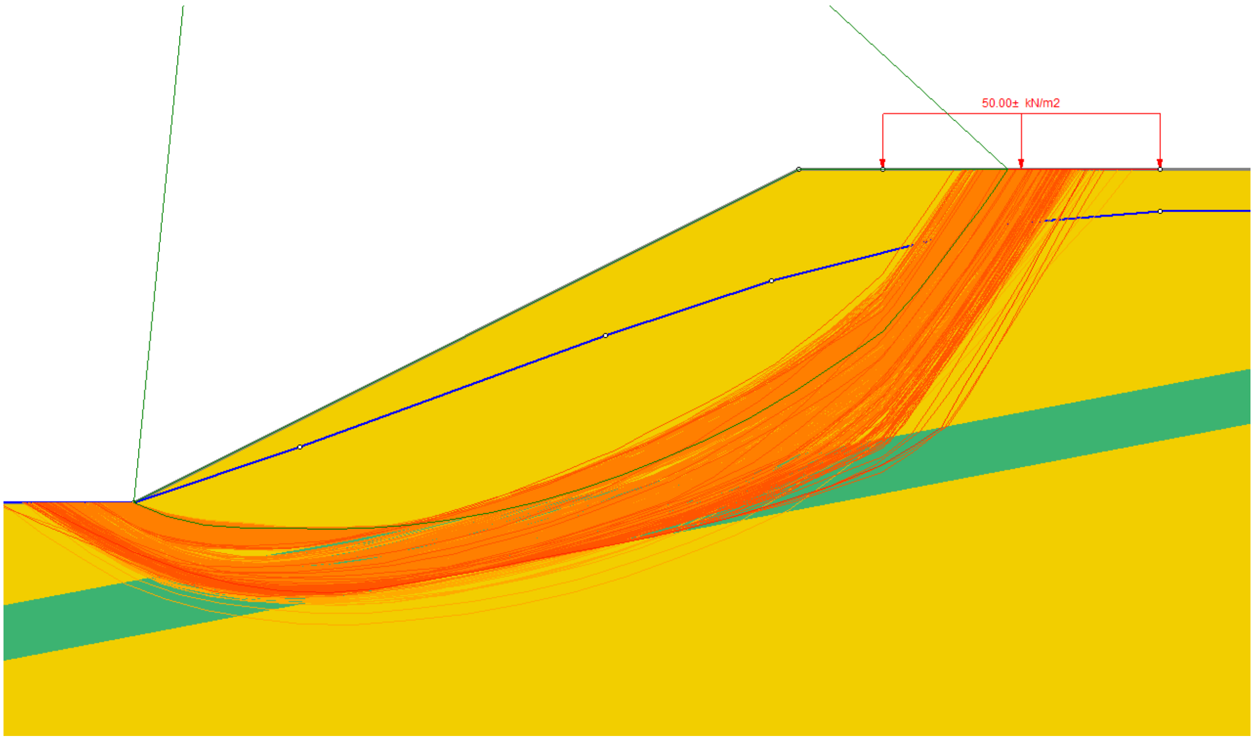
Overall Slope – Bishop				
Method	**Num Samples**	**PF (%)**	**Mean FS**	**Time (sec)**
Latin-Hypercube	100	9.000	2.139	20.7
Response Surface	100 (20)	9.278	2.125	5.0

In this case 100 samples were considered, so the Latin-Hypercube case indicates that 0.09*100 = 9 samples were found to be less than 1. Meanwhile with Response Surface, 3 samples were discarded with error codes and so 0.09278*97 = 9 samples were found to be less than 1. The plot below shows the FS calculated by Latin-Hypercube and predicted by Response Surface for the same 100 samples.



In short, this is a worst-case example where the range is very wide with a uniform distribution and the response surface must be created with 20 samples. Even so, the predictions are in good agreement for the majority of samples, resulting in probability of failure values that are in agreement. Now we will move to more realistic examples.

Example 2 Weak Layer with Variable Load

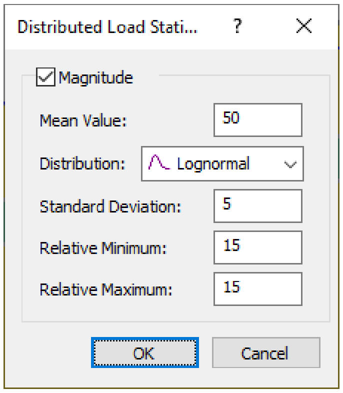


This example considers three random variables (two material properties and one load magnitude) with three different types of distributions. It also considers two modes of failure, one above the weak layer, and one inside of the weak layer.

Surface Type	Non-Circular
Search Method	Auto Refine Search
Optimization	Surface Altering
Method	GLE/Morgenstern-Price
Deterministic FS (GLE)	0.989

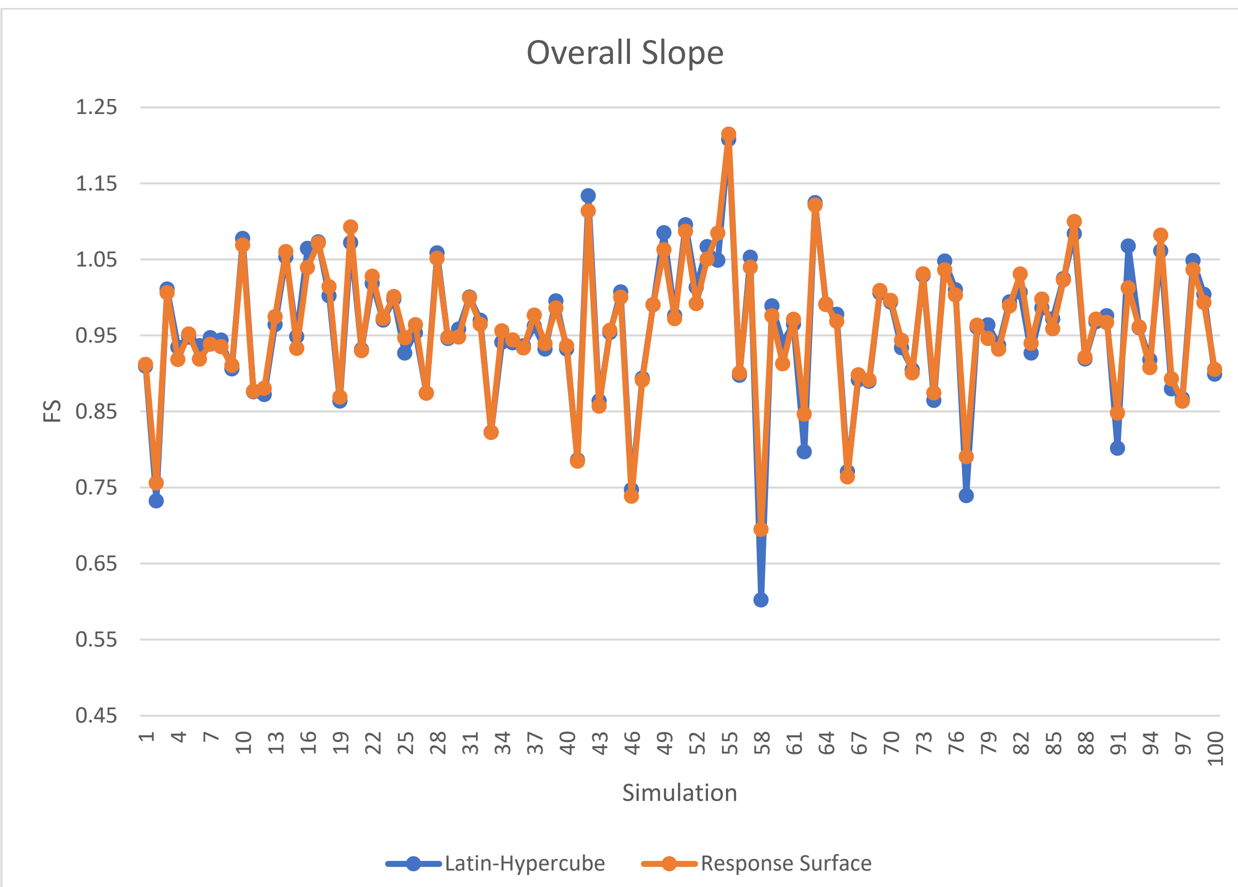
All Material Stats

#	Name	Property	Distribution	Mean	Std. Dev.	Rel. Min	Rel. Max
1	Soil	Cohesion	Normal	12	3	12	200
2	weak layer	Phi	Gamma	30	7	30	59.9



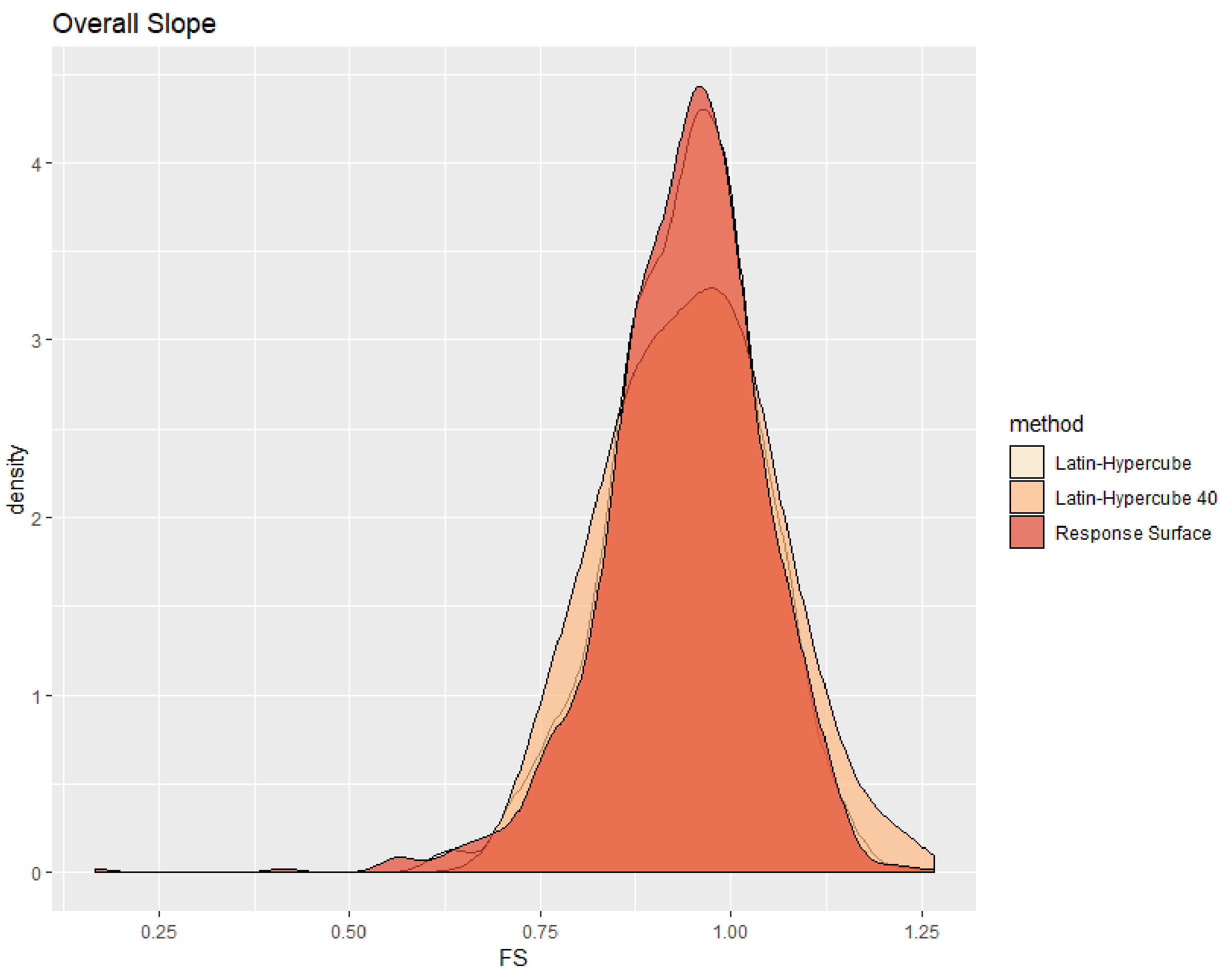
Overall Slope - GLE				
Method	**Num Samples**	**PF (%)**	**Mean FS**	**Time (min)**
Latin-Hypercube	1000	72.800	0.941	6.9
Response Surface	1000 (40)	73.200	0.939	0.3
Latin-Hypercube	40	65.000	0.948	0.3

A look at the first 100 samples for both methods shows that they are in good agreement:



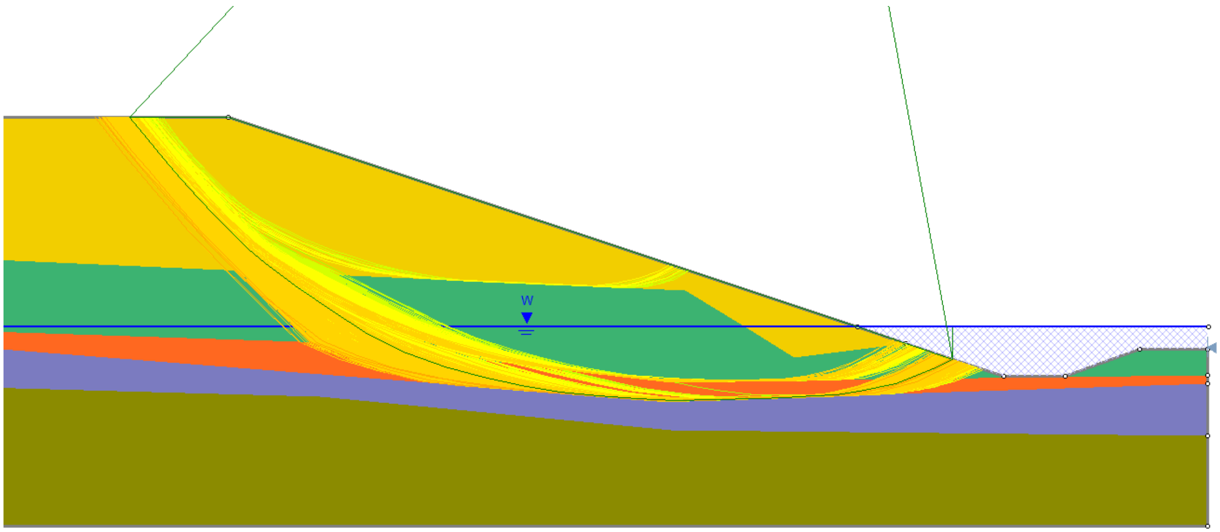
The figure below shows the histogram of FS values found by Latin-Hypercube and Response Surface. The histograms are in good agreement.

It is also notable that running only 40 Latin-Hypercube simulations on their own, gives a very different PF value and FS distribution.



Example 3 Sugar Creek

This example has six random variables, five of them material properties, and one the location of the water table. Three modes of failure are easily distinguishable in the figure below. Such a complicated model would require a large number of samples; 10,000 are considered here.

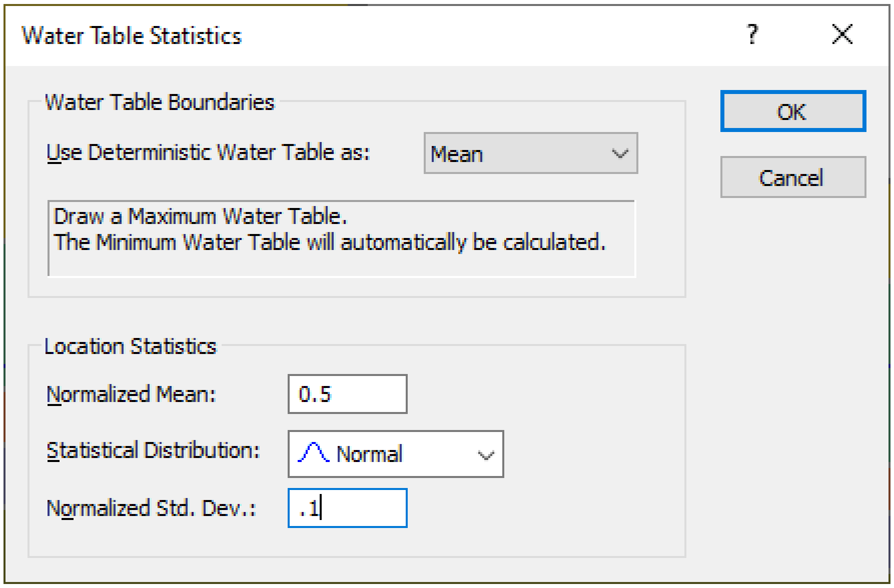


Surface Type	Non-Circular
Search Method	Auto Refine Search
Optimization	Surface Altering
Method	GLE/Morgenstern-Price
Deterministic FS (GLE)	1.59

All Material Stats

#	Name	Property	Distribution	Mean	Std. Dev.	Rel. Min	Rel. Max
1	Compacted fill	Phi	Lognormal	12	2.4	12	77.9
2	Alluvium(Sandy Clay)	Phi	Lognormal	16.5	3.465	16.5	73.4
3	Highly Weathered Shale	Phi	Lognormal	12.8	4	12.8	77.1
4	Moderately Weathered Shale	Cohesion	Lognormal	97	30	97	1000
5	Slightly wathered shale	Cohesion	Lognormal	675	200	675	10000

OK

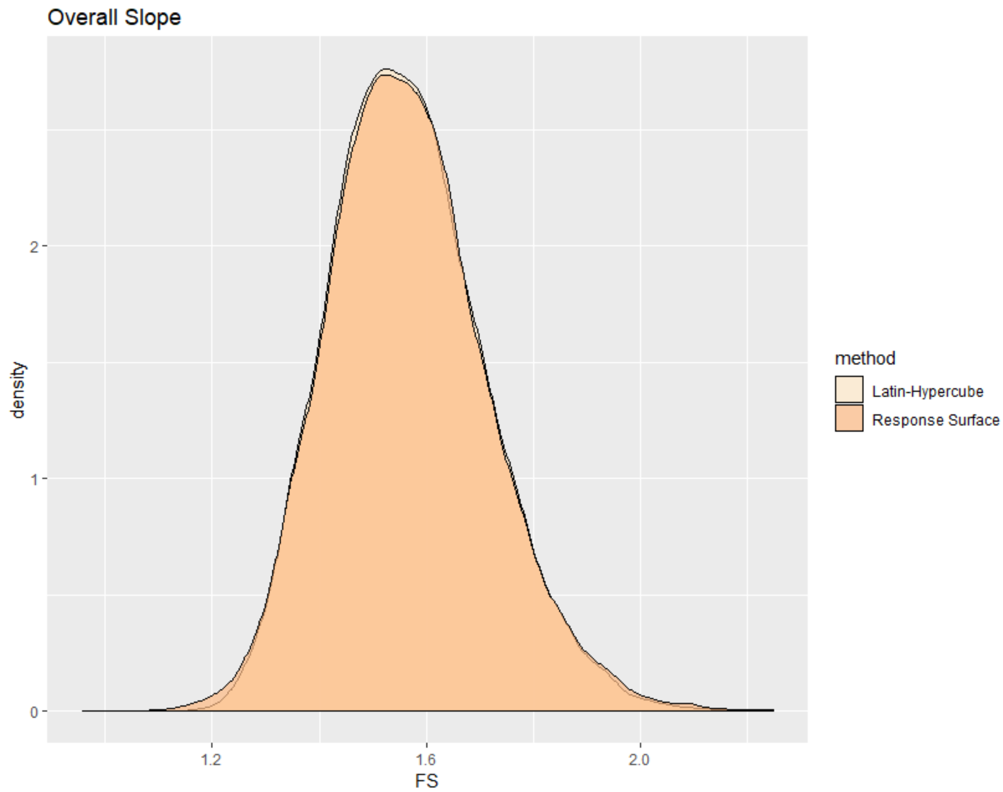


The probabilistic analysis results are found below:

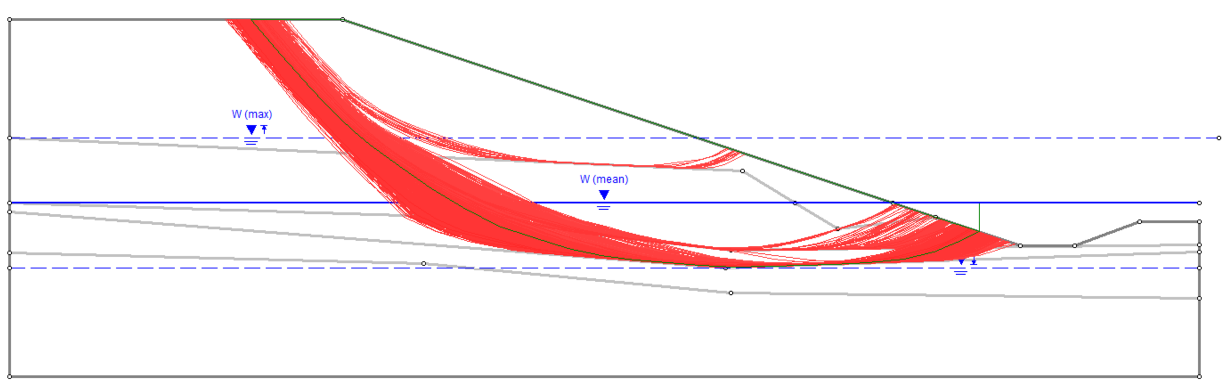
Overall Slope - GLE				
Method	**Num Samples**	**PF (%)**	**Mean FS**	**Time (min)**
Latin-Hypercube	10000	0.000	1.565	63.4
Response Surface	10000 (168)	0.010	1.566	1.1

Latin-Hypercube found no simulations with an FS<1 while Response Surface found 1 simulation (0.010*10000 = 1). With a one simulation difference, the PF values are in good agreement. The difference in computation time is notable.

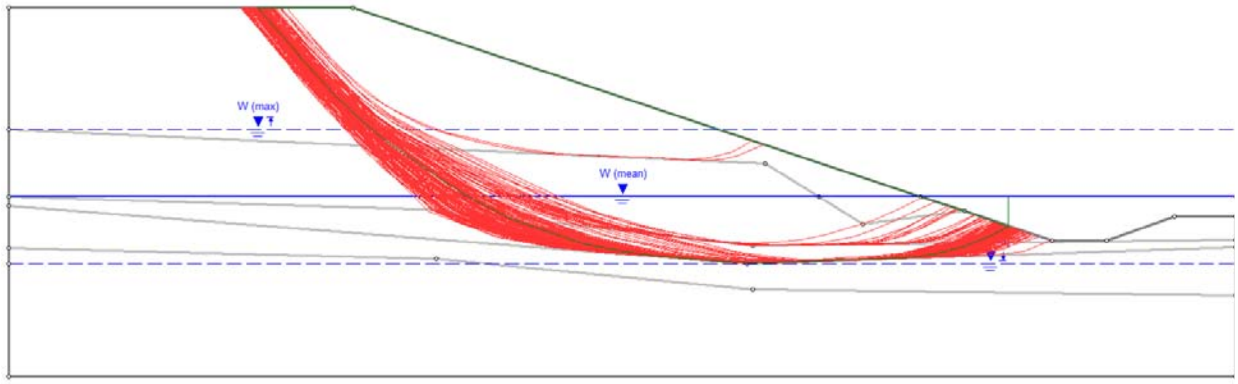
The figure below shows the histogram of FS values found by Latin-Hypercube and Response Surface. The histograms are in good agreement.



Finally, using the “Show All Surfaces” button allows us to see the original computations and surfaces that are used to train the Response Surface model for this example, and compare them to the surfaces found using Latin-Hypercube. It can be seen that although less surfaces are located with Response Surface on account of the smaller number of simulations, the strategically selected samples have ensured that all three failure modes are considered.



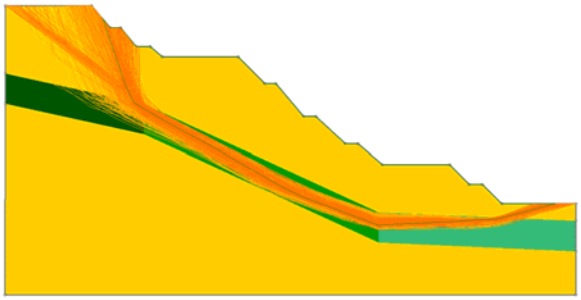
Latin-Hypercube with 10000 simulations



Response Surface with 168 simulations

Example 4 Correlation

This example considers Tutorial 27 Advanced Correlation. The tutorial is changed to an Overall Slope probabilistic analysis and the file is computed with both Latin-Hypercube and Response Surface.

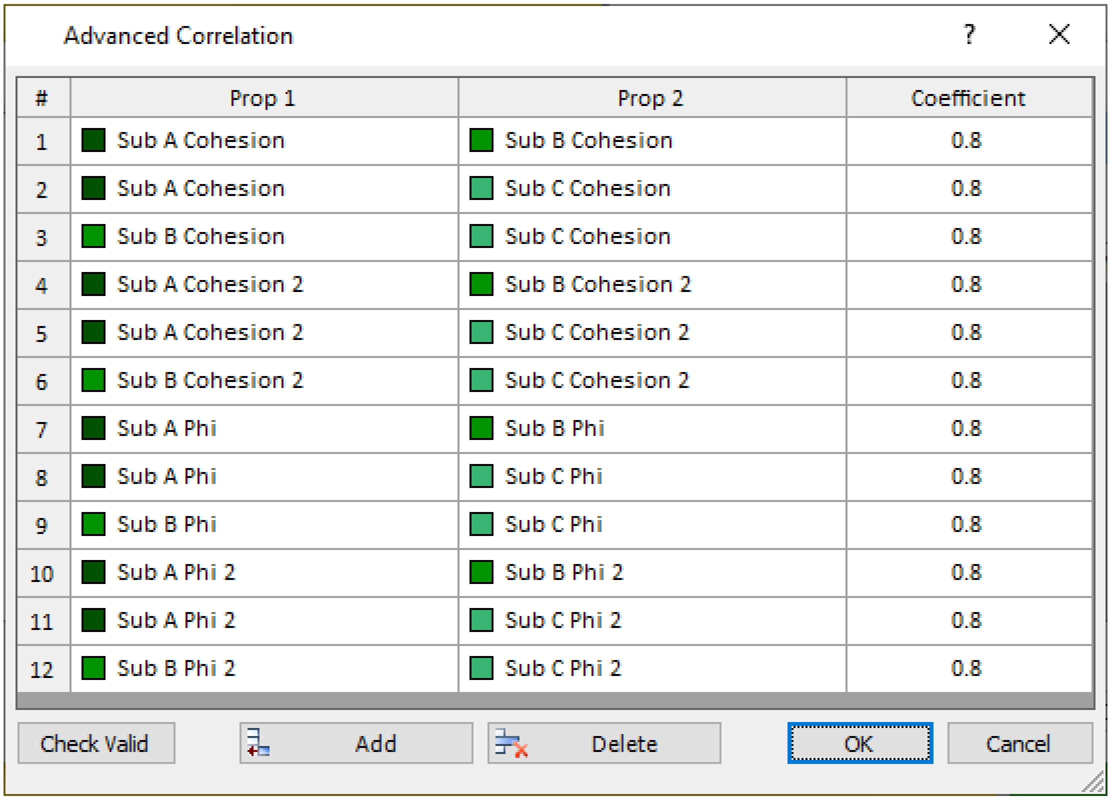


Surface Type	Non-Circular
Search Method	Cuckoo Search
Optimization	Surface Altering
Method	Spencer
Deterministic FS (Spencer)	1.088

All Material Stats

#	Name	Property	Distribution	Mean	Std. Dev.	Rel. Min	Rel. Max
1	Sub A	Cohesion	Normal	10	3	9	9
2	Sub A	Cohesion 2	Normal	20	3	9	9
3	Sub A	Phi	Normal	15	2	6	6
4	Sub A	Phi 2	Normal	25	2	6	6
5	Sub B	Cohesion	Normal	10	3	9	9
6	Sub B	Cohesion 2	Normal	20	3	9	9
7	Sub B	Phi	Normal	15	2	6	6
8	Sub B	Phi 2	Normal	25	2	6	6
9	Sub C	Cohesion	Normal	10	3	9	9
10	Sub C	Cohesion 2	Normal	20	3	9	9
11	Sub C	Phi	Normal	15	2	6	6
12	Sub C	Phi 2	Normal	25	2	6	6

OK

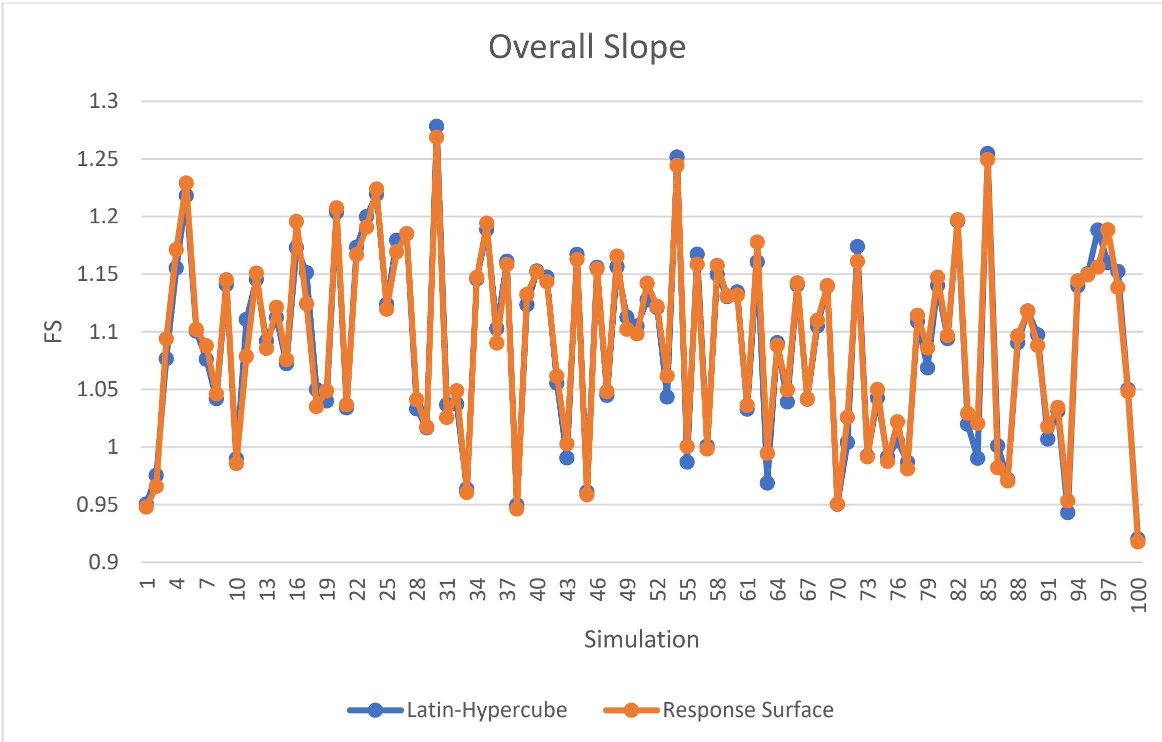


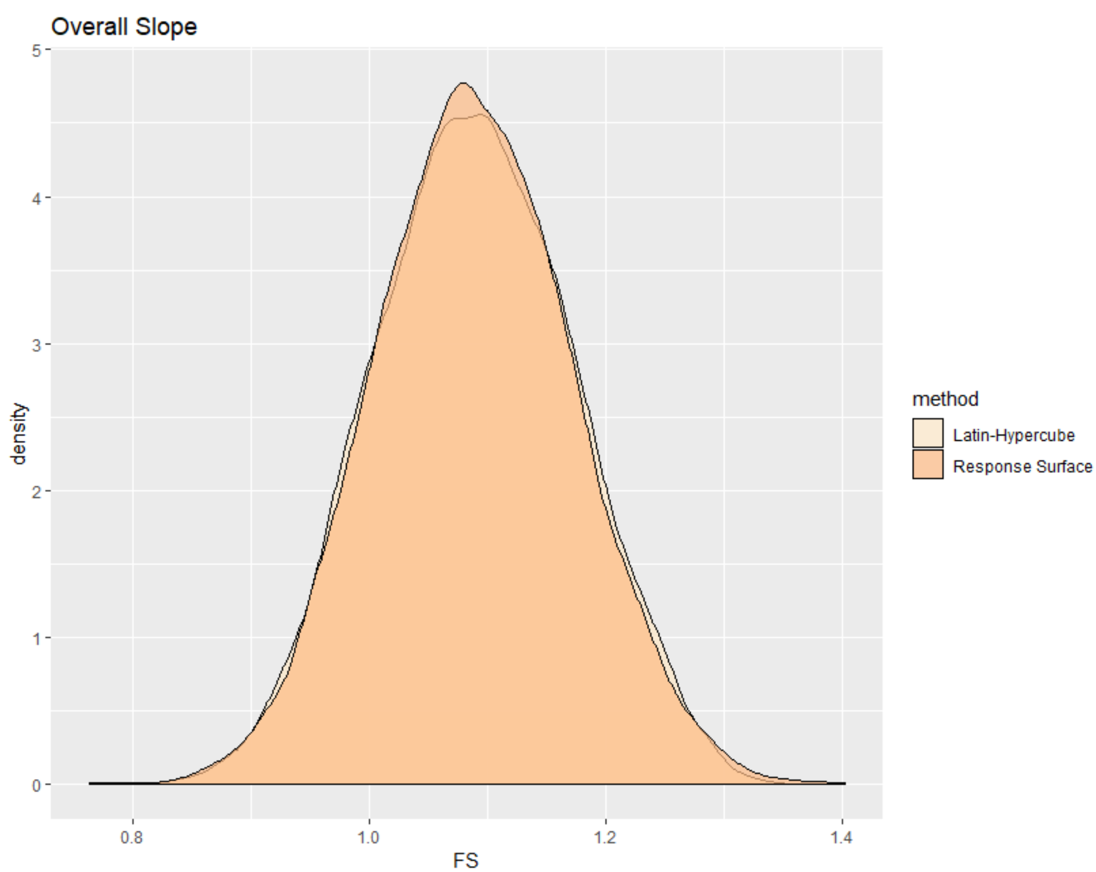
#	Prop 1	Prop 2	Coefficient
1	Sub A Cohesion	Sub B Cohesion	0.8
2	Sub A Cohesion	Sub C Cohesion	0.8
3	Sub B Cohesion	Sub C Cohesion	0.8
4	Sub A Cohesion 2	Sub B Cohesion 2	0.8
5	Sub A Cohesion 2	Sub C Cohesion 2	0.8
6	Sub B Cohesion 2	Sub C Cohesion 2	0.8
7	Sub A Phi	Sub B Phi	0.8
8	Sub A Phi	Sub C Phi	0.8
9	Sub B Phi	Sub C Phi	0.8
10	Sub A Phi 2	Sub B Phi 2	0.8
11	Sub A Phi 2	Sub C Phi 2	0.8
12	Sub B Phi 2	Sub C Phi 2	0.8

The probabilistic analysis results are found below and are in good agreement:

Overall Slope – Spencer				
Method	**Num Samples**	**PF (%)**	**Mean FS**	**Time (hrs)**
Latin-Hypercube	10000	14.970	1.089	4.3
Response Surface	10000 (910)	14.320	1.089	0.4

The first 100 simulations are shown to be in good agreement:


Overall Slope
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
FS
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70 73 76 79 82 85 88 91 94 97 100
Simulation
Latin-Hypercube
Response Surface

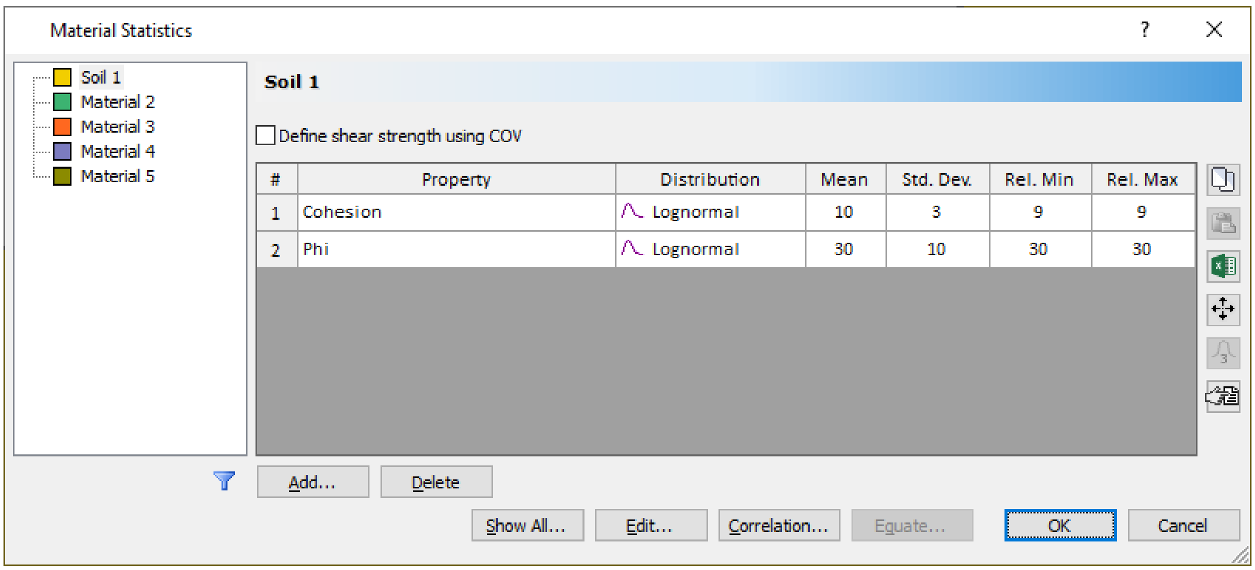

Overall Slope
5
4
3
2
1
0
density
0.8
1.0
1.2
1.4
FS
method
Latin-Hypercube
Response Surface

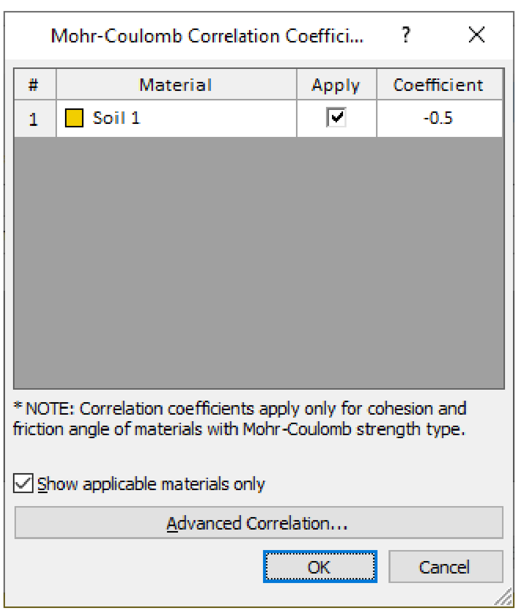
Example 5 Simple Correlation

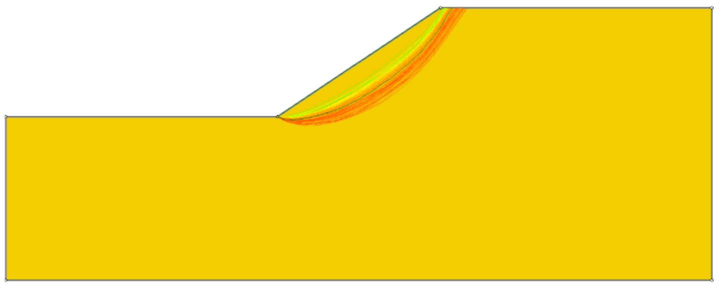
The previous example has the advantage of having 12 random variables, hence requiring Response Surface to use 910 training simulations. More random variables means more training simulations, and a more accurate result from Response Surface. Since Response Surface is useful for its time savings, it is in any case only of use when many random variables are defined.

A simpler correlation example, where Response Surface wouldn't be necessary (a more challenging one for Response Surface) is presented below.

The slope from Tutorial 1 is defined with two random variables with and without correlation:







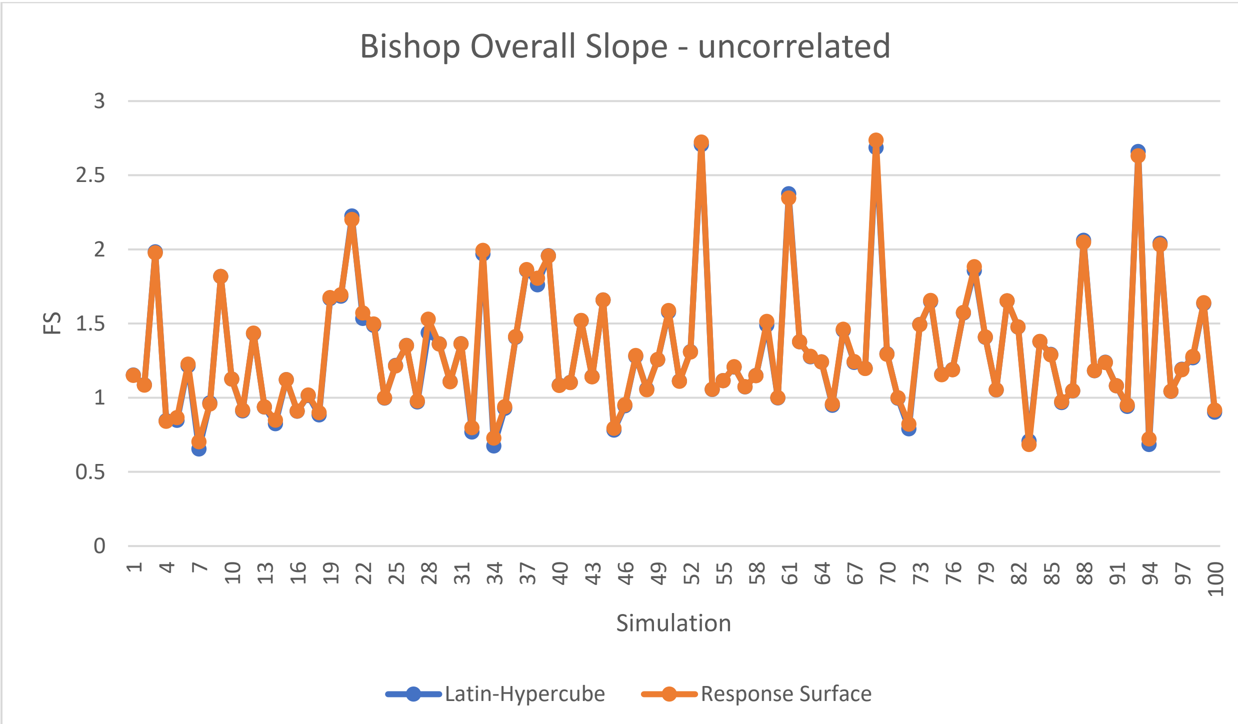
Surface Type	Non-Circular
Search Method	Particle Swarm
Optimization	Surface Altering
Method	Bishop Simplified; Janbu Simplified
Deterministic FS (Bishop)	1.294
Deterministic FS (Janbu)	1.224

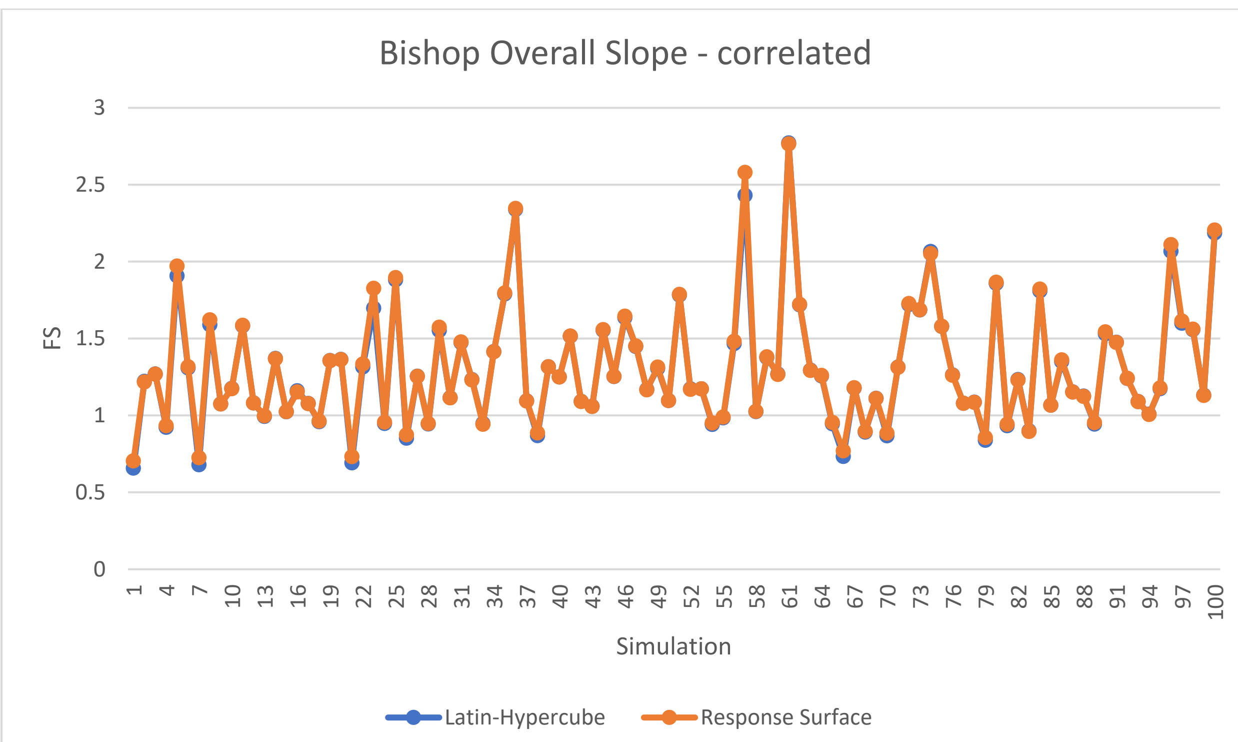
The results are found below:

	Overall Slope - Bishop				
	Method	Num Samples	PF (%)	Mean FS	Time (sec)
Uncorrelated	Latin-Hypercube	100	25.0	1.308	26.7
	Response Surface	100 (20)	24.0	1.314	6.6
Correlated	Latin-Hypercube	100	21.0	1.303	26.7
	Response Surface	100 (20)	21.0	1.312	6.6

	Overall Slope – Janbu				
	Method	Num Samples	PF (%)	Mean FS	Time (sec)
Uncorrelated	Latin-Hypercube	100	30.0	1.238	26.7
	Response Surface	100 (20)	31.0	1.245	6.6
Correlated	Latin-Hypercube	100	25.0	1.234	26.7
	Response Surface	100 (20)	25.0	1.244	6.6

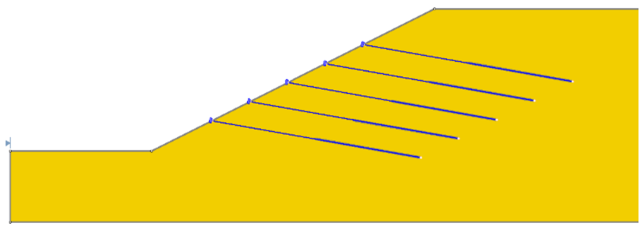
The Latin-Hypercube and Response Surface results are in good agreement. The plots below also show the simulations for both methods, for uncorrelated and correlated, respectively:



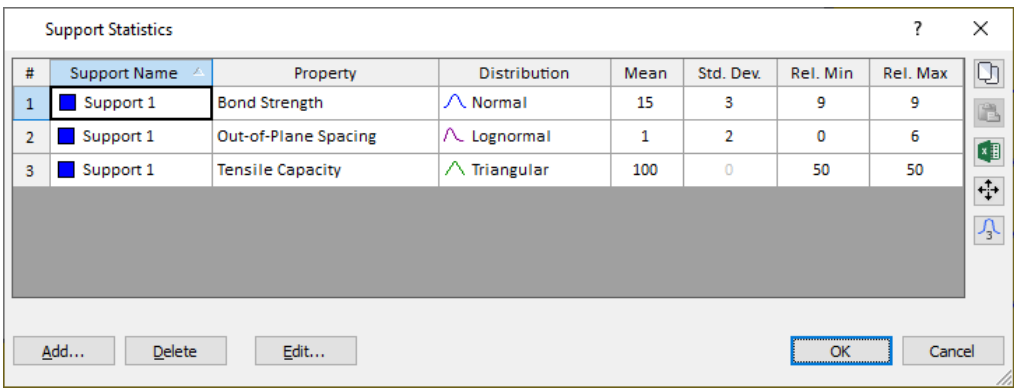


Example 6 Random Variables in Support

This example considers grouted tiebacks with variability in bond strength, out-of-plane spacing, and tensile capacity, with three different types of distributions.



Surface Type	Non-Circular
Search Method	Cuckoo Search
Optimization	Surface Altering
Method	Bishop simplified
Deterministic FS (Bishop)	1.39



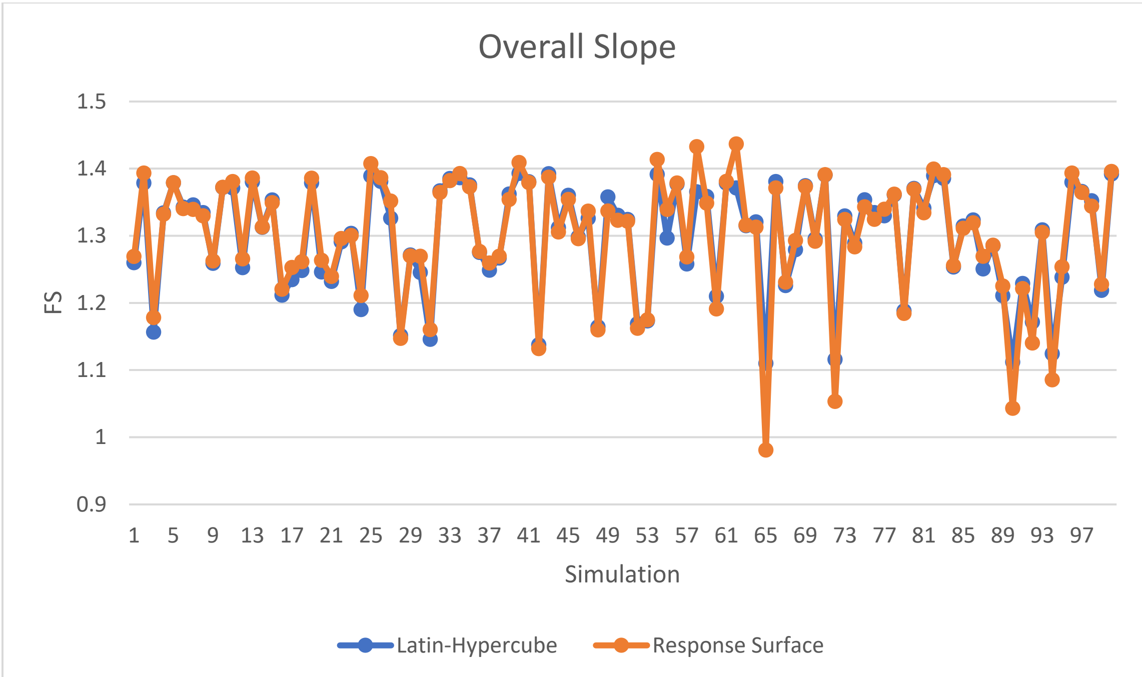
Support Statistics

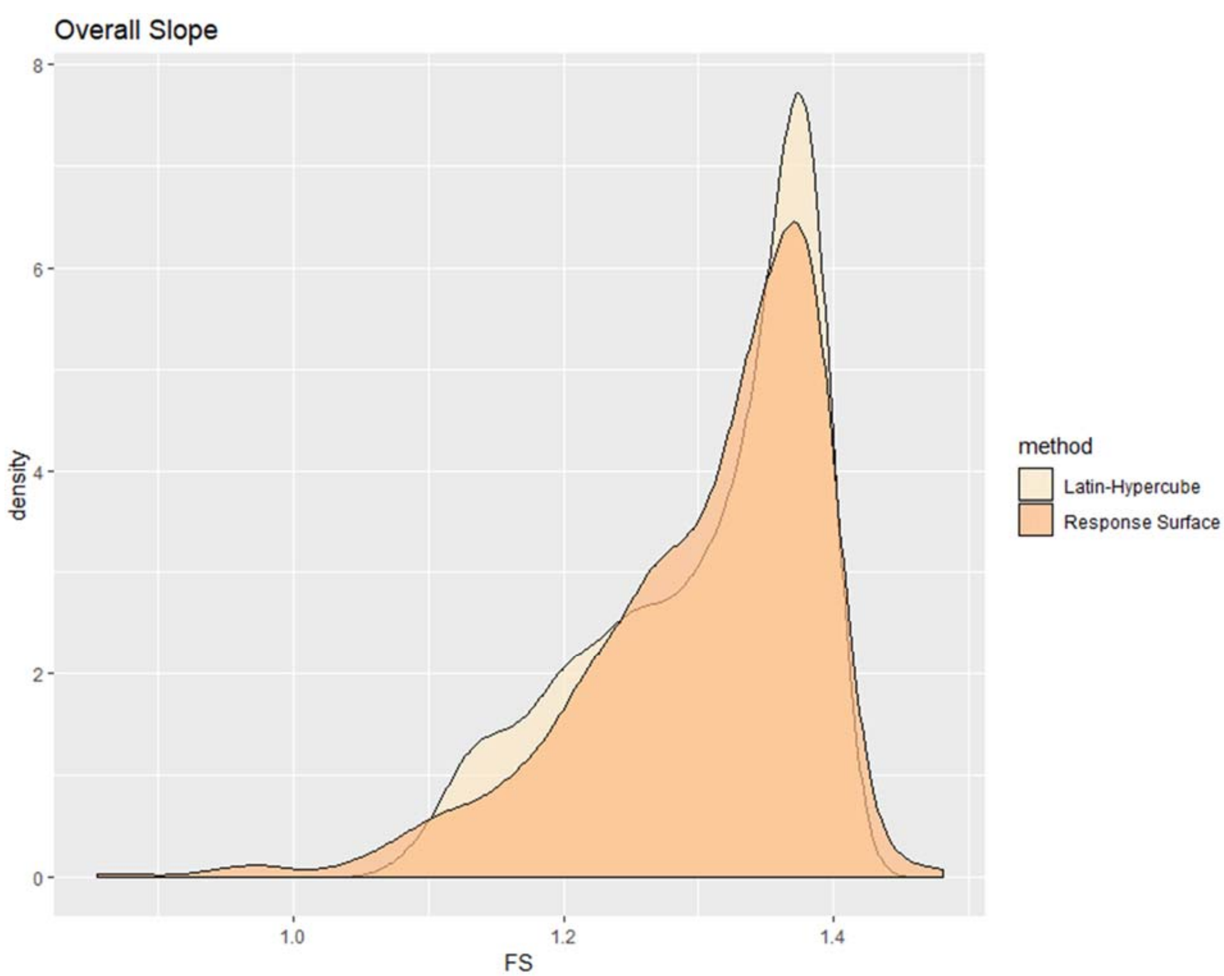
#	Support Name	Property	Distribution	Mean	Std. Dev.	Rel. Min	Rel. Max
1	Support 1	Bond Strength	Normal	15	3	9	9
2	Support 1	Out-of-Plane Spacing	Lognormal	1	2	0	6
3	Support 1	Tensile Capacity	Triangular	100	0	50	50

Add... Delete Edit... OK Cancel

The results are found below:

Overall Slope - Bishop				
Method	**Num Samples**	**PF (%)**	**Mean FS**	**Time (min)**
Latin-Hypercube	1000	0.000	1.303	7.1
Response Surface	1000 (40)	1.000	1.303	0.3


Overall Slope
1.5
1.4
1.3
1.2
1.1
1
0.9
FS
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73 77 81 85 89 93 97
Simulation
Latin-Hypercube
Response Surface


Overall Slope
8
6
4
2
0
density
1.0
1.2
1.4
FS
method
Latin-Hypercube
Response Surface

With three types of distributions, the FS histograms are certainly not identical, however they are very close. This can be confirmed by calculating the Probability of Failure, at different thresholds as shown:

Threshold FS	Latin-Hypercube PF (%)	Response Surface PF (%)
1.0	0.00	1.00
1.15	6.30	6.70
1.38	82.8	81.2